EFFECT OF FLEXURAL NSM-FRP BARS ON THE SHEAR STRENGTH OF
REINFORCED CONCRETE BEAMS

by

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**Approval Signatures**

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Dedication

To my parents...
Abstract

Strengthening of reinforced concrete (RC) members in flexure and shear, using fiber-reinforced polymer (FRP) laminates and near surface mounted (NSM) bars has been developed over the last few decades. Conventionally, FRP composite laminates are attached to the vertical sides of RC beams to increase their shear strength. However, in some cases, the sides of the beam might be inaccessible or shallow. According to the ACI 318-14 guidelines and other codes of practice, longitudinal reinforcement contributes to the shear strength of RC beams. The objective of this study is to assess experimentally the effect of longitudinal NSM bars, which are mounted to the beam’s sides, on the shear strength of RC beams. The variables of the experimental program included the beam’s depth to evaluate the size effect, the concrete compressive strength, and the flexural FRP reinforcement ratio. A total of 18 shear deficient RC beams were built, strengthened with longitudinal NSM bars, and tested under three-point bending until failure. It was observed that the strengthened beam specimens exhibited an increase in shear capacity that ranged from 10 to 35% over the control beams. It was also observed that the increase in the concrete shear strength with longitudinal FRP bars for the high strength concrete specimens was not as effective as that with normal concrete strength specimens. The results have also revealed that the size effect phenomena has a significant effect on the test results, in which the increase in shear strength has decreased from around 35% to 10%, with the increase of beam’s height from 230 to 650 mm, respectively. It was concluded that equation 22.5.5.1 of the ACI 318-14 code, which gives a simplified expression for the concrete contribution to the shear strength of beams, is not conservative for beams with large depths. Thus, other published analytical shear strength models that account for size effect were utilized in this study to predict the strength of the tested specimens. This study utilized 10 different analytical models in predicting the shear strength of the tested RC beam specimens. The models that showed the closest agreement with the tested data were the University of Houston method and the model based on the second order simplified modified compression field theory. It can be concluded that flexural longitudinal NSM bars could be used as a viable solution to enhance the shear strength of RC beams.

Key words: FRP, Concrete, Beams, Shear Strengthening, CFRP, NSM, Size Effect, High Strength Concrete, Flexural Strengthening
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Chapter 1: Introduction

Several existing reinforced concrete (RC) structures are deteriorating all over the world leading to a major reduction in load carrying capacity of these structures over time. Examples of major causes of deterioration for these structures include corrosion of steel reinforcement, design errors, severe environmental conditions and increase of dead and live loads. Therefore, these structures need to be strengthened and repaired.

Slabs and beams are designed in RC structures to ensure a ductile failure. However, shear failures in beams are more brittle, compared to flexural failures. Therefore, design codes usually have a larger safety margin shear strength, compared to flexural strength.

Over the last 50 years, strengthening existing structures with Carbon Fiber Reinforced Polymers CFRP composite materials have gained popularity over other strengthening materials. This is due to the fact that they can be applied quickly and have good corrosion resistant properties and high strength to weight ratio. Several studies have been conducted over the years, proving the efficiency of strengthening using FRP strengthening materials [1–26].

Typical flexural strengthening is done by bonding Carbon Fiber reinforced Polymer (CFRP) composite sheets or laminates to the tension face of RC beams surface via epoxy adhesives. A great body of research has shown that bonding CFRP laminates to the tensile surface (soffit) of RC beams can increase their flexural strength. Numerous experimental studies have been conducted on RC beams that were strengthened with CFRP laminates in flexure. Results have shown that the load carrying capacity of the strengthened specimens increased up to 70% over the control specimen [27]. The major drawback of using CFRP sheets was the de-bonding failure mode, which was resolved by using near surface mounted (NSM) bars. Such bars were mounted to the beam’s soffit; the results showed an increase in the load-carrying capacity of strengthened specimens, and beam specimens did not fail by de-bonding. Therefore, external reinforcement in the form of CFRP laminates or bars showed contribution to the enhancement of moment strength similar to the internal flexural steel reinforcement.

Numerous studies have shown that the internal longitudinal steel reinforcement ratio ($\rho_s$) has contributed to the concrete shear strength ($V_c$), and it played a vital role in increasing the shear strength of RC beams [15–18]. Thus, external reinforcement in the
form of CFRP laminates or bars can increase the shear strength of RC beams and can have the same effect on $V_c$ as that of the internal steel reinforcement. Limited studies have been conducted to examine the flexural contribution of CFRP laminates or sheets to the $V_c$ [1,11,19]. However, the literature lacks information about the contribution of flexural NSM bars on the concrete shear strength of RC beams. Accordingly, this study will investigate the combined effect of internal steel and external flexural NSM bars on the shear strength of RC beams which are deficient in shear. According to the ACI 318-14 code [33], increasing the depth of RC beams is linearly proportional to the increase in beam’s shear strength. Yet, research and experimental data [21–23] have shown that when the depths exceed certain limits, the shear strength increases, but at a decreasing rate, due to a phenomena called “size effect”. Therefore, the beam’s size effect will also be investigated in this study. Figure 1 illustrates strengthening of RC beams with CFRP laminates and NSM bars.

![Figure 1](image)

**Figure 1**: Flexural strengthening using (a) NSM bars and (b) FRP laminates

In order to strengthen RC beams for shear, the sides have to be accessible; strengthening with FRP is shown in Figure 2. Moreover, shear strengthening using FRP is bonded to the sides to imitate the effect of transverse reinforcement (stirrups). There are different methods of strengthening RC beams in shear using FRP laminates including: full wrapping, U-wrapping, and side bonding.

Increasing the steel reinforcement ratio of RC beams has an effect on the beam’s concrete shear strength $V_c$. The latter has been proven in many studies and is shown as a parameter in many code equations for $V_c$ and other prediction models [7, 9, 18, 20, 24 – 27]. It is suspected that strengthening specimens with CFRP in flexure would have a
similar effect to increasing the reinforcement ratio on \( V_c \). The contribution is carried out by increasing the area in the compression zone and through dowel action. Experimental studies in the literature [8, 10, 28–30] have shown the contribution of flexural external CFRP composite sheets in increasing the beams shear strength with promising results. However, available literature lacks studies on the effect of increasing the flexural reinforcement ratio on \( V_c \) on larger beams (size effect), increase of the concrete compressive strength (\( f'_c \)) and the use of NSM-FRP bars. Accordingly, this study aims to examine experimentally these effects, which will help to gain a better understanding on the shear behavior of flexurally strengthened beams. Moreover, this is expected to provide a solution in cases where the sides of RC beams are inaccessible to provide conventional shear strengthening with the side-bonding technique.

1.1. Research Significance

Strengthening RC structures using FRP composite materials has emerged as a viable solution in the last few decades and has been the interest of many researchers. Many designers are using FRP to strengthen structural elements such as beams, slabs and columns, using different design codes of practice [14,24]. Many beams that need strengthening are deficient in shear. Conventionally, those beams are strengthened by bonding FRP vertically at the beam’s sides with epoxy adhesives. However, the sides may not be accessible for strengthening due to certain obstacles. The amount of flexural steel reinforcement in RC beams contribute to the beam’s shear capacity of concrete \( (V_c) \). Conceptually, strengthening beams with flexural NSM-FRP bars should have a positive contribution on \( V_c \).
There are a few studies that investigated the concept of using flexural CFRP laminates to increase $V_c$ [17–20]. However, variations in the parameters tested, such as using NSM-FRP bars, varying sizes of cross-sections, and using high strength concrete, have not been studied. It is important to investigate these parameters to check the applicability of this method and assess its shortcomings.

1.2. Research Objectives

The goal of this study is to understand the behavior of flexural strengthening, using NSM-CFRP technology on the shear strength of RC beams ($V_c$) while varying several parameters contributing to $V_c$. A total of 18 RC beams will be cast and tested under three-point bending. The main objectives of this study are to:

1- Investigate the effect of longitudinal NSM bars mounted at the beam’s soffit on the shear strength of RC beams $V_c$

2- Test RC beams for shear with heights of 230, 380, 550 and 650 mm and study the size effect on $V_c$ for flexurally strengthened RC beams

3- Investigate the effect of using high strength concrete ($f'_c = 70$MPa) with flexural CFRP-NSM and comparing the results with that of normal strength concrete beams ($f'_c = 30$ MPa) in terms of the concrete shear strength $V_c$

4- Compare the load-deflection response curves, stiffness and the load carrying capacity of the strengthened specimens with that of the control un-strengthened beams

5- Evaluate and predict the concrete shear strength results with analytical and empirical models published in the literature
Chapter 2: Literature Review

2.1. Overview on Shear Strength of RC Beams

The shear strength in RC beams can be divided into two parts including section behavior before and after cracking. The first part can be studied using mechanics of materials rules and basic theories, while the second part is more complex and involves many variables.

Upon the cracking of an RC beam section, the main contributing factors to the shear strength of RC beams without stirrups are shown in Figure 3.

![Figure 3: Factors contributing to the concrete shear resistance of RC beams](image)

Figure 3: Factors contributing to the concrete shear resistance of RC beams

The shear carried by the compression zone, aggregate interlock, and reinforcing steel dowel action are denoted in Figure 3 as $V_{cy}$, $V_a$ and $V_d$, respectively. At initial cracking, it can be approximated that 40 to 60% of the shear forces are resisted by $V_a$ and $V_d$ [47], [48]. When the crack becomes wider, the contribution of $V_a$ is minimized, leading to an increase in the fraction of shear carried by $V_d$ and $V_{cy}$. Afterwards, $V_d$ leads to a splitting crack along the reinforcement, leaving the compression region $V_{cy}$ to have all the resistance. After the beam section reaches its full capacity, the compression region fails and crushes or buckles upwards.

2.2. Reinforced Concrete Contribution to Shear Strength ($V_c$)

There is a considerable debate in the literature on the shear capacity carried by the concrete in RC members. But even though researchers have been studying $V_c$ for over a hundred years, no agreement has been reached on a theory that completely and accurately predicts such brittle and complex behavior. Researchers have studied
different contributing factors to the shear strength such as longitudinal reinforcement ratio ($\rho_s$), shear-span to depth ratio ($\frac{a}{d}$), concrete compressive ($f'_{c}$) and tensile strength of reinforcing steel ($f_y$), size effect ($d$), aggregate interlock and the dowel action. Many of these factors are explained in the proceeding subsections.

2.2.1. Longitudinal reinforcement ratio ($\rho_s$). The increase of reinforcement ratio ($\rho_s$) leads to a significant increase in the shear strength of RC beams [8], [47], [49]. This could be due to the increase of the neutral axis depth ($c$) as flexural reinforcement increases. Tureyen and Frosch [30] argued in a new model that the shear stress is carried only by the beam’s compression region, and the concrete below the neutral axis is cracked and doesn’t contribute to the shear resistance. It should be noted that the shear stress in a cracked RC beam section depends only on the compression region. Increasing the neutral axis depth results in an increase in the area of the compression region, and thus an increase in the concrete shear strength. Having small reinforcement ratios can lead to un-conservative values by the ACI 318-14 equations [20, 41, 43]. Tureyen and Frosch [30] explained that small reinforcement ratios lead to flexural cracks that extended higher and wider into the beam’s depth. Therefore, they decrease the shear strength components by the dowel action.

Hoult et al. [23] conducted a study on a database that consisted of 146 beams failing in shear. The beams had no transverse reinforcement and were cast with different types of longitudinal reinforcement such as mild steel, CFRP, GFRP and AFRP bars. It was concluded that beams reinforced with steel or FRP bars exhibit fundamentally the same shear behavior to that of shallow members. Bentz et al. [51] added to this conclusion by testing 11 FRP reinforced concrete members to study the size and strain effect. The beams showed a similar behavior to mild steel when correctly addressing the Young’s modulus and the area of reinforcement.

Nawaz et al. and Hawileh et al. [17–19] conducted a full scale study on 24 beams divided into 3 groups. The first group had a no flexural steel reinforcement; the second had a low reinforcement ratio, and the third had high flexural reinforcement. The beams in the groups are strengthened with CFRP sheets and laminates in the flexural direction. The study also investigated the effect of flexural longitudinal CFRP laminates on increasing the shear strength of RC beams. Since longitudinal steel and FRP reinforcement had the same effect on the shear strength of concrete; they have
unified the reinforcement and strengthening materials with an effective reinforcement ratio and an effective depth based on stiffness and area. Increasing the layers of CFRP composite sheets in the flexural direction had a direct effect on the effective reinforcement ratio. That had led to an increase in shear capacity that ranged from 10 to 70%. The shear strength of the tested specimens was also predicted using obtained experimental results: the ACI shear formulae [33], the simplified modified compression field theory (SMCFT) [37], a model developed by Tureyen and Frosch [30], and the University of Houston model [52]. It was concluded that SMCFT yielded the closest predictions.

El-Sayed [25] conducted a similar study on the effect of longitudinal CFRP sheets on the shear strength of RC beams. The experimental program was composed of 7 beams, five of which were strengthened using FRP sheets and laminates with different strengths. El-Sayed took into account that applying strengthening materials in the longitudinal direction increases the reinforcement ratio, which was computed by finding an equivalent stiffness between the FRP and steel reinforcement. El-Sayed also adjusted the depth of the beam and used different prediction models. The models used were: ACI prediction formulae [33], SMCFT using CSA 2004 [37], CEP FIP model [53] and Zsutty’s model [39]. The strengthened specimens had shown an increase in the concrete shear strength up to 35% over the control beams. El-Sayed concluded that the SMCFT model exhibited the best prediction results.

Saqan and Frosch [50] investigated the effect of partially prestressed concrete members with longitudinal mild steel bars. Their experimental program consisted of nine beams separated into three series. Each series contained three beams: a control prestressed beam without mild steel reinforcement and two beams with varied mild and prestressed steel reinforcement. Their test results showed that the increase in mild steel reinforcement ratio increased the beam’s shear capacity, which was recorded at the first major shear crack. In each beam, strain gauges were applied on the reinforcement, and the strength of the concrete was calculated using the Hognistad stress strain curve [50]. A study on the neutral axis depth decreasing with the increase of moment was conducted. They concluded that the total amount of reinforcement (prestressed or mild) controls and increases the shear capacity of RC beams until the formation of the major shear crack.
2.2.2. Size effect. It is commonly observed that increasing the size of members does not have a proportional effect on the increase of the concrete shear strength in RC beams. According to Hoult et al. [23], explaining the MCFT [54], the aggregate interlock contribution depends on the width of the shear crack at failure. Another perspective in explaining the size effect is Bazant and Sener [55] approach in terms of energy. Shear failures are brittle and not simultaneous and ductile like flexural failures. The major shear crack propagates until failure, while flexural failure occurs with various cracks and does not depend mainly on the tensile capacity of concrete. Previously failed regions release elastic energy into failing regions, which explains the propagation in shear failures. Enlarging the sections increases the energy released, which explains the size effect. Modern theories and studies [47], [54], [56] assumed that a portion of the size effect depends on energy release, yet the crack width analysis has closer results.

Alam and Hussein [36] conducted a study, using normal strength concrete with CFRP, GFRP and mild steel as longitudinal reinforcement. The aim of the study was to compare the size effect of FRP reinforced beams with steel reinforced beams. The experimental program was composed of 12 beams with depths of 350, 500, 650 and 800 mm, respectively. The beams had different lengths of 2840, 3540 and 4040 mm in order to have a constant \( \frac{a}{d} \) ratio of 2.5. The axial stiffness (EA) was held constant in order to compare steel with FRP reinforcement. The major shear crack in most beams had a similar angle (approximately 42 degrees), which is due to having a constant \( \frac{a}{d} \) ratio. They also compared these results with Bazant’s law of size effect and had a slight variation, which can be attributed to the different materials used and different failure modes (Shear tension failure, shear compression failure, and diagonal compression failure). Their crack spacing was directly proportional to the size of the beams, which agrees with the MCFT assumption [54]. It was concluded that the size effect was higher in FRP reinforced beams than that in steel reinforced beams.

Godat et al. [35] studied the size effect on beams strengthened with side CFRP strips. The experimental program consisted of 7 beams divided into three groups. All the beams had a constant reinforcement ratio and used either U-wrapping or fully wrapping strengthening techniques. The depths of the beams were varied at sizes of 200, 400, and 600mm, respectively, while the corresponding widths were 100, 200, 300
mm, respectively. A finite element model was developed that predicted the load vs deflection and strain response curves effectively. It was concluded that increasing the depths of beams increased the spacing between cracks, which agrees with the MCFT explanation of size effect [54]. It was also concluded that the strains in the CFRP strips in smaller beams were higher than the larger beams, and therefore provided less improvement in shear capacity.

2.2.3. Concrete compressive strength. One of the important factors effecting the shear capacity of concrete in RC beams is its compressive strength. Experimentally, a correlation between the shear capacity $V_c$ and the square root of the compressive strength ($\sqrt{f'_c}$) is found. Nevertheless, the relationship over-estimates the $V_c$ when $f'_c$ exceeds 70 MPa (10,000 Psi) for the reason that in high strength concrete, the shear crack forms between the aggregates and not through them. The ACI code 318-14 $V_c$ equation [33] does not account for high strength of concrete and is not conservative for this case. While in the MCFT [54], it accounted for by assuming the maximum aggregate size to be zero. Frosch’s prediction equation has a different approach to this phenomenon. Tureyen and Frosch’s prediction equation [30] uses the depth of the neutral axis instead of the depth of the beam. An increase in the compressive strength of concrete decreases the depth of the neutral axis, which decreases the area of which shear is transferred [15-17, 38, 42].

Alam and Hussein [58] conducted an experimental program on six RC beams reinforced with either CFRP or GFRP bars. The targeted compressive strength was 70 MPa, and the depth of the beams were 350, 500, 650 mm. The aim of the study was to investigate if there is a size effect on high strength concrete (HSC) beams reinforced with FRP bars. The angle of the shear crack were fairly similar in all the beams (approximately 41 degrees), which indicated that the angle is dependent only on the shear span to depth ratio ($\frac{a}{d}$) which was 2.5. The results of increasing the size decreased the shear strength, indicating that there is a size effect in FRP reinforced beams. The size effect in their case agreed well with Bazant’s law of size effect.

2.2.4. Shear span to depth ratio ($\frac{a}{d}$). It is known that the shear span ($a$) to depth ($d$) ratio ($\frac{a}{d}$) has an effect on the shear crack and strength of RC beams. The deep beam behavior causes the crack to act as an arch in which the crack is dependent on the
compressive stresses in the concrete (shear compression failure), whereas the shallow beams have failed in shear by an inclined crack having an angle close to 45° with the beam longitudinal axis and are dependent various parameters (shear tension failure). As a rule of thumb, an RC beam with $\frac{a}{d}$ ratio less than 2.5 is considered a deep beam, and for a higher value the beam is considered a shallow beam.

Tang et al. [44] studied the shear behavior of sedimentary light-weight aggregate without shear reinforcement. A total of 24 beams were tested, and the parameters of the experimental matrix were the $(\frac{a}{d})$ ratio and concrete compressive strength $f'c$. The $(\frac{a}{d})$ ratio variations were 1.5, 2.0, 2.5 and 3.0. It was observed that RC beams with ratios less than 2.5, showed an inclined shear strut and had a higher shear capacity for both lightweight and normal weight concrete.

Li et al. [20] studied the effect of $(\frac{a}{d})$ ratios on the performance of FRP strengthened RC beams. A total of 12 beams were cast and tested, of which 6 were control and the remaining were strengthened with FRP laminates. The key variation of the specimens was the $(\frac{a}{d})$ ratio, that varied from 1.0 to 3.5. The tested specimens with low $(\frac{a}{d})$ ratios showed the least effective use of the FRP strengthening materials since the concrete carries most of the shear stress in compression.

2.3. Overview on Fiber Reinforced Polymers (FRP)

When combining small diameter fibers with polymeric matrices at a microscopic level, a synergistic material called fiber reinforced polymers (FRP) is produced [59]. These fibers were limited to applications in the fields of aerospace, automotive and marine industries. In 1954, the use of FRPs as an alternative to steel was introduced by Goldsworthy [60], where structural elements highly benefited from the chemical inertness of the material and encouraged its use in structures with high corrosive environments [60]. One of the old methods of strengthening was the use of steel plates bonded with epoxy or bolts to the tension side of structural elements. However, durability studies showed that corrosion is a major restriction in the use of steel plates for strengthening [59]. A major advancement in FRP strengthening was introduced by Meier in 1987 [61]. Meier’s approach was to strengthen bridges with epoxy bonded FRP materials, where laminates have excellent resistance to corrosion, high strength to weight ratio, low specific gravity, and efficient in construction [61].
Ever since, extensive research has been conducted and various methods has been developed in strengthening RC members (slabs, beams, columns, walls... etc.) using FRP laminates.

Over the decades, many technologies of strengthening, using FRP materials, have been developed to enhance flexural and shear capacity of RC beams. The main methods commercially available are strengthening via FRP sheets, pre-cured plates and near surface mounted NSM systems. FRP Sheets and plates are available commercially as unidirectional and multidirectional. Unidirectional laminates are used more often due to their cheaper price and availability. FRP Sheets are installed using the wet lay-up technique, by impregnating dry sheets with a saturating resin on site [4]. However, for the case of pre-cured plates, an adhesive with primer and putty are used to bond the procured laminates to concrete surfaces [4]. In addition, RC beams can be strengthened with NSM bars and strips. The strengthening reinforcement is typically made of FRP bars inserted to the near surface of the tension side. The surface is grooved and filled with a resin matrix, and then FRP bars are pressed into the resin and left to cure for a few days [2].

Carbon Fiber reinforced polymers (CFRP) are considered one of the most commonly used FRP materials in the strengthening industry. Carbon fibers have the highest tensile strength and modulus of elasticity, compared to other FRP materials. Furthermore, the fibers have high strength to weight ratio and has high resistance to harsh alkalinities and environmental exposure. FRPs usually have higher strengths than mild steel, but are brittle and not ductile as steel bars. Figure 4 shows a comparison in the mechanical properties of different FRP materials and conventional steel.

![Figure 4: Typical steel and FRP stress-strain diagrams [62]](image-url)
2.4. Using FRP for Shear Strengthening

There are many studies in the literature that investigated the effect of side-bonded strengthening of FRP sheets and bars on the shear capacity of RC beams. A summary of selected studies published in the literature are presented in this subsection.

Triantafillou [43] conducted an experimental program on 11 shear deficient RC beams with CFRP strips bonded to the beams vertical side. The beams failed due to an inclined brittle shear crack, and the strengthened specimens showed an excellent improvement over the control specimens. The increase in strength was in a range of 65 to 95%, and it was observed that the CFRP sheets debonded from the concrete substrate at failure.

Taljsten [41] has conducted a full scale study on seven shear deficient beams to investigate the effect of inclination angle of the CFRP sheets when attached to the beam’s side at 0, 90 and 45 degrees, respectively. The CFRP was perpendicular to the crack orientation; the strengthened specimens had shown a great improvement in the range from 98 to 169%. It was concluded that inclined sheets are superior in shear strengthening over ones with 90 degrees.

Another technique of shear strengthening is mounting NSM bars on the vertical side of RC beams. Barros and Dias [63] conducted an experimental program on 4 groups of specimens deficient in shear. Each group had 5 beams: one with no shear reinforcement, one reinforced with steel stirrups, one strengthened with CFRP sheets, and two beams strengthened with NSM bars oriented at 90 and 45 degrees, respectively. NSM bars oriented at 45 degrees had shown the most increase in shear strength and provided higher deformations than the other strengthened beams specimen. The beams with inclined NSM bars had shown an increase in capacity over control specimens in the range of 57 to 162%.

Godat and Chaallal [22] conducted an experimental program on 14 beam specimens strengthened in shear, using the U-wrap technique with different spacing between the stirrups. The purpose of the study was to validate that the strut-and-tie model is capable to predict the performance of shear strengthened beams. The model basically accounted for the strength of the section as a truss, where the compression members are based on the concrete compressive strength (struts), and the tension
members are from the steel stirrups and FRP (ties). Their modified analysis technique had shown good accuracy in predicting the shear strength increase due to FRP strengthening.
Chapter 3: Shear Strength Models

This chapter includes information about available analytical and empirical models found in the literature to predict the shear strength of RC beams. Many of those equations were taken from design codes from already published literature and were modified to include the effect of flexural strengthening reinforcement on the shear strength \( V_c \) of RC beams. The following is a brief description of the equations and models used in this study to compare with the experimental results.

3.1. ACI 318-14 Equations

The American Concrete Institute (ACI) codes for design and strengthening [27], [33] state that the shear strength of RC beams should be computed by including the effect of the shear strength of concrete \( V_c \), transverse shear reinforcement \( V_s \) and transverse FRP reinforcement \( V_f \). The values for \( V_c \) and \( V_s \) are those given in the ACI 318-14 [33] equations, while \( V_f \) is taken from the ACI 440.2R.08 [27] document. To compute \( V_c \), two equations are provided by ACI 318-14 code:

3.1.1. The general equation. The main equation used to predict the shear strength of the concrete in RC beams without stirrups was developed in the 1960’s. The equation was based on empirical data and took into account the concrete compressive strength, longitudinal reinforcement ratio and shear span to depth ratio.

\[
V_c = \left[ 0.16 \sqrt{f'_c} + 17 \rho_w \frac{V_u}{M_u} \right] b_w d \leq 0.29 \sqrt{f'_c} b_w d
\]

(1)

where:

\( f'_c \) = concrete compressive strength (MPa)

\( \rho_w \) = the longitudinal reinforcement ratio at the section under consideration

\( V_u \) = the applied factored shear at the section under consideration (N)

\( M_u \) = the applied factored moment at the section under consideration (N-mm)

\( b_w \) = width of the web (mm)

\( d \) = depth of the beam from the extreme compression fiber to the center of the longitudinal steel (mm)
To account for the strength of the concrete, the square root of $f'_c$ fits well with the empirical data curve. However, it shows un-conservative predictions for concrete strengths higher than 70 MPa. The shear span to depth ratio is accounted for in the relationship $\left(\frac{V_u d}{M_u}\right)$ as it works with different loading conditions.

3.1.2. The simplified equation. Denoted as equation 22.5.5.1 in ACI-318-14 code, the equation is widely used for design purposes. Equation (2) is considered a simplification of equation (1) since the factor $(0.01 \sqrt{f'_c})$ was considered to be conservatively equal to $(0.01 \sqrt{f'_c})$.

$$V_c = 0.17 \sqrt{f'_c} b_w d$$

(2)

Both ACI equations do not account for the size of the beam and provide un-conservative estimations for large beams [21-23], [26], [27].

To calculate the equivalent effective depth of the beam based on both areas of FRP and steel reinforcement, equations (3) and (4) are used. Equations (3), (4) and (5) serve to convert FRP into steel reinforcement using modular ratios and center of mass so that models in the literature can be applicable to the studied parameters.

$$d_{eff} = h - x$$

(3)

where $x$ is measured from the bottom of the cross section with the following:

$$x = \frac{d'}{1 + n \left(\frac{A_f}{A_s}\right)}$$

(4)

where:

$d'$ = concrete cover

$n = $ modular ratio between FRP and steel (\(\frac{E_f}{E_s}\))

The effective reinforcement ratio is equal to:

$$\rho_{eff} = \frac{A_s + n A_f}{b \ d_{eff}}$$

(5)
3.2. The Simplified Modified Compression Field Theory

The Canadian code CSA A23.3-14 [37] is based on the simplified modified compression field theory. The shear strength of concrete $V_c$ is computed as follows:

$$V_c = \beta \sqrt{f'_c} b_w d_v$$

(6)

where:

$\beta$ = Factor for the contribution of the tensile stresses in cracked concrete

d$_v$ = shear depth taken as the greater of 0.9d or 0.72h, where h is the height of the beam’s cross-section

The factor $\beta$ is dependent on the crack width, aggregate interlock and the concrete compressive strength. $\beta$ is calculated, using the following formula:

$$\beta = \frac{0.40}{1 + 1500\varepsilon_x} \frac{1300}{1000 + s_{ze}}$$

(7)

where:

$\varepsilon_x$ = the longitudinal strain in the web

$s_{ze}$ = equivalent crack spacing parameter in mm

To account for size effect and crack spacing, equation (8) is developed for the maximum aggregate size. The aggregate size should be taken equal to zero when the concrete compressive strength is equal to 70 MPa or higher. When the concrete strength is between 60 and 70MPa, the aggregate size shall be taken with linear interpolation between the actual size and zero.

$$s_{xe} = \frac{35 s_z}{15 + a_g} \geq 0.77 d$$

(8)

where:

$a_g$ = the maximum aggregate size (mm)

$s_z$ = crack spacing parameter which is taken as the value of $d_v$ (mm)
The strain in the web is one of the important factors used when calculating \( \beta \). It can be used as the actual value of a strain gauge in the middle of the web or approximated using equation (9).

\[
\varepsilon_x = \frac{M_f + V_f}{2 * (E_s A_s + E_f A_f)}
\]

(9)

where \( M_f \) is the moment at the location of the section, \( V_f \) is the ultimate shear force calculated at distance \( d_v \) from the support, \( E_s \) and \( A_s \) are the modulus of elasticity of steel and the area of longitudinal steel, respectively, \( E_f \) is the elastic modulus of the FRP material used, and \( A_f \) is the area of the longitudinal FRP reinforcement.

3.3. 2nd Order Simplified Modified Compression Field Theory

Hoult et al. [23] derived an equation based on the compression field theory that presents more reliable information. The same parameters of the first order equation are used. However, the end equation for \( V_c \) is different in terms of \( \beta \).

\[
\beta = \frac{0.3}{0.5 + (1000 \varepsilon_x + 0.15)^{0.7}} \times \frac{1300}{(1000 + s_{xe})}
\]

(10)

A study done by Bentz et al [51] showed that the average of aggregated data provides a \( \frac{V_{test}}{V_{pred}} \) of 1.04 and 1.22 for the second order and the first order equations, respectively. Moreover the coefficient of variation of the second order equation was 12.2%, compared to 15.8% which is the value of the first order. This indicates that the second order equation provided more precise values.

3.4. Shear Strength Based on Tureyen and Frosch Model

Tureyen and Frosch [30] conducted a study on the contribution of concrete in shear resistance and presented a model to compute the shear strength of concrete \( V_c \) reinforced with FRP. The model assumes that the concrete below the neutral axis has no contribution to the shear strength, and that the shear is dependent on the compression zone. The proposed model for \( V_c \) by Tureyen and Frosch is given by equation (11).
\[ V_c = \frac{2}{5} \sqrt{f'_c} b_w c \]  

(11)

where

\[ c = \text{the distance from the extreme compression fibers to the neutral axis (mm)}. \]

\[ c = kd \]  

(12)

The factor “c” is to be calculated using linear analysis for the cracked moment of inertia. The value k should be calculated using equation (13) for beams reinforced with steel only. The value k for beams with external FRP is given by equation (14) provided by ACI 440.2R-17 code [27].

\[
k = \sqrt{\left(\rho_s \frac{E_s}{E_c}\right)^2 + 2\rho_s \frac{E_s}{E_c}} - \left(\rho_s \frac{E_s}{E_c}\right)
\]

(13)

\[
k = \sqrt{\left(\rho_s \frac{E_s}{E_c} + \rho_f \frac{E_f}{E_c}\right)^2 + 2(\rho_s \frac{E_s}{E_c} + \rho_f \frac{E_f}{E_c})\left(\frac{d_f}{d}\right) - \left(\rho_s \frac{E_s}{E_c} + \rho_f \frac{E_f}{E_c}\right)}
\]

(14)

The model was created in order to provide conservative predictions when the reinforcement ratios are small. The ACI equations do not provide accurate results when used on beams with large sections. Furthermore, the equation is not applicable to concrete strengths over 70MPa. The distance from the extreme compression fibers to the neutral axis “c” accounts for size effect, as it varies linearly with the shear strength.

3.5. **The University of Houston’s Model (UH)**

Laskar et. al [52] and Hsu et. al. [65] from the University of Houston conducted an experimental study on the shear strength of reinforced concrete post-tensioned beams. The aim of both parts of the study was to create a simple and general model that predicts the shear strength of RC beams without stirrups. The model was the offspring of different published theories in the literature. It was based on the neutral axis depth which was taken from Tureyen and Frosch [30]. The factor for shear span to depth ratio was developed from accumulation of many tests and curve fitting. The size effect was accounted for by using Bazants approach [34]. The model shows excellent predictions and is a candidate to replace the ACI-318 code formulae.
\[ V_c = 2 \left( \frac{M_u}{V_u d} \right)^{-0.7} \times \left( \frac{1}{\sqrt{1 + \frac{h}{300}}} \right) \times \sqrt{f'_c b_w c} \]  

(17)

where \( h \) is equal to the actual height of the beam (mm)

### 3.6. CSA Standard S806-12 (CSA 2012)

The Canadian code for the embedded FRP as longitudinal reinforcement [38] provides a reliable equation that takes into account reinforcement ratio, deep beam effect and size effect. The equation was modified in order to fit the requirements of the study FRP, as strengthening material increases the reinforcement ratio.

\[ V_c = 0.05 K_r K_m d^{1/3} b_w f'_c \]  

(18)

\[ K_m = \sqrt{\frac{d}{a}} \leq 1 \]  

(19)

\[ K_r = 1 + (E_s \rho_s + E_f \rho_f)^{\frac{1}{3}} \]  

(20)

where:

\( K_m = \) is the deep beam effect

\( K_r = \) takes into account the reinforcement ratio

A study done by Kim and Jang [66] compiles test values of 60 specimens without stirrups. The same equation was used to predict 24 beams with lightweight aggregates. The results showed a \( \frac{V_{\text{test}}}{V_{\text{pred}}} \) value of 1.09 and a standard deviation of 0.19 showing the accuracy and precision of the equation.

### 3.7. Zsutty Model

Zsutty [67] combined statistical regression with dimensional analysis which has led to equation (21):

\[ V_c = 2.138 \sqrt[3]{\frac{f'_c \rho_{eff} d}{a}} b_w d_{eff} \]  

(21)
where:

\[ a = \text{shear span (mm)} \]

\[ d_{\text{eff}} = \text{is as per equations (3) and (4)} \]

\[ \rho_{\text{eff}} = \text{is the ratio of steel and FRP using their modular ratios} \]

### 3.8. Multi Action Model

Mari et al. [21] proposed a shear equation called “Multi action model” to predict one way shear without stirrups. When tested on a database of 144 specimen, the equation showed a mean of \( \frac{v_{\text{test}}}{v_{\text{pred}}} \) equal to 1.09 and a coefficient of variation of 14.9%. The equation takes into account size effect, deep beam effect and beam reinforcement ratio.

\[
V_c = 6 \ k \ \zeta \ \sqrt{f'_c} \ b \ d_{\text{eff}}
\]  

(22)

where:

\[
\zeta = \frac{2}{\sqrt{1 + \frac{d}{200} \left(\frac{d}{a}\right)^{0.2}}} \quad \text{(23)}
\]

\( k = \text{is equal to equation (14)} \)

\( \zeta = \text{is the slenderness and size effect factor} \)

### 3.9. CEB-FIP Equation

The Comité Euro-International du Béton in the CEB-FIP code [53] recommends an equation to predict one way shear that studies size effect, slenderness effect and reinforcement ratio. The reinforcement ratio to be used in this study represents the horizontal strengthening material and the reinforcing steel. Moreover, the equation uses the cubic root of \( f' \) to represent the concrete strength in shear.

\[
V_c = 0.15 \left(\frac{3d}{a}\right)^\frac{1}{3} \left(1 + \sqrt{\frac{200}{d}}\right) \left(100 \ \rho_{\text{eff}} f'_c\right)^\frac{1}{3} b \ d_{\text{eff}}
\]  

(24)
Chapter 4: Experimental Program

The test matrix is composed of 18 RC beams that will be strengthened, using different configurations, to study the effect of flexural NSM-FRP on the shear capacity of concrete \( (V_c) \); the variables of the experimental program are:

1- Normal (30 MPa) and high strength (70 MPa) concrete
2- Height of the beam cross-sections (230, 380, 550 and 650 mm) to study the size effect on strengthened specimens

In order to compare samples, other parameters such as steel reinforcement ratio \( (\rho_s) \), shear span to depth ratio \( \left( \frac{a}{d} \right) \), and FRP reinforcement ratio \( (\rho_f) \) were held constant. However, \( \rho_f \) is varied on samples with depth of 230mm in order to check the effect of increasing the FRP to increase the shear strength \( V_c \). FRP bars are placed on the sides for larger samples in order to have enough spacing between bars. The steel reinforcement ratio was set to be at about 0.6%, which is between the minimum and maximum reinforcement ratios in the ACI318-14 code [15]. The reinforcement was varied with the depth of the beams in order to maintain the reinforcement ratio to be constant. The beams were cast without stirrups in the shear span, in order to solely study the effect on various parameters on \( V_c \). The shear span to depth ratio \( \left( \frac{a}{d} \right) \) was taken to be 2.5 in order to insure shear failure without arch action (deep beam behavior). Moreover, it was not chosen to be larger in order to limit the spans of the beams specimens. All the beams were designed to have a shear capacity smaller than its flexural capacity to ensure shear failure. The test matrix of this study is summarized in Table 1.

The beams were designated with the letters L and H to indicate low and high strength concrete, respectively. The letters C and N are to indicate if a beam is a control beam or an NSM-strengthened beam. The numbers 230, 380, 550 and 650 are to indicate the height of the cross-sections. For example, beam designation HN650 indicates that the beam is made of high strength concrete, strengthened with NSM, and has a total height of 650 mm. The beam’s depth \( (d) \), shear span \( (a) \), span length \( (L) \), area of steel \( (A_s) \), area of FRP bars \( (A_f) \), \( (\rho_s) \) and \( (\rho_f) \) are provided in table 1 for each beam specimen.
Table 1: Test Matrix

<table>
<thead>
<tr>
<th>Beam</th>
<th>$f'_c$ (MPa)</th>
<th>$a/d$</th>
<th>d (mm)</th>
<th>a (mm)</th>
<th>L (mm)</th>
<th>$A_s$ (mm$^2$)</th>
<th>$A_f$ (mm$^2$)</th>
<th>$\rho_s$ (%)</th>
<th>$\rho_f$ (%)</th>
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<td>187</td>
<td>475</td>
<td>1250</td>
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<td>0.6</td>
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<td>187</td>
<td>475</td>
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<td>226</td>
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<td>0.6</td>
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<td>2000</td>
<td>402</td>
<td>0</td>
<td>0.6</td>
<td>0</td>
</tr>
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<td>2.5</td>
<td>334</td>
<td>848</td>
<td>2000</td>
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<td>0.6</td>
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<tr>
<td>HN550</td>
<td>70</td>
<td>2.5</td>
<td>502</td>
<td>1250</td>
<td>2800</td>
<td>628</td>
<td>0</td>
<td>0.6</td>
<td>0.36</td>
</tr>
<tr>
<td>HC650</td>
<td>70</td>
<td>2.5</td>
<td>600</td>
<td>1500</td>
<td>3300</td>
<td>942</td>
<td>0</td>
<td>0.6</td>
<td>0</td>
</tr>
<tr>
<td>HN650</td>
<td>70</td>
<td>2.5</td>
<td>600</td>
<td>1500</td>
<td>3300</td>
<td>942</td>
<td>0</td>
<td>0.6</td>
<td>0.33</td>
</tr>
</tbody>
</table>

where:

$$\rho_s = \frac{A_s}{bd}$$

$$\rho_f = \frac{A_f}{bd_f}$$

4.1. Beam Design Details

The proposed beams have four different lengths with nine different cross-sections as shown in Figures 6-12. An anchorage distance was taken at the ends of the beam equal to 150 mm. A clear cover of 30 mm was taken to ensure adequate space for grooves of NSM bars. The beams were tested under three point bending, with the load applied at the beam’s mid-span using a hydraulic jack. One deflection transducer was
used to measure the mid-span deflection as shown in Figures 6 and 7. The distance $d_{\text{eff}}$ is calculated using equations (3) and (4).

Figure 5: Beam specimen detailing and test layout
Figure 6: Beam testing set-up
a) Beams with $h=230\text{mm}$

b) Beams with $h=380\text{mm}$

c) Beams with $h=550\text{mm}$

d) Beams with $h=650\text{mm}$

Figure 7: Beams reinforcement detailing and experimental setup
a) LC230 and HC230  

b) L1N230 and H1N230  
c) L2N230 and H2N230

Figure 8: Cross-section detailing for RC beams with \( h = 230\text{mm} \)

a) LC380 and HC380  

b) LN380 and HN380

Figure 9: Cross-section detailing for RC beams with \( h = 380\text{mm} \)

a) LC550 and HC550  

b) LN550 and HN550

Figure 10: Cross-section detailing for RC beams with \( h = 550\text{mm} \)
4.2. Materials Properties

4.2.1. Concrete. The concrete compressive strength was targeted at 30 MPa and 70 MPa for beams of Group 1 and Group 2, respectively. A total of ten cylinders were tested at the day of testing the beams for each of the two mixes, and their results were averaged. All the tests were conducted in accordance with ASTM C 39M-99 [68]. The test results are summarized in Table 2.

The majority of the cylinders yielded a well-formed cone shape after concrete crushing which is the recommended failure mode as specified by ASTM. Figure 13 demonstrates failure mode close to type 3 by high-strength sample 5. An important observation made was that the high-strength concrete cylinders experienced a significantly brittle failure which is not an uncommon observation for high strength concrete specimens.

In addition, one cylinder had a different mode of failure; type 4. This could be due to some errors while concreting. Figure 13 shows the failure mode in high-strength
concrete sample 4, where the failure mode is similar to type 4 as specified by ASTM standards which was removed from the study as an outlier specimen.

Table 2: Concrete compressive strength results summary

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Normal Strength Concrete (L) $f'_c$ (MPa)</th>
<th>Sample No.</th>
<th>High Strength Concrete (H) $f'_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40.8</td>
<td>1</td>
<td>74.3</td>
</tr>
<tr>
<td>2</td>
<td>26.4</td>
<td>2</td>
<td>71.6</td>
</tr>
<tr>
<td>3</td>
<td>39.2</td>
<td>3</td>
<td>67.6</td>
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<tr>
<td>4</td>
<td>36.1</td>
<td>4</td>
<td>62.6</td>
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<td>5</td>
<td>35.8</td>
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</tr>
<tr>
<td>8</td>
<td>41.6</td>
<td>8</td>
<td>75.3</td>
</tr>
<tr>
<td>9</td>
<td>36</td>
<td>9</td>
<td>72.4</td>
</tr>
<tr>
<td>10</td>
<td>37.6</td>
<td>10</td>
<td>68.4</td>
</tr>
<tr>
<td>$f'_c$ (average)</td>
<td>36.6</td>
<td>$f'_c$ (average)</td>
<td>69.9</td>
</tr>
</tbody>
</table>

Figure 12: Cone failure mode in high-strength concrete
4.2.2. **Steel Reinforcement.** Normal Mild steel bars were used as reinforcement for the RC beam specimen. The yield strength, tensile strength and modulus of elasticity of the bars were to be 560 MPa, 620 MPa and 200 GPa, respectively as reported by the manufacturer.

4.2.3. **V-wrap CFRP bars.** The used NSM-CFRP bars used are provided commercially by Structural Technologies [69]. The bars had a diameter of 12 mm with a design area of 122 mm$^2$. The bars have a tensile strength and modulus of elasticity of 2,068 MPa and 131 GPa, respectively. In addition, the ultimate strain at failure of the bars was 1.58%. Using the rule of mixtures, the FRP fiber fraction should be around 70% resin, while the matrix is around 30%.

4.2.4. **V-wrap adhesive resin.** The matrix used is epoxy V-Wrap™ 778 similarly provided by Structural Technologies [62]. The tensile strength and elastic modulus of the epoxy adhesives are 62 MPa and 3,450 MPa, respectively. Moreover, the flexural strength is 110 MPa, and the flexural modulus is 3,100 MPa. The elongation of the epoxy at failure is 7%, as provided by the supplier. The epoxy must have at least 15 minutes as setting time and has to be applied at a temperature range of 4°C to 38°C.
Chapter 5: Experimental Results

The results of this chapter include the load-deflection response curves for 18 tested beams specimens. The shear strength of the strengthened beams is compared to that of the control beams, and the ratio of the increase is provided. In addition, the results are supplemented with photos of the specimens to demonstrate the failure modes and crack propagation patterns of the tested beam specimens.

5.1. Load-Deformation Curves

This section presents the load-deformation results of the 18 specimens divided into two subsections, one for each group. In each section, the results are presented for each strengthened specimen with its corresponding control beam of the same beam size in one graph to demonstrate the behavior of the NSM-FRP strengthened specimens in comparison with the control specimens.

5.1.1. Group 1. Which represents specimens with different cross sections and normal concrete compressive strength ($f'_c = 36.6$ MPa), as provided in section 4.2.1. Table 3 summarizes the Group’s ultimate attained load $P_u$, shear failure load $P_{us}$ at the occurrence of the first major diagonal shear crack and designated by a circle on the load versus mid-span displacement graphs, beam’s shear strength ($V_c = P_{us}/2$), and the load capacity increase over the control specimen as a ratio, $(V_c/V_{u,c})$, where $V_{u,c}$ is the shear strength of the control beam for each size of specimens.

As expected, there was a noticeable increase in shear strength for the NSM-FRP strengthened RC beams as shown in Table 3. The percentage increase varies from 16 to 34%. Additionally, test sample L2N230 experienced an increase in shear strength, compared to L1N230 by 15.3%, since the FRP reinforcement ratio is higher. A summary of the results is shown in Table 3. This proves that the amount of flexural longitudinal reinforcement contributed to the shear strength of RC beams.

Figure 15 shows the load-midspan deformation response curves of the three beams with depth of 230 mm from group 1. The graph shows that NSM-FRP reinforced beams experienced an increase in shear strength, compared to the control beam. However, they also experienced an increase in post-stiffness shown by the failure which happened at a relatively smaller deformation. The major shear crack for LC230, L1N230, and L2N230 occurred at a loading of 112, 130, and 149 kN, respectively. The results
indicate that the range of increasing the FRP reinforcement ratio ($\rho_s$) from 0 to 0.65% increased the shear capacity of the beams by 34%.

Table 3: Summary of test results of Group 1

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>$P_u$ (kN)</th>
<th>$P_{us}$ (kN)</th>
<th>$V_c$ (kN)</th>
<th>$V_c/(V_{u,c})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC230</td>
<td>119.0</td>
<td>112</td>
<td>56</td>
<td>-</td>
</tr>
<tr>
<td>L1N230</td>
<td>129.7</td>
<td>130</td>
<td>65</td>
<td>1.16</td>
</tr>
<tr>
<td>L2N230</td>
<td>146.7</td>
<td>149</td>
<td>75</td>
<td>1.34</td>
</tr>
<tr>
<td>LC380</td>
<td>161.3</td>
<td>158</td>
<td>79</td>
<td>-</td>
</tr>
<tr>
<td>LN380</td>
<td>199.8</td>
<td>199</td>
<td>100</td>
<td>1.27</td>
</tr>
<tr>
<td>LC550</td>
<td>203.7</td>
<td>200</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>LN550</td>
<td>244.0</td>
<td>243</td>
<td>122</td>
<td>1.22</td>
</tr>
<tr>
<td>LC650</td>
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<td>295</td>
<td>135</td>
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</tr>
<tr>
<td>LN650</td>
<td>338</td>
<td>332</td>
<td>166</td>
<td>1.13</td>
</tr>
</tbody>
</table>

Figure 14: Load-Deformation for low-strength beams with 230 mm depth

Figure 16 shows the load-deformation graphs of the control and NSM-FRP reinforced beams, with a depth of 380 mm. Shear cracking for LC380 and LN380 occurred at 158 kN and 199 kN, respectively. The results indicate that the range of increasing ($\rho_f$) from 0 to 0.36% resulted in an increase in the shear capacity of the beams by 27%.
The load-deformation results of the 550 mm depth beams is shown in Figure 17. There is also a clear increase in shear strength. The obtained shear cracking loads for LC550 and LN550 are 200 and 243 kN, respectively. The results reveal that the range of increasing ($\rho_s$) from 0 to 0.36% again resulted in an increase in the shear capacity of the beams by 22%.

Figure 16: Load-Deformation for low-strength beams with 550 mm depth
Lastly, Figure 18 presents the load-deformation graphs of the two beams with depth of 650 mm. The shear crack in beams LC650 and LN650 occurred at 295 and 332 kN, respectively. The results indicate that the range of increasing $\rho_f$ from 0 to 0.36% helped in increasing the shear capacity of the beams by 13%.

![Load-Deformation Graphs for LC650 and LN650](image)

Figure 17: Load-Deformation for low-strength beams with 550 mm depth

Overall, when strengthening the beams with the same amount of longitudinal NSM-FRP (same $\rho_s$ and $\rho_f$) while increasing the beams’ size, the increase in shear strength will be lower. The latter can be linked to the size effect phenomena that will be discussed in chapter 6.

5.1.2. **Group 2.** Group 2 represents specimens with different cross sections and high compressive strength ($f'_c = 69.9$ MPa), as obtained from section 4.1. The maximum attained load, shear strength, and $V_c/V_{uc}$ are summarized in Table 4.

Table 4 clarifies the noticeable effect of using NSM-FRP reinforcement in enhancing the shear strength of the tested RC beam specimens. The percent increase in shear strength for group 2 for the NSM-FRP strengthened specimens ranged from 10 to 35%. However, test specimen H2N230 did not experience any significant increase in shear strength, as compared to H1N230 (only 2% strength increase). It is possible that the concrete shear has reached a limit where increasing the NSM-FRP material is deemed ineffective. Further discussion will be provided in Chapter 6. Figures 17-20
display the load-deflection response of each NSM-FRP reinforced specimens with their corresponding control specimen.

Table 4: Summary of test results of Group 2

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>( P_u ) (kN)</th>
<th>( P_{us} ) (kN)</th>
<th>( V_c ) (kN)</th>
<th>( V_u/(V_{u,c}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC230</td>
<td>140</td>
<td>135</td>
<td>67.5</td>
<td>-</td>
</tr>
<tr>
<td>H1N230</td>
<td>179.6</td>
<td>179</td>
<td>90.0</td>
<td>1.33</td>
</tr>
<tr>
<td>H2N230</td>
<td>180.3</td>
<td>180</td>
<td>91.0</td>
<td>1.35</td>
</tr>
<tr>
<td>HC380</td>
<td>175.0</td>
<td>175</td>
<td>88.0</td>
<td>-</td>
</tr>
<tr>
<td>HN380</td>
<td>218.15</td>
<td>212</td>
<td>110.0</td>
<td>1.25</td>
</tr>
<tr>
<td>HC550</td>
<td>221.0</td>
<td>210</td>
<td>111.0</td>
<td>-</td>
</tr>
<tr>
<td>HN550</td>
<td>260.3</td>
<td>250</td>
<td>130.0</td>
<td>1.17</td>
</tr>
<tr>
<td>HC650</td>
<td>324.1</td>
<td>316</td>
<td>162.0</td>
<td>-</td>
</tr>
<tr>
<td>HN650</td>
<td>361.3</td>
<td>355</td>
<td>177.5</td>
<td>1.10</td>
</tr>
</tbody>
</table>

For the beam specimens with depth 230 mm, two NSM-FRP reinforced specimens were strengthened, one with a single CFRP-NSM bar, and the other with two CFRP-NSM bars. Figure 19 shows the load-deformation response curve of the two specimens and their corresponding control specimen HC230. The shear cracking points are indicated with a black circle. The major shear crack for HC230, H1N230, and H2N230 specimens occurred at a loading of 135, 179, and 180 kN, respectively. The results indicate that the range of increasing \( \rho_f \) from 0 to 0.6% resulted in an increase in the shear capacity of the beams by 35%.

Figure 18: Load-Deformation for high-strength beams with 230 mm depth
For the high strength concrete specimens with depth of 380 shown in Figure 19, the major shear crack in beams HC380 and HN380 occurred at 175 kN and 212 kN, respectively. The results indicate that the range of increasing $\rho_f$ from 0 to 0.35% increased the shear capacity of the beams by 25%.

![Figure 19: Load-Deformation for high-strength beams with 380 mm depth](image)

Figure 19: Load-Deformation for high-strength beams with 380 mm depth

Figure 20 shows the major shear crack occurred at 250 and 210 kN in beams HC550 and HN550, respectively. The results again indicate that the range of increasing $\rho_f$ from 0 to 0.35% increased the shear capacity of the beams by 17%.

![Figure 20: Load-Deformation for high-strength beams with 550 mm depth](image)

Figure 20: Load-Deformation for high-strength beams with 550 mm depth
Figure 22 shows that the major shear crack occurred at 316 and 355 kN in beams HC650 and HN650, respectively. However, due to the specimens’ relatively large size, the effect of increase in shear strength is less significant proportionately, which is attributed to size effect phenomena. The results indicate that the range of increasing $\rho_f$ from 0 to 0.3% helped in increasing the shear capacity of the beams by 10%.

Overall, when strengthening the beams with the same amount of longitudinal NSM-FRP (same $\rho_s$ and $\rho_f$), while increasing the beams’ size, the increase in shear strength will be lower. The latter can be linked to the size effect phenomena that will be discussed in the chapter 6.

![Graph showing Load-Deformation for high-strength beams with 650 mm depth](image)

**Figure 21: Load-Deformation for high-strength beams with 650 mm depth**

### 5.2. Observations of Cracking and Failure Modes

This section will present observed cracking and failure modes of the tested specimens in Group 1 and 2.

#### 5.2.1. Group 1. The mechanism of the shear failure, along with crack propagation and failure modes, are highly dependent on the tensile stresses that are generated in the specimens by the applied loads. The typical mode of failure observed in the tested specimens was the conventional “diagonal tension” failure. Prior to the beams’ failure by diagonal crack, some vertical flexural cracks were also observed at the center of the beams where maximum moment occurred. As the load gradually increased, the crack
initiated from the support and propagated towards the loading point. Figure 23 shows the modes of failure and crack propagation patterns for Group 1 specimens.
e) LN380
f) LC550
g) LN550
h) LC650
Figure 22: Crack propagation and failure mode of Group 1 specimens

Note: Drawings are not to scale

5.2.2. Group 2. The failure mode and crack propagation patterns are shown in Figure 24. The specimens of this group generally experienced a more sudden failure due to the high strength concrete in this group. The specimens failed with one main diagonal shear crack which propagated towards the loading point. Similar to Group 1 specimens, some vertical flexural cracks also occurred during loading. Furthermore, NSM-FRP specimens had a more sudden failure mode which occurred right after the appearance of the diagonal tension crack.
b) H1N230

c) H2N230

d) HC380

e) HN380

f) HC550
g) HN550

h) HC650

i) HN650

Figure 23: Crack propagation and failure mode of Group 2 specimens

Note: Drawings are not to scale
Chapter 6: Discussion of Results

In this chapter, the test results are summarized and then compared against each other. In addition, the results are compared in terms of size effect to demonstrate the relative decrease in concrete shear strength with the increase of beam size. Lastly, the results are compared with the theoretical results predicted by the prediction equations discussed in Chapter 2.

6.1. Key Parameters of Test Specimens

Table 5 summarizes the key parameters of the test specimens along with the test results of the nominal concrete shear strength $V_c$ and normalized with respect to $\frac{V_c}{bd_{eff}\sqrt{f'_c}}$ ratio. Moreover, Table 5 includes $\rho_{eff}$ and $d_{eff}$ which can be calculated using equations 3-5.

Table 5: Summary of test results

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>$f'_c$ at the day of testing (MPa)</th>
<th>$d_{eff}$ (mm)</th>
<th>$\rho_{eff}$ (%)</th>
<th>$V_c$ (kN)</th>
<th>$\frac{V_c}{bd_{eff}\sqrt{f'_c}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC230</td>
<td>40.8</td>
<td>189.0</td>
<td>0.598</td>
<td>56.0</td>
<td>0.246</td>
</tr>
<tr>
<td>L1N230</td>
<td>37.6</td>
<td>199.6</td>
<td>1.091</td>
<td>65.0</td>
<td>0.334</td>
</tr>
<tr>
<td>L2N230</td>
<td>39.2</td>
<td>205.9</td>
<td>1.583</td>
<td>75.0</td>
<td>0.310</td>
</tr>
<tr>
<td>LC380</td>
<td>36.1</td>
<td>339.0</td>
<td>0.593</td>
<td>79.0</td>
<td>0.198</td>
</tr>
<tr>
<td>LN380</td>
<td>35.8</td>
<td>350.6</td>
<td>1.142</td>
<td>100.0</td>
<td>0.246</td>
</tr>
<tr>
<td>LC550</td>
<td>33.6</td>
<td>509.0</td>
<td>0.617</td>
<td>100.0</td>
<td>0.173</td>
</tr>
<tr>
<td>LN550</td>
<td>40.0</td>
<td>520.3</td>
<td>1.166</td>
<td>122.0</td>
<td>0.189</td>
</tr>
<tr>
<td>LC650</td>
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<td>609.0</td>
<td>0.619</td>
<td>135.0</td>
<td>0.154</td>
</tr>
<tr>
<td>LN650</td>
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<td>166.0</td>
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</tr>
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<td>0.215</td>
</tr>
<tr>
<td>HN230</td>
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<td>199.6</td>
<td>1.091</td>
<td>90.0</td>
<td>0.281</td>
</tr>
<tr>
<td>H2N230</td>
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<td>91.0</td>
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<td>0.163</td>
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<td>0.189</td>
</tr>
<tr>
<td>HC550</td>
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<td>0.617</td>
<td>111.0</td>
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<tr>
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<td>609.0</td>
<td>0.619</td>
<td>162.0</td>
<td>0.123</td>
</tr>
<tr>
<td>HN650</td>
<td>72.4</td>
<td>619.3</td>
<td>1.108</td>
<td>177.5</td>
<td>0.139</td>
</tr>
</tbody>
</table>
6.2. Overall Load-Deformation Response Curves for Normal and High Strength Concrete Specimens

Figure 25 demonstrates the load-deflection graphs of the low strength concrete specimens, whose ID starts with L. Figure 26 compares the load-deformation graphs of the high strength specimens. It can be clearly indicated from Figures 25 and 26 that as the CFRP reinforcement ratio increases, the shear strength of the beam specimen increases as well.

Figure 24: Load-deformation graphs of low concrete strength specimens

Figure 25: Load-deformation graphs of high strength concrete specimens
6.3. Comparison between Strengthened and Control Specimens by Size

6.3.1. Comparison between all specimens with depth of 230 mm. Figures 27 and 28 show the load vs deflection graphs for the low and high concrete strength beams having a size of 230mm. It was noticed experimentally that the shear crack propagation was more sudden for the high strength specimen at higher loads. Beams H1N230 and H2N230 showed similar shear capacity of about 90kN. The results can indicate that the beams have reached a limit by which increasing the CFRP reinforcing materials becomes ineffective and has little effect on increasing the beam’s shear capacity. For beams L1N230 and H1N230, the beams’ capacities were 65 and 90kN, respectively. Moreover, beams L2N230 and H2N230 had a shear capacity of 75 and 91kN, respectively. The increase in $f^c$ from about 40MPa to 70MPa, results in an increase in strength of 38% and 21% for beam specimens reinforced with 1 and 2 NSM-CFRP bars, respectively. This indicates that the increase in $V_c$ for high strength concrete specimens is not as effective as in normal (low) strength concrete specimens.

Figure 26: Load-deformation graph for test specimens with depth of 230 mm
**6.3.2. Comparison between all specimens with depth of 380 mm.** Figures 29 and 30 show the load vs deflection graphs for the beams for low and high concrete strength having a size of 380mm. When comparing 380mm beams to the 230mm, the increase in strength is not remarkably significant. The percentage increase in strength of the 380mm beams was 27% compared to 34% for the 230mm counterpart. For beams LN380 and HN380, the beams’ capacities were 100 and 110kN, respectively. When increasing in $f'_c$ from about 40MPa to 70MPa, the increase in shear strength is about 10%. When comparing it to the 230mm beams, increasing the $f'_c$ in larger beams has a lesser effect to smaller beams. This indicates that the increase in $V_c$ for high strength concrete specimens is not as effective as in normal (low) strength concrete specimens and rather decreases with the increase in size.
Figure 28: Load-deformation graph for test specimens with depth of 380 mm

Figure 29: 380 mm specimen groups side-by-side comparison

6.3.3. Comparison between all specimens with depth of 550 mm. Figures 31 and 32 show the load vs deflection graphs for the beams for low and high concrete strength having a size of 550mm. When comparing 550mm beams to the 380mm, the increase in strength is not as significant. The percentage increase in strength of the
550mm beams was 22% compared to 27% for the 380mm counterpart. For beams LN550 and HN550, the beams’ capacities were 122 and 130kN, respectively. When an increase in $f'$, from about 40MPa to 70MPa, takes place, it results in an increase in shear strength of about 6.6%. When comparing it to the 380mm beams, increasing the $f'$ in larger beams has a lesser effect that that of smaller beams. This indicates that the increase in $V_c$ for high strength concrete specimens is not as effective as in normal (low) strength concrete specimens and rather decreases with the increase in size.

![Figure 30: Load-deformation graph for test specimens with depth of 550 mm](image1)

![Figure 31: 550 mm specimen groups side-by-side comparison](image2)
6.3.4. **Comparison between all specimens with depth of 650 mm.** Figures 33 and 34 show the load vs deflection graphs for beams of low and high concrete strength, having a size of 650mm. When comparing 650mm beams to the 550mm, the increase in strength is not as significant. The percentage increase in strength of the 650mm beams was 13% compared to 27% for the 550mm counterpart. For beams LN650 and HN650, the beams’ capacities were 166 and 177.5kN, respectively. An increase in $f'_c$ from about 40MPa to 70MPa, results in an increase in shear strength of about 6.9%. When comparing it to the 550mm beams, increasing the $f'_c$ in larger beams has a similar effect to smaller beams. This is an indication that the increase of size with varying $f'_c$ increases in a decreasing matter.

![Graph showing load vs deflection for beams of different strengths](image1)

*Figure 32: 650 mm specimen groups side-by-side comparison*

![Graph showing load vs deflection for beams of different strengths](image2)

*Figure 33: 650 mm specimen groups side-by-side comparison*
6.3.5. **Summary.** Figure 35 summarizes the test results in the form of bar charts to demonstrate the effect of NSM-CFRP reinforcement on the test specimens. Figure 36 shows the ratio increase of strengthened specimens over their control counterpart. There is a noticeable increase in the shear strength of the NSM-CFRP reinforced specimens as shown in figure 36. However, this increase becomes less prominent as the size of the specimen increases. In particular, the increase in strength for low strength specimens having a size of 230mm and reinforced with 1 and 2 NSM-CFRP bars was 8.3 and 21.7%, respectively. For the larger specimens, the increase in strength for that group was about 27, 22 and 13%. A similar case can also be seen in the high strength specimens as per Figure 36. The observation of the decreasing effect in shear strength with increasing size of the beam gives evidence to the presence of size effect in NSM-CFRP strengthening. Moreover, increasing $f_c'$ in larger beams strengthened with longitudinal CFRP reinforcement is not as effective as increasing it for the smaller counterparts. The latter can also be related to the size effect phenomena which will be discussed in section 6.4.

Figure 34: Bar chart summary of shear strength in all test specimens
6.4. **Comparison in Size Effect**

6.4.1. **Overview.** Figure 32 shows the normalized \( \frac{V_c}{b d_{eff} \sqrt{f'_c}} \) ratio versus the beam effective depth depth \((d_{eff})\). The effect of the specimen size on the test results, mainly the effectiveness of the NSM-CFRP bars on concrete shear strength, was noticeable in the results. This is shown in Figure 36, where there is a noticeable pattern of convergence of the results, showing the diminishing strength gained by the CFRP reinforcement. With the increase in beam size, Figure 36 shows that the ACI-318 code overestimates the strength of beams with larger depth. These results demonstrate the presence of the size effect as an important parameter in determining the concrete shear strength \( V_c \) of the beam, meaning that the ACI-318 constant of 0.17 for the shear strength design equation \( V_c \) does not capture all the different parameters necessary for determining the beam shear strength.
6.4.2. **Low strength concrete specimens.** In the low strength concrete specimens, the size effect can be seen in Figure 38, where the increase in the shear strength in CFRP reinforced specimens becomes less prominent as the depth of the beam increases. Another important observation made from this graph is that for
specimen LC650, the results showed a lower normalized shear ratio (0.15) than that provided by ACI-318 shear provision (0.17). This shows the significance of the size effect and the importance of considering it when predicting $V_c$.

![Figure 37: Size effect in low strength concrete specimens](image)

6.4.3. **High strength concrete specimens.** Similar to the low strength concrete specimens, the high strength specimens, shown in Figure 38, also show a significant presence of the size effect on their shear strength. The effect is more critical in higher strength concrete beams since most of the specimens fall under the 0.17 line, indicating that the ACI-318 equation overestimates the shear strength for such beams.

![Figure 38: Size effect in high strength concrete specimens](image)

Thus, it can be concluded that the size effect has a significant influence on the shear capacity of RC beams. The same conclusion can be drawn to beams strengthened with
longitudinal NSM-CFRP reinforcement. Such size effects are not captured in the ACI-318 simplified equation factor of 0.17, which shows the need to modify the ACI-318 shear design equation to incorporate the size effect (d) factor in predicting the shear strength for normal and high strength RC beams. In addition, other models in the literature should be studied to examine if they predict the size effect on concrete shear strength of the tested specimens.
Chapter 7: Comparison of Test Results and Analytical Models

7.1. Analytical Models

The analytical models presented earlier show a variety of approaches used for calculating the shear strength provided by concrete. Additionally, the equations provided by these models range in their level of conservativeness and accuracy. Table 6 provides a comparison of the test results with all the models presented.

Since the different equations have different levels of accuracy, it is important to look at the statistical parameters which can assess the validity of the predictions. The values of standard deviation, mean, and coefficient of Variation (COV), normalized mean average error (MAE) and root mean square deviation (RMSE) of all $V_{test}/V_c$ are presented in Table 7.

The Multi-Action Model was able to predict the test results to the least coefficient of variation of 0.1. However, the mean $V_{test}/V_c$ is equal to 1.52 which does not show a high level of accuracy. The Canadian Code equation showed the lowest $V_{test}/V_c$ of 1.16 but with a COV equal to 0.26, which is one of the highest values among the tested models.

Equations such as UH and the 2nd order SMCFT show coherent results in terms of accuracy and precision. The mean $V_{test}/V_c$ is equal to approximately 1.2 for both equations. However, the UH model showed one of the least COV among the tested models. Moreover, the MAE and RMSE for the UH model 13.5 and 17.2, respectively. However, the 2nd order SMCFT shows values of 35.2 and 36.4, respectively.

Tureyen and Frosch shows the most conservative results with $V_{test}/V_c$ equals to 2.07 and COV of 0.24.
Table 6: Comparison of test results with the analytical models

<table>
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<tr>
<th>Beam ID</th>
<th>$V_{\text{test}}$ (kN)</th>
<th>ACI Simplified $V_{\text{test}}/V_c$</th>
<th>ACI General $V_{\text{test}}/V_c$</th>
<th>Tureyen and Frosch $V_{\text{test}}/V_c$</th>
<th>UH $V_{\text{test}}/V_c$</th>
<th>SMCFT $V_{\text{test}}/V_c$</th>
<th>SMCFT $V_{\text{test}}/V_c$</th>
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Table 7: Statistical data of the analytical models

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7.2. **Comparison Between Predicted and Test Data for All Models**

Looking at the mean and the coefficient of variation of the different $V_{\text{test}}/V_c$ can help estimate the level of accuracy and precision of each equation. However, the full picture can be captured visually when the graphs of $V_{\text{test}}$ vs $V_c$ are plotted. The graph reveals aids that help in showing the conservativeness of the equations which predict values above and below the line of unity. Figure 39 shows the distribution of the different test points for the specimens. The dashed lines in Figure 40 show a deviation of 10% to provide a visual range of which the points lay in.

Tureyen and Frosch along with the SMCFT can be observed to be significantly conservative, while the two ACI equations and the CEB-FIP reveal a number of points falling under the line, meaning that the equations overestimate the shear capacity of the beams. The overestimation is due to the size effect and the high strength concrete use in the tested specimens in this study. A size effect factor is included in the UH model,
and hence, the points are falling on the line or above showing the conservativeness and accuracy of the UH model.

The average value of the 2\textsuperscript{nd} order SM CFT is comparatively lower than the rest of the models. However the equation shows values that are not conservative. A similar case can be seen in the Canadian code equation. The average $\frac{V_{\text{test}}}{V_c}$ is low due to the points that aren’t conservative.

The high COV of CEB-FIP can be shown in the scatter of the points of the graph, and many of its values are not conservative. Zsutty equation shows 3 values falling below the unity line. Moreover, the mean value is 1.3, and the COV is 0.25 which is high compared to the UH equation.
(f) SMCFT 2ND

(g) CANADIAN

(h) CEB-FIP
7.2.1. **Summary.** Tureyen and Frosch, along with the SMCFT show the most conservative results of the analytical models. However, the SMCFT shows a lesser mean of $V_{test}/V_c$ and COV (1.6 and 0.19 compared to 2.07 and 0.24, respectively).

The ACI general and simplified equations show a mean $V_{test}/V_c$ of 1.26 and 1.24, respectively. Moreover, the COV of the general and simplified equations are 0.28 and 0.29. The general equation shows a better performance than the simplified one.
However, both equations show results that are not conservative, which can be related to the size effect and high strength concrete test RC beams.

The Canadian code shows the least average $V_{\text{test}}/V_c$ of 1.16, which corresponds to a COV of 0.26. Moreover, the multi-action model shows the least COV of 0.1, which corresponds to a $V_{\text{test}}/V_c$ average of 1.52. However, both equations show results that are not conservative according to Figure 40 (g and i).

CEB-FIP and Zsutty models show average values of $V_{\text{test}}/V_c$ (1.22 and 1.3, respectively). However, the CEB-FIP show a high value of COV equals to 0.4, an effect which can be seen in the scatter of Figure 40 (h). The coefficient of variation of the Zsutty model is 0.25 which is average, compared to the rest of the models. Moreover, when analyzing Figure 40 (j), values that are not conservative are shown in the graph.

The UH and 2nd order SMCFT models show excellent results in terms of the accuracy of prediction. The average $V_{\text{test}}/V_c$ for the UH and 2nd order SMCFT are 1.2 and 1.19. The COV of both equations is low compared to the rest of the studied models which are 0.14 and 0.19, respectively. However, the 2nd order SMCFT in figure 40 (f) shows values below the unity line, while the UH model shows excellent performance overall.

### 7.3. Proposed Analytical model

The university of Houston model shows the best prediction data compared with testing. However, the equation can be optimized in order to produce more accurate data with respect to flexural strengthening, using NSM-FRP bars effect in shear strength. Equation (17) will be optimized by varying the factor $(x = 2)$, using the testing data from this study as shown in equation (25).

\[
V_c = x \left( \frac{M_u}{V_{ud}} \right)^{-0.7} \times \left( \frac{1}{\sqrt{1 + \frac{h}{300}}} \right) \times \sqrt{f' c b w c} \tag{25}
\]

Using excel solver to minimize the sum of squared errors between the tested data and the predictions by changing $x$, the average value of $x$ was 2.32 with a standard deviation of 0.32. When testing the equation with the data versus the UH model, the results in table 8 are obtained. Moreover, a comparison of the data is shown in figure
All the statistical data shows a better fit for the proposed model versus the UH model. However, the equation yields some data that is not conservative, which falls between ranges of ±10%.

Table 8: Proposed equation and UH model statistical data

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Figure 40: Comparison between test and predicted data for (a) UH model and (b) the proposed model
Chapter 8: Summary and Conclusions

The aim of this study was to address the effect of flexural strengthening of RC beams with NSM CFRP bars on the shear strength of the beams contributed by concrete. The beams were divided into two groups, based on the concrete compressive strength, which are 40 MPa and 70 MPa. Another parameter of the test matrix was the size effect where the heights of the beams varied from 230 to 650 mm. Other variables were held constant such as the effective reinforcement ratio with the increase in size and the shear span to depth ratio($\frac{a}{d}$). Based on the experimental results, the following conclusions can be drawn:

1- All beams showed an increase in shear strength when strengthened flexurally using NSM-FRP bars. The increase in strength is similar to increasing the normal reinforcement ratio to an RC beam.

2- The beams showed an increase in shear strength from 16% to 34% for group 1 and 10% to 35% for group 2.

3- The size effect had an impact on the percentage increase for the strengthened specimens, which is consistent for both groups. The ACI 318-14 factor of 0.17 shows results that are un-conservative for large beams, indicating the need to study other analytical models from the literature that take into account the size effect.

4- Strengthening group 2 beams had a lesser effect when compared to group 1. That can be contributed to the high $f'_{c}$ used in group 2. Furthermore, the size effect in group 2 is more significant in reducing the shear strength of reinforced concrete beams.

5- Prediction equations from various sources in the literature and design codes were used. Predictions from the University of Houston equation, Second order modified compression field theory showed excellent accuracy when compared to other models. The mean $\frac{V_{\text{test}}}{V_{c}}$ shear strength values and coefficient of variation for the university of Houston method are 1.2 and $\pm0.14$, respectively. Moreover, the mean test to predicted shear strength values and coefficient of variation for the 2nd order SMCFT were 1.19 and $\pm0.19$, respectively. However, the UH model shows the least tendency to be un-conservative, compared to other analytical models.
6- A proposed model based on the test data is provided in this study, which relies on optimizing the UH model. The model shows an average \( \frac{V_{test}}{V_c} = 1.03 \) and a COV of 0.14. Moreover, the model shows lesser values of MAE and RMSE (5.05 and 16, respectively) than the UH model. The proposed model shows promising results, but it needs to be tested on a larger cluster of data to prove its validity.

7- Overall, flexural longitudinal NSM bars could be used as a viable solution to enhance the shear strength of RC beams
References


[29] R. J. Frosch, “Concrete Contribution to Shear Strength,” in *Structures Congress 2009 : Don’t Mess with Structural Engineers - Expanding our Role*, 2009, pp. 1579–1586.


Appendices
Appendix A
A. Testing Setups and failure modes

A.1. Group 1:

A.1.1. LC230:

![Figure 41: Beam LC230 setup](image)

![Figure 42: Beam LC230 failure cracks](image)

A.1.2. L1N230:

![Figure 43: Beam L1N230 setup](image)
Figure 44: Beam L1N230 failure cracks

A.1.3. L2N230:

Figure 45: Beam L2N230 setup

Figure 46: Beam L1N230 failure cracks
A.1.4. LC380:

Figure 47: Beam LC380 setup

Figure 48: Beam LC380 failure cracks

A.1.5. LN380:

Figure 49: Beam LN380 setup
Figure 50: Beam LN380 failure cracks

A.1.6. LC550:

Figure 51: Beam LC550 setup

Figure 52: Beam LC550 failure cracks
A.1.7. LN550:

Figure 53: Beam LN550 setup

Figure 54: Beam LN550 failure cracks

A.1.8. LC650:

Figure 55: Beam LC650 setup
Figure 56: Beam LC650 failure cracks

A.1.9. LN650:

Figure 57: Beam LN650 setup

Figure 58: Beam LN650 failure cracks
A.2. Group 2:

A.1.1. HC230:

Figure 59: Beam HC230 setup

Figure 60: Beam HC230 failure cracks

A.1.2. H1N230:

Figure 61: Beam H1N230 setup
Figure 62: Beam H1N230 failure cracks

A.1.3. H2N230:

Figure 63: Beam H2N230 setup

Figure 64: Beam H2N230 failure cracks
A.1.4. HC380:

Figure 65: Beam HC380 setup

Figure 66: Beam HC380 failure cracks

A.1.5. HN380:

Figure 67: Beam HN380 setup
A.1.6. HC550:

Figure 68: Beam HN380 failure cracks

Figure 69: Beam HC550 setup

Figure 70: Beam HC550 failure cracks
A.1.7. HN550:

Figure 71: beam HN550 setup

Figure 72: Beam HN550 failure cracks

A.1.8. HC650:

Figure 73: Beam HC650 setup
Figure 74: Beam HC650 failure cracks

A.1.9. HN650:

Figure 75: Beam HN650 setup

Figure 76: Beam HN650 failure cracks
Vita

Rayyan Saleh was born in 1993, in Riyadh, Kingdom of Saudi Arabia. He studied and received his high school degree from Al Khaleej Secondary School in Riyadh, KSA. He enrolled in the American University of Sharjah and graduated with a BS in Civil Engineering in 2015. After that in the same year, he enrolled in the American University of Sharjah for a master’s degree in Civil Engineering with a concentration on structures. Whilst studying, he worked as a graduate teaching assistant for various courses and labs such as bridge engineering, computer methods of structural engineering, geotechnical lab, surveying lab. He is currently working at Engeprot engineering and post tension as a structural design engineer.