AUTONOMOUS VEHICLES DELIVERY SYSTEMS: ANALYZING VEHICLE ROUTING PROBLEMS WITH A MOVING DEPOT

by

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Approval Signatures

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Most importantly, I am very thankful to Allah for all the opportunities I had and for giving me the strength and the patience to make it through all the challenges I faced.

Nobody has been more important to me in the pursuit of this research than the members of my family. I must express my very profound gratitude to my parents and my siblings for providing me with unfailing support and continuous encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them.

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Dedication

To my parents ...
Abstract

The vast growth in the e-commerce market has increased the attention to resolving the problem of Last Mile Delivery that has significant challenges such as reducing operational cost or ecological impact and increasing supply chain performance. The inclusion of new technologies such as drones and robots help tackle these challenges by developing new distribution systems to improve from traditional deliveries methods. However, the use of these technologies brings new operational challenges. This research deals with the impact of using autonomous vehicles in logistics. We first present a technological review of the use autonomous vehicles in logistics and use it to introduce a classification of the delivery systems based on the parcel handover at the time of the last handling before delivery to customers. We describe three categories of handovers, namely, machine-to-person, machine-to-machine, and person-to-machine, and characterize for each of them the type of vehicle routing optimization that it implies. Moreover, we study a truck-drone system, where the truck serves as a depot from where we load the product to the drone for final delivery to customers. The depot is now moving unlike in a traditional Vehicle Routing Problem for which we always assume a fixed depot. Therefore, we present a new class of Vehicle Routing Problems with a moving depot for a truck-drone system and formulate six Integer Linear Programming formulations to minimize the total operational cost through sequencing the deliveries to different customers and optimizing the locations for the truck to release and collect the drones. The problem is NP-hard, thus developing heuristic solutions is more appropriate for large size instances. The proposed models are first solved using the General Algebraic Modeling System software to find the optimal solutions and study their characteristics. Furthermore, a Clarke and Wright Savings heuristic is developed using C++ language to solve large-size problems. The algorithm returned solutions that are within the known quality, 20%. The solutions provided 8% to 20% deviation from the optimal solutions. The algorithm returned solutions for 80 nodes within 1200 seconds. Different real-life applications can adopt the proposed models such as the UPS truck-drone and Amazon airborne fulfilment centre.

Keywords: Autonomous vehicles; last mile delivery; Traveling Salesman Problem; Vehicle Routing Problem, moving depot
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List of Abbreviations

AV    Autonomous Vehicle
AV2AV Autonomous Vehicle to Autonomous Vehicle
AV2D  Autonomous Vehicle to Drone
B2C   Business to Consumer
CW    Clarke and Wright Heuristic
GAMS  General Algebraic Modelling System
ILP   Integer Linear Programming
LMD   Last Mile Delivery
M2M   Machine to Machine
M2P   Machine to Person
MmTSP Multi-Depots Multiple Travelling Salesmen Problem
mTSP  Multiple Travelling Salesmen Problem
P2M   Person to Machine
S2S   System to System
T2AV  Truck to Autonomous Vehicle
T2D   Truck to Drone
TSP   Travelling Salesman Problem
VRP   Vehicle Routing Problem
VRP-MD Vehicle Routing Problem with Moving Depot
Chapter 1. Introduction

In this chapter, we provide an introduction about electronic commerce, Last Mile delivery, autonomous vehicles, and the vehicle routing problem. Then, we present the problem investigated in this study as well as the thesis contribution. Finally, we present the general organization of the thesis.

1.1. Overview

The rapidly growing technology of the internet has infused in almost every aspect of our life. It is providing unlimited opportunities for companies and customers interactions resulting in adopting electronic commerce as a trending business transaction. Shin [1] defines electronic commerce as an undertaking in which the internet is used first as a platform to establish the terms of trades (e.g., price, availability, and order processing time to delivery) among the participants in a marketing channel [1]. With the rise of business-to-consumer transactions, the demand for parcel handling and deliveries has increased resulting in creating a significant number of smaller orders fulfillment. This rapid evolution has intensified the need for logisticians to resolve the problem of conveying goods from transportation hubs to their final destinations. This distribution is known as the Last Mile Delivery problem (LMD) [2-4]. It is the last leg of a product’s journey nevertheless the most challenging, with the highest transportation cost across all distribution networks [5].

With e-commerce, customers are looking for instant deliveries at no additional cost. This attitude affects the performance of the supply chain and puts pressure on logistics management to meet customers’ expectations [6]. Also, these deliveries result in higher pollutant emissions such as CO₂ and NO₂ [7]. Recent advent in technology such as drones, robots, electric cars, artificial intelligence and the internet of things has given new opportunities to design improved logistics systems. The logistics industry makes use of these technologies to improve the traditional delivery methods (delivery person, postal/delivery boxes). One illustrative example is a combined truck-drone delivery system, which allows parcel deliveries from a mounted launch pad on the truck. Such new logistics system comes with new operational challenges in which the truck (depot) from where the product is last handed over to the drone for delivery to
the costumers is now moving, allowing faster deliveries, unlike a traditional system which has a fixed depot.

1.2. Electronic Commerce

During the last decade, the internet has grown tremendously to the extent that it permeated almost every aspect of our life. When the internet was established, it was considered as a medium for communication, but now it becomes a platform for purchasing products and services. People from different ages tend to shop from different e-stores as it has become simple to find anything on them. The Internet has provided unlimited opportunities for companies and customers as it has shaped various businesses, resulting in adopting electronic commerce in business transactions and being in a continuous phase. There are many definitions of e-commerce. It is broadly defined as a transaction in which the Internet is first used as a platform to establish the terms of trades (e.g., price, availability, and order processing time to delivery) among the participants in a marketing channel [1]. According to Kalakota and Whinston [8], e-commerce is recognized as the transmission of information, products, or services using computer networks or telephones from a communication perspective. E-commerce can also be defined as the transaction of goods and services through electronic communications [6].

E-commerce differs from traditional business in many ways, one of the most distinct ways is that e-commerce makes it much easier to reach a global market for various kinds of goods and services with the flexible communication between producers, suppliers, and customers [9]. It can be divided into five types: business-to-business (B2B), business-to-consumer (B2C), business-to-government (B2G), consumer-to-consumer, and mobile consumer [10]. Among the five types, B2C is the most popular one. It is a business conducted directly between the company and the consumer who are the end users of the company’s products and services via electronic media and by eliminating the process in the middle. As stated by Nisar and Prabhakar [10], the growth of Internet users purchasing products online is increasing annually. With the development of e-commerce specifically the B2C type, more and more customers tend to purchase almost all their needed products online from various websites that cover a wide range of products, e.g., electronics, books, furniture, clothes, and even food.
1.3. **Last Mile Delivery**

B2C transactions raised the demand for home deliveries resulting in creating a larger number of smaller orders. This enormous growth in the market of e-commerce increased the need of supply chains to solve the problem of Last Mile Delivery (LMD), which refers to the process of conveying goods from transportation hubs or fulfillment centers to a final destination in the supply chain management [2, 3]. In the supply chain, the Last Mile is the last leg of a product’s trip before it arrives at the customer and in an e-commerce environment, and it is the problem of transport planning for delivering goods from e-retailers hubs to their final destination [4]. Also, it is considered the most expensive, most polluting, and the least efficient part of the e-commerce supply chain. It accounts for 13% - 75% of the total supply chain cost [5]. Figure 1 illustrates the Last Mile.

![Diagram illustrating the Last Mile](image)

**Figure 1: The last mile**

According to Morganti et al. [6], the problems experienced with online shopping are mostly related to the delivery rather than the product itself. The paper reports that 39% of e-consumers have experienced problems such as: delivery at home when nobody was there (15%), a delay in the delivery (13%), delivery costs that were too high (7%), the lack of a way of tracking delivery status (5%), and the need to collect the product from a distant collection point (3%). The challenges in LMD are summarized as follows:
1. **Ecological aspect.** The vast number of daily deliveries including the repeated deliveries that result from the unavailability of customers at their place of residence during the standard working hours of the couriers can all result in increasing the emission of air pollutants (CO$_2$) through the increment of the car mileage of the courier. The problem of repeated deliveries concerns about 20% - 30% recipients [7].

2. **Cost.** It can be considered as the most important challenge when it comes to LMD. It is due to the considerable portion of the cost that goes to managing the delivery of the goods to their final destinations. The cost of fuel needed for traveling from one location to another and repeated deliveries can increase the amount of money spent that eventually cause a massive problem in the B2C delivery [5].

3. **Supply chain performance.** With the rise of e-commerce and the high demand for online shopping, the complexity of the supply chain increases due to the smaller size of orders and the high number of deliveries. Therefore, the traditional supply chain network should change to cope with the new design of supply chain networks. For instance, distribution networks should be restructured and reorganized to meet the high demands. E-commerce represents the driver of change in retail physical distribution networks. According to Morganti et al. [6], the growth of online shopping led to the need for new demand for e-fulfillment facilities directed by retailers. There are five types of e-commerce facilities:

   1. Mega e-fulfillment centers - where merchandises are stored and selected to make up the order.
   2. Parcel sorting centers (hubs) - where parcels are sorted before being forwarded to local parcel delivery centers.
   3. Local parcel delivery centers - for ‘last mile’ fulfillment.
   4. Local urban logistics depots - to ensure rapid order fulfillment.
   5. Return processing centers - to process returned items that customers decide they do not want.

For instance, Amazon has eight facilities in Germany totaling meter 762,000 sqm, including four mega centers of 110,000 sqm each [6]. Figure 2 shows the
transformation of logistics due to e-commerce in which this evolution has passed through multiple phases.

![Diagram of logistics transformation due to e-commerce](image)

**Figure 2:** The transformation of retail logistics due to e-commerce

### 1.4. Autonomous Vehicles

The autonomous vehicle (AV) revolution is about to become the world-changing technology as it is developing rapidly in every field: consumer, logistics, aerospace, agriculture, and automotive industry. This technology can have many advantages but faces many challenges.

In this section, we discuss the definition of autonomous vehicles, the recent technologies, and their challenges.

#### 1.4.1. Definition of autonomous vehicles.

The autonomous vehicle is considered a self-driving vehicle that operates without the need for direct input from the driver to control the steering, acceleration, and braking [11]. Autonomous vehicles are controlled during motion by a computer, along with various electronic subsystems and components rather than a human driver [12].

The US National Highway Traffic Safety Administration has classified autonomous vehicles by the division between automatic control and driver intervention [12]. There are six levels and can be listed as follows:
1. Level 0 - the automated system has no vehicle control but may issue warnings.

2. Level 1 - the automated system may include features such as autonomous cruise control (ACC), parking assistance with automatic steering, and lane keeping assistance (LKA). It is essential to have the driver ready to take control at any time.

3. Level 2 - the driver is obliged to detect objects, events and respond in case the automated system fails to respond correctly. The automated system executes accelerating, braking, and steering. Also, the automated system can deactivate immediately when taken over by the driver.

4. Level 3 - within known and limited environments (such as freeways), the driver can safely turn their attention away from driving tasks.

5. Level 4 - the driver must enable the automated system only when it is safe to do so. When enabled, driver attention is not required.

6. Level 5 - no human intervention is required. The automatic system can drive to any location where it is legal to drive.

To achieve a vehicle capable of driving itself, four primary functions are required:


2. Situational analysis to monitor the environment through which the vehicle is moving.

3. Motion planning using sensors for determining a precise course of motion within a defined period.

4. Trajectory control to manage the execution of pre-planned changes in speed and directions [11].

1.4.2. Key benefits of AVs. The rapid development of urbanization and car ownership impacts the transport systems, leading to challenges, including demand management, traffic signalization, traffic safety, and public transport [13].

Compared to the conventional modes of transport, automated vehicles are proved to offer a huge improvement in service quality and efficiency. Self-driving vehicles have important key benefits such as improving safety. According to [11], up to 90% of road traffic accidents are caused by drivers. The autonomous systems can
make better and faster decisions than humans because vehicles will always monitor and adapt to traffic and weather conditions as well as avoiding road obstacles with more diligence, speed, and safety than human drivers. In addition, autonomous vehicles can intelligently avoid busy routes using their vehicle-to-vehicle communication technology.

Also, passengers can have greater comfort, in which the driver becomes a passenger, and he or she can rest or enjoy doing other activities. Thus, it can make AVs an attractive form of transportation method for elderly, underage, and people with physical disabilities. Also, it can lower the environmental impact by achieving lower pollutant emissions [11].

In dense urban cities, labor costs are high. Manpower is one of the significant elements in any logistics system, which undoubtedly leads to a high level of inputs into the logistics system. Additionally, fatigue driving and bad driving behaviors lead to potential risks of transportation safety; however, automated vehicles are free from such risks. Thus, the application of automated vehicles in logistics can efficiently cut down the operation costs, improve the efficiency of freightage, and eliminate the potential risks caused by human beings [13].

1.4.3. Current technologies of AVs. Autonomous vehicles have been already deployed in a wide range of applications across different industries and fields such as aerospace, agriculture, consumer applications, automotive applications, and public transport applications. Also, AVs have been applied in logistics such as autonomous loading and transport, assisted order picking and linehaul transportation. Table 1 summarizes the recent applications of autonomous vehicles.

1.4.4. Truck-drone system. Trucks have been used to handle goods distribution across logistics networks. However, drones are recently considered as a method for achieving distribution tasks. According to Amazon, 86% of their order is less than 5 pounds which it is the ideal for drone delivery, and that 82% of customers are willing to pay for drone delivery [22]. Hence, drones can provide new opportunities to improve home delivery processes. Moreover, drones have four advantages: they operate without a human pilot and avoid the congestion of traditional road networks by flying over them. Also, they are faster than trucks and have much
lower transportation costs per kilometer [23].

Table 1: The deployment of autonomous vehicles in different industries

<table>
<thead>
<tr>
<th>Industry</th>
<th>Application</th>
<th>Type</th>
<th>Description</th>
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<tr>
<td>Aerospace</td>
<td>The Mars Rover Curiosity</td>
<td>Autonomous extraterrestrial vehicle</td>
<td>Uses autonomous wayfinding routine. Evaluates the area in advance. Decides independently which route is the safest [14].</td>
</tr>
<tr>
<td>Agriculture</td>
<td>Fendt GuideConnect</td>
<td>Two tractors – one driver</td>
<td>The system connects two tractors via satellite navigation and radio communication to form one unit. One of the two vehicles is unmanned and performs the same working procedure as the manned vehicle [15].</td>
</tr>
<tr>
<td>Consumer Application</td>
<td>The Homerun vacuum cleaner</td>
<td>Autonomous vacuum cleaner</td>
<td>Runs autonomously through the house and vacuums the dust and dirt beneath it into a built-in receptacle [16].</td>
</tr>
<tr>
<td>Automotive Industry</td>
<td>Bosch Park Assist</td>
<td>Parking assistant system</td>
<td>Parks the car automatically with great accuracy and within a few seconds and into tight parking spaces [11].</td>
</tr>
<tr>
<td>Automotive Industry</td>
<td>Volvo’s Autonomous Valet Parking</td>
<td>Parking assistant system</td>
<td>Enables a vehicle to be parked once the driver has stepped out of the vehicle. The driver can communicate with the system via a mobile device such as a smartphone, directing it to a preferred parking place and summoning the parked vehicle to leave the car park to collect them when required [17].</td>
</tr>
<tr>
<td>Automotive Industry</td>
<td>Google Self Driving Car</td>
<td>Self-driving vehicle</td>
<td>Powered by Google Chauffeur software and capable of driving itself on both highways and urban streets [18].</td>
</tr>
<tr>
<td>Warehousing Operation</td>
<td>Open Shuttle</td>
<td>Autonomous Loading and Transport</td>
<td>Designed for transport and picking activities including cartoons and containers [19].</td>
</tr>
<tr>
<td>Warehousing Operation</td>
<td>MoveBox</td>
<td>Autonomous Loading and Transport</td>
<td>Can automatically position itself to pick up and deliver pallets as directed [11].</td>
</tr>
<tr>
<td>Warehousing Operation</td>
<td>Kiva</td>
<td>Assisted order picking</td>
<td>Kiva can optimize the picking efficiency by mobilizing the shelves using Kiva autonomous vehicles [20].</td>
</tr>
<tr>
<td>Warehousing Operation</td>
<td>Multi-Shuttle Move</td>
<td>Autonomous Loading and Transport</td>
<td>Vehicles can handle small load carriers and pallets. They can communicate with each other to determine the tasks among them [11].</td>
</tr>
<tr>
<td>Public Transport</td>
<td>Park Shuttle</td>
<td>Self-driving Vehicle</td>
<td>It finds the way automatically, moves on a simple ground-level asphalt road, and it is ideally suited to short distance public transport [21].</td>
</tr>
<tr>
<td>Public Transport</td>
<td>Automated Public Mover</td>
<td>Autonomous Vehicle</td>
<td>It runs as an on-demand nonstop transportation system between any two points on a network [21].</td>
</tr>
</tbody>
</table>

However, drones have a limited travel distance and parcel size. While in the opposite, the truck has long-range travel capability and can carry large and heavy cargo with different sizes, but it has disadvantages such as being heavy, slow and having higher transportation costs [24]. Consequently, the advantages of trucks offset the disadvantages of drones and similarly for the other way around. These complementary capabilities are the foundation of the “Last Mile delivery with drone” [25]. According to Campbell et al. [22], “Using the drones is preferred as long as drone cost per mile is about 35% or less of the truck cost per mile”. Therefore, drones can deliver to more customers at a lower cost than the truck.
The truck drone system works as follows: the truck transports the drone close to the customer's locations, allowing the drone to serve customers while remaining within its flight range, effectively increasing the usability and making the schedule more flexible for both drones and trucks. In other words, a truck departs from the depot carrying the drone and all the customer parcels. As the truck makes deliveries, the drone is launched from the truck to serve a nearby customer with a parcel. While the drone is in service, the truck continues its route to further customer locations. The drone then returns to the truck at a location different from its launch point [24]. UPS Company is testing this system, and according to them, drone delivery can help in lowering the cost, specifically in rural locations where cars must drive miles between single deliveries. This system can save up to $50 million per year by cutting a mile off of every driver’s route each day [26]. However, the Federal Aviation Administration prohibits till now commercial drone from flying beyond the sight of their pilots [27]. Mercedes Benz is also interested in this system; it is pouring $562 million into delivery van-drone. Mercedes Benz van is with a range of up to 168 miles. The van has a fully automated cargo space. It consists of a mechanical shelving system that can load packages and autonomously know where packages are going. The driver can get a notification when reaching a drop of a package. The drone is resting on a landing station on top, and when reaching a drop off location, the shelving system will push the package to the drone that will do the rest of the job and deliver to customers [28].

1.4.5. AVs Challenges. Automated vehicles are still on the test bed in which the defects and instabilities in the technology limit the scale of applications in the initial period. The complexity of the driving environment, congestion, consumer acceptance, regulations, and technical performance are discussed below.

1.4.5.1. Detection. A substantial challenge for AVs rests in making sense of the complex and dynamic driving environment. AVs operate on three-phases known as “sense-plan-act”. Trajectory control and motion planning are the most important functions in autonomous vehicles in which they manage the execution of the pre-planned changes in speed, direction, or maintaining the stability as well as monitoring the vehicle’s movement. The real challenge is to make an accurate and detailed identification and prediction in the self-driving vehicle’s environment such as the
movement of objects, weather changes, or anything that could happen in a blink of an eye. The more dynamic an object is, the more difficult it is to predict its future environment. The prediction of where pedestrians and bicyclists are going to turn next is one important challenge that needs to be discussed. Harshitha and Manikandan [29], are proposing in their research paper a real-time pedestrian detection system for autonomous vehicles. This system uses a camera to capture the front scene, and each frame is processed to extract features using Histogram of Oriented Gradients (HOG) followed by Support Vector Machine (SVM) classifier to differentiate between pedestrians and backgrounds. The pedestrians are identified, and their locations are marked. This proposed system is useful for smarter mobility and would serve the purpose of a driver assistance system. The detection system can detect pedestrians with an accuracy of 98.31%.

1.4.5.2. Congestion. AVs are supposed to provide an easing of traffic circulation and reduce the travel cost. However, it may induce additional travel demand. This demand could be an opportunity and a threat. The additional travel demand may worsen traffic congestion. This concern is becoming severe, and thus, researchers are trying to manage the congestion resulting from injecting the autonomous vehicles into the transportation system. Currently, most of the research papers focus on finding the optimal routes for AVs to maximize traffic throughput. One research paper is employing the Dynamic Lane Reversal (DLR) to enable the automatic lane reversal. It is used to optimize the travel schedules of connected autonomous vehicles (CAVs) based on DLR for performance improvements in which it collects the travel requests from the CAVs and determines the optimal schedules and routes on dynamically reversal lanes [30].

1.4.5.3. Consumer acceptance. Despite the advantages of AVs, the public could see the disadvantages outweigh the advantages such as:

− Control: People prefer being in control and perform the tasks at hand. Taking this away would make them feel more susceptible to any risk.
− System failure: Technology could fail, especially in the early stages of the development and the system of autonomous vehicles could fail in many areas in which predicting them can be unforeseen such as bad weather, digital traffic jam, data loss, and overall system crashing [31].
Many efforts have surveyed consumer perception of automation technology. MIT AgeLab performed a study and asked around 3,000 people about their interest in self-driving vehicles. The study was repeated twice. In the first year, results showed that 48% of the participants would never purchase this type of vehicles, as they are uncomfortable with the idea of trusting the technology and losing control of the vehicle. In the second year, the research showed a decline in consumer interest in AVs across all age groups. In 2016, 40% of people in the age of 25-34 were comfortable with the idea of AVs while in 2017, this percentage was decreased by 20% [32].

Moreover, according to a report of public views on automated vehicles from the American Automobile Association [13], 75% of interviewees hold pessimistic views on unmanned technology: 54% of drivers maintain that automated vehicles on real roads increase the potential risk of a traffic accident and intensify public anxiety.

1.4.5.4. Regulations and legislation. AVs present a concern. For instance, many restricted regulations and barriers face drone systems from being adopted in the commercial sector. In the United States, the Federal Aviation Administration (FAA) requires drones to be operated under a ceiling of 400-feet and prohibits them from flying beyond the sight of their pilots [27].

1.4.5.5. Technical challenges. Drones have technical challenges in terms of endurance, reliability, and safety. They have limited battery capacity that impacts the flight endurance of such unmanned aerial vehicles [33]. In addition, they may require redundant systems such as additional motors and sensors that further reduce flight endurance. Furthermore, they rely on GPS, which has a limited accuracy of about 10 meters without corrective technologies. Therefore, drones that are operating in heavily forested areas or so-called urban canyons may lose contact with a GPS signal [34].

1.5. Vehicle Routing Problem

Vehicle Routing Problem (VRP) has been studied intensely in the last four decades. It was formally introduced in 1959 by Dantzig and Ramser. It is considered an important problem in the fields of transportation, and logistics, and holds an important place in distribution management. The problem has different forms, and that is due to the variety of constraints encountered in practice. It has attracted the attention of operational research community because of the economic importance of it
as well as the methodological challenges that it poses [35]. VRP is defined as a set of routes for a fleet of vehicles based on one, or multiple depots that must be determined for a set of customers [36]. The problem can also be defined as determining a set of vehicle routes to perform all or some transportation requests with the given vehicle fleet at the minimum cost, particularly, deciding which vehicle handles which requests in which sequence so that all vehicle routes can be feasibly executed [37]. In other words, the objective is to deliver a set of customers with known demands on minimum cost vehicle routes originating and terminating at a depot [38].

There are two types of VRP, the conventional static VRP, and the dynamic VRP. Psaraftis [39] used the following classification of the static routing problem; “If the output of a certain formulation is a set of preplanned routes that are not re-optimized and are computed from inputs that do not evolve in real-time.” While he refers to a problem as dynamic if “the output is not a set of routes, but rather a policy that prescribes how the routes should evolve as a function of those inputs that evolve in real-time.” In other words, the dynamic VRP is a problem where the planner does not know all the relevant information of planning the routes in which information can be continuously changing and updating once initial routes have been constructed. The followings represent different variants of VRP:

• **Travelling Salesman Problem (TSP)**

  The Travelling Salesman Problem is one of the well-known problems in combinatorial optimization in which many researchers have solved it with different schemes. It refers to a salesman visiting a set of cities and returning to the city he started in; the objective is to minimize the total distance traveled [40]. It can also be formulated as following: “Given a set of cities along with cost of traveling between each pair of them, the problem is to find the cheapest way of visiting all cities and returning to the starting point in which the way of visiting the cities is the order in which cities are visited; the order is called a tour or circuit through the cities” [41].

• **Multiple Travelling Salesmen Problem (mTSP)**

  The multiple traveling salesman problems (mTSP) is considered as the generalization of the TSP in which more than one salesman is allowed to travel. It is the core of VRP, which are central to logistics management. The mTSP consists of
determining a set of routes for m salesmen whom all start from and return to the same depot [42]. The problem can be defined as follows: “Given a set of nodes, and m salesmen located at a single depot node. Nodes to be visited are called intermediate nodes. The mTSP solution is to find the tours for all m salesmen whom all start and end at the same depot in which each intermediate node is to be visited only once, the objective function is to minimize the total cost of visiting all nodes” [43]. Compared to the TSP, the mTSP is more suitable for real-life situations, because it is capable of handling more than one salesman. The following represents some of the possible variations of mTSP:

- Single vs. Multiple depots: In a single depot case, all salesmen should start from and end their tours at a single node while in multiple depots, the salesmen can either return to their original depot after completing their tour or return to any depot with the restriction that the initial number of salesmen at each depot remains the same after all travel.

- Several salesmen: The number of salesmen may be bounded as a variable or a fixed number.

- Fixed charges: Each salesman has an associated fixed cost accounted whenever the salesman is used. Thus, minimizing the number of salesmen is a concern.

- Time windows: In such a case, certain nodes need to be visited in a specific period, which refers to the TSP with a time window. It is used in applications such as school bus, ship, and airline scheduling applications.

- Other variations could be added to the mTSP including a number of nodes each salesman should visit as well as maximum or minimum distances traveled by a salesman [42].

The TSP and mTSP can be solved using different approaches such as:

- Exact Solutions: The exact methods such as mathematical modeling, dynamic programming, and branch and bound are all capable of giving exact solutions [44]. But, to find the exact solution, the computational time can take too long which makes it unacceptable in real life [45].

- Heuristics: The TSP requires a large computational time. Thus, heuristics are designed to overcome the drawback of the exact solutions [45].
• Multi-Depot Multiple Travelling Salesmen Problem

The multi-depot multiple traveling salesmen problem (MmTSP) is a generalization of single depot mTSP. It consists of finding tours for all salesmen such that all customers are visited only once, the number of customers visited by a salesman lies between a predetermined interval, and the total cost of all tours is minimized. The MmTSP can have two categories, fixed destination MmTSP, and non-fixed destination MmTSP. If the problem is to determine a total of m tours such that all salesmen should return to their original depot, then it is the fixed destination case. On the other hand, if the salesman does not need to return to his original depot, but the number of salesmen at each depot should be the same at the end as it was at the beginning, then this is the case of non-fixed destination MmTSP [43].

1.6. Problem Statement

As e-commerce business continues to grow every day, the responsibility of logistics for solving the Last Mile delivery problem is increasing. As mentioned before, the Last Mile delivery is the most expensive, most polluting, and least efficient part of the e-commerce supply chain as it accounts for 13% - 75% of the total supply chain cost [46]. Logistics companies are always trying to find new solutions to minimize the total cost and increase delivery efficiency.

In this work, we aim to study the Last Mile Delivery problem that is characterized by multiple challenges such as high operation cost, high ecological impact and the complexity in the performance of the supply chain. To tackle the challenges, the inclusion of new technologies such as drones, robots, and the internet of things can help by developing new distribution systems to improve from traditional deliveries methods. However, the use of these technologies brings new operational challenges that add new characteristics to the configurations (the depot) of the delivery process. Therefore, this research deals with the impact of using Autonomous Vehicles (AV) on the logistics of the LMD. We first present a technological review on the use of AVs in logistics and use it to introduce a classification of the delivery systems based on the parcel/product handover at the time of the last handling before delivery to customers. We describe three categories of handovers, namely, machine-to-person, machine-to-machine, and person-to-machine, and characterize for each of them the type of vehicle routing optimization that it implies.
Furthermore, we present a new class of VRP that is characterized by a moving depot and formulate six variants of a Discrete Vehicle Routing Problem with a Moving Depot (VRP-MD). The problem under consideration can be described as follows: a vehicle moves on a particular path to release self-driving vehicles to serve a set of customers from different areas. The self-driving vehicles can handle multiple deliveries given their load capacities and operating time. Then, the self-driving vehicles can be collected by the vehicle at a certain location for refilling before being released to serve a new set of customers. Once all customers are served, the self-driving vehicle is collected by the vehicle. In our work, we will focus on the case where the vehicle is a truck, and the self-driving vehicles are drones. Figure 3 illustrates the problem and explains the delivery process as follows:

- First, the customer is scheduled for delivery.
- As a second step, the company sends the truck to serve customers from different areas.
- The truck releases the drones to serve a set of customers.
- After serving customers, the truck collects the drones.
- In the case of more deliveries, drones are re-launched to serve a new set of customers.
- Finally, the truck collects the drones.

Figure 3: Problem illustration
The objective is to find the optimal path for the drones and truck, along with the optimal locations to release and collect the drones to minimize the total operational cost. The total operational cost is composed of the traveling costs for the drones and the truck. The output of the optimization will give the sequencing of the deliveries to different customers as well as the optimized locations for the truck to release and collect the drones. The problem is solved using the General Algebraic Modeling System (GAMS) software. Also, a Clarke and Wright savings heuristic method will be developed to handle large-size instances. For the testing of the algorithm, we used data from an existing problem from the online TSP library.

1.7. Research Objectives and Contribution

The problem of Last Mile delivery has been studied widely especially with e-commerce growth. Recently, the inclusion of the AVs is used to reduce the effects of the LMD challenges, in which the effective deployment of the AVs can be the solution for the LMD problem.

In this work, we approach the problem of LMD by providing a detailed review of the current applications of AVs in LMD. Also, we provide a review of the use of TSP in solving problems of the delivery systems. Then, through the technological review, we introduce a classification of the system-to-system (S2S) handover, namely, person to machine handover, machine to machine handover, and machine to person. Consequently, we introduce a new class of VRP with a moving depot and formulate variants of a Discrete Traveling Salesman Problem (TSP) with a Moving Depot (TSP-MD) to solve the problem of the Last Mile. Moreover, we provide Integer Linear Programming formulation to optimize the delivery efficiency by minimizing the total truck-drone operational cost. We determine the optimal path for the drones and truck, along with the optimal locations to release and collect the drones. Finally, we develop a Clarke and Wright savings heuristic method to test the ability of the developed model to solve large instances.

The main contributions of this research are the following:

- Review the literature on TSP in the area of the applications of AVs in LMD.
- Provide a technological review of the recent applications of AVs technology in the design of LMD systems.
• Introduce a classification of the system-to-system (S2S) handover that is at the time of the last handling before reaching the customer.

• Introduce a new class of Vehicle Routing Problem with a moving depot and formulate an Integer Linear Programming to perform the Last Mile.

• Minimizing the total truck-drone system’s operational cost by finding the optimal path for the drones and truck, along with the optimal locations to release and collect the drones.

• Develop a Clarke and Wright savings heuristic methodology to deal with large number of instances.

1.8. Research Significance

From 2013 to 2018, the e-commerce retail sales increased dramatically, and it is predicted to be in a continuous increase [10]. Nowadays, all businesses are offering online shopping. E-retailers increased the complexity of the supply chain due to the small order sizes and the large number of deliveries that delivery companies need to handle. With online shopping, customers are expecting to have fast home deliveries. As a result, the Last Mile is currently regarded as the most expensive, and most pollutant part in the entire supply chain. Thus, the implemented solutions should address the LMD challenges in terms of replacing the existing solutions with ones that can overcome its problems. For instance, the traditional delivery methods such as delivery cars can increase the operational cost and air pollutions. Thus, they should be replaced by other alternatives. The effective deployment of autonomous vehicles in the problem of the Last Mile can be a considerable solution. Autonomous vehicles like drones can reduce the operational cost as well as the ecological impact. From an application point of view, many companies are considering the use of the autonomous vehicle as the promising alternative for an efficient delivery at the Last Mile. For example, UPS is supporting the idea of using truck-drone systems [26, 27]. Also, Amazon patents train mounted mobile hubs for drone delivery fleet in which the system allows for the use of drones to fulfill customers’ orders while the train is in motion [47].

The delivery service offered by e-retailers is one of the fundamental factors influencing the customers’ decision to shop from them, so the ability to deliver customers’ orders on time reflect the success of businesses. Thus, companies should
start working on replacing their traditional ways of deliveries using autonomous vehicles. The proposed research helps address the delivery of customers’ orders using a combine truck-drone system.

1.9. Methodology

The following steps will be followed to achieve the outcomes of this research:

Step 1: Update the literature review in the topics of Last Mile delivery, drone systems, and the multiple traveling salesmen problems.

Step 2: Identify objective functions, decisions variables, and constraints based on model assumptions.

Step 3: Formulate a mathematical model to find the optimal path and locations.

Step 4: Code the formulated model using appropriate optimization software.

Step 5: Develop a Clarke and Wright savings heuristic method to test the ability of the developed model to solve large instances.

1.10. Thesis Organization

The rest of the thesis is organized as follows: Chapter 2 is dedicated to the literature review on e-commerce, LMD, the use of autonomous vehicles, and TSP. Moreover, related works of this research are discussed. The introduction of the S2S handover classification is discussed in Chapter 3 along with the applications of each handover type as well as its importance in introducing new types of logistics problems. Chapter 4 presents the different variants of TSP-MD for a truck-drone system with their formulation. Chapter 5 illustrates the proposed Clarke and Wright savings Heuristics Algorithm. Chapter 6 concludes the thesis and outlines future work.
Chapter 2. Background and Literature Review

This chapter presents the related theoretical knowledge about Last Mile, autonomous vehicles, and the technical knowledge about traveling salesman problem. It also presents the recent related studies and work that has been conducted on the above topics. Furthermore, the aim is to review the fundamental concepts and some up to date and relevant information, which will be used as the basic conceptual line in further analysis.

2.1. Last Mile Delivery Problem

As mentioned earlier, the enormous growth in the market of e-commerce has driven explosive growth in the logistics demand for online shopping in which the Last Mile distribution plays an important role in serving the final customer. Home delivery is considered the most common and convenient method among customers in delivering the purchased products to their homes directly. They are usually conducted by an external courier service.

Besides the direct home deliveries, many solutions are initiated to rationalize the Last Mile Delivery. According to Iwan et al. [48], the key solutions to this type of problem are as follow:

- **Reception boxes**, fixed outside the customer’s home, access is possible using a key or an electronic code; the customer is alerted of the delivery by mobile phone or email.
- **Delivery boxes**, owned by the delivery company; filled with the goods at the distribution depot, and then temporarily attached to the home via a locking device fixed on the wall in a secure place at the customer’s home.
- **Controlled access systems**, provide the delivery driver with the means of gaining access to a locked area to leave the goods in.
- **Collection points** are based on the use of locations other than customers’ homes for delivery. The retailer or the carrier delivers to the collection point, and the customer is informed that their order is ready for collection.
- **Locker points** are groups of reception box units, sited in apartment blocks, workplaces, car parks, and railway stations. Lockers have electronic locks with a variable opening code and can be used for different
customers on different days. Customers are notified by a message about when their delivery will arrive, the box number, location, and the code to open the box. Locker banks require the customer to make the final step of the journey [48]. Table 2 shows a comparison between the key solutions of last mile delivery in terms of many important parameters.

Table 2: A comparison of last mile delivery systems [48]

<table>
<thead>
<tr>
<th>Who covers the last mile?</th>
<th>Attended delivery</th>
<th>Reception box</th>
<th>Controlled access systems</th>
<th>Locker-bank</th>
<th>Collection point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer present?</td>
<td>Delivery company</td>
<td>Delivery company</td>
<td>Delivery company</td>
<td>Customer</td>
<td>Customer</td>
</tr>
<tr>
<td>Types of products</td>
<td>Any</td>
<td>Packages, groceries</td>
<td>Packages, groceries</td>
<td>Packages</td>
<td>Packages</td>
</tr>
<tr>
<td>Failed deliveries</td>
<td>High</td>
<td>Virtually none</td>
<td>Virtually none</td>
<td>Virtually none</td>
<td>Virtually none</td>
</tr>
<tr>
<td>Delivery window</td>
<td>Fixed delivery hours</td>
<td>Delivery company operating hours</td>
<td>Delivery company operating hours</td>
<td>Delivery company operating hours</td>
<td>CP opening times</td>
</tr>
<tr>
<td>Times at which goods can be collected</td>
<td>Not appropriate</td>
<td>24 hours</td>
<td>24 hours</td>
<td>24 hours</td>
<td>CP opening times</td>
</tr>
<tr>
<td>Retrieval time for customer</td>
<td>None</td>
<td>Very short</td>
<td>Very short</td>
<td>Short-Long</td>
<td>Short-Long</td>
</tr>
<tr>
<td>Drop-off time</td>
<td>Long</td>
<td>Short</td>
<td>Short</td>
<td>Very short</td>
<td>Very short</td>
</tr>
<tr>
<td>Initial investment</td>
<td>Low</td>
<td>High / Medium</td>
<td>Medium</td>
<td>Medium</td>
<td>Low / Medium</td>
</tr>
<tr>
<td>Delivery Costs</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
<td>Lowest</td>
<td>Lowest</td>
</tr>
<tr>
<td>Possible operational problems</td>
<td>High failed deliveries. Poor use of vehicle capacity</td>
<td>A large number of boxes needed. Need to collect boxes</td>
<td>Customer concerns about safety. Need for a suitable delivery location</td>
<td>The customer has to travel to collect</td>
<td>The customer has to travel to collect</td>
</tr>
<tr>
<td>Potential reduction in goods vehicle activity compared to attended delivery</td>
<td>-</td>
<td>Some reduction</td>
<td>Some reduction</td>
<td>Greatest reduction</td>
<td>Greatest reduction</td>
</tr>
</tbody>
</table>

From the table, we can see that locker banks (parcel lockers) and collection points are considerable solutions in terms of reducing the delivery cost and the possible operation problems. In terms of the ecological aspect, parcel lockers can play an essential role in reducing pollutant emissions. Table 3 compares the ecological aspects between deliveries performed by a courier company and parcel machine.
Table 3: A comparison of ecological aspects of deliveries performed by a courier and a company via parcel lockers [7]

<table>
<thead>
<tr>
<th></th>
<th>Touchstone</th>
<th>Parcel delivery company</th>
<th>24/7 parcel lockers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parcels daily per one courier</td>
<td>60</td>
<td>600</td>
<td></td>
</tr>
<tr>
<td>Kilometers daily per one courier</td>
<td>150</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>CO2 emission per parcel</td>
<td>300g</td>
<td>14g</td>
<td></td>
</tr>
<tr>
<td>Fuel consumption per parcel</td>
<td>0.231</td>
<td>0.011</td>
<td></td>
</tr>
</tbody>
</table>

As it can be seen from the above table, the direct home delivery causes a huge increase of the pollutants such as CO2 and NO2 while the current solutions particularly the parcel lockers can reduce the emissions of such pollutants as it reduces the traffic jams and that is because the delivery trucks serving the collecting points run a smaller mileage than regular delivery method.

Based on the three delivery modes – attended home delivery, reception box, and collection points, Wang et al. [49] undertook a quantitative study of the competitiveness of the three modes through analyzing the delivery cost structure and operation efficiency in different scenarios, which helps identify the most suitable mode for different customer distribution densities. Also, Hayel et al. [50] proposed a queuing model to describe the Last Mile delivery system with attended home delivery and collection point options. Considering the monetary and congestion effect of two options, a game-theoretical approach was designed for determining the optimum option a consumer would make.

Many delivery companies are implementing the parcel lockers concept, as it is a convenient solution. For instance, in Germany, there are 3000 parcel lockers implemented by DHL [6]. Moreover, Aramex and InPost are using a similar solution to facilitate their businesses [7]. According to Zhang and Lee [51], parcel lockers / shared reception boxes (SRB) can release the time constraint for the customer and the courier. It also can protect customer privacy in which customers do not need to use their home address for their online purchasing, but instead, they can use the location of SRB as the consignment address. Therefore, they proposed scheduling of flexible vehicles for the Urban Last Mile Logistics (ULML) problem. This paper is aimed at investigating the integration of the attended home delivery (AHD) and the shared
reception box (SRB). It addressed the problem of online shopping and e-commerce as a huge challenge to ULML as it accounts for a large proportion of logistics cost due to city traffic, including labor cost and the cost of the used vehicles. The analysis is performed using Ant Colony Optimization (ACO) to solve the proposed system [51]. Furthermore, a cost simulation tool was developed by Gevares et al. [46] to simulate the Last Mile delivery costs and according to their research work; changes within Last Mile characteristics significantly influence the cost. Customer density and delivery window size are considered the factors that affect the delivery cost. These factors were further studied by Boyer et al. [52] in which the simulation results showed that greater customer density and longer delivery windows benefit the delivery efficiency.

The last Mile puts a significant pressure on the logistics management to meet customers’ expectations and reduce the complexity of the supply chain performance. As a result, other alternatives should be considered to tackle the problems of the Last Mile. The following section discusses the importance and implementations of autonomous vehicles in logistics.

2.2. Autonomous Vehicles

The focus on autonomous vehicles and the Last Mile Delivery problem arises day by day. Yu and Lam [2] proposed a logistics system that accommodates logistics demands for smart cities and determines the optimal routes for the governed AVs by taking into consideration the various requirements imposed by the vehicles, logistics requests, renewable generation, and transportation system. This was performed by coordinating routes and charging schedules as well as formulating joint routing and charging problem in the form of quadratic constrained mixed integer linear program.

Nowadays, the truck-drone system is studied widely, as it is used as a delivery system to reduce the effect of the Last Mile. The truck and the drone need to interact to determine the optimal routing. In this regard, many types of research have investigated the routing problem of a combined truck-drone system. Murray and Chu [33] introduced the problem of Flying Sidekick Travelling Salesman Problem (FSTSP). FSTSP considered a set of c customers in which each of them should be served once by either a truck or a drone that is in coordination with the truck. Because the drone cannot fulfill some customers’ orders, the truck will only serve these orders. In FSTSP, the truck and the drone must depart from and return to a single point.
exactly once in which they may depart or return independently, or the drone might be transported by the truck to reduce battery power consumption. The drone is collected at the customer node; the truck cannot connect with the drone at an intermediate place. A Mixed Integer Linear Programming (MILP) formulation and heuristic are proposed. The heuristic is based on “Truck First, Drone Second” idea, in which they construct first routes for the truck by solving a TSP and then run a reallocation procedure to reduce the objective value. Moreover, the reallocation procedure iteratively checks each node from TSP tour and considers if the node is suitable for use as a drone node. The change is applied immediately when this is true, and the current node is never checked again. Otherwise, the node is relocated to other positions to improving the objective value. The relocation procedure is designed in a “best improvement” fashion; it evaluates all the possible moves and executes the best one. The proposed methods are tested only on small-sized instances with up to 10 customers.

Agatz et al. [53] studied a slightly different problem - called the “Traveling Salesman Problem with Drone” (TSP-D), in which the drone has to follow the same road network as the truck. Moreover, in TSP-D, the drone may be launched and return to the same location, while this is forbidden in the FSTSP. This problem is modeled as a MILP formulation and solved by a “Truck First, Drone Second” heuristic in which drone route construction is based on either local search or dynamic programming. Bouman et al. [54] extended this work by proposing an exact approach based on dynamic programming that can solve larger instances. Additionally, Wang et al. [55] in recent research introduced a more general problem called “The vehicle routing problem with drone” (VRP-D) that deals with multiple trucks and drones to minimize the completion time. The analysis was conducted on several worst-case scenarios, from which they propose bounds on the best possible savings in time when using drones and trucks instead of trucks alone where the drones can be dispatched from and picked up only at the nodes; customer locations and the depot. Further development of this research was studied by Poikonen et al. [56], where they extended the worst-case bounds to more generic distance/cost metrics as well as explicitly consider the limitation of battery life and cost objectives. Moreover, Dorling et al. [57] addressed some existing problems in the planning of drone deliveries such as not allowing multiple trips to the depot which leads to excessive use
of drones and the in-consideration of the effect of battery and payload weight on energy consumption that leads to costly or infeasible routes. They proposed two multi-trip VRP’s that address the two problems mentioned above, one to minimize costs subjected to the delivery time limit, while the other minimizes the overall delivery time subjected to budget constraint. A mixed integer linear program was used to solve the problem.

To extend the literature further, Ham [58] studied the truck – drone system but added new configuration on the traditional system. The drone has a single unit capacity and can perform two different tasks: drop and pickup. That is the drone has two options after it delivers to the customer. It either returns to the depot for delivering to the next customer, or it can travel directly to a customer for a parcel pickup. Also, the time window is addressed in which the customer can order multiple products with different time priorities (which product should be shipped first). Beside drone’s delivery, the truck can serve customers along its route. Finally, constraint programming approach is used for modeling the truck-drone system. Additionally, Kim et al. [59] addressed the use of drones in the healthcare sector for delivery and pickup planning of medications in rural areas where the patients must visit clinics for health testing and medicine fill-up. In this paper, the drone can carry more than one package. Approximately, the drone can deliver three packages per route. Also, the paper introduced two models: the first model is to determine the optimal number of drone center locations using the set covering approach. The second model is to minimize the operational cost of the drones that are resulted from delivering medicines to customers and picking-up exam kits on their way back in which the drone can deliver to more than one patient. The solution is developed using a preprocessing algorithm, a Partition method, and a Lagrangian Relaxation method.

Furthermore, an innovative system was developed by Dayarian et al. [60] in which the drones are used to resupply the delivery vehicles with customers’ packages. The resupply can occur when the delivery vehicle is not moving and when the drone is handed on the roof of the vehicle. The purpose of this paper is to investigate the advantage of the resupplying configuration and introduce a VRP with a drone. The problem is solved using different algorithms and the performance of the algorithms
compared. Lastly, Figliozzi [61] provided different characteristics for the commercial use of the truck and drone systems such as:

- Drone’s most speeds are in the range of 16 to 20 meters per second (35 to 45 miles per hour) and the flying time is about 20 to 30 minutes. In terms of payload, it ranges from 1.8 kg to 6.4 kg (4 to 14 lbs).
- Truck’s maximum speed is 30 miles per hour (13 to 14 meters per second), and its maximum payload is 1890 kg. Hence, the drone is considered more attractive when it cooperates with the truck.

Deviating from the truck-drone system, Boysen et al. [62] studied a system that replaces the drones with autonomous robots to deliver to customers. In this system, the truck starts from a warehouse loaded with customers’ packages as well as small autonomous robots. The truck fills in the robots with customers’ packages and then releases the robots in which each robot is dedicated to a single customer. After the delivery is made, the robots can return to decentralized robot depots where the truck can refill them by delivery packages. The process continues until all the customers are served. Therefore, the paper develops scheduling procedures that aim at finding the truck route and the launching schedule of the robots that minimizes the weighted number of late deliveries. In this, a mixed integer programming and a local search heuristic are used.

Drones are affected by many factors such as loadable capacity, speed, battery charging, and body weight. Regarding these aspects, Lim and Jung [63] performed a simulation that focuses on charging speed, weight, and battery capacity. Results show that recharging speed is the most important factor among the others for increasing the delivery amount. Hence, the recharging speed of the battery should be the focus for researchers as it has a significant impact on increasing the delivery amounts.

In brief, autonomous vehicles have acquired great attention. Specifically, recent studies have focused on the truck-drone system as a way of tackling the problem of the Last Mile. However, most of the studies deal with a single drone. To the best of our knowledge, the number of researches that addresses the handling of multiple drones is scarce. Also, the collection of the drones is located at the customer location, which implied that no location optimization for the truck is performed in the
previous studies. The next section discusses the approaches used to solve the routing problem.

2.3. The Traveling Salesman Problem

A widely studied problem that falls under VRP is the TSP and lot of researches highlighted this problem. The TSP has a large number of variants such as TSP with time window [64], TSP with pickup and delivery [65], TSP with profits [66], maximal based TSP [67], kinetic based TSP [67], and TSP with drones [68]. According to Ha et al. [68], TSP with drones is the main focus in which drones are deployed alongside trucks to deliver goods to customers to achieve service quality improvements. The paper gave rise to a new variant of TSP with drones (TSP-D) that aims to minimize the operational cost including transportation cost using two algorithms. The first algorithm (TSP-LS) was adapted from the approach proposed by Murray and Chu [33], in which the optimal TSP solution is converted to a feasible TSP-D solution by local search. The second algorithm, a Greedy Randomized Adaptive Search Procedure (GRASP), is based on a new split procedure that optimally splits any TSP tour into a TSP-D solution. After a TSP-D solution is generated, it is then improved using local search operators.

TSP deals with one salesman, and it is limited to a certain number of applications. However, many applications in the context of LMD deal with multiple trucks and more than one salesman is used. Thus, mTSP is more useful than TSP. Angel et al. [69] investigated the buses scheduling to obtain a bus loading pattern in which the number of routes is minimized, total distance traveled by all buses is kept at a minimum, no bus is overloaded, and the time required to traverse any route does not exceed a maximum allowed policy. Kencana et al. [70] studied the technique of ant system (AS) to solve the mTSP in which this technique was simulated to determine the total shortest path for m salesmen who have to visit n cities. Arya et al. [71] stated that “the amount of computation time needed to solve the mTSP grows exponentially as the number of cities.” Thus, they proposed a modified genetic algorithm that generates a population of solutions in each iteration and the best point in the population approaches an optimal solution. Kara and Bektas [43], considered the single and multi-depot cases of mTSP by proposing integer linear programming for both cases with new bounding and the elimination of sub tour constraints.
Finally, the MmTSP is one possible variant of the mTSP where several depots exist, and at each depot, one salesman locates. References [72-75] studied this type of variant. Ghafurian and Javadian [73], proposed an ant colony algorithm to solve a fixed destination MmTSP to find the routes for all the salesmen with minimizing the total cost of all routes.

The TSP/mTSP and its variations been studied in many applications especially where the scheduling and routing problem is applied. However, these variants do not include the characteristics of the depot. To the best of our knowledge, there is no research considered the problem of mTSP with a moving depot.

The next chapter highlights the different applications of AVs in logistics and its importance in bringing new logistics issues and introducing definitions.
Chapter 3. System to System Handover

In this chapter, we perform a technological review of the recent application of autonomous vehicles in Last Mile Delivery, and we introduce the classification of system-to-system handover with illustrating examples. Also, we address the importance of vehicle routing in logistics. Finally, we introduce the effect of emerging the new vehicles in on bringing new logistics problems.

3.1. Technological Review of AVs in Logistics

Autonomous vehicles play an important role in reducing the problem of LMD. Consequently, we performed an intensive internet search of the recent applications of AVs technology in the design of LMD systems; some of them are still prototypes and patents. Tables 4, 5, and six present a descriptive summary and characteristics of each of them.

Table 4: Applications of autonomous vehicles in logistics – part a

<table>
<thead>
<tr>
<th>Technology (Company)</th>
<th>Description</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime Air Delivery Drone (Amazon)</td>
<td>It can deliver orders to customers within 16 km radius of Amazon’s fulfillment centers (It departs from warehouse and travel to customer location) [76]. It navigates through the onboard global positioning system (GPS) [77].</td>
<td>It can ship under 2.3 kg (5 pounds) [78].</td>
</tr>
<tr>
<td>Starship Vehicle (Starship)</td>
<td>It is powered by an electric motor, and it is a six-wheeled intelligent robot [79]. It is for use on the sidewalk (capable of identifying pedestrians, bicyclists). [80]. It took the trail in Redwood City and Postmates in Washington, DC [80]. It delivers in less than 30 minutes. It can travel 4 miles per hour, and each shipment cost less than $1.40 [81].</td>
<td>It can handle 40 pounds max.</td>
</tr>
<tr>
<td>Carry (Dispatch)</td>
<td>It has four compartments, and it is designed to make multiple deliveries per trip. It is connected to a 4G network for accurate location tracking so that it can be tracked, and once it reaches a destination, people get notified and can unlock and access their package through using their phones. The company launched a pilot program at Menlo College to deliver students their mail and packages [82]</td>
<td>It can carry 100 pounds.</td>
</tr>
<tr>
<td>Technology (Company)</td>
<td>Description</td>
<td>Differences</td>
</tr>
<tr>
<td>---------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Cargo Pod (Ocado &amp; Oxbotica)</td>
<td>It has been tested in a residential area in London. The customer can place his order through Ocado app that it is loaded to the CargoPod at a hub station. When the vehicle pulls up, Ocado delivery person confirms the customer is the same person who placed the order[83].</td>
<td>It can hold up to 282 pounds of groceries in its eight boxes.</td>
</tr>
<tr>
<td>Ford Autonomous Car (Ford &amp; Domino’s Pizza)</td>
<td>Customers place an order on Domino’s app and then an autonomous car will deliver the order. They use a unique code to access a heated container. It has been tested in Michigan, US. [84].</td>
<td>It can handle one order.</td>
</tr>
<tr>
<td>Electric PostBOT (DHL)</td>
<td>It is 150 cm tall, can hold up to 150 kg worth of mail, and scurries along at up to 6 km/hr. It has a sensor to navigate obstacles and track legs of human mail carrier to follow him safely. It lightens the load for the mail carrier and frees up their hands from carrying the packages[85].</td>
<td>-</td>
</tr>
<tr>
<td>Parcelcopter Skyport (DHL)</td>
<td>It can fly 70 km/h and it can do deliveries within 8 minutes and tested in mountain areas. Packages are inserted in the parcel locker, drone swoops into action (through a helipad on top of the locker), grab the package, and deliver[86].</td>
<td>It can carry 5 pounds for 8.3 km</td>
</tr>
<tr>
<td>UPS Truck-Drone System (UPS)</td>
<td>Trucks have an inside space to store and load drones in which the top of it can slide off. Drones can fly for 30 minutes and are recharged while docking on the electric delivery vehicle. UPS drivers can load packages into the drone and confirm flight path by truck’s dashboard [27].</td>
<td>It can carry 10 pounds.</td>
</tr>
</tbody>
</table>
The applications of AVs have different characteristics and ways of delivering to customers. In other words, these systems perform differently in the way they last handle the parcel/product before the final delivery to the customer. Moreover, the systems use different combinations of technology, people, and machines that results in classifying the ways of handling customers’ parcels in terms of the handling process and who is handling the parcels. The following section discusses the classification of parcels handling.

### 3.2. Handover Classification

As mentioned previously, drones, robotic machines, and cars are one of the possible autonomous vehicles that can be used to tackle the challenge of LMD. Precisely, these systems can make the final step of the last mile journey and make the delivery to the customers. Therefore, we introduce a classification for the type of handover between the logistics systems to make a differentiation between them. The handover occurs at the time of the last transfer of the parcel between any two systems and then the systems can make the delivery of the parcel to the customer. Also, the
handover is considered as the last handling of the parcel before the customer. Defining the system-to-system (S2S) handover classification is significant as it eases the characterization of the associated vehicle routing problems. We propose three classes of the handover namely, person-to-machine, machine-to-machine, and machine-to-person. The following represents the definition and characteristics of each class.

3.2.1. **Person-to-machine.** It is defined as the transfer of parcels from a person to a machine. The machine can be an autonomous vehicle that could handle the delivery to the customer. For instance, the person loads the delivery package in the AV, which will then make the delivery to the customer.

3.2.2. **Machine-to-machine.** In this handover, the possible systems are a truck - drone system (T2D), truck - autonomous vehicle system (T2AV), autonomous vehicle - autonomous vehicle system (AV2AV), and an autonomous vehicle - drone system. (AV2D). The most common type of S2S handover is the T2D. This system has been broadly studied to improve efficiency and system reliability [33, 89, 90]. The AV2AV could be considered in which an autonomous truck offloads a fleet of autonomous vehicles close to their destination and can communicate with each other to determine their routes. AV2D is like T2D, but instead of using a regular truck, an autonomous truck could be used as it can decrease pollution and increase the efficiency of the delivery service. T2AV also has the same concept of the AV2AV system, but in this system, a regular truck is used. These four systems can all be implemented, and the selection depends on location, technology, and objectives.

3.2.3. **Machine-to-person.** This system can use a machine, or a driverless vehicle to assist the delivery person in carrying the parcels during the delivery. M2P handover can be used to address the long walking distance that the delivery person should cover when he fails to find a parking space close to the recipient’s door and thus, force the delivery person to park anywhere and cover the distance on foot which results in using extra time especially when packages are heavy. DHL is using M2P, where the vehicle follows the person to the destinations [11].

Table 7 shows the different logistics systems that were mentioned in Tables 4, 5, and six along with classifying these systems based on the type of handover. The
Table combines the different technologies that fall under the same handover class. For example, the UPS truck-drone system, Amazon fulfillment centers, Amazon airborne fulfillment center, and Amazon train hubs fall under the M2M class in which the product is handed to the drone which will later handle the delivery to the customer.

Traditionally, the M2P handover is the most common delivery type in which the delivery person picks the package from a truck/car and hand it to the customer. Alternatively, she/he could now pick the package from a robotic machine. In contrast, M2M and P2M are recent types of handover that result from the newly developed technologies like Amazon drone systems patents such as a fulfillment center towers [87] and an airborne fulfillment center [88]. The proposed systems can handle multiple deliveries with multiple handovers all carried out by a moving system. In such a case, the system also needs proper scheduling of the handovers and a proper routing optimization of both system components.

<table>
<thead>
<tr>
<th>Handover Type</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Person to Machine</td>
<td>(1) Prime Air Delivery Drone (Amazon)</td>
</tr>
<tr>
<td></td>
<td>(2) Starship Vehicle (Starship)</td>
</tr>
<tr>
<td></td>
<td>(3) Carry (Dispatch)</td>
</tr>
<tr>
<td></td>
<td>(4) Cargo Pod (Ocado &amp; Oxbotica)</td>
</tr>
<tr>
<td></td>
<td>(5) Ford Autonomous Car (Ford &amp; Domino’s Pizza)</td>
</tr>
<tr>
<td>Machine to Person</td>
<td>(6) Electric PostBOT (DHL)</td>
</tr>
<tr>
<td>Machine to Machine</td>
<td>(7) Parcelcopter Sky port (DHL)</td>
</tr>
<tr>
<td></td>
<td>(8) UPS Truck-Drone System (UPS)</td>
</tr>
<tr>
<td></td>
<td>(9) Fulfillment Center Towers with Drone Delivery (Amazon)</td>
</tr>
<tr>
<td></td>
<td>(10) Train-Mounted Mobile Hubs for Drone Delivery (Amazon)</td>
</tr>
<tr>
<td></td>
<td>(11) Amazon Airborne Fulfillment Center</td>
</tr>
</tbody>
</table>

3.3. Emerging of New Vehicles in Logistics

The different logistics systems mentioned in Table 7 reveals that the depot now does not need to be in one place and can be moving from one place to another as we deliver. The Amazon patent of the train-mounted mobile hubs for drone delivery is an illustration of the moving depot. The system consists of a train, and a drone in which the drone can pick up packages returns to the train for charging or for further pickup while the train is in motion [47]. From the different logistics systems and their
characteristics mentioned previously, we performed a further classification for the handover classes. The handover structure can be fixed/moving and single/multiple. The following defines each type:

- **Fixed Handover**, systems are fixed at the time of the package/product delivery handover.
- **Moving Handover**, systems are moving when the handover of the delivery package is performed.
- **Single Handover**, a single delivery package is allowed at a time, The system schedules the handovers to handle them one at a time
- **Multiple Handover**, more than one delivery package is allowed at a time.

The handover structure corresponds to how the logistics systems are oriented. A key example is the fulfillment centers tower with drone delivery in which the tower is served as a drone charging hub in which it allows the drones to pick up the packages and then make the delivery to the customers. Such a system can be recognized as a fixed – single type of handover. The handover is taken under fixed handling of package/product. Besides the fixed handover, the drone is handled one package at a time. Nevertheless, the handover structure leads to the further classification of the routing considerations regarding the depot location. The depot can have two possible situations:

- **Fixed depot TSP/mTSP**, when the handover is fixed.
- **Moving depot TSP/mTSP**, when the handover is moving.

The introduced classifications are addressed in different logistics systems. Table 8 summarizes the above classifications and categorizes the type of handover (P2M, M2M, and M2P). The different applications that were mentioned in Tables 4, 5, and six are used here to highlight the corresponding S2S type, handover structure, routing considerations (TSP type), and finally related references from the literature. The following is an illustration of how the table is organized. In the M2P handover, the robotic machine follows the delivery person, and when he/she reaches the destination, the machine will stop and hand over the parcel to the delivery person. In such a situation, the handover is single and fixed. In terms of routing situation, this will result in a fixed depot TSP.
As can be seen from Table 8, many examples deal with the moving handover. Consequently, the classification highlights new variants of VRP in which the routing and optimization decisions will involve both systems concurrently, unlike a fixed handover in which the routing decisions come strictly after the handover. These new variants bring challenges that logistics companies need to address. Taking a T2D system as an example, we need to define the optimal path of the truck while determining the optimal route of the drone. If we take into consideration the availability of customers, the possibility of rerouting deliveries, drones power limitations, and other operational factors, then the routing problem becomes more dynamic and therefore more challenging to address.

The introduced S2S handover and its different classes have proved the significance of the new technologies in bringing new logistics problems that researchers and logistics companies need to address and solve to convoy with what is currently implemented to tackle the last mile delivery. Therefore, in the next chapter, we introduce a new formulation for a VRP with a Moving Depot (VRP-MD) to address the moving systems according to different conditions and situations.

Table 8: System to system handover classifications

<table>
<thead>
<tr>
<th>Handover</th>
<th>Related articles</th>
<th>Handover Structure</th>
<th>VRP</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fixed</td>
<td>Moving</td>
<td>Single</td>
</tr>
<tr>
<td>P2M</td>
<td>[59], [91], [92]</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>[2] [62] [11]</td>
<td>✓</td>
<td>✓</td>
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</tr>
<tr>
<td></td>
<td>[33], [53], [24], [62, 93]</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>M2M</td>
<td></td>
<td>✓</td>
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</table>
Chapter 4. Vehicle Routing Problem with a Moving Depot

There are several variants of the VRP in the literature that differentiate themselves from the number of stops on route, customer service demand quantity, onsite service/waiting times, time window structure of customers, time horizon, location of customers, number of points of origin, number of vehicles, capacity constraint, travel time and objective [94]. These variants do not include the characteristics of the depot. Indeed, its location is usually given and set for the entire delivery process. In this chapter, we propose different variants for the VRP-MD taken into consideration the release location of the drone, the collection location of the drone, the number of the operating drones, and the need for the truck to reach the collection location before the drone in different situations. We present the problem definition and notations and provide the ILP formulations. Also, the characteristics of each model are studied. Finally, we test and validate the models.

The VRP-MD considers a truck-drone system in which the truck moves on a path and releases the drone to serve a set of customers, each of whom must be served exactly once. When all customers are served by the drone, the truck must collect the drone. The way the problem was modeled is that we constructed an $M$ by $M$ matrix in which the first nodes, $L$ are the locations from where the truck can release and collect the drone. The second part of the matrix is customers’ nodes, $N$. This matrix is carried out in all models. Figure 4 shows how the distance matrix is developed.

![Distance matrix](image)

Figure 4: Distance matrix
The followings present the assumptions and delimitations used to reduce the complexity of the problem.

- **Assumptions**
  - The system deals with a moving M2M handover.
  - The truck is assumed to have predetermined locations where it can launch and collect the drone.
  - The traveling distances for the drones are calculated using the Euclidean distance equation that is:
    \[ d(P, Q)^2 = (q_1 - p_1)^2 + (q_2 - p_2)^2 \]  \hspace{1cm} (4 - 1)
  - The traveling distances for the truck are approximated using the Euclidean distance.
  - Truck’s and drones’ fixed costs are not taken into consideration.

- **Delimitations**
  - The number of drones that the truck can handle is limited.

4.1. **Model I: Single Drone, Single Trip with fixed Starting Location**

The first model handles a single drone and a single trip to cover all deliveries. The truck is assumed to launch the drone from a predetermined location (first location of the set) from where the drone can serve \( N \) customers. Afterward, the truck collects the drone at an optimized location along the truck’s path.

4.1.1. **Problem definition.** In this problem, let \( N \) represents the number of customers nodes and \( L = \{0, \ldots, l\} \) is the set of the predetermined locations for the truck to launch and collect the drone. The truck starts and launches the drone from \( l = 0 \). Afterwards, the drone delivers to the customers, \( N = \{L+1, \ldots, n+L\} \). In the end, the truck collects the drone at an optimized location that lies within \( \{1, \ldots, l\} \) in which the truck cannot return to the original launching location \( l = 0 \). The followings are the added assumptions to our problem.

  - The truck should launch the drone from the starting location \( l = 0 \).
  - The truck cannot return to the starting location \( l = 0 \).
  - The drone delivers to all customers on a single trip and then returns to the truck.
4.1.2. **Model I formulation.** We modelled the problem as $M$ by $M$ matrix in which $l = 0$ is the fixed starting location, the second $N$ nodes are customers’ nodes, and the last $L$ nodes are truck’s locations at where it collects the drone. Figure 5 shows how the system operates.

![Model I illustration](image)

Figure 5: Model I illustration

The followings are the problem parameters and variables used to assess in formulating the ILP.

- **Problem parameters**
  - $d_{ij}$: Distance travelled by the drone between node $i$ and node $j$
  - $T_{ij}$: Distance travelled by the truck between node $i$ and node $j$
  - $N$: Number of customers’ nodes
  - $L$: Set of predetermined locations
  - $F_d$: Drone unit cost (cost per unit distance)
  - $F_t$: Truck unit cost (cost per unit distance)

- **Problem decision variables**
  - $x_{ij}$: \(\begin{cases} 1; & \text{if the drone travels from node } i \text{ to node } j \\ 0; & \text{otherwise} \end{cases}\)
  - $y_{ij}$: \(\begin{cases} 1; & \text{if the truck travels from node } i \text{ to node } j \\ 0; & \text{otherwise} \end{cases}\)
  - $u_j$: Number of nodes visited by the drone from the depot to node $j$

We present in Figure 6 the ILP formulation for model I. Equation (4-2) presents the objective function that minimizes the total traveling cost for the truck and the drone. Constraint (4-3) ensures that the drones start its route from the predetermined location only once.
Constraints (4-4) and (4-5) are to ensure that customers’ nodes are entered and departed from only once. Constraint (4-6) ensures that the drone enters the collection location only once. Constraint (4-7) ensures that the drone does not return to the initial predetermined location. Constraint (4-8) indicates that the variable $y_{0j}$ exists if the drone enters the collection region. Finally, constraint (4-9) is a sub-tour elimination constraint.

4.1.3. Model I validation. For the validation of the proposed model, an example is constructed to show the validity of the model. Example 4.1.1 is solved manually to confirm the results obtained from GAMS. Figure 7 shows the possible locations of the truck and the customers’ locations. This example will be used to validate the rest of the proposed formulations.
Figure 7: Coordinates of the customers’ and truck’s nodes in example 4.1.1

In this example, the unit cost of the drone is assumed to be equal to $1, and the truck is assumed to be equal to $0.

The followings are the possible truck drone paths with the corresponding total cost.

- Option 1: 0-3-4-5-0. Total cost = $ 6.650
- Option 2: 0-4-5-3-0. Total cost = $ 7.886
- Option 3: 0-5-4-3-0. Total cost = $ 6.650
- Option 4: 0-3-4-5-1. Total cost = $ 5.828
- Option 5: 0-4-5-3-1. Total cost = $ 7.472
- Option 6: 0-5-4-3-1. Total cost = $ 6.623
- **Option 7: 0-3-4-5-2. Total cost = $ 5.414**
- Option 8: 0-4-5-3-2. Total cost = $ 8.30
- Option 9: 0-5-4-3-2. Total cost = $ 6.472

Option 7 shows the minimum total cost. The model is solved using GAMS and its results correspond to the obtained result manually.

4.1.4. **Model I illustration.** We solved Model I using example 4.1.2 that has the following inputs:

- It consists of 6 possible locations for the truck to collect the drone and seven customers to deliver to by the drone.
− The unit cost of the truck and drone are $1, $0.30 respectively.
− The coordinates of the truck’s possible locations and customers’ nodes are shown below in Table 9.

Table 9: Coordinates of the customers’ and truck’s nodes in example 4.1.2

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>20</td>
<td>10</td>
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<tr>
<td>y</td>
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<td>90</td>
<td>60</td>
<td>100</td>
<td>40</td>
<td>100</td>
<td>60</td>
<td>120</td>
</tr>
</tbody>
</table>

Figure 8 illustrates the solution. The drone is launched from location $l = 0$. Then, it enters the customer's nodes in which it follows the following path: 7-6-10-12-8-11-9. After that, the truck collects the drone at $l = 1$. The resulted total cost is **$103**.

This initial formulation looks very similar to the open path TSP in which the delivery person returns to a different (not necessarily predetermined) depot. If the movement of the truck and the drone collection points are known in advance, then this formulation will reduce to an open path TSP. However, given the nature of the problem, the release/starting location of the drone should be part of the optimization. Therefore, we introduce next the TSP-MD with an unknown starting point.
4.2. **Model II: Single Drone, Single Trip with Unknown Starting Locations**

In this model, the truck starts its path from the warehouse/origin \((l = 0)\). However, the truck is not forced to launch the drone from the warehouse/origin in which, it can release the drone from any location along the truck’s path. Afterward, the drone serves \(N\) customers. At last, the truck collects the drone from an optimal collection location.

**4.2.1. Problem definition.** In this problem, the releasing location is part of the optimization process. Let \(N\) represents the number of customers nodes, and \(L\) is the set of the predetermined locations for the truck to launch and collect the drone. The truck starts its trip from the origin/warehouse \(l = 0\) and can launch the drone from the origin or travel a distance and then launch the drone at any location \(L = \{0, \ldots, l\}\). Then, the drone delivers to the customers that locate in \(N = \{L+1, \ldots, n+L\}\). In the end, the truck collects the drone at an optimized location that is within \(L = \{0, \ldots, l\}\).

**4.2.2. Model II formulation.** In this problem, the system has a new configuration. Figure 9 shows the systems operations.

![Model II illustration](image)

Figure 9: Model II illustration

The following added variables and constraints are to address the new configurations to start the route from an unknown location.

- **Added variable**

  \[
  z_{0j} = \begin{cases} 
  1 & \text{if the truck travels from origin to node } j \\
  0 & \text{otherwise}
  \end{cases}
  \]

  We present in Figure 10 the ILP formulation for model II.
Figure 10: Proposed ILP, model II

Equation (4-10) presents the objective function that minimizes the total traveling cost for the truck and the drone including the traveling cost from the warehouse/origin to the launching location of the drone. Constraints (4-11) and (4-12) ensure that the drone departs from the launching location and enters the collection
location at most once. Constraints (4-13) and (4-14) guarantee that customers’ nodes are entered and departed from only once. Constraints (4-15), (4-16), and (4-17) are to ensure that $y_{ij}$ exists when the drone leaves the releasing location and enters the collection location (contingent on $x_{ij}$). Constraint (4-18) ensures that the truck departs from the origin only once. Constraint (4-19) shows that $z_{0j}$ exists when the drone is launched from the truck. Finally, constraint (4-20) is a sub-tour elimination.

4.2.3. Model II validation. For the validation of the proposed model, for example 4.1.1 is carried out here. The example is solved manually to confirm the results obtained from GAMS. The followings are the possible truck drone paths with the corresponding total cost that are obtained by solving the problem manually.

- Option 1: 0-3-4-5-0. Total cost = $6.65
- Option 2: 0-3-4-5-1. Total cost = $5.828
- Option 3: 0-3-4-5-2. Total cost = $5.414
- Option 4: 1-3-4-5-1. Total cost = $5.414
- **Option 5: 1-3-4-5-2. Total cost = $5.00**
- Option 6: 2-3-4-5-2. Total cost = $5.414

Option 5 shows the minimum total cost. The model is solved using GAMS, and its results correspond to the obtained here manually.

4.2.4. Model II illustration. Referring to example 4.1.2. Figure 11 shows that the truck launches the drone from the origin/warehouse $l = 0$. Then, the drone starts its route, and it is founded to be: 9-11-8-12-10-6-7. In the end, the drone is collected by the truck at the same location where it launched ($l = 0$). Hence, the truck location did not split, and the truck launched and collected the drone from the origin/warehouse. The resulted total cost is $88.

To compare the first and the second variant, the total cost dropped from $103 to $88. This can indicate that optimizing the truck’s releasing and collection locations can result in decreasing the total cost and enhance the optimization solution. Hence, determining the truck’s movement should be included in the optimization process.

4.2.5. Model characteristics. The total cost is considered an important contributing factor in studying the behavior of the model, in which it is sensitive to the configuration of the truck-drone system including different parameters.
The unit cost of the drone and the truck affects the movement of the truck including the splitting of its location, meaning that releasing location differs from the collection location. Therefore, three different scenarios are going to be tested.

1. Truck’s unit cost is greater than the drone’s unit cost
2. Truck’s unit cost is less than the drone’s unit cost
3. Truck’s unit cost is equal to the drone’s unit cost

In studying the relationship between the unit costs and the configuration of the truck’s location, example 4.1.2 was used. Table 10 summarizes the results for each one of the scenarios according to its total cost, drone route, and truck path.

Table 10: System configuration when changing unit costs

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Truck and Drone Unit Costs</th>
<th>Total Cost</th>
<th>Drone’s Route</th>
<th>Truck’s Path</th>
<th>Splitting/Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>$1 and $0.3</td>
<td>$73.3</td>
<td>0-5-9-11-7-10-8-6-0</td>
<td>0-0</td>
<td>No Split</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>$0.1 and $0.3</td>
<td>$77.0</td>
<td>0-6-5-9-11-7-10-8-2</td>
<td>0-2</td>
<td>Split</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>$0.3 and $0.3</td>
<td>$88</td>
<td>0-6-8-10-7-11-9-5-0</td>
<td>0-0</td>
<td>No Split</td>
</tr>
</tbody>
</table>

From the above table, it can be stated that the splitting of the truck’s location is sensitive to the unit costs of both the truck and the drone. As the unit cost of the truck decreases, the truck might release and collects the drone at the same location.
4.2.6. **Model II testing.** In this part, we tested mode II to study the behavior of the computational time with increasing the number of customers. The test was performed in GAMS 24.8.5 using Intel® Xeon® CPU E5-2630 v2 @2.60 GHz machine. The used data set (problem eil101) was taken from the online TSP library [95]. Among the 101 locations, 20 locations are set as predetermined locations for the truck. The rest are considered as customers’ nodes. Figure 12 shows the $x$ and $y$ coordinates of both truck and customers’ nodes.

![Figure 12: The coordinates of the truck’s and customers’ nodes](image)

We increased the number of customers with an increment of 10 nodes, starting with 11 nodes up to 81 nodes. Also, five different random samples were taken, and then the average was calculated to have a better insight of the behaviour. Table 11 shows the computational time for the five samples along the average.

By plotting the number of customers’ nodes versus the average of the computational time, the computational time increased exponentially with the increase in the number of customers’ nodes as shown in Figure 13.

Also, we performed another test to find the total computational time. The test was conducted on the data of [vm1084] from the online TSP library [95]. Two hundred nodes are selected, in which 20 of them are used as predetermined locations for the truck, and the rest represents the customers’ nodes.
Table 11: Computational time for the different sets of customers' nodes

<table>
<thead>
<tr>
<th>Number of Customers' Nodes</th>
<th>Sample 1</th>
<th>Sample 2</th>
<th>Sample 3</th>
<th>Sample 4</th>
<th>Sample 5</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.97</td>
<td>0.59</td>
<td>0.91</td>
<td>0.59</td>
<td>0.98</td>
<td>0.808</td>
</tr>
<tr>
<td>21</td>
<td>1.97</td>
<td>0.97</td>
<td>1.83</td>
<td>1.74</td>
<td>1.03</td>
<td>1.508</td>
</tr>
<tr>
<td>31</td>
<td>2.22</td>
<td>1.66</td>
<td>2.66</td>
<td>2.45</td>
<td>1.19</td>
<td>2.036</td>
</tr>
<tr>
<td>41</td>
<td>4.98</td>
<td>2.83</td>
<td>2.98</td>
<td>26.16</td>
<td>11.17</td>
<td>9.624</td>
</tr>
<tr>
<td>51</td>
<td>4.47</td>
<td>4.58</td>
<td>4.75</td>
<td>5.08</td>
<td>101.16</td>
<td>24.008</td>
</tr>
<tr>
<td>61</td>
<td>101.13</td>
<td>27.16</td>
<td>17.19</td>
<td>3.63</td>
<td>119.67</td>
<td>53.756</td>
</tr>
<tr>
<td>71</td>
<td>261.66</td>
<td>436.25</td>
<td>32.19</td>
<td>6.16</td>
<td>15.91</td>
<td>150.434</td>
</tr>
<tr>
<td>81</td>
<td>145.16</td>
<td>384.5</td>
<td>464.52</td>
<td>219.09</td>
<td>384.67</td>
<td>319.588</td>
</tr>
</tbody>
</table>

Figure 13: The computational time versus the number of customers

The results have shown that the needed computational time is 7383.4 seconds that is approximately 2 hours. Our problem is considered an NP-hard class of problems. Indeed, when the drone returns to the origin/warehouse, the problem is reduced to a TSP which tends to be an NP-hard problem. The NP-hard problems are incapable of reaching the optimality within a reasonable computational time. Results from Table 11 suggest that a heuristic solution is needed to solve large instances’ problems in a shorter time.

The drone has a limited operational time that should be taken into consideration as it can change the configuration of the solution. The next problem adds the time dimension and precedence constraints.
From now on, we will refer to Single Drone Trip with an Unknown Starting Location as Single Drone Trip.

4.3. Model III: Single Drone, Single Trip with time and Precedence Constraints

This problem considers the previous scenario but with time and precedence constraints. The precedence constraints encounter the traveling times of both vehicles from one node to another. The main objective of the precedence constraints is to ensure that the truck precedes the drone’s arrival at the collection location. These constraints can tackle the drone’s operational time limitations and can reduce the unnecessary usage of the drone’s operation.

4.3.1. Problem definition. The problem is considered as an extension of model II. However, time and precedence constraints are added. To cope with time configurations, the followings are the added assumptions:

- Customers are available during the time of the delivery.
- The truck is always preceding the drone at a drone collection point. However, the trucks waiting time cost is not considered.
- The drone has a maximum operating time.

4.3.2. Model III formulation. The following added parameters, variables, and constraints are to address the new configurations to start the route from an unknown location.

- **Added parameters**
  
  \( M \) Big number
  
  \( t_L \) Operational time required by the truck to prepare and launch the drone (launching time)
  
  \( t_R \) Time required by the truck to collect the drone (rendezvous time)
  
  \( \tau_{ij} \) Traveling time by the truck from node \( i \) to node \( j \)
  
  \( \tau'_{ij} \) Traveling time by the drone from node \( i \) to node \( j \)
  
  \( T'_{\text{max}} \) Maximum operating time for the drone

- **Added variables**
  
  \( t_j \) The time at which the truck arrives at node \( j \)
  
  \( t'_{j} \) The time at which the drone arrives at node \( j \)
We present in Figure 14 the ILP for model III.

$$t'_{ij} \geq t_j - M \left(1 - \sum_{i=L+1}^{N+L} x_{ij}\right), \quad j = 0, \ldots, L \quad (4-21)$$

$$t'_i + \tau'_{ij} - M \left(1 - x_{ij}\right) \leq t'_j, \quad i, j = L + 1, \ldots, N + L \quad (4-22)$$

$$ST'_i + \tau'_{ij} + t_L - M \left(1 - x_{ij}\right) \leq t'_j,$$

$$i = 0, \ldots, L, j = L + 1, \ldots, N + L \quad (4-23)$$

$$t'_i + \tau'_{ij} + t_R - M \left(1 - x_{ij}\right) \leq t'_j,$$

$$i = L + 1, \ldots, N + L, j = 0, \ldots, L \quad (4-24)$$

$$ST_i + \tau_{ij} + t_R + t_L - M \left(1 - y_{ij}\right) \leq t_j, \quad i, j = 0, \ldots, L \quad (4-25)$$

$$ST'_i = \tau_{0i}z_{0i}, \quad i = 0, \ldots, L \quad (4-26)$$

$$ST'_i = ST_i, \quad i = 0, \ldots, L \quad (4-27)$$

$$\sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \tau'_{ij}x_{ij} \leq T'_\text{max}, \quad (4-28)$$

Figure 14: Proposed ILP, model III

Constraint (4-21) is to ensure that the arrival time of the drone at the collection location $j$ is greater than the arrival time of the truck at the collection location $j$. Constraint (4-22) ensures that the arrival time of the drone at node $j$ is greater than the departing time from node $i$ including the traveling time from node $i$ to $j$. Constraint (4-23) ensures that the starting time of the drone incorporates the drone’s preparation time ($t_L$) while constraint (4-24) ensures that the time at the collection location incorporates the rendezvous time ($t_R$). Constraint (4-25) ensures that the arrival time of the truck at node $j$ is greater than its starting time at node $i$ incorporating the traveling time between $i$ and $j$, the launching time of the drone, and the rendezvous time of the drone. Constraint (4-26) indicates that the starting time of the truck is considered when the truck travels from the warehouse/origin to the launching location of the drone. Constraint (4-27) states that the truck and the drone have the same
starting time. Lastly, constraint (4-28) shows that drone traveling time should not exceed its maximum operating time.

4.3.3. Model III validation. For the validation of the proposed model, an example is constructed to show the validity of the model. The example is solved manually to confirm the results obtained from GAMS. The model used for validation in section 4.1.1 is carried out here

- In this example, the followings are assumed:
- The unit cost of the drone is equal to 0.5, and the truck is equal to 1.
- The traveling time by the drone from node \( i \) to \( j \) \( \tau'_{ij} \) is 1 minute.
- The traveling time by the truck from node \( i \) to \( j \) \( \tau_{ij} \) is 3 minutes.
- The time required by the truck to release the drone \( t_L \) is 1 minute.
- The time required by the truck to collect the drone \( t_R \) is 1 minute.
- The maximum operation time of the drone \( T'_{max} \) is 20 minutes

The followings are the possible truck drone paths with the corresponding total cost.

- **Option 1: 0-3-4-5-0. Total cost = $3.325**
  - Traveling time by truck, drone = 2, 14 minutes
- Option 2: 0-3-4-5-1. Total cost = $3.914
  - Traveling time by truck, drone = 5, 14 minutes
- Option 3: 0-3-4-5-2. Total cost = $4.707
  - Traveling time by truck, drone = 8, 14 minutes
- Option 4: 1-3-4-5-1. Total cost = $3.707
  - Traveling time by truck, drone = 5, 17 minutes
- Option 5: 1-3-4-5-2. Total cost = $4.50
  - Traveling time by truck, drone = 8, 17 minutes
- Option 6: 2-3-4-5-2. Total cost = $4.704
  - Traveling time by truck, drone = 8, 17 minutes

Option 1 shows the minimum total cost. The model is solved using GAMS, and its results correspond to the obtained here manually.

4.3.4. Model III illustration. Example 4.1.2 is used to illustrate this model. However, some inputs are added: the design of this model follows the same design for
the previous model.

- The time required by the truck to prepare the drone $t_L$ is 1 minute [33].
- The time required by the truck to collect the drone $t_R$ is 1 minute [33].
- The operating time of the drone is 30 minutes.
- The truck’s speed is 25 mile per hour (11 meters per second).
- The drone’s speed is 45 mile per hour (20 meters per second).

Figure 15 shows the results of the example used to illustrate the proposed constraints. The results show that the truck launches the drone from the origin $l=0$. Then, the drone starts its route, and it is founded to be: 9-11-8-12-10-6-7. In the end, the drone is collected by the truck at the same location where it launched ($l=0$). Hence, the truck location did not split. Moreover, the figure demonstrates the traveling time from one node to another. The drone takes 1 minute to be prepared and launched by the truck and from the truck and 3.7 minutes to visit the first customer reaching a total of 26.2 minutes to return to the truck. Also, the drone takes 24.2 minutes to deliver to all customer. On the other hand, the truck spends 2 minutes for launching and collecting the drone, that it the addition of $t_L$ and $t_R$ as the truck does not move. The resulted cost is $88.

4.3.5. Model III characteristics. Different parameters can affect this model. One possible parameter is changing the drone’s maximum operational time. It can
change the configuration of the system. Figure 16 shows the results for the same example but with a different maximum operation time that is 22 minutes. As shown, the truck moves from the origin to \( l = 2 \) to launch the drone \((z_{02} = 1)\). Then the drone enters its route that is 9-7-6-10-12-8-11. Afterward, the drone returns to \( l = 3 \) to be collected by the truck. In other words, the truck moves from \( l = 2 \) to \( l = 3 \) for launching and collecting the drone \((y_{23} = 1)\). In terms of the traveling times of both vehicles, their starting time is 5.7 minutes. The truck needs a total of 2 minutes to prepare, launch, and collect the drone that results eventually in a total traveling time of 10.9 minutes. On the other hand, the drone takes 21.8 minutes to cover all the customers. Also, the arrival time of the truck is ensured to be before the drone’s arrival. The change in the maximum operation time results in a total cost of $139.

Hence, as the drone’s maximum operation time decreases, the drone would be forced to find a shorter path to take and the truck would need to move to further locations in order to cover the drone’s traveling time. As the truck moves, the total cost will increase.

![Figure 16: Impact of changing the drone’s operational time](image.png)

Another parameter is considering the operational time that the drone needs at each customer to land and take off. Therefore, a new parameter is introduced.

\[ O_{t_j} \quad \text{Operational time of the drone at customer } j \]

Constraints (4-22) and (4-28) should be modified to account for the new parameter.
\[ t'_i + t'_{ij} + Ot_j x_{ij} - M \left(1 - x_{ij}\right) \leq t'_j, \quad i,j = L + 1, \ldots, N + L \]

\[ \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} (t'_{ij} + Ot_j) x_{ij} \leq T''_{max}, \]

The values of the new parameter vary between 1 minute to 2 minutes. By modifying the constraints, the system configuration and the total cost change in which it increases from $88 to $157.3. Regarding the operational time of the drone during its trip, it takes 29.98 minutes. Concerning the truck’s movement, its location must split to cover the limited operating time of the drone in which it releases the drone at \( l = 2 \) and collects it at \( l = 4 \). Figure 17 shows the results.

![Figure 17: Impact of considering the drone’s operational time at customers](image)

To conclude, adding the precedence constraints allow the truck to reach the collection location before the drone. Moreover, the traveling time from one node to another is calculated concerning the operating times for the truck to launch and collect the drone. Furthermore, changing the operational time and considering the operational

The previous models consider the delivery of the drone to all customers on a single trip. However, the limited capacity of the drone should be considered to deal with real-life situations. Therefore, with considering this limitation, the truck is obliged to make multiple trips for releasing and collecting the drone to serve the customers within the drone’s capacity. As mentioned earlier, the drone is capable of
carrying more than one parcel [59]. The following variant deals with multiple trips with an unknown starting location.

4.4. Model IV: Single Drone, Multiple Trips

Given that the drone has a load capacity limitation, it should make multiple routes to serves all customers. Thus, the truck needs to stop many times to release and collect the drone. This situation deals more with real-life situations as the drone cannot deliver to all customers on a single trip.

4.4.1. Problem definition. This problem deals with multiple trips situation. \( M = \{1, ..., m\} \) represents the number of trips used to accomplish the delivery for all customers. In this problem, we assume that the number of deliveries per trip is limited by the load capacity of the drone.

4.4.2. Model IV formulation. The truck-drone system is designed as shown in Figure 18. The truck starts its path from the origin/warehouse \((l = 0)\) that might launch the drone from that location or another location along the truck’s path. Then, the drone delivers to a certain number of customers. Then, it returns to the truck at an optimized location to be refilled with customers’ packages. The drone will continually get released, deliver, and return to the truck until all customers are served.

The following added parameters and variables are to address the new configurations.

- Problem parameters
  
  \( DL \)  Number of deliveries/trip
• Problem variables

\[ x_{ijk} \begin{cases} 1; & \text{if the drone travels from node } i \text{ to node } j \text{ in trip } k \\ 0; & \text{otherwise} \end{cases} \]

\[ y_{ijk} \begin{cases} 1; & \text{if the truck travels from node } i \text{ to node } j \text{ in trip } k \\ 0; & \text{otherwise} \end{cases} \]

\[ u_{jk} \text{ Number of nodes visited by the drone from the depot to node } j \text{ in trip } k \]

\[ z_{ijk} \begin{cases} 1; & \text{if the truck moves from node } i \text{ to node } j \text{ before the beginning of trip } k \\ 0; & \text{otherwise} \end{cases} \]

\[ \text{trip}_k \begin{cases} 1; & \text{if trip } k \text{ is made} \\ 0; & \text{otherwise} \end{cases} \]

Figures 19 and 20 show the mathematical formulation for this problem.

\[
\begin{align*}
\text{Min} & = \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \sum_{k=1}^{m} F_d d_{ij} x_{ijk} & + & \sum_{i=0}^{L} \sum_{j=0}^{L} \sum_{k=1}^{m} F_T T_{ij} (y_{ijk} + z_{ijk}) \\
& & + & \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \sum_{k=1}^{m} \sum_{\ell=0}^{L} F_T T_{ij} (y_{ijk} + z_{ijk}) \\
& & & - \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \sum_{k=1}^{m} \sum_{\ell=0}^{L} F_T T_{ij} (y_{ijk} + z_{ijk}) \\
& & & \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \sum_{k=1}^{m} \sum_{\ell=0}^{L} F_T T_{ij} (y_{ijk} + z_{ijk}) \\
& & & \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \sum_{k=1}^{m} \sum_{\ell=0}^{L} F_T T_{ij} (y_{ijk} + z_{ijk})
\end{align*}
\] (4-29)

\[
\sum_{j=L+1}^{N+L} x_{ijk} \leq 1, \quad i = 0, \ldots, L, \forall k 
\] (4-30)

\[
\sum_{i=L+1}^{N+L} x_{ijk} \leq 1, \quad j = 0, \ldots, L, \forall k 
\] (4-31)

\[
\sum_{i=0}^{N+L} \sum_{k=1}^{m} x_{ijk} = 1, \quad j = L + 1, \ldots, N + L, \quad i \neq j
\] (4-32)

\[
\sum_{j=0}^{N+L} \sum_{k=1}^{m} x_{ijk} = 1, \quad i = L + 1, \ldots, N + L, \quad i \neq j
\] (4-33)

\[
\sum_{i=0}^{N+L} \sum_{j=0}^{N+L} x_{ijk} = \sum_{i=0}^{N+L} x_{jik}, \quad j = L + 1, \ldots, N + L, \quad \forall k
\] (4-34)

\[
\sum_{i=0}^{N+L} \sum_{j=0}^{N+L} x_{ijk} \leq (DL + 1) \text{trip}_k, \quad \forall k
\] (4-35)

Figure 19: Proposed ILP, model IV
Equation (4-29) gives the cost of traveling distance for the truck and the drone. Constraints (4-30) and (4-31) ensure that drone must depart from the releasing
location and enters the collection location in trip \( k \) at most once. Constraints (4-32) and (4-33) ensure that customers’ nodes should be visited only once. Moreover, constraint (4-34) ensures that the number of times that the drone enters a node is equal the number of times the drone exits a node. Constraint (4-35) ensures that the number of visited customers per trip should be at most \( DL+1 \). In other words, the drone cannot make more than \( DL+1 \) arc, in which the last arc is the returning to the truck. Also, constraints (4-36) and (4-37) order the number of needed trips. By adding these constraints, the restriction on defining the number of needed trips as a fixed input is eliminated. Constraint (4-38) ensures that the drone cannot move along the truck’s path and cannot travel to the same node. Furthermore, constraints (4-39), (4-40), and (4-41) are contingent constraints that ensure that \( y_{ijk} \) exists when the drone leaves the releasing location and enters the collection location in trip \( k \).

Constraints (4-42), (4-43), and (4-44) are contingent constraints that ensure \( z_{ij(k+1)} \) exists if the drone is collected in trip \( k \) and released in trip \( k+1 \). Constraint (4-45) indicates that the truck should start from the origin location/warehouse where it might launch the drone from it. Constraint (4-46) ensures that \( z_{0j1} \) of \( k = 1 \) exists when the drone is launched by the truck. Finally, constraint (4-47) is a sub-tour elimination constraint.

### 4.4.3. Model IV validation

For the validation of the proposed model, for example 4.1.1 is used. However, some modifications are made:

- Truck’s unit cost is $1 and the drone’s unit cost is $1.
- The number of deliveries/trips is assumed to be 2.

The example is solved manually to confirm the results obtained from GAMS. The followings are the possible truck drone paths with the corresponding total cost that are obtained by solving the problem manually.

- **Option 1:**
  - Trip 1: 0-3-4-0
  - Trip 2: 0-5-0 \hspace{1em} Total cost = $9.122

- **Option 2:**
  - Trip 1: 0-3-4-1
  - Trip 2: 1-5-1 \hspace{1em} Total cost = $7.656
• Option 3:
  – Trip 1: 0-3-4-2
  – Trip 2: 2-5-2  Total cost = $7.414
• Option 4:
  – Trip 1: 1-3-4-1
  – Trip 2: 2-5-2  Total cost = $7.414
• Option 5:
  – Trip 1: 1-5-1
  – Trip 2: 1-3-4-1  Total cost = $7.242
• Option 6:
  – Trip 1: 1-3-4-2
  – Trip 2: 2-5-2  Total cost = $7.00

Option 6 shows the minimum total cost. The model is solved using GAMS and its results correspond to the obtained here manually. Therefore, the model is validated.

4.4.4. Model IV illustration. This model is illustrated using example 4.4.1 carried to experiment the model and it has the following inputs:

  – It consists of 6 possible truck’s locations and ten customers’ nodes.
  – The unit cost of the truck and drone are $1, $0.3 respectively.
  – The drone can make three deliveries per trip (DL=3).
  – Figure 21 shows the coordinates of the used example.

![Coordinates of customers' and truck's nodes in example 4.4.1](image-url)
Figure 22 demonstrates the solutions of the drone’s routes.

![Diagram showing drone routes and truck's predetermined locations](image)

**Figure 22: Solution for model IV**

Table 12 shows the route taken by the drone on each trip and the truck’s path.

<table>
<thead>
<tr>
<th>Trip number</th>
<th>Drone route $x_{ijk}$</th>
<th>Truck's path</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y_{ij}$</td>
<td>$z_{ijk}$</td>
</tr>
<tr>
<td>1</td>
<td>1 - 6 - 7 - 9 - 2</td>
<td>1 - 2</td>
</tr>
<tr>
<td>2</td>
<td>2 - 12 - 13 - 14 - 2</td>
<td>2 - 2</td>
</tr>
<tr>
<td>3</td>
<td>2 - 15 - 10 - 11 - 2</td>
<td>2 - 2</td>
</tr>
<tr>
<td>4</td>
<td>2 - 8 - 2</td>
<td>2 - 2</td>
</tr>
</tbody>
</table>

The solution resulted in a total cost of $91 with a total number of 4 trips in which the drone is delivering from 1 to 3 deliveries per trip.

**4.4.5. Model III characteristics.** The configuration of the system is affected by many parameters such as truck’s and drone’s unit costs, the load capacity of the drone, the location of the truck path, and the number of truck’s predetermined locations. Therefore, each is evaluated to determine its impact on the system. Example 4.4.1 is used.

**Unit cost:** The truck’s and drone’s unit costs are comparable, $F_d = 0.3$ and $F_t = 0.5$. The total cost becomes $80.4$. Figure 23 shows the new set-up of the system.
Figure 23: The solution for comparable unit costs

**Load capacity:** With using $DL = 2$, the total cost is $112.1. The number of trips taken by the drone is 6. Figure 24 shows the number of routes taken by the drone along the truck’s movement.

![Diagrams showing comparison of routes and costs](image)

Figure 24: The solution for drone’s capacity of 2

On the other hand, if $DL = 4$, the cost is $83.4. The number of trips taken by the drone is 6. Figure 25 shows the routes taken by the drone along the truck’s movement.

From the above, it can be concluded that as the capacity of the drone’s increases, the total cost will decrease as the drone will have fewer routes to make.

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Figure 25: The solution for the drone’s capacity of 4

**Position of the truck’s path:** A comparison between the cost and the location of the truck’s path is performed. The original truck’s path is displaced along the y-axis. Figure 26 illustrates the different displacements done on the truck’s path concerning its original location.

Figure 26: The coordinates of the possible truck’s path and the customers’ nodes

Figure 27 shows that the cost is sensitive to the truck’s path location and it decreases as the truck’s path is close to customers (from +10 to +30). The minimum cost is reached when the location of the truck’s path is displaced by 30. To sum up, the cost is reduced when the truck’s path location is around the customers’ nodes.
The obtained result above is used in the following comparison. In this comparison, the total cost and the number of predetermined locations of the truck are evaluated to find their relationship with each other. The existing truck’s locations are discretized around the solution. Figure 28 shows the discretization of the initial truck’s location around the optimal location. As the predetermined locations are more discretized, the total cost decreases. This is due to the increase in the number of options for the truck to stop which will give a better optimization solution. Part (g) and part (h) show that discretizing the truck’s path away from the initial solution does not affect the cost.

4.5. **Model V: Single Drone, Multiple Trips with Time and Precedence Constraints**

This problem is an extension of the previous problem (multiple trips with an unknown starting location). New constraints are added to explore the behavior of truck’s and drone’s path in terms of time. Also, this problem ensures that the arrival time of the truck at the collection location should be less than that of the drone. This aspect should be ensured as the drone has limited operating duration.

4.5.1. **Problem definition.** In this problem, the main concern is to guarantee that the drone does not reach the collection location before the truck on each trip. The following assumptions are added:
- The drone has a maximum operating time.
- The truck provides new batteries (or recharge the batteries) for the drones during each preparation of launching of the drones.

Figure 28: Number of truck’s predetermined locations and the corresponding cost

**4.5.2. Model V formulation.** The followings are the added variables for considering time and precedence constraints.

\[ t_{jk} \] The time at which the truck arrives at node \( j \), trip \( k \)
The time at which the drone arrives at node \( j \), trip \( k \) is denoted by \( t'_{jk} \). The starting time of the drone at node \( i \), trip \( k \) is denoted by \( ST'_{ik} \). The starting time of the truck at node \( i \), trip \( k \) is denoted by \( ST_{ik} \). The end time of the drone at node \( j \), trip \( k \) is denoted by \( ET'_{jk} \). The traveling time from \( i \) to \( j \) in trip \( k \) is denoted by \( Bet_{ijk} \).

Figure 29 shows the ILP formulation for time and precedence constraints.

\[
\begin{align*}
t'_{jk} & \geq t_{jk} - M \left( 1 - \sum_{i=L+1}^{N+L} x_{ijk} \right), \quad j = 0, \ldots, L, \forall k \quad (4 - 48) \\
t'_{ik} + \tau'_{ij} - M \left( 1 - x_{ijk} \right) & \leq t'_{jk}, \quad i, j = L + 1, \ldots, N + L, \forall k \quad (4 - 49) \\
ST'_{ik} + \tau'_{ij} + t_L - M \left( 1 - x_{ijk} \right) & \leq t'_{jk}, \quad i = 0, \ldots, L, \forall k \quad (4 - 50) \\
ST_{ik} + \tau_{ij} + t_R + t_L - M \left( 1 - y_{ijk} \right) & \leq t_{jk}, \quad j = 0, \ldots, L, \forall k \quad (4 - 51) \\
ET'_{jk} & \geq t'_{ik} + \tau'_{ij} + t_R - M \left( 1 - x_{ijk} \right), \quad i = L + 1, \ldots, N + L, \forall k \quad (4 - 52) \\
Bet_{ijk} & = \tau_{ij} z_{ijk}, \quad i, j = 0, \ldots, L, \forall k \quad (4 - 54) \\
ST_{jk} & = ET'_{ik-1} + Bet_{ijk}, \quad i, j = 0, \ldots, L, \forall k \quad (4 - 55) \\
ST'_{ik} & = ST_{ik}, \quad i = 0, \ldots, L, \forall k \quad (4 - 56) \\
\sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \tau'_{ij} x_{ijk} & \leq T'_{max}, \quad \forall k \quad (4 - 57)
\end{align*}
\]

Figure 29: Proposed ILP, model V

Constraint (4-48) is to ensure that the arrival time of the drone at the collection location \( j \) is greater than the arrival time of the truck at the collection location \( j \) in trip \( k \). Constraint (4-49) ensures that the arrival time of the drone at node \( j \) is greater than the departing time from node \( i \) including the traveling time from node \( i \) to \( j \) in trip \( k \).
Constraint (4-50) ensures that the arrival time at node \( j \) incorporates the drone’s preparation time \( (t_L) \), the traveling time between node \( i \) and \( j \), and the starting time at node \( i \) in each trip \( k \) while constraint (4-51) ensures that the time at the collection location at node \( j \) incorporates the rendezvous time \( (t_R) \), the time at node \( i \), and the traveling time between node \( i \) and \( j \) in each trip \( k \).

Constraint (4-52) ensures that in trip \( k \), the arrival time of the truck at node \( j \) is greater than its starting time at node \( i \) incorporating the traveling time between \( i \) and \( j \), the launching time of the drone, and the rendezvous time of the drone. Constraint (4-53) is to account for the end time of the drone when it is collected by the truck in trip \( k \). Note that the end time of the truck is not accounted because the end time will be calculated only when the truck collects the drone. Before that, the truck will wait for the drone. Constraint (4-54) is to ensure the accounting for the traveling time between two trips including the traveling time from the origin to the releasing location of the drone on the first trip. Constraint (4-55) shows that the starting time of the truck is equal to the end time of the drone and the traveling time between trip \( k \) and \( k + 1 \). Constraint (4-56) shows that the starting time of both vehicles in trip \( k \) is equal. Finally, constraint (4-57) shows that drone’s traveling time cannot go beyond its maximum operating time in each trip \( k \).

If the operational time of the drone at customers is included, then the following parameter should be added.

\[ Ot_{jk} \quad \text{Operational time of the drone at customer } j \text{ in trip } k \]

Constraints (4-49) and (4-57) should be modified to account for the new parameter.

\[
t'_{ik} + t'_{ij} + Ot_{jk}x_{ijk} - M \left( 1 - x_{ijk} \right) \leq t'_{jk},
\]

\[
i, j = L + 1, \ldots, N + L, \forall k
\]

\[
\sum_{i=0}^{N+L} \sum_{j=0}^{N+L} (t'_{ij} + Ot_{jk})x_{ijk} \leq T'_{max}, \quad \forall k
\]

4.5.3. Model V illustration. Example 4.4.1 is used. However, some inputs are added:

- The time required by the truck to prepare/release the drone \( t_L \) is 1 minute.
The time required by the truck to collect the drone $t_R$ is 1 minute.
The operating time of the drone is 20 minutes.
The truck’s speed is 25 mile per hour (12 meters per second).
The drone’s speed is 45 mile per hour (20 meters per second).

Figure 30 shows the resulted drone’s route on each trip.

![Figure 30: Solution for model V](image)

Table 13 shows the drone’s route in each trip as well as the truck’s path. On the other hand, Table 14 displays the traveling times from one node to another for the drone and the truck.

**Table 13: Solution for model V (routes)**

<table>
<thead>
<tr>
<th>Trip number</th>
<th>Drone route $x_{ijk}$</th>
<th>Truck’s path</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$y_{ijk}$</td>
</tr>
<tr>
<td>1</td>
<td>2 - 8 - 2</td>
<td>2 - 2</td>
</tr>
<tr>
<td>2</td>
<td>2 - 12 - 13 - 14 - 2</td>
<td>2 - 2</td>
</tr>
<tr>
<td>3</td>
<td>2 - 15 - 10 - 11 - 2</td>
<td>2 - 2</td>
</tr>
<tr>
<td>4</td>
<td>2 - 6 - 7 - 9 - 2</td>
<td>2 - 2</td>
</tr>
</tbody>
</table>

The solution resulted in a total cost of $92 with a total number of 4 trips in which the drone is delivering from 1 to 3 deliveries per trip. At first, the truck is departing from the origin and launching the drone at $l = 2$. In other words, $z_{021} = 1$.  

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Then, the truck launches and collects the drone at the same location \( l = 2 \). In terms of time considerations, the drone’s operational time is not exceeding the maximum time (20 minutes) in each trip. Furthermore, the truck is arriving before the drone’s arrival at the collection location.

### Table 14: Solution for model V (time)

<table>
<thead>
<tr>
<th>Trip number</th>
<th>Drone’s traveling time (minutes) ( t'_{jk} )</th>
<th>Truck’s traveling time (minutes) ( t_{jk} )</th>
<th>Drone’s operational time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.6 - 4.0 - 5.4</td>
<td>3.6 - 4.6</td>
<td>0.85</td>
</tr>
<tr>
<td>2</td>
<td>6.4 - 7.3 - 9.0 - 10.0 - 13.0</td>
<td>6.4 - 7.4</td>
<td>5.7</td>
</tr>
<tr>
<td>3</td>
<td>14.0 - 17.3 - 18.7 - 19.6 - 24.4</td>
<td>14.0 - 15.0</td>
<td>9.3</td>
</tr>
<tr>
<td>4</td>
<td>25.4 - 26.2 - 27.5 - 28.6 - 31.2</td>
<td>25.4 - 26.4</td>
<td>4.8</td>
</tr>
</tbody>
</table>

#### 4.5.4. Model characteristics.

As in model IV, the arrangement of the system changes with unit costs. When the truck’s and drone’s unit costs are comparable, \( F_d = $0.3 \) and \( F_t = $0.5 \), the total cost becomes $82.2. Figure 31 shows the new set-up.

![Figure 31: The solution for comparable costs](image)

The above problem can be generalized by considering multiple drones instead of a single drone. This generalization can increase the number of delivered customers and reduces the drone’s operational time. The next problem suggests the multiple drones’ scenario.
4.6. Model VI: Multiple Drones, Multiple Trips with Time and Precedence Constraints

The model merges all the previous conditions and configurations. However, it adds the multiple drones’ scenario where the truck handles multiple drones. Given the limited load capacity of the drone and its limited operational time, the truck should handle multiple drones to cover larger set of customers.

4.6.1. Problem definition. In this problem, multiple drones are used to serve customers. \( R = \{1, \ldots, r\} \) represents the number of drones that the truck can handle. The followings are the added assumptions:

- The truck can deal with multiple handovers where it can handle different drones independently.
- The truck can handle two drones.
- All drones are launched from the same location.
- The drones are collected from the same location.

4.6.2. Model VI formulation. The truck-drone system is designed as shown in Figure 32. The truck starts its path from the origin/warehouse \((l = 0)\) that might launch the drone from that location or another location along the truck’s path. Then, the drone delivers to a certain number of customers. Then, it returns to the truck at an optimized location to be refilled with customers’ packages. The drone will continually get released, deliver, and return to the truck until all customers are served.

![Figure 32: Model VI illustration](image)

The followings are the added parameters and variables.

- Added problem parameters
\textbf{R} Number of drones
\textbf{DLT} Number of deliveries/trip
\textbf{DLD} Number of deliveries/drone

- Problem variables

\[ x_{ijkl} \begin{cases} 
1; & \text{if drone } l \text{ travels from node } i \text{ to node } j \text{ in trip } k \\
0; & \text{otherwise} 
\end{cases} \]

\[ y_{ijk} \begin{cases} 
1; & \text{if the truck travels from node } i \text{ to node } j \text{ in trip } k \\
0; & \text{otherwise} 
\end{cases} \]

\[ u_{jkl} \text{ Number of nodes visited by drone } l \text{ from depot to node } j \text{ in trip } k \]

\[ z_{ijk} \begin{cases} 
1; & \text{if the truck moves from node } i \text{ to node } j \\
0; & \text{before the beginning of trip } k \\
0; & \text{otherwise} 
\end{cases} \]

\[ trip_{kl} \begin{cases} 
1; & \text{if trip } k \text{ is made for drone } l \\
0; & \text{otherwise} 
\end{cases} \]

\[ t_{jk} \text{ The time at which the truck arrives at node } j, \text{ in trip } k \]

\[ t'_{jkl} \text{ The time at which drone } l \text{ arrives at node } j, \text{ in trip } k \]

\[ ST'_{ikl} \text{ The starting time of drone } l \text{ at node } i, \text{ in trip } k \]

\[ ST_{ik} \text{ The starting time of the truck at node } i, \text{ in trip } k \]

\[ ET'_{jkl} \text{ The end time of drone } l \text{ at node } j, \text{ in trip } k \]

\[ Bet_{ijk} \text{ The traveling time from node } i \text{ to node } j, \text{ in trip } k \]

Figures 33, 34, and 35 show the ILP formulation for this problem. Equation (4-58) gives the cost of traveling distance for the truck and the drones. Constraints (4-59) and (4-60) ensure that each drone \( l \) must depart from the releasing location and enters the collection location in trip \( k \) at most once. Constraints (4-61) and (4-62) ensure that customers’ nodes should be visited only once.

Moreover, constraint (4-63) ensures that for each drone \( l \), the number of arriving at every customer and collection location is equal to drone’s number of leaving at every customer and releasing location or (ensures that if the drone enters a node, then it should depart from it). Also, constraints (4-64) and (4-65) are counter constraints to determine the number of needed trips given the number of served customers for each drone in each trip. Constraint (4-66) ensures that drones cannot move along the truck’s path and cannot travel to the same node.
\[ \text{Min} = \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \sum_{k=1}^{m} \sum_{l=1}^{r} F_d d_{ij} x_{ijkl} \]
\[ + \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \sum_{k=1}^{m} F_T T_{ij} (y_{ijk} + z_{ijk}) \]  \tag{4 – 58}

\[ \sum_{j=L+1}^{N+L} x_{ijkl} \leq 1, \quad i = 0, \ldots, L, \quad \forall k, \quad \forall l \]  \tag{4 – 59}

\[ \sum_{i=L+1}^{N+L} x_{ijkl} \leq 1, \quad j = 0, \ldots, L, \quad \forall k, \quad \forall l \]  \tag{4 – 60}

\[ \sum_{i=0}^{N+L} \sum_{k=1}^{m} \sum_{l=1}^{r} x_{ijkl} = 1, \quad j = L + 1, \ldots, N + L, \quad i \neq j \]  \tag{4 – 61}

\[ \sum_{j=0}^{N+L} \sum_{k=1}^{m} \sum_{l=1}^{r} x_{ijkl} = 1, \quad i = L + 1, \ldots, N + L, \quad i \neq j \]  \tag{4 – 62}

\[ \sum_{i=0}^{N+L} x_{ijkl} = \sum_{i=0}^{N+L} x_{jikl}, \quad j = L + 1, \ldots, N + L, \quad \forall k, \quad \forall l \]  \tag{4 – 63}

\[ \text{Trip}(k+1)l \leq \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} x_{ijkl}, \quad \forall k, \quad \forall l \]  \tag{4 – 64}

\[ \text{Trip}_{kl} \leq \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} x_{ijkl}, \quad \forall k, \quad \forall l \]  \tag{4 – 65}

\[ \sum_{i=0}^{L} \sum_{j=0}^{L} x_{ijkl} + \sum_{i=0}^{N+L} x_{ijkl} = 0, \quad \forall k, \quad \forall l \]  \tag{4 – 66}

\[ \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} x_{ijkl} \leq (DLD + 1) \text{Trip}_{kl}, \quad \forall k, \quad \forall l \]  \tag{4 – 67}

\[ \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \sum_{l=1}^{r} x_{ijkl} \leq (DLT + 2) \text{Trip}_{kl}, \quad \forall k \]  \tag{4 – 68}

Figure 33: Proposed ILP, model VI
\[ y_{ijk} \geq \sum_{w=L+1}^{N+L} x_{iwlk} + \sum_{w=L+1}^{N+L} x_{wjkl} - 1, \quad i, j = 0, \ldots, L, \forall k, \forall l \] (4-69)

\[ y_{ijk} \leq \sum_{w=L+1}^{N+L} x_{iwlk}, \quad i, j = 0, \ldots, L, \forall k, \forall l \] (4-70)

\[ y_{ijk} \leq \sum_{w=L+1}^{N+L} x_{wjkl}, \quad i, j = 0, \ldots, L, \forall k, \forall l \] (4-71)

\[ z_{ij(k+1)} \geq \sum_{w=L+1}^{N+L} x_{wjkl} + \sum_{w=L+1}^{N+L} x_{iwl(k+1)} - 1, \quad i, j = 0, \ldots, L, \forall k, \forall l \] (4-72)

\[ z_{ij(k+1)} \leq \sum_{w=L+1}^{N+L} x_{wjkl}, \quad i, j = 0, \ldots, L, \forall k, \forall l \] (4-73)

\[ z_{ij(k+1)} \leq \sum_{w=L+1}^{N+L} x_{iwl(k+1)}, \quad i, j = 0, \ldots, L, \forall k, \forall l \] (4-74)

\[ \sum_{i=0}^{L} z_{0i1} = 1, \] (4-75)

\[ z_{0i1} \leq \sum_{w=L+1}^{N+L} x_{iwl1}, \quad i = 0, \ldots, L, \forall l \] (4-76)

\[ u_{ikl} - u_{jkl} + N x_{ijkl} \leq N - 1, \quad \forall i \geq L + 1, \forall j, \forall k, \forall l, i \neq j \] (4-77)

\[ t'_{jkl} \geq t_{jk} - M \left(1 - \sum_{i=L+1}^{N+L} x_{ijkl}\right), \quad j = 0, \ldots, L, \forall k, \forall l \] (4-78)

\[ t'_{ikl} + t'_{ij} - M \left(1 - x_{ijkl}\right) \leq t'_{jkl}, \quad i, j = L + 1, \ldots, N + L, \forall k, \forall l \] (4-79)

\[ ST'_{ikl} + t'_{ij} + t_{l} - M \left(1 - x_{ijkl}\right) \leq t'_{jkl}, \quad i = 0, \ldots, L, j = L + 1, \ldots, N + L, \forall k, \forall l \] (4-80)

Figure 34: Proposed ILP, model VI
Figure 35: Proposed ILP, model VI

Constraints (4-67) and (4-68) ensure that the total number of visited customers per drone should be at most $DLD + 1$ (maximum number of acres per drone is $DLD+1$) and the total number of visited customers per trip should not exceed $DLT + 2$ (maximum number of acres per trip for the two drones is $DLT + 2$). Furthermore, constraints (4-69), (4-70), and (4-71) are contingent constraints that ensure that $y_{ijk}$ exists when the drones leave the releasing location and enter the collection location in trip $k$. Constraints (4-72), (4-73), and (4-74) are contingent constraints that ensure $z_{ij(k+1)}$ exists if the drones are collected in trip $k$ and released in trip $k+1$. Constraint (4-75) indicates that the truck should start from the origin location/warehouse where it might launch the drones from it. Constraint (4-76) ensures that $z_{0j1}$ of $k = 1$ exists when the drones are launched by the truck and constraint (4-77) is a sub-tour elimination constraint.

In terms of time considerations, Constraint (4-78) is to ensure that the arrival time of drone $l$ at the collection location $j$ is greater than the arrival time of the truck.
at the collection location $j$ in trip $k$. Constraint (4-79) ensures that the arrival time of drone $l$ at node $j$ is greater than the departing time from node $i$ including the traveling time from node $i$ to $j$ in trip $k$. Constraint (4-80) ensures that the arrival time at node $j$ incorporates the drone’s $l$ preparation time ($t_L$), the traveling time between node $i$ and $j$, and the starting time at node $i$ in each trip $k$ while constraint (4-81) ensures that the time at the collection location at node $j$ incorporates the rendezvous time ($t_R$), the time at node $i$, and the traveling time between node $i$ and $j$ in each trip $k$. Constraint (4-82) ensures that in trip $k$, the arrival time of the truck at node $j$ is greater than its starting time at node $i$ incorporating the traveling time between $i$ and $j$, the launching time, and the rendezvous time. Constraint (4-83) is to account for the end time of drone $l$ when it is collected by the truck in trip $k$. Note that the end time of the truck is not accounted because the end time will be calculated only when the truck collects the drone. Before that, the truck will wait for the drone.

Constraint (4-84) is to ensure the accounting for the traveling time between two trips including the traveling time from the origin to the releasing location of the drones on the first trip. Constraint (4-85) shows that the starting time of the truck is equal to the end time of drone $l$ and the traveling time between trip $k$ and $k + 1$. Constraint (4-86) shows that the starting time of both vehicles in trip $k$ is equal. Also, constraint (4-87) shows that drone’s $l$ traveling time cannot go beyond its maximum operating time in each trip $k$. Finally, constraints (4-88) and (4-89) guarantee that the ending time of the trip is the maximum of the ending times of the two drones.

The operational time of the drone at customers can be considered by including the following parameter.

$O_{t_{jkl}}$ Operational time of the drone at customer $j$ in trip $k$ for drone $l$

Constraints (4-79) and (4-87) should be modified to account for the new parameter.

$t'_{ikl} + \tau'_{ij} + O_{ijkl}x_{ijkl} - M \left(1 - x_{ijkl}\right) \leq t'_{jk'l}$,

\[ \sum_{i=0}^{N+L} \sum_{j=0}^{N+L} \left(\tau'_{ij} + O_{ijkl}\right)x_{ijkl} \leq T'_{max}, \quad \forall k, \forall l \]
Also, the drones’ and truck’s fixed costs can be considered for such problems by adding decisions variables that determine the use of the drones. However, in our case, the drones are assumed to be always in operation. Therefore, the added fixed costs are not taken into consideration as mentioned at the beginning of this chapter.

4.6.3. Model VI illustration. Example 4.6.1 is used here has the following inputs:

- It consists of 6 possible locations for the truck and ten customers’ nodes.
- The unit cost of the truck and drone are $1, $0.3 respectively.
- The time required by the truck to prepare / release the drone $t_L$ is 1 minute.
- The time required by the truck to collect the drone $t_R$ is 1 minute.
- The operating time of the drone is 20 minutes.
- The truck’s speed is 25 mile per hour (12 meters per second).
- The drone’s speed is 45 mile per hour (20 meters per second).
- The drone can make three deliveries per trip ($DL=2$).
- The coordinates of the truck’s possible locations and customers’ nodes are shown in Figure 36 and Figure 37 shows the solution.

![Figure 36: Coordinates of customers' and truck’s nodes for example 4.6.1](image)

Table 15 shows the drones routes in each trip as well as the truck’s path. On the other hand, Table 16 displays the traveling times from one node to another for the drones and the truck. The solution resulted in a total cost of $98 with a total number of 3 trips in which the drone is delivering from 1 to 3 deliveries per trip.

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In terms of time considerations, the truck is arriving before the drone’s arrival at the collection location. Lastly, the entire delivery process took around 20 minutes.

Figure 37: Solution for model VI

Table 15: Solution for model VI (routes)

<table>
<thead>
<tr>
<th>Trip number</th>
<th>Drone’s Route ($x_{ijkl}$)</th>
<th>Truck’s path</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Drone #1</td>
<td>Drone #2</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>1 - 11 - 12 - 1</td>
</tr>
<tr>
<td>3</td>
<td>1 - 13 - 9 - 1</td>
<td>1 - 10 - 1</td>
</tr>
</tbody>
</table>

Table 16: Solution for model VI (time)

<table>
<thead>
<tr>
<th>Trip number</th>
<th>Drone's traveling time (minutes) $t'_{ijkl}$</th>
<th>Truck's traveling time (minutes) $t_{jk}$</th>
<th>Drone's operational time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Drone #1</td>
<td>Drone #2</td>
<td>Drone #1</td>
</tr>
<tr>
<td>1</td>
<td>1.0 - 1.47 - 2.78 - 5.48</td>
<td>1.0 - 1.5 - 2.95</td>
<td>1.0 – 2.95</td>
</tr>
<tr>
<td>3</td>
<td>16.9 - 18.5 - 19.4 - 22.9</td>
<td>16.9 - 17.8 - 19.7</td>
<td>16.9 – 17.9</td>
</tr>
</tbody>
</table>
4.6.4. Model characteristics. In this model, changing the number of operated drones can affect the system set-up. Referring to example 4.6.1, the total cost between single and two drones’ scenarios is compared. By operating with a single drone, the total cost is $96.1. Figure 38 demonstrates the solution. The drone takes five trips and the total operational time is 35 minutes.

![Figure 38: Solution for single drone](image)

Using a single drone reduced the cost in which it reduced from $98 to $96.1. However, the total operational time increased from 20 minutes to 35 minutes.

4.7. Summary

This section summarizes all the conclusions obtained from the different models as well as the applications that be linked to the proposed models. Table 17 shows the summary. All the applications are motivational examples for adopting the proposed models.

Exact methods can return optimal solutions within a reasonable time frame for small size problems. But its efficiency reduces with expanding the problem size. Despite that the optimal solutions in heuristics cannot be predicted, heuristics can provide solutions with good quality and within a rational time limit. Therefore, a Clarke and Wright savings heuristic is developed in the next chapter.
<table>
<thead>
<tr>
<th>Model Number</th>
<th>Summary</th>
<th>Application</th>
</tr>
</thead>
</table>
| **Model I** | • Including the releasing location in the optimization process reduces the total operational cost.  
  • Unit cost has an impact on truck’s movement. When unit costs are comparable, truck turns to move from its starting location. | The Amazon Fulfillment Center Towers with Drone Delivery [92] is a direct application. The drones are launched from the center to make the deliveries. After they serve the customers, the truck may collect them from any point along its path. |
| **Model II** | • Total cost increases with decreasing the maximum drone’s operational time.  
  • The total cost increases with the inclusion of the drone’s operational time at customers.  
  • This is due to the truck’s movement as it will travel a longer distance to compensate for the drone’s limited operational time. | In precision agriculture drones are used for spraying pesticides and that can cover many different areas in a single trip [96]. |
| **Model III** | • The system’s set-up is sensitive to the drone’s and truck’s unit cost.  
  • As the capacity of the drone increases, the drone will make fewer routes and thus cost is less.  
  • The location of the truck’s path has a major impact on the total cost that affects the traveling distance made by the drone. When the location of the truck’s path is closer to customers, the drone will travel a shorter distance. Hence, the total cost will drop.  
  • The number of predetermined locations was tested. As the locations are discretised more, the total cost drops. | - |
| **Model IV** | • The two drones’ scenario costs more than the single drone scenario. However, the total system operational time is less. Thus, there is a trade-off between time and cost. | UPS truck-drone system [28] is an illustrative application for the case of single drone and multiple trips. In this application, the drone is launched from the truck to serve customers and then returns to the truck. Carry, Dispatch [82] has four compartments, and it is designed to make multiple deliveries per trip. |
| **Model V** | - | - |
| **Model VI** | The two drones’ scenario costs more than the single drone scenario. However, the total system operational time is less. Thus, there is a trade-off between time and cost. | It can be used in Amazon Airborne Fulfillment Centre application [88]. In this application, multiple UAVs are released from the airborne and multiple airships are used to replenish the airborne inventory. |
Chapter 5. Clarke and Wright Savings Algorithm for the Proposed Models

VRP problems require enough computational time to reach an optimum solution that cannot be practical for real-life situations. The exact solution methods are approaches that find optimal solutions for VRP. They are classified into three major categories: dynamic programming, branch and bound, and mathematical programming [97]. VRP are NP-hard problems, and this means that exact solutions cannot provide optimal solutions to medium to large instances. This due to the long required computational time. Therefore, heuristics can be the only solution to tackle large and difficult combinatorial optimization problems as they can adopt additional tasks and constraints. They can produce several solutions that give the flexibility for the user to choose one or more solution [45]. There are two primary methods for building VRP solutions, namely, merging existing routes using a saving criterion, and gradually assigning vertices to vehicle routes using an insertion cost [98]. In other words, heuristic methods can mostly be categorized into the following classifications:

- Constructive methods: tours are built up by adding nodes to partial tours or combining sub tours with respect to the capacities and costs.
- The two-phase method consists of clustering of vertices and route construction. The order of these phases can be clustering first, route later procedures or the route first, clustering later procedures [99].

5.1. **Clarke and Wright Savings Heuristic**

The savings heuristic is a constructive heuristic, and it is widely used to solve the VRP. It provides relatively good solutions. The basic savings concept demonstrates the cost savings obtained by joining two routes into one route [99]. Figure 39 illustrates its concept, in which point 0 represents the depot. Figure 39 part (a) shows that customers $i$ and $j$ are visited on separate routes. However, Figure 39 part (b) shows when both customers are visited on the same route, in which savings will be generated when the two routes are combined.

The Clarke and Wright (CW) savings algorithm for the capacitated VRP (CVRP) is the most used heuristic for solving this problem. Generally, it is the best-known heuristic in which its simplicity, intuitive appeal, and the quality of the produced solutions contribute to the algorithm’s acceptance in the research
community [100]. Generally, the quality of this algorithm is approximately 22% [101]. The general algorithm of the Clarke and Wright method is shown in Figure 40.

![Figure 39: Illustration of the savings concept](image)

1. Starting solution: each of the n vehicles serves one customer.
2. For all pairs of nodes $i, j, i \neq j$, calculate the savings for joining the cycles using edge $[i,j]$:
   \[
   s_{ij} = c_{0i} + c_{0j} - c_{ij}
   \]
3. Sort the savings in decreasing order.
4. Take edge $[i,j]$ from the top of the savings list. Join two separate cycles with an edge $[i,j]$, if
   i. The nodes belonging to separate cycles
   ii. The maximum capacity of the vehicle is not exceeded
   iii. $i$ and $j$ are first or the last customer on their cycles.
5. Repeat step 4 until the savings are handled or the capacities don’t allow more merging.

![Figure 40: Steps of the CW savings heuristic](image)

There are two versions of CW algorithm: a sequential version that allows only one route at a time to be constructed, and a parallel version that is all routes are constructed simultaneously. The parallel version is more common as it provides better results than the sequential version [99]. In the parallel version, each time a pair of customers are connected, the cost of the solution is decreased with the saving obtained by the customer pair. At last, the algorithm proceeds down the savings list until no
more customer pairs can be connected [100].

In the implementation of this algorithm, Solomon [101] used approximation methods to solve the problems of vehicle routing and scheduling with a time window. The parallel savings method was implemented using list processing and heapsort structures. The method resulted in a 22% deviation from the best average solution value, which is considerably good. Altinel and Oncan [102], introduced a new enhancement of the original Clarke and Wright algorithm. The enhancement is performed by considering customer demands while calculating the savings. This inclusion can increase the opportunity of obtaining higher opportunities. The paper concluded that adding the new parameter increases the search effort. However, it increases the accuracy of the original remarkably with up to 5.32% of relative improvement.

Another modification of the savings heuristic was performed by Anbuudayasankar et al. [103]. The paper focused on the problem of optimizing the process of replenishing money in the automated teller machines (distribution of logistics problem). Therefore, a bi-objective vehicle routing problem with forced backhauls was formulated. Two savings heuristics were modified, namely, modified savings heuristics with arc removal procedure (MASR), and modified savings heuristic with node swap procedure (MSNS). The first heuristic (MASR) considers the objectives of minimizing the travel distance and the span of travel tour. In contrast, the MSNS heuristic moves each node in a maximal tour to a set of nodes in other tours that results in solving the algorithm twice. The results showed that both heuristics provide good quality near-optimal solutions with relatively short computing time.

Furthermore, Stanojevic et al. [104] proposed a new way of combining routes and its corresponding formula. Also, they developed a new heuristic – Extended Savings Algorithm (ESA) that can dynamically calculate savings during iterations and merge routes in more different ways than the CW. Also, the ESA can explore more convenient neighbors while still having the same computational complexity. Hence, the proposed heuristic provides better solutions than the original CW algorithm. In a more recent paper, Li et al. [105] studied the problem of rollon-rolloof vehicle routing
that focuses the concern on waste material logistics. To solve the problem, a two-stage heuristic was followed involving a modified CW algorithm and a local search phase. The modified algorithm accounts for the short distance span and the number of required vehicles that are not covered by the original CW algorithm. The effectiveness of the heuristic was examined using randomly-generated small scale instances and benchmark instances, in which both proved the good performance of the introduced algorithm.

Using heuristics is extremely beneficial for adapting real-life situations as it gives a near optimum solution with an acceptable computational time comparing to the exact solutions. The savings procedure is to be followed as it is considered the simplest type of heuristics. In our work, we implement a savings heuristic to deal with a single drone and multiple drone scenario. The proposed savings algorithm is implemented in the following phases:

5.1.1. Initial solution. In this phase, each customer is visited in a separated route. The initial routes will be used then in forming the final routes. The initial solutions are constructed with regards to the different combinations of depots. In our problem, the truck has multiple predetermined locations/depots for launching and collecting the drone. Figure 41 shows the different possible combination for a drone’s route to visit a single customer. In this figure, the drone can be launched and collected from three depots “D1, D2, and D3”. Thus, the drone’s route can be formed in six different configurations. Figure 42 illustrates the construction of the initial solutions.

![Initial solution](image)

Figure 41: Initial solution
**Inputs:** Total number of nodes (nodes), The first customer node (customers), Number of initial routes (num_cycles), Depots (depot1, depot2), Demand

**Outputs:** Initial cycles

```plaintext
for (m=0 to depot1) do
    for (n=0 to depot2) do
        if (n >= m) do
            for (i=customers to nodes) do
                Store the values of depot1, customer i, depot 2, position of last visited customer and demand
                index++
                i++
            end for
            n++
        end for
    m++
end for
```

Figure 42: Construction of initial solution

5.1.2. **Savings calculations.** In this phase, each customer is joined with another customer with respect to the different combination of depots. Afterward, the resulted saving from joining customers $i$ and $j$ in one route is calculated. Figure 43 shows the different scenarios that are taken into consideration in the calculations of the savings. The classical CW algorithm accounts for the first scenario in Figure 43 part (a). However, our proposed algorithm will consider all scenarios.

Figure 43: Customers’ routes
Equation (5-1) shows the traditional method of calculating the savings.

\[ s_{ij} = \text{drone's unit cost} \times \left( d(i, \text{depot2}(i)) + d(\text{depot1}(j), j) - d(i, j) \right) \]  \hspace{1cm} (5 - 1)

An alternative way of calculating the savings is to consider the truck’s movement. For instance, in Figure 43 part (d), the truck must move from D1 to D2 to collect the drone. Therefore, we penalize the savings by considering the cost of the distance traveled by truck. Equation (5-2) considers the penalty.

\[ s_{ij} = \text{drone's unit cost} \times \left( d(i, \text{depot2}(i)) + d(\text{depot1}(j), j) - d(i, j) \right) - \text{truck's unit cost} \times \left( d(\text{depot1}, \text{depot2}) \right) \]  \hspace{1cm} (5 - 2)

A further improvement that can be done is by penalizing the savings through including the cost of the truck’s traveling distance that is from the origin to depot2. The improved savings is described in equation (5-3).

\[ s_{ij} = \text{drone's unit cost} \times \left( d(i, \text{depot2}(i)) + d(\text{depot1}(j), j) - d(i, j) \right) - \text{truck's unit cost} \times \left( d(0, \text{depot2}) \right) \]  \hspace{1cm} (5 - 3)

The above three equations are taken into consideration to explore their impact on the quality of the solution. Figure 44 shows the calculations of the savings. The generated number of savings will be stored in arrays. Afterwards, savings will be sorted in a decreasing order.

| Inputs: | Total number of nodes (Nodes), The first customer node (Customers), Number of edges, Number of depots (depot1, depot2), distance between customers, Drone’s unit cost (Fd), Truck’s unit cost (Ft) |
| Outputs: | Edges |
| for (i=customers to Nodes) do | for (m=0 to depot1) do |
| for (n=0 to depot) do | if (n >= m) |
| for (j=customers to Nodes) do | if (j > i) |
| for (l=0 to depot1) do | if (l >= n) do |
| if (k >= 1) do | if (s(i,j) > 0) do | (This it to eliminate any saving with non-positive value) | Store the value of savings, customers (i,j), depots |
| end for all loops |

Figure 44: Savings calculations

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5.1.3. **Savings sorting.** In this phase, the edges/savings are sorted in decreasing order. Later, the phase of joining routes will be constructed from the highest savings to the lowest savings. Figure 45 demonstrates the phase of sorting savings.

![Figure 45: Savings sorting](image)

5.1.4. **Merge cycles.** In this phase, the ordered savings are taken and checked whether they can be connected or not. The connection of two cycles/routes can be feasible if the cycles are separated, if the nodes are not interior and if the vehicle’s capacity is not exceeded. Each of the three conditions is explained below.

**5.1.4.1. Check for interior nodes and cycles separation.** In this part, the edges are tested to check if the nodes $i$ or $j$ are interiors. In other words, the customers should be either the first or the last customer in a given cycle. Also, the cycles of an edge $[i,j]$ are tested to check if they fall in separate cycles. If one of the nodes in edge $[i,j]$ is interior or the two nodes fall in the same cycles, then the edge is skipped, and the next edge will be examined. Figure 46 explains the procedure.

**5.1.4.2. Check for capacity.** After the above conditions are satisfied, the capacity is checked. If the limit is not exceeded, then a new node can be added to the route. If not, the edge is ignored. Once the capacity condition is satisfied, the initial routes for a customer should be deleted, and only the current route should be kept. This is to ensure that the customer is not visited more than once. In this phase, the sequence of the routes should be taken into consideration. This is to eliminate backtracking and reduce the total cost. Figure 47 explains how capacity is checked.

**5.1.5. Single-customer route construction.** In this phase, the constructed routes are checked to look for any route with a single customer.

---

**Inputs:** Number of edges, Elements stored in each edge (7)

**Outputs:** Edges in descending order

```plaintext
for (i=0 to number_edges) do
  for (j=0 to number_edges) do
    if (Edges[i][0] > Edges[j][0])
      Sort the edges in a decreasing order
      j++
    end for
  i++
end for
```

96
**Inputs:** Total number of cycles (num_cycles), Total number of edges (num_edges)

```plaintext
for (i=0 to num_edges) do
    next edge:
        if (i < num_edges) do
            for (j=0 to num_cycles) do
                if (cycle[j] has the same starting and ending depots of the first element in edge [i]) do
                    look for cycle[j] that corresponds to first customer in edge[i]
                    if it is found, do
                        check whether it is not interior or not
                        if interior
                            go to next edge
                        else
                            check second customer in edge[i]

                else if (cycle[j] has the same starting and ending depots of the second element in edge [i]) do
                    look for cycle[j] that corresponds to second customer in edge[i]
                    if it is found, do
                        check whether it is not interior or not
                        if interior
                            go to next edge
                        else
                            go to check capacity

                j++
            end for
        else
            break
    i++
end for
```

Figure 46: Check for conditions

**Inputs:** Demands, Drone’s capacity (Capacity), Total number of cycles (num_cycles), Total capacity for combining two demands (curr_capacity)

**Outputs:** Final routes

```plaintext
Curr_capacity = the summation of demands when combining two cycles
if (curr_capacity <= Capacity) do
    join the two cycles
    for (i=1 to num_cycles) do
        if (starting depot of cycle[i] <= ending depot of current cycle &&
            cycle[i] is not equal to constructed cycles) do
            delete cycle[i]
        i++
    end for
    for (n=1 to num_cycles) do
        keep current cycle and delete all initial cycles that consist of the same customers
        n++
    end for
end if
Reset parameters and go to next edge
```

Figure 47: Check capacity
In a regular savings algorithm, any customer that is left in a single route, the same initial route will be taken. However, due to the structure of our problem, there is a different combination of depots. In other words, the same customer can have multiple initial solutions for the different combination of depots. Therefore, only one route should be taken as the customer cannot be visited more than once. The selection is based on the route that gives the minimum traveling distance. Figure 48 shows the implementation of this phase.

**5.1.6. Total cost calculations.** In this phase, the algorithm calculates the operational cost for each constructed route which is the sum of the drone’s traveling cost and the truck’s traveling cost that account for the distance traveled between routes and within routes. Figure 49 illustrates the total cost calculations.

The above-detailed algorithm is tested on several problems to evaluate the resulted solutions in terms of quality and computational time. Next section shows the performed experimentations.

![Algorithm Implementation](image)

**Figure 48: Check for single – customer route**

### 5.2. Savings Heuristic Experimentation

In this section, we present the experimentations done to evaluate the performance of the proposed algorithm by comparing the results with the exact solutions provided by GAMS.

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Figure 49: Total cost calculations

For testing the solution obtained from savings algorithm and evaluating the quality of solution in terms of cost and time, the experiment is performed on different data sets from TSP Library. The algorithm is coded using a C++ programming language, and all the experiments were conducted using Intel(R) Xeon(R) CPU @ 2.60GHz with 64 GB RAM machine under Windows 10 (64 bit).

Table 18 shows a comparison between the objective values for the three ways of calculating savings. Equation (5-3) provides the least objective value.

Table 18: Savings objective values for different savings equations

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Using Eq. (5-1)</th>
<th>Using Eq. (5-2)</th>
<th>Using Eq. (5-3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$19,687</td>
<td>$18,667.2</td>
<td>$18,667.2</td>
</tr>
<tr>
<td>2</td>
<td>$21,305.8</td>
<td>$26760.2</td>
<td>$17,698.5</td>
</tr>
<tr>
<td>3</td>
<td>$27,170.9</td>
<td>$21,098.8</td>
<td>$10,085.1</td>
</tr>
</tbody>
</table>

Similarly, Table 19 shows the elapsed time taken to compute the total cost using the different savings equations. By using equation (5-3), the elapsed time is the minimum.

From Table 19 results, the proposed algorithm will be based on the improved savings, equation (5-3) as it gives better near-optimal solutions. Table 20 illustrates...
the results obtained from finding a solution to the proposed model using our algorithm for different number of depots, number of customers and the different total number of nodes.

Table 19: Elapsed time (sec) for different savings equations

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Using Eq. (5-1)</th>
<th>Using Eq. (5-2)</th>
<th>Using Eq. (5-3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.38</td>
<td>0.98</td>
<td>0.76</td>
</tr>
<tr>
<td>2</td>
<td>2.16</td>
<td>1.52</td>
<td>1.06</td>
</tr>
<tr>
<td>3</td>
<td>9.77</td>
<td>1.98</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Table 20: Improved savings objective values and elapsed time

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Problem</th>
<th>Number of Depots</th>
<th>Number of Customers</th>
<th>Total Number of Nodes</th>
<th>Objective Value</th>
<th>Elapsed Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vm1084</td>
<td>4</td>
<td>9</td>
<td>13</td>
<td>$18,667.2</td>
<td>0.76</td>
</tr>
<tr>
<td>2</td>
<td>Rd100</td>
<td>5</td>
<td>11</td>
<td>16</td>
<td>$17,698.5</td>
<td>1.06</td>
</tr>
<tr>
<td>3</td>
<td>Att48</td>
<td>7</td>
<td>13</td>
<td>20</td>
<td>$10,085.1</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Figure 50 shows the time taken by the algorithm to find the solution for different problems in which as the number of total nodes increases, the elapsed time increases. Figure 51 demonstrates a comparison between the values obtained from the exact solution through GAMS and the algorithm.

Likewise, Table 21 shows a comparison between GAMS and savings algorithm in terms of elapsed time. The algorithm is considered very fast compared to GAMS solutions. For 20 nodes, GAMS took 14 hours to find the optimal solution while the algorithm took 1.6 seconds to solve the same problem.

Savings algorithm can give near-optimal solutions within a short time. The comparison could not be extended due to GAMS limitations. Next, the algorithm is experimented to solve larger instances.

To experiment further, another data set from the online TSP Library is used. Table 22 shows the effect of increasing the total number of nodes on the objective value and the elapsed time using eil101 problem. Likewise, Figure 52 shows the
relationship between the elapsed time and number of customers.

Figure 50: Elapsed time, improved savings

Figure 51: Improved CW savings percentage deviation from GAMS

Table 21: Elapsed time, improved savings vs. GAMS

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Elapsed Time using Savings (sec)</th>
<th>Elapsed Time Using GAMS (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.76</td>
<td>911.5</td>
</tr>
<tr>
<td>2</td>
<td>1.06</td>
<td>38291.1</td>
</tr>
<tr>
<td>3</td>
<td>1.56</td>
<td>49535.4</td>
</tr>
</tbody>
</table>
Table 22: Improved savings objective values and elapsed time

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Problem</th>
<th>Number of Depots</th>
<th>Number of Customers</th>
<th>Total Number of Nodes</th>
<th>Objective Value</th>
<th>Elapsed Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Eil 101</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>$111.1</td>
<td>1.9</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>$246.54</td>
<td>11.68</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>10</td>
<td>30</td>
<td>40</td>
<td>$343.4</td>
<td>48.2</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>10</td>
<td>40</td>
<td>50</td>
<td>$466.4</td>
<td>140.4</td>
</tr>
<tr>
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<td></td>
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<td>50</td>
<td>60</td>
<td>$548.8</td>
<td>329.4</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>10</td>
<td>60</td>
<td>70</td>
<td>$632.6</td>
<td>670.8</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>10</td>
<td>70</td>
<td>80</td>
<td>$761.3</td>
<td>1288.8</td>
</tr>
</tbody>
</table>

Figure 52: Elapsed time, improved clarke and wright savings

Generally, the proposed algorithm can give good quality solutions within a reasonable time frame compared to GAMS longs runs.
Chapter 6. Conclusion and Future Work

The vast growth in the e-commerce market has increased the attention to resolving the problem of Last Mile Delivery to reduce the operational cost. The inclusion of new technologies is offering different solutions to tackle the challenges of delivering products to customers. In this thesis, we studied the inclusion of drones in the delivery process to form a truck-drone system. For this system, we studied six different scenarios, and they were formulated as Integer Linear Programming problems to minimize the total operational cost. The modeling environment considered the use of a truck to release and collect the drone at optimized locations, in which the drone is used to serve customers. The different scenarios are subject to different constraints that restrict the drone’s capacity, drone’s maximum operational time, number of drones, and precedence constraints. The output of the optimization is the sequencing of the deliveries to different customers and the optimized locations for the truck to release and collect the drones. GAMS software was used to provide exact solutions to different data sets from the online TSP library. Results have shown that drone’s load capacity, drone’s operational cost, unit traveling costs of drones and truck, the location of truck’s path, and the number of truck’s locations affect the configuration of the system and change the operational cost. Despite the optimal solutions that GAMS can provide, it cannot give an optimal solution within a short time. GAMS computational time increased exponentially with the increase in the size of the problem. Therefore, a Clarke and Wright savings heuristic was developed to give a solution with good quality in a reasonable time duration.

Clarke and Wright savings algorithm is an effective method for getting good quality solutions for large size problems. In our work, we implemented an improved Clarke and wright that considers multiple depots, in which there are different depots combinations. It also reflects the different scenarios for joining customers that is unlike the traditional savings algorithm. Also, the savings equation was modified by including the cost for the distance traveled by truck. The proposed model has been coded using a C++ programming language, and experimentations were conducted to test the effectiveness of the algorithm against GAMS exact solutions using data sets from the online TSP library. The improved savings returned solutions that are within the known quality of savings heuristics, 20%. The solutions provided 8% to 20%
deviation from the optimal solutions. The algorithm was further experimented by testing larger sets, and it returned solutions for 80 nodes within 1200 seconds.

In terms of limitations, the proposed models consider the availability of the customers at the delivery without taking into consideration the time window. Also, the truck’s waiting time is not part of the optimization process. Moreover, the drone’s power consumption is not addressed in which the truck is assumed to provide new batteries for the drones during each preparation of launching of the drones. Finally, the different launching and collection locations in the multiple drones’ model is neglected, in which the launching and collection points for all drones are assumed to be the same.

As a future work, the proposed models can be modified to consider the drones’ characteristics such as power consumption. Another modification is to consider our problem with a time window. Moreover, the truck’s role can be extended by allowing it to serve customers that are along its path. Furthermore, the Clarke and Wright savings algorithm can be further improved to enhance the performance of the algorithm.
References


Vita

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