A FRAMEWORK FOR PROJECT TIME-COST OPTIMIZATION CONSIDERING FLOAT CONSUMPTION IMPACT

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ABSTRACT

The main objectives of construction projects include completing the project on time and within budget. There is always a tradeoff between time and cost. Time loss is costly and time savings can provide benefits to all the parties involved in the project. Time-cost optimization is essential for construction projects. The objective of time-cost optimization is to determine the optimum project duration corresponding to the minimum total cost. This is accomplished through shortening the duration of critical activities in order to reduce the overall project duration. Time-cost optimization techniques were developed to accelerate the project schedule by expediting the construction process and reducing the activities' durations to meet owner's convenience and expectations, or to recover the lost time when the project performs behind schedule or exhibits a negative time variance. Since the 1960's, several methods for time-cost optimization were developed with the aim of minimizing the project cost and duration without paying close attention to the effect of float loss resulting from schedule compression. The float is an important element in the project schedule that can be used by contractors to change the start of noncritical activities for resource management purposes, and by owners to accommodate change orders. Although total float is defined as a time contingency against project delays, the consumption of float can lead to serious increase in project risk and cost. Time-cost optimization techniques result in reducing the available float for noncritical activities and thus reducing the schedule flexibility. The main objective of this research is to develop a new time-cost optimization framework that can provide an optimum cost-time value for a project taking into consideration the effect of float loss.

This thesis presents two new frameworks that are developed to solve the time-cost optimization problem taking into account the float loss impact: a stochastic framework and a Non-Linear Integer Programming (NLIP) framework. The stochastic framework uses Monte Carlo Simulation (MCS) to calculate the effect of float loss on risk. This is later translated into an added cost to the optimization problem. The Non-Linear Integer Programming (NLIP) framework uses

What’s Best solver to find an efficient solution to the optimization problem while incorporating the float loss cost calculated according to the float commodity approach.

An application example of the frameworks is presented. The deterministic solution, using classical time-cost tradeoff techniques, shows the optimum duration of 23 days at a minimum cost of $12,490. Using the proposed stochastic framework, the optimum duration is 25 days at a minimum cost of $12,709. The Non-Linear Integer Programming (NLIP) framework shows an optimum duration of 24 days at a minimum cost of $12,659. The results from both proposed frameworks confirmed the research hypothesis, which states that the new optimum solution will be at a higher duration and cost. This is due to the fact that the proposed frameworks incorporate the effect of float loss on project cost and risk. This presents a new tradeoff between time, cost and flexibility (represented in the amount of float). The results obtained using the two frameworks; in comparison with the deterministic time-cost tradeoff, are better in terms of schedule flexibility, activities’ criticality index, and probability of finishing the project on time. The probability of completing the project on time is 0.28 and 0.33, using the nonlinear-integer programming framework and the stochastic framework, respectively, as compared to 0.23 in the deterministic scenario. Five examples, selected from literature, are solved via the two proposed frameworks. Overall, the results of the examples used to validate the developed frameworks have justified them as valid, time-saving and reliable methods against float loss oriented risks. The results are significant and allow project managers to exercise new tradeoffs between time, cost and flexibility. This will ultimately improve the chances of achieving project objectives.

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TABLE OF CONTENTS

ABSTRACT iii
LIST OF FIGURES viii
LIST OF TABLES ix
ACKNOWLEGMENTS xi
DEDICATION xii
CHAPTER ONE: INTRODUCTION 1
1.1 Chapter Overview 1
1.2 General Introduction 1
1.3 Problem Statement 3
1.4 Thesis Objectives 5
1.5 Thesis Scope 5
1.6 Research Methodology 5
1.7 Research Significance 7
1.8 Thesis Organization 8
CHAPTER TWO: REVIEW OF LITERATURE 9
2.1 Chapter Overview 9
2.2 Project Time-Cost Optimization 9
2.2.1 Classical or Manual Time-Cost Optimization Technique / TCT Techniques 10
2.2.2 Mathematical Optimization Techniques 13
2.2.3 Meta-Heuristic Techniques 18
2.3 Free Float and Total Float 19
2.4 Delays in Construction Projects 21
2.5 Float Consumption Impact 22

2.6 Float Allocation and Float Ownership ... 23
2.7 Chapter Conclusion ... 26
CHAPTER THREE: MANUAL-PROBABILISTIC OPTIMIZATION FRAMEWORK CONSIDERING FLOAT CONSUMPTION IMPACT ... 28
3.1 Chapter Overview ... 28
3.2 Proposed Framework ... 28
3.2.1 Assumptions ... 28
3.2.2 Manual-Stochastic Framework Steps ... 28
3.3 Application Example ... 32
3.3.1 Normal Project Compression (Deterministic Approach) ... 32
3.3.2 Project Compression Considering Float Consumption Impact ... 40
3.4 Analysis and Discussion of the Results ... 48
3.5 Chapter Conclusion ... 51
CHAPTER FOUR: PROJECT COMPRESSION CONSIDERING FLOAT CONSUMPTION IMPACT VIA NONLINEAR-INTEGER PROGRAMMING ... 52
4.1 Chapter Overview ... 52
4.2 Proposed Optimization Model ... 52
4.2.1 Assumptions ... 53
4.2.2 Model Formulation ... 53
4.3 Application Example ... 58
4.3.1 Project Compression Considering Float Consumption Impact (Manual Approach) ... 58
4.3.2 Project Compression via Nonlinear-Integer Programming Considering Float Consumption Impact ... 67
4.4 Results and Discussion ... 70
4.5 Chapter Conclusion ... 73
CHAPTER FIVE: FRAMEWORKS VALIDATION ... 74

5.1 Chapter Overview 74
5.2 Frameworks Validation: Results and Analysis 74
5.2.1 Example One 74
5.2.2 Example Two 77
5.2.3 Example Three 81
5.2.4 Example Four 86
5.2.5 Example Five 89
5.3 Chapter Conclusion 94
CHAPTER SIX: SUMMARY, CONCLUSION & RECOMMENDATIONS 95
6.1 Summary and Conclusion 95
6.2 Recommendation for Future Research 96
REFERENCES 97
VITA 102

LIST OF FIGURES

Figure Page
1. Project Cost-Duration Graph 3
2. Conventional vs. New Time-Cost Curve 4
3. Continuous Linear Model of Utility Curve of an Activity 11
4. Example illustrating a concave function and a convex function 16
5. Framework Flowchart 31
6. Cycle Zero: Normal Schedule 33
7. Cycle One: Crashed Schedule 1 34
8. Cycle Two: Crashed Schedule 2 35
9. Cycle Three: Crashed Schedule 3 36
10. Cycle Four: Crashed Schedule 4 37
11. Cycle Five: Crashed Schedule 5 38
12. Project Time-Cost Tradeoff 39
13. Project Total Cost vs. Duration Curve 39
14. Total Cost Curves Comparison 50
15. NLIP Framework Flowchart 57
16. Cycle Zero: Project Normal Schedule 59
17. Cycle One: Crashed Schedule 1 60
18. Cycle Two: Crashed Schedule 2 62
19. Cycle Three: Crashed Schedule 3 64
20. Project Time-Cost Tradeoff Considering Float Consumption Impact 66
21. Project Total Cost vs. Duration Curve Considering Float Consumption Impact 66
22. Example of Model Building using Excel and What's Best 69
23. Comparison of Total Cost Curves 72
24. Schedule Network and Activities Relations in Example One 75
25. Total Project Cost vs Project Duration in all Cases in Example One 77
26. Schedule Network and Activities Relations in Example Two 78
27. Total Project Cost vs Project Duration in all Cases in Example Two 81
28. Schedule Network and Activities Relations in Example Three 83
29. Total Project Cost vs Project Duration in all Cases in Example Three 85
30. Schedule Network & Activities Relations in Example Four 86
31. Total Project Cost vs Project Duration in all Cases in Example Four 88
32. Schedule Network & Activities Relations in Example Five 89
33. Total Project Cost vs Project Duration in all Cases in Example Five 92

LIST OF TABLES

Table Page

1. Project normal & crashed costs and durations 32
2. Activities total float at cycle zero 33
3. Activities total float at cycle one 34
4. Activities total float at cycle two 35
5. Activities total float at cycle three 36
6. Activities total float at cycle four 37
7. Activities total float at cycle five 38
8. Project crashing results 39
9. Simulation results for the baseline schedule at cycle zero 40
10. Simulation results after crashing activity B at cycle one 40
11. Simulation results after crashing activity F at cycle one 41
12. Simulation results after crashing activity H at cycle one 42
13. Simulation results after crashing activity K at cycle one 42
14. Simulation results after crashing activity B at cycle two 43
15. Simulation results after crashing activity F at cycle two 43
16. Simulation results after crashing activity H at cycle two 44
17. Simulation results after crashing activity K at cycle two 45
18. Simulation results after crashing activity B at cycle three 45
19. Simulation results after crashing activity H at cycle three 46
20. Simulation results after crashing activity K at cycle three 46
21. Comparison of remaining TF between deterministic compression method & new compression framework 48
22. Activities critical index at 25 and 23 days durations 49
23. NLIP framework parameters 54
24. NLIP framework decision variables 55
25. Activities costs and durations 58
26. Noncritical activities total float at cycle zero 59
27. Noncritical activities total float at cycle one 60
28. Float loss cost at cycle one 61
29. Noncritical activities total float at cycle 2 62
30. Float loss cost at cycle 2 63
31. Noncritical activities total float at cycle 3 64
32. Float loss cost at cycle 3 65
33. Project crashing results (crashing considering the float consumption impact) 65

34. Comparison of TF between deterministic compression method & new proposed NLIP framework 70
35. Activities' critical indices at 23 and 24 days duration 71
36. Isidore & Back project data 74
37. Comparison between all cases in terms of remaining total float in Example One 76
38. Activities critical indices in Example One 76
39. Oxley & Poskitt project data 78
40. Comparison between all cases in terms of remaining total float in Example Two 79
41. Activities critical indices in Example Two 80
42. Zeinalzadeh project data 82
43. Comparison between all cases in terms of remaining total float in Example Three 84
44. Activities critical indices in Example Three 85
45. Elbeltagi project data 86
46. Comparison between all cases in terms of remaining total float in Example Four 87
47. Activities critical indices in Example Four 88
48. F. Gould project data 90
49. Comparison between all cases in terms of remaining total float in Example Five 91
50. Activities critical indices in Example Five 92
51. Summary of the five examples results 93

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This Thesis is dedicated specifically to my dear family; my husband, my dad, my mom, my brothers, and my husband's family. My mere expressions of thanks can't suffice their endless love and support.

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CHAPTER ONE: INTRODUCTION

1.1 Chapter Overview

Chapter one presents an introduction about the work undertaken in this Thesis. It starts first with the statement of the problem followed by the Thesis objectives and scope, and the research methodology. Since time is of the essence of any project and since delays can affect the project's cost and schedule, the significance section discusses the advantage of the new framework that will account for the float loss risks associated with project compression in terms of time and money and will serve in meeting the benefits that the optimization process can provide for construction project stakeholders. At the end, the organization of the Thesis is further explored in detail to summarize the content of each chapter.

1.2 General Introduction

During the last years, the construction industry has been on the rise all over the world. This increase in demand for construction was the main motive to adopt construction management and rank it as a crucial element in the construction process that is needed to handle challenges and risks associated with projects. The construction field refers to the word "project" generally as "a temporary endeavor undertaken to create a unique product or service" [1]. For all construction projects, the main objectives are to handover the project within the required time and cost. Each project has its planned budget and schedule. The budget is a measure of the cost that the project will consume in order for the final deliverable to be finished. Generally, the cost is subdivided into direct and indirect cost. The direct cost is simply the cost of the resources such as direct labors, materials and equipment and is calculated by summing up the resources' cost of all the activities or work packages. The indirect cost is the summation of general overhead and job overhead; or in other words, it's a cost other than the direct cost that can be assigned to a specific activity.

Project objectives cannot be attained unless proper management of the construction is implemented. Employing effective planning in terms of scheduling, budgeting, safety and quality at the early stages of the project is very important since it allows control over the process from its initiation phase to its close out phase, minimizes delays and cost over-runs, and assists in achieving the project objectives efficiently. Nowadays, construction project participants are becoming more aware of the high impact associated with the delays in terms of cost and litigations. In fact, meeting compulsory deadlines of projects are necessary due to the following reasons [2]:

- Contract agreements and owner's needs (imposed deadline)
- Project launching time (time to market demand)
- The need for committing the resources for other projects that are in need of extra resources; the competence of projects over resources available and the need for efficient resource utilization
- Incentives or bonuses rewarding the early completion of a project
- The desire to avoid unexpected unforeseen conditions or risks

One way to achieve the delivery of the project at the required completion date and with the least cost associated is by the employment of the least-cost scheduling technique or the time-cost tradeoff techniques. Time-cost tradeoffs are one of the most frequent and critical decisions that project managers usually make. Analyzing the time and the cost is essential in order to obtain an optimum schedule that maintains the project deadline while having the lowest cost. Optimization as a word refers to the determination of a highest or lowest value over some range, either to maximize the profit or minimize the loss. According to Charoenngam and Popescu [3], time-cost optimization or trade-off is defined as "a scheduling technique using the critical path method by which the project duration is shortened with a minimum of added cost". In general, project time and cost are linked via a relationship. As the project schedule or the project time is shortened, the direct cost of critical activities increase, while the indirect cost (overheads) of the project decreases. Figure 1 demonstrates the project's time-cost tradeoff curve. When there is a need for crashing the activities and accelerating project completion, Gray & Larson [2] mention several options based on the resources constraint. Options when resources are not constrained include outsourcing the work like by subcontracting, having overtimes or multiple shifts at work, or adding additional resources such as extra labors or extra machineries. On the other hand, the available options when resources are constrained can include reducing the overall scope of the project, or go with the fast tracking option by re-changing the logical relationships between the activities in a way the critical activities are performed in a parallel rather than in a sequential basis. Several suggested models for optimizing construction schedules were developed over the previous decade, but none of the reviewed techniques took into account the effect of float loss over the project cost. The proposed framework over here is suggesting the implementation of such impact into the optimization process.

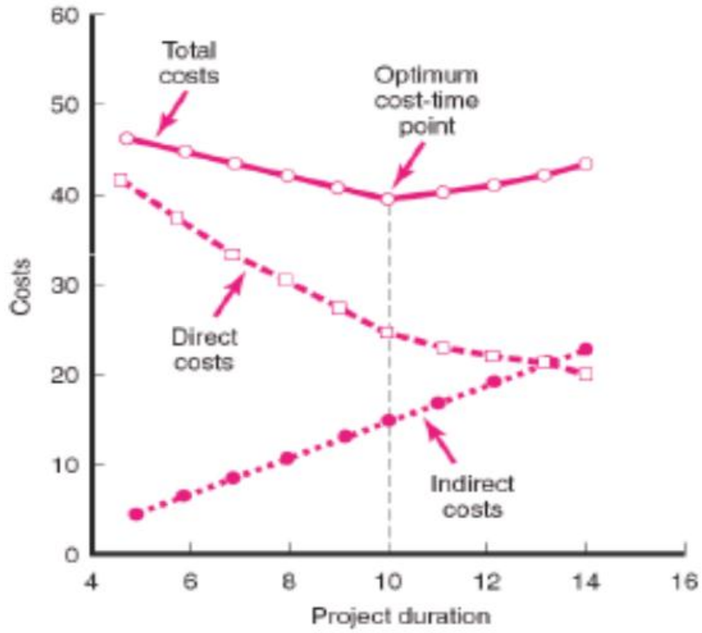


Figure 1: Project cost-duration graph **[2]**

1.3 Problem Statement

Optimizing the project's duration while maintaining the least crashing cost is usually a needed issue in order to complete the project activities earlier than originally scheduled and planned or to meet the project deadline with the least additional cost. When the project duration is reduced, the total float available for noncritical activities is reduced as well. Typically, the optimization process involves deterministic procedure that is carried out until an optimum value is reached. Available time-cost optimization methods are based on the concept of shortening the duration of the critical activities in the network progressively while observing the decrease in total project cost until the optimal solution that provides the shortest project duration with the minimum total cost is reached. The deterministic optimization technique doesn't consider the impact of the float lost within the noncritical activities when the project duration is being crashed or reduced. Such losses in total float can impact the project cost and schedule, and may lead to delays in activities that are in a path; causing a ripple effect on the downstream activities of that path; and therefore, losing the chance of early finish for these activities. To solve the problem and provide a more accurate and realistic results, two frameworks are developed in this Thesis that cope the effect of float loss in terms of cost and time with the optimization technique. Talking from a manual optimization perspective, the proposed framework presented in this Thesis proposes the inclusion of float loss cost into the compression method by finding the daily trade-off value of total float or the daily change in project risk using the commodity approach proposed by De La Garza et al. [4] for the nonlinear-integer framework and by @risk simulation for the probabilistic framework. Then for each crashing cycle, the float cost (cost impact) value found previously for a specified crashed duration will be added to the direct cost generated through that crashing cycle, and then by adding that to the assumed indirect cost, total direct cost can be found for that project duration. The process of crashing continues until reaching an optimum solution (duration) with least possible total cost/ total extra cost. Theoretically, the resulting time-cost optimization curve based on the proposed framework will have the same shape as that of the time-cost optimization curve without float cost, but it will be shifted further above due to the increased total cost and the optimum point is assumed to be found at a higher cost and duration compared to the original optimum point. This assumed resulting curve is illustrated in Figure 2. The frameworks details are presented in Chapters Three and Four.

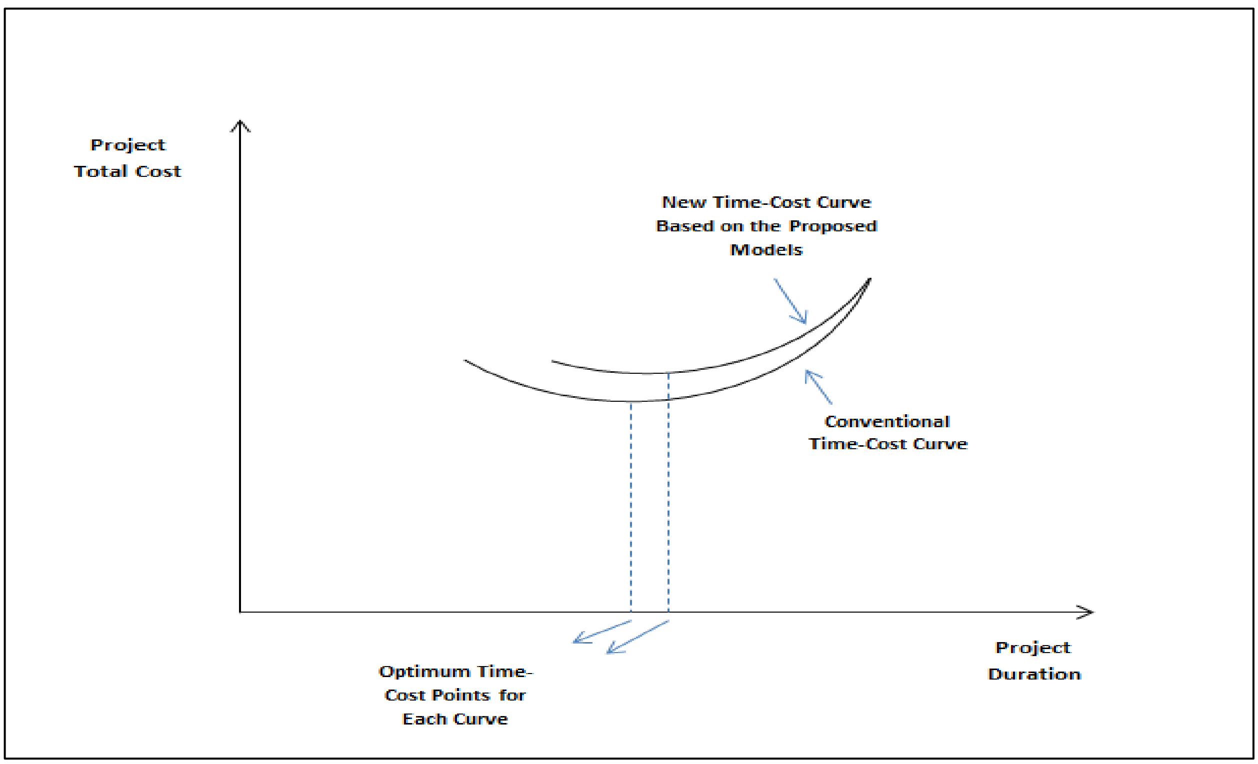


Figure 2: Conventional vs. new time-cost curve

1.4 Thesis Objectives

This Thesis seeks refining the project trade-off analysis by incorporating the float loss cost in the optimization technique. The main objectives of the Thesis are as follows:

- Develop a manual-probabilistic framework for project time-cost optimization considering the float loss impact in non-critical activities.
- Develop a Nonlinear-Integer Programming (NLIP) framework to solve the optimization problem while considering the float consumption impact.

1.5 Thesis Scope

Based upon the previous objectives, the Thesis in general aims on highlighting the importance of introducing the effect of float loss to the time-cost tradeoff analysis. Float consumption can impact the project risk by increasing the chances of having potential delays. The Thesis therefore is limited to a manual-probabilistic framework and a nonlinear-integer programming framework that can account for the float loss effect in terms of cost. The manual framework is developed with the aid of @risk simulation, while the nonlinear framework is approached via What's Best solver. What's Best 11.0 solver is used due to its unlimited capacity, the great reliability and speed and the ease of use. The nonlinear programming framework (mathematical approach) is chosen to make it a user friendly framework that can produce an efficient solution for the optimization problem.

1.6 Research Methodology

The development of the Thesis in general is done over three phases. In order to meet the Thesis objectives, the following methodology and tasks are implemented:

Phase One: Preliminary Work

In this phase, the following two tasks are accomplished:

- Task one: Determining the statement of the problem and the research objectives

- Task two: Performing literature review that covers similar, to-date work (like journals, books, online papers,...etc) over the topics of optimization techniques, time-cost tradeoffs, delays and float.

Phase Two: Framework Development

In this phase, the following two tasks are accomplished:

- Task one: Collecting hypothetical examples that contain crashing data (cost and durations) from previously published papers and books
- Task two: Formulating and developing the framework that can link between the float consumption impact and the time-cost tradeoff problem. The two frameworks developed are as follows:
  - First framework: Calculates stochastically the float loss impact using Monte Carlo Simulation (@risk) based on the idea that when a project is compressed, the probability of finishing on time decreases; as well as the total float available for noncritical activities. The framework calculates the duration difference between the probabilistic compressed duration and the deterministic compressed duration and quantifies the float loss cost as a product of the duration difference and cost per day.
  - Second framework: Solves the optimization problem using nonlinear-integer programming approach (while considering the float loss impact calculated using total float traded as a commodity method [4]). The initiation of the NLIP follows the stages of defining the objective function, decision variables and constraints, writing the mathematical function for the objective, writing some description, RHS, and LHS for each constraint, and finally determining the relations (<, =, or >) and the coefficients for decision variables in each constraint. Then using What's Best solver, the developed problem is solved and an efficient solution is found.

Phase Three: Validation of the Framework

In this phase, the following tasks were performed:

- Task One: Comparing the results found using the frameworks against the results obtained using conventional deterministic approaches that doesn’t consider float loss impact to show the significance of the proposed framework.
- Task Two: Verification of the framework by applying it over five examples selected from literature.

1.7 Research Significance

Completing any project within the required budget and schedule are the main objectives for any project manager. Construction projects are risky by nature, and risks associated with project phases can cause delays in project duration and cost overruns.

Over the last two decades, the time-cost trade-off problem was approached in different aspects for the purpose of facilitating the process of optimization and improving the reliability of the results, as finding the least costly method to shorten the schedule practically is complicated. This research proposes a new framework for project’s time-cost optimization considering a new criterion, which is float consumption, that wasn’t adequately considered in the process of optimization before. Uncertainties or risks are high during the project life cycle; especially at the early phases of the project, and they should be considered when minimizing project cost and duration [5]. Such risks can arise from project compression whenever total float is lost, and such a loss can be costly depending on the project characteristics and case. Float losses can cause huge effects over the project cost and schedule, and may lead to delays in activities that are in a path, resulting on adverse impacts over project quality, labor performance and moral, and the resulting disputes. However, conventional optimization techniques examine only the decrease in total project cost until a minimum cost is reached and neglects the risk impacts associated with the reduction in project duration; or in other words, the impact of within-total float losses in noncritical activities. Due to such losses that can happen during the compression cycle, total float can be consumed and non-critical activities may become critical. Taking into consideration the float consumption or the float loss in non-critical activities while crashing will assist in providing a more realistic optimized schedule with the least cost needed and will develop a new promising technique. The new proposed framework provides decision makers with a better, more efficient tool to solve the time-cost optimization problem with the least possible risks associated with float loss in terms of time and money, besides serving the management team in meeting the benefits that the optimization process can provide for construction projects.

1.8 Thesis Organization

The Thesis is divided into six chapters. Each chapter contains the following:

Chapter One

- Provides a brief introduction about the nature of work by stating the objectives, the scope, the statement of the problem, the research methodology and the significance of the new proposed framework

Chapter Two

- Explores a brief review about the time-cost optimization techniques: the manual, the mathematical and the meta heuristic approches
- Presents the defenition of each technique and a review of previous works conducted in the same area of time-cost trade-off analysis
- Defines total float and free float concepts, and presents some of the papers that approched this topic.
- Highlights the effects of delays in construction projects and how float consumption can have a potential impact over the project schedule and budget.
- Explains the float allocation methods avilable from literature and explain why the float comodity approach was selected in this Thesis to quantify the float cost per day for each crashing cycle

Chapter Three

- Explains in detail the deterministic manual solution for the selected time-cost optimization problem
- Explains the new -manual, probabilistic solution for the optimization problem using @risk simulation
- Compares the results between the determinstic approach and the new-developed framework

Chapter Four

- Explains how the float cost per day was found for each activity via the total float tradded as a comodity approach
- Expresses manually the optimization solution considering the float loss cost for the selected optimization problem
- Expresses the optimization solution using a mathematical approach; nonlinear-integer programming using What's Best solver, and the development of the framework
- Compares the results between the diffrent approaches

Chapter Five

- Validates the two new developed frameworks by applying them over several examples and analyzing and discussing the results obtained

Chapter Six

- Provides concluding remarks, recommendations and thoughts for future research

CHAPTER TWO: REVIEW OF LITERATURE

2.1 Chapter Overview

This chapter presents an insight review of literature related to the schedule compression and float concepts in construction projects. The first part of the chapter illustrates what is meant by project time-cost optimization, and explains later the available time-cost optimization techniques available including the classical or manual approaches, the mathematical approaches, and the meta-heuristic approaches and a review of the main studies done by several researchers over these techniques. The second part of the chapter explains the float concept and the delays in construction projects and how float is used to measure the flexibility and criticality of the project schedule in terms of the ability of extending the duration of a certain activity. Moreover, the chapter presents how float loss can have an impact over the project in terms of cost and schedule overruns, adverse outcome on quality of work, performance, and moral of labors, in addition to resulting claims and disputes. In order to demonstrate the effect of float consumption impact on noncritical activities due to delays over the project schedule and cost, the chapter explains Sakka & El-Sayegh [6] method to control the risks associated with such delays. The last part of the chapter highlights the main approaches used to allocate and manage the float and the float ownership issue. Some of the discussed approaches include the float commodity approach, the total risk approach, the use of safe float approach and the pre-allocation of total float approach.

2.2 Project Time-Cost Optimization

The Construction Industry Institute (CII) identifies two critical schedule mechanisms: schedule reduction and schedule compression. Schedule reduction indicates a reduction in project time without increasing the cost inhabited through the use of some techniques such as freezing the project's scope; on the other hand, schedule compression signifies a time reduction with an escalated cost [7]. Finding optimum or near optimum solution for the time-cost tradeoff problem implies the use of several techniques such as manual time-cost tradeoff (TCT) techniques, mathematical techniques or Meta-heuristic techniques. The following is a literature review performed over the previously mentioned three major techniques.

2.2.1 Classical or Manual Time-Cost Optimization Technique / TCT Techniques

Time-cost optimization aims to find the least cost point; which is the optimum point between the normal activity time-cost point and the crash activity time-cost point. The traditional time-cost optimization technique is based on the critical path method (CPM) and has been used in the construction industry over the previous fifty years. This technique "requires that all operations in a project be represented in activities, each of whose start is dependent upon completion of other activities" [8]. Moreover, an "input time-cost curve is required of each activity that describes the relationship between activity duration and direct cost for alternative plans for performing the activity" [8].

Time-cost tradeoff analysis usually assumes that the project duration is shortened through critical activities only and the amount of crashing time for each activity is limited. Moreover, it assumes that the cost of overheads is invariable during project duration, while the direct costs are linear between the normal cost and the crash cost for an activity. The planned duration in time-cost tradeoff analysis is assumed to be any point between the normal duration and crash duration for an activity [9].

Time-cost optimization is based on the idea of shortening the critical activities with the minimal increase in cost per unit of time. It is usually performed through the following steps:

1. Developing the schedule based on the normal duration and normal cost of project's activities.
2. Estimating the crash cost and duration for each activity and finding the crashing slope. The crashing slope, which is a constant cost per unit of time, can be found through the following formula:

   Crash cost per time (Slope) = (Crash cost – Normal cost) / (Normal time – Crash time)

   Figure 3 represents the continuous linear model of utility curve of an activity including the crashing slope.

3. Identifying the critical activities on the critical path, then identifying the critical activity that can be crashed with the least cost; that is, the activity with the least crashing slope. If there is more than one critical path, a critical activity from each path should be selected and crashed as long as the two selected activities can still be crashed and the total crash cost of the selected activities is the smallest.

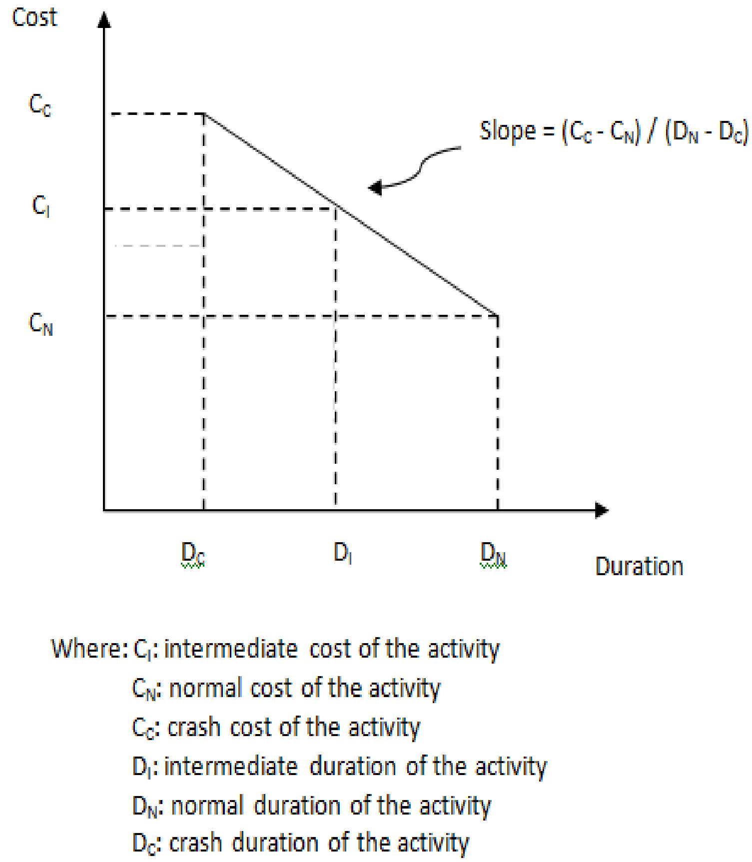


Figure 3: Continuous linear model of utility curve of an activity

4. Shortening the activity by the units required (maximum crashing units = normal time – crashing time).
5. The new cost and duration of the project should be calculated at this stage, and then steps through 3 to 5 should be repeated until the optimum point, where “the overhead cost savings are greater than the increased direct costs” [3], is achieved.

Hinze [10] explained a logical method for crashing that considers the schedule network as a rigid frame. This technique is based on the use of “Link lag values” that helps in determining the possible number of times the activity can be crashed. The method involves the following steps:

1. The early start and early finish for each activity in the network and their corresponding duration-cost data are computed.

2. Link lag values are found by subtracting the early start value of each successor activity from the early finish time of its predecessor activity. Generally, zero and non-zero link lag values are distinguished. Zero link lag values have to be distinguished graphically from the non-zero ones while compressing the schedule (graphically, zero link lags will be represented through double lines connecting the activities, while non-zero links will be presented as a one connecting line). An important value to be computed and updated at each crashing cycle for each activity is the network interaction limit (NIL). The NIL represents "the number of days an activity can be shortened before some other link lag value becomes zero" [10]. If the vertical line is to pass through the last activity in the network, The NIL value for that last activity is limitless or infinity.
3. At each compression cycle, the activity with least crashing cost is selected.
4. After deciding the activity (or activities depending on the critical path) to be crashed, a vertical line down through the network is drawn. The line will pass through the activity or activities being crashed and through any non-zero link lags that may be in the path of the line. Zero links can't be passed because otherwise their value will be reduced. This line will show the activities that might be affected with each crashing cycle.
5. To determine the number of days an activity can be crashed, one has to compare the number of days an activity can be shortened with the number of days in the crossed NIL's. The one with the smallest value will give the number of compressing days of the crashed activity.
6. At each crashing cycle, network and NIL values have to be updated.
7. Crashing iterations continue until the network can't be shortened further. This point is reached when "all the activities on a critical path can no longer be shortened, and while there are other activities that can be shortened and the project duration would not be altered" [10].

Maximum flow-Minimal cut is another manual optimization method for construction schedules that was discussed by Jinming & Rahbar [11]. This method is based on maximal flow-minimal cut theory that states that "in the network from start to end the maximal flow is equal to the minimum cut set capacity" [11]. This method is an improvement over the previous classical optimization technique as it is possible to solve it through programming approach. This optimization method is helpful when several compression sets are required for review (when multiple critical paths with interrelated activities occur).

Isidore & Back [7] proposed an improved model of the least-cost scheduling technique or the crashing technique in which variability of activities in terms of cost and time is taken into consideration. In this model, range estimating and probabilistic scheduling is applied over the data then the resulting data is analyzed in a statistical way to reach an optimum project cost and duration at higher confidence level. Yang [12] proposed a chance-constrained programming model to incorporate the variability of funding then translate it into a corresponding deterministic at pre-defined confidence level. The equivalent deterministic is then incorporated to the traditional time-cost optimization technique.

2.2.2 Mathematical Optimization Techniques

Mathematical programming generally refers to selecting the best element from some set of available alternatives. It is solved in a way that the objective function is maximized or minimized and the real or integer optimized solution is picked. Some of the mathematical approaches that can serve in decision making problems include linear programming, nonlinear programming, integer programming, dynamic programming, simulation models, stochastic models and inventory models. In these approaches, the relationship between time and cost of an activity is assumed to be either: linear/nonlinear, concave/not fixed, discrete/ continuous, or a hybrid of the previously mentioned.

Three main types of mathematical optimization techniques are distinguished in the construction management field:

- Linear Programming: linear programming is an optimization technique used with linear functions subject to linear constraints (constraints can be equalities or inequalities). This technique is first developed by Leonid Kantorovich in 1939, and then used in World War Two for military optimization problems. In fact, linear programming is helpful in many other applications other than the military field. It served the applications of "transportation and distribution, scheduling, production and inventory management, telecommunication, agriculture and more". [13].

  Each linear programming (LP) problem has an objective function, constraints and decision variables. An LP problem can be bounded feasible, unbounded feasible, or infeasible, where feasible LP problem is said to be unbounded if the objective function assumes randomly large positive or negative values at feasible vectors; or else it is called a bounded problem. The feasible vector that achieves the required value of the objective function is the optimum solution [14].

Moreover, the following characteristics apply for linear programming problems:

- All the relations between variables are linear, and variables must be continuous.
- Single objective function applies: the main objective is to find the maximum or minimum output.
- Constraints need to be maintained to indicate the feasible vectors.
- Decision variables are assumed to be continuous and of any number.

Therefore, linear programming, when used in typical time-cost tradeoff problem, aims in minimizing total project cost subject to project deadline constraint. Linear programming problems are solved generally using one of the following approaches: graphical approach, simplex method, transportation method or assignment method. The simplex method is the most used method for solving LP problems. This method, which is created in 1947, examines in sequence (iterations) the vertices of the solution. This method selects the variables that will generate the maximum (or minimum) change at each iteration until a final solution is reached, and it can predict if no solution case is present. Generally speaking, initiating a linear programming problem involves the following steps [15], [16]:

- Identifying the decision variables.
- Listing the objective functions in terms of the function to be optimized and the expression that describes the performance measure (money profit, number of labors, duration, etc).
- Listing the resource restrictions and the boundary of constraints.

A standard LP problem can be of the following form (adopted from Module for linear programming- the simplex method [17]):

The standard form of the linear programming problem is to Maximize F(x) of n variables $x = (x_1, x_2, \ldots, x_n)$

(1) Maximize $z = F(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

$= \sum_{j=1}^{n} c_j x_j$

Where $c_j \geq 0$ for $j = 1,2,\ldots,n$

(2) Subject to the m constraints: $a_{i,1} x_1 + a_{i,2} x_2 + \ldots + a_{i,n} x_n \leq b_i$,where $b_i \geq 0$ for i= 1,2,…,n

(3) With the primary constraints $x_j \geq 0$ for j= 1,2,…,n. The coefficients c_j and $a_{i,j}$ can be any real number. It is often the case that m > n, but the cases m = n or m < n can occur

- Nonlinear Programming: represents the mathematical optimization technique used to solve equality or inequality systems to maximize or minimize an objective function subject to nonlinear constraints (or sometimes the objective function can be nonlinear). Nonlinear programming has been widely used in several applications such as resource allocation, production planning, routing data networks, computer aided design, solution of equilibrium models, data analysis and least squares formulation and modeling human or organizational behavior [18]. This programming technique is inspired from the study of "calculus of variations" during the eighteenth and nineteenth centuries. The general form of a typical nonlinear optimization problem can be stated as followed [19]:

$$\text{Minimize } f(x)$$

$$\text{Subject to } g_i(x) = 0, \quad i \in \mathcal{E}$$

$$g_i(x) = 0, \quad i \in I$$

Where;

$\mathcal{E}$: index set for the equality constraints

I: index set for the inequality constraints

Nonlinear programming generally has to consider and analyze all the solutions (local maxima and local minima) in the feasible region not only the solutions on the boundary, in which the global maxima or minima is considered to be the optimal solution. The local optimum point is a point at which its value; in case of maximizing for example, exceeds the value of all surrounding points but may not exceed that of distant points. The second derivative of the function can describe the function's shape whether its concave or convex; and therefore, can imply whether the optimum solution is local or global as the second derivative is defined as the rate of change in the first derivative. For a multivariate function (a function with several variables) with a stationary point XA, it can have a) a local maximum at XA if the function is concaved locally, b) a global maximum at XA if the function is strictly concaved via the considered domain, c) a local minimum at XA if the function is of a convex shape locally, d) a global minimum at XA if the function is strictly of a convex shape, e) a saddle point if the function is neither concave nor convex [20] , [21], [22].

To restate the previously mentioned words mathematically, the following conditions can guarantee having any local optimum as a global optimum:

- For the function f(x); $\frac{d^2f}{dx^2} \leq 0$ for all x $\longrightarrow$ f(x) is a concave function (a function that is always curving down or not curving at all).
- For the function f(x); $\frac{d^2f}{dx^2} \geq 0$ for all x $\longrightarrow$ f(x) is a convex function (a function that is always curving upward or not curving at all). Figure 4 illustrates an example of a concave and a convex function.

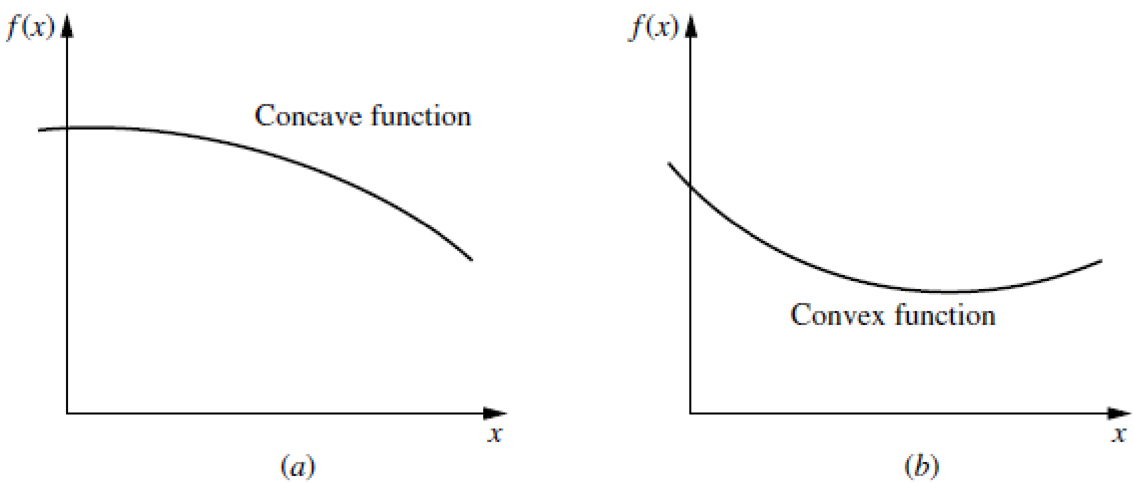


Figure 4: Example illustrating a concave function and a convex function **[19]**

- Integer Programming: a class of mathematical optimization techniques that restricts the decision variables to be integers. Such a techniques can be helpful in many applications such as capital budgeting (when selecting best potential investment), scheduling, and modeling distribution systems.
- Dynamic Programming: is a mathematical method that solves complex optimization problems by breaking them into simpler treads. It is first invented by Richard Bellman in the early 1950s. This technique identifies a collection of subproblems

and solves them one by one by using the answers of the small subproblems to find the larger ones. “The process starts with a small portion of the original problem and finds the optimal solution for this smaller problem. It then enlarges the problem finding the current optimal solution from the preceding one until the original problem is solved entirely” [23].

Several researches adopted the mathematical techniques to solve optimization problems. Islam et al. [24] proposed a linear algorithm (LP) to solve the optimization problem, while Klansek & Psunder [25] presented the cost optimal scheduling using an NLP algorithm. Soliman & Ezeldin [23] proposed a composite technique that combines genetic algorithms with dynamic programming to solve uncertain optimization problems of nonserial repetitive projects, such as multiunit housing projects and retail network development projects. Chassiakos & Sakellaropoulos, [26] suggested the incorporation of project characteristics into the optimization process to produce more realistic results. Such characteristics include generalized precedence relationships between activities, external time constraints, activity planning constraints, and bounces/penalties. To solve the generated optimization problem, linear programming or linear/integer programming can be used to find the exact optimum solution. Haksever & Moussourakis [27] used the mixed integer programming technique to solve optimization problems while accounting for any type of cost function (linear, piecewise, or discrete). Haksever & Moussourakis [28] proposed three mixed-integer linear programming models to solve project compressing problems while assuming nonlinear activity time-cost functions. Perera [29] made a use of linear programming technique to solve projects time-cost tradeoff problems with overlapping precedence networks. His model was valid for any large network, part of a network and to multi-fragnet networks. Liu et al. [30] suggested the use of linear and integer (LP/ IP) programming models to solve optimization problems. Massourakis & Haksever [31] proposed a zero-one mixed integer model to solve the optimization problem while finding the correct early and late start times of the schedule activities, along with a “what if” analysis to account for different perspectives of the project cases. Deckro et al. [32] presented a series of nonlinear time-cost tradeoff models to solve the optimization problem in construction projects by representing the time-cost relationships via nonlinear quadratic functions depending on the project case. Finally, Coskunoglu [33] used a probabilistic version of LP called chance constrained linear programming (CCLP) to find the optimal compression of project schedule considering the probabilistic nature of activities durations.

2.2.3 Meta-Heuristic Techniques

Meta-heuristic techniques are classified into three types: Evolutionary Algorithms (EA), Genetic algorithm (GA), and Genetic Programming (GP) techniques. A genetic algorithm in optimization is a global heuristic examination technique that evolved from a particular class of evolutionary algorithms and used to compute near exact or approximate optimum solutions. EA algorithms are stochastic search methods that imitate the biological evolution and/or the social behavior of species kinds [34]. This method proved to have some advantages over the previous mathematical techniques as it can offer an intermediate solution at any iteration and it can search the whole space of the solution in a shorter time. From the EA algorithms, three distinguished types are used in the construction management area: Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), and modified Shuffled Frog Leaping (SFL). Genetic algorithms (GA) are heuristic techniques that are used to solve single and multi-objective optimization problems. The formation of a GA problem requires generating a possible set of solutions or a "population" for the problem, then the reasonable solutions are combined to generate new solutions to the next iteration (generation). Later, in each generation, and to form a new population, the fitness for individual and multiple solutions in the population are evaluated and modified. The process continues at each generation and new populations are formed. Eventually, the new modified solutions replace the poorer solutions of the original initial population or solution and the process is repeated until a near optimum solution is reached [35]. The accuracy of GA technique depends highly on the number of generations developed and the fitness level [35]. GP algorithms are the automated extension of GA. The GP algorithms are helpful in many complicated applications such as automatic design, pattern recognition, robotic control and time scheduling. Due to uncertainty and vagueness in some applications such as optimization, control, decision making and approximate reasoning, fuzzy logic was developed. This type of logic uses sets of normal and convex numbers and solves the problem using special methods and shapes like triangular and trapezoidal shapes. The GA, EA, GP, and fuzzy logics have been increasingly used in the field of construction management to solve optimization and decision making problems. Many researchers have tackled such techniques to solve schedule compressing problems. Castro-Lacouture et al. [36] used fuzzy logic to produce construction schedules with restrictions on time, cost and material and incorporated the time-cost tradeoff into the schedule assuming linear fuzzy relations. Fayek & Oliveros [37] integrated the daily site reporting of activity progress and delays with a schedule updating and forecasting system for construction project controlling using fuzzy logic model. Ng & Zhang [38] validate the use of ant colony optimization technique to solve optimization or schedule compressing papers. Li & Love [39] suggested some improvements over the basic GA for the purpose of increasing the efficiency and reducing the computational cost involved when finding the optimum solution for the time-cost tradeoff. Que [40] proposed a model that uses the GA optimization technique that takes care of all scheduling parameters to ensure realistic results. Zahraie & Tavakolan [41] used two concepts of time-cost trade-off and resource leveling and allocation in a stochastic multiobjective optimization model which minimizes the total project time, cost, and resource moments. Leu & Yang [42] used GA technique to propose a multi-criteria computational optimal scheduling model that integrates the time-cost tradeoff, resource limited and resource leveling models.

2.3 Free Float and Total Float

The float is a measure of the flexibility and criticality of the project schedule. It measures flexibility in a matter of the ability of a certain activity to have its performance time extended. On the other hand, criticality of an activity is established through the following statement: the more float an activity has, the less critical the activity is and vice versa. Alternatively, critical activities can be defined as activities with zero or no total float.

According to Charoenngam and Popescu [3], free float or slack is defined as "the amount of time by which the finish time of an activity may exceed its earliest finish time without increasing the earliest start time of any other activity immediately following". The total float indicates by how many time units is the activity path away from the late finish of the project. Total float should be distinguished from free float as total float belongs to the activities path (it is shared by all the activities), while the free float belongs to a particular activity. Farr & Griffis [43] labels the total float through Equation 1

$$TF(i, j) = LS(i, j) - ES(i, j) = LF(i, j) - EF(i, j) \quad (1)$$

Where;

TF: The total time that an activity may be postponed without delaying project completion

LS: The latest time at which an activity may start

ES: The earliest time at which an activity may start

LF: The latest time at which an activity may finish

EF: The earliest time at which an activity may finish

While the free float can be calculated using Equation 2

$$FF(i, j) = Min\ [ES(j, k)\ \text{for all}\ k] - EF(i, j) \tag{2}$$

Where;

FF: The maximum time that an activity may be postponed without delaying the earliest start or earliest finish of any following activity

Free float can occur in certain cases when merge activities arise. "Free float can occur only when more than one arrow goes into a node; one or more of the multiple arrows entering the node can possess free float" [44]. Since free float can't be shared within the activities, its importance compared to the total float is minimal. Free float can be useful when selecting activities for resource leveling.

As float is a key element in project scheduling, several studies were raised over this topic over the years. Ziegler [45] introduced the minimal and maximal float concept and proposed computation approaches; MAXF and MINF, to calculate the independent (minimal) and total (maximal) floats. The concept of minimal float represents the float present in "worst case" while maximal float represents the float available in the "best case" [45]. Raz and Marshall [46] proposed a new definition and calculation method for total and free float that is related to the availability of resources in the project. The method improves the traditional ways of calculating the total and free float but using two new introduced concepts: the early scheduled dates and the late scheduled dates. The early scheduled start date represents the earliest date an activity can begin at provided all prerequisites and resources are available, while the late scheduled finish represents "the latest the activity can finish without delaying the project completion and without exceeding resource availability constraints" [46]. Gong D. [47] realized the effect of uncertainties in non-critical activities over project cost and schedule; therefore, he developed a method to find the optimum float use in a project network. Optimum float was defined as the "point at which the sum of the cost resulting from float use and the cost resulting from project delay due to float use is the lowest" [47].

The optimization can be achieved by integrating the time disturbance analysis and the time dependent cost (TDC) that is defined as the varying part of project cost that is dependent on activities' duration. Lucko and Orozco [48] originated a mathematical method to compute float in repetitive projects with linear schedules using singularity functions and equations that define the activities and their buffers "over a continuous range" [48] . In their paper, they were able to calculate and define the total float, safety float, free float, independent float and interfering float in LSM. Lim et al. [49] refined the float concept in resource-constrained projects in an attempt to overcome the limitations of the previous methods used in such projects. They were able to develop an algorithm to compute float and identify critical activities using the introduced concepts of group float, float, critical activity, critical set, and float graphs. Moreover, they introduced the notion "negative float" along with the negative critical activity, negative group float, negative float set and negative critical set in order to "investigate the effect on the minimum completion time resulting from reduced activity duration" [49].

2.4 Delays in Construction Projects

The need for time-cost optimization models arises from the possible delays that can happen and affect the project schedule; as well as, the project overall cost. Construction delays are any event occurring throughout the project planning or execution phases that may extend the project duration and require additional time, cost and work (revision of plans, addition of works, more time for decision making and material re-sourcing) than what initially is agreed on in the contract. According to Assaf and Al-Hejji [50], construction delays are defined as "the time overrun either beyond completion date specified in a contract, or beyond the date that the parties agreed upon for delivery of a project." Causes of delays vary depending on the delay kind. Abd El-Razek et al. [51] did a comprehensive study about the construction delays in Egypt and were able to classify the overall delays into six groups: financing, manpower, changes (design errors and change orders, contractual relationships, environment, equipments, rules and regulations, materials and scheduling and control. Faridi & El-Sayegh [52] analyzed the top 10 most major causes of delays in the United Arab Emirates. They found that around 50% of the projects in the UAE construction projects encounter delays in the completion time, and that preparation and approval of drawings ranks as the first delay cause. Some other distinguished delay causes in the UAE were the insufficient early planning of the project, slow decision making process by the owner, inadequate number of skilled manpower, poor site management, financing problems and slow process of obtaining permits or approvals from municipality or other government authorities. The effects of such delays are not confined to the delay of the completion date and cost overruns only; their effect may extend to have an adverse effect on quality of work, adverse effect on performance and moral of labors, beside the resulted claims and disputes. Moreover, delays can adversely influence the overall economy of the countries.

2.5 Float Consumption Impact

Sakka & El- Sayegh [6] developed a method to control the risks associated with the float loss due to delays in construction projects and its effect on noncritical activities. Their study is based on the fact that conventional CPM method cannot measure the impact of within float delays of noncritical activities on the total cost and time of the project. In fact, float is considered as a by-product of the CPM computation, and CPM theoretically assumes that duration and cost of activities in construction projects are deterministic; and so is the project total duration and cost. However, in real life, project's total cost and duration are not fixed due to the uncertainty resulting from several risks associated with each project. "The overall compression characteristics of the network model [CPM] are too vague to provide other than an approximate forecast of the time-cost behavior" [33]. Sakka & El- Sayegh method predicts the safe float loss level for any activity in a given project schedule. The method uses the Multiple Simulation Analysis Technique (MSAT) to combine the results of cost estimates and stochastic scheduling using Monte Carlo simulation, and then converts the stochastic results using a least-squares linear/ nonlinear regression into a polynomial function that specifies the float impact by relating directly the float consumption value to the project duration and cost at a certain confidence rank. Their analysis method suggests six major stages [6]:

Stage One: Preliminary Analysis: In this stage the CPM computations are preformed to find the critical path and the project's total duration and cost

Stage Two: Stochastic Analysis of Baseline Schedule: The use of Monte Carlo simulation using at-Risk is applied over the baseline schedule to find the mean project duration and standard deviation in this stage. The simulations produce a set of different durations with their corresponding total costs, indicating that for a project's duration there isn't a mutually exclusive cost values. The critical index (CI) for each activity is then calculated from the results obtained from the simulation analysis; where the critical index is a percentage measure of the probability of an activity to be on the critical

Stage Three: Development of Scenarios: Based on the CI values developed in the previous stage for all the activities, delay scenarios for the activities with the highest CI values can be produced using the simulation.

Stage Four: Stochastic Analysis of Scenarios: For each activity, delay scenarios are investigated to show the affected project duration at different values of float loss. All the scenarios are then analyzed in order to monitor the change in each activity's CI and the critical path

Stage Five: Project Duration Impact Model: At this stage the float loss value and the corresponding mean project duration found in Stage 4 for each activity are plotted, and the best fit equation (regression relationship) is found.

Stage Six: Cost Impact Model: The aim of this stage is to select a cost that is directly related to the delay in a given activity at a sufficient confidence level such that there is a small possibility of exceeding that value. To do so, the MSAT [53] is used at this stage to calculate the impact of within-float loss in noncritical activities. MSAT, in general, relates the results of cost range estimates and stochastic scheduling such that high confidence level values can be selected for a project cost and schedule. At this stage, for each cost data-set generated in Stage two, percentile level is determined, then the costs and their associated percentile level at all project durations are plotted and fitted into a polynomial. Then a confidence level is chosen and its corresponding cost is calculated for all values of float loss in all the activities. Then, the costs found at the specified confidence level are fitted with the float loss values in a linear/ nonlinear model to produce another polynomial that can quantify the total cost impact at a certain confidence level.

2.6 Float Allocation and Float Ownership

According to Wickwire et al. [54], float is recognized as an expiring resource that doesn't belong to any party but at the same time it is available to be used by the project parties on a fair basis. Nowadays, several approaches are used in the construction industry to allocate float. A summary of the most known approaches is presented below:

- Owner ownership of float: this approach is based on the claim that the owner can own the float as he is responsible for the costs and risks associated with the project in cases when the owner accepts to bear the cost of project risk [55], [56].
- Contractor ownership of float: this approach states that the contractor has the right to own the float since he has to have a control over the project labors, equipment and cash flow to deliver the project on time and avoid cost overruns especially in lump-sum contracts [56].
- Project float approach: in this approach the float is kept free for use to whoever requests it first from the project parties. This approach provides ability to the owner to issue change orders and ability to the contractor to re-arrange the resources when needed, but one of its main disadvantages is the disagreements between the project parties when non-compensable delays occur [56].
- Bar approach: the bar approach established by Ponce de Leon [57] represents the total float of each activity as a bar in the bar chart to observe the critical and noncritical path delays. This method; unlike the project float approach, restricts the disentitled float
- Allocating Float to individual activities along a path of activities: this is an approach used to distribute the float to individual activities along a path using quantitative and qualitative selection criteria. The shortcoming of this method is that it doesn’t solve clearly the float ownership problem between the project parties [56].
- Day by day approach: inspired by the total float management method; the day by day approach is a systematic method to record and control the float consumption due to the owner and the contractor delays in the project and place a cost for the lost total float [56].
- Using safe float approach: this approach is developed by Gong and Rowings [58] to illustrate the concept of safe float to project scheduling. This method specifies the range of safe float to be used by the project parties in general so as to logically minimize the risks associated with delays in noncritical activities, based on a time-disturbance analysis over the project schedule.
- Contract risk approach: developed by Householder and Rutland [59], the contract risk approach specifies that the float is owned by the party that assumes full responsibility for the project risk. In some cases float can be shared between the owner and the contractor based on an agreed percent of share for project risk.

- Total float traded as commodity approach: this method assumes that the total float of each activity is a product or a "commodity" that is traded between the owner and the contractor based on the relationship presented in Equation 3 [4]:

$$\text{Float cost per day} = \frac{(\mathrm{LFC}) - (\mathrm{EFC})}{(\mathrm{TF})} \tag{3}$$

Where;

EFC: early finish cost of the activity in question

LFC: Late finish cost of the activity in question

TF: Activity Total Float

The commodity method is in favor of the contractor as it provides for him a good regulation over float besides offering a clear view about the time money value when negotiations about delays and change orders are issued. As the owner's opportunities are saved via liquidated damages, the commodity method can save contractor's right for compensation for the potential impacts associated with increased overhead costs and acceleration costs, lost learning rates, lost moral, resource mobilization problems and lost opportunities. According to De La Garza et al. [4]: "the model perceives total float as a time contingency for both owners and contractors, and as an incentive for contractors to finish early". This method is selected in this Thesis to quantify, in number, the risk cost per day when float is consumed in noncritical activities as "flexible time taken away from the schedule needs to be replaced with monetary contingencies" [4]. Using this method, the user will be able to identify which activities have more float cost than others to compare between opportunities and decide which one to crash when needed. Moreover, the paper presents a sample of contract language to be used in accordance with the proposed commodity approach to insure the value of total float and allow the trade-in process on demand so as to mitigate any corresponding uncertainty while using float.

- Preallocation of total float: this method is developed by Garza et. al [60] to overcome the shortcomings of the previously discussed approaches. In this method the project total float is distributed between the project parties based on a pre-agreed ratio (also called allowable total float) to be stated in the contract clauses. The simplest ratio is the 50/50 ratio in which the owner and the contractor each have 50% of the total float for use and if one party didn't use his share the other party can have the chance of using that share, while if one party consumes more than his allowable total float and causes delays that impacts the critical path then this party is responsible for that delay and any damages or incurred costs.

- Total risk approach: Al-Gahtani [56] developed the total risk approach to introduce the total risk point of view to the float ownership. This method in general assigns the float to parties based on the amount of risk they encounter in the project. Moreover, it uses the commodity concept and the day by day approach to allocate the float. "The approach is based on the basic concept that the party who has the greatest risk in a project should be entitled to float ownership and deserves compensation from other project parties who increase the risk associated with project by consuming the float" [56]

2.7 Chapter Conclusion

The concept of time-cost optimization is one of the main topics that were addressed by the researchers over the years. In general, two schedule mechanisms are identified in the construction industry: schedule reduction and schedule compression. Schedule compression is the main concept to be identified in this Thesis, in which the project duration will be shortened with an associated incurred extra cost.

In order to find the optimum or near-optimum solutions for schedule compression, techniques such as manual, mathematical and meta-hierastic techniques can be used.

The manual time-cost tradeoff involves finding the optimum duration and cost based on the CPM method by a standard systematic computation method that uses activity crashing slopes. Linear programming is a mathematical optimization approach that uses linear functions subject to linear constraints, while on the other hand; the non-linear optimization maximize or minimize an objective function subject to nonlinear constraints (or sometimes the objective function can be nonlinear).The Meta-heuristic techniques are global heuristic techniques that can analyze and solve stochastically the optimum or near optimum solutions.

Several meanings are interpreted to define the project float. Some defined float as an expiring resource that is used to measure the criticality and flexibility of project schedule. Generally, float is a by-product of CPM that represents the amount of time available for noncritical activities to be delayed without extending the project duration.

Since time is money, and since time is of the essence in any project contract, there is a need to address the float consumption and the float allocation in construction projects. Several approaches for float allocation and management are discussed previously in the chapter. The approaches are the owner ownership approach, the contractor ownership approach, the bar approach, the day by day approach, the float commodity approach, the total risk approach, the pre-allocation of total float approach, the contract risk approach, the use of safe float approach, the project float approach, and allocating float to individual activities along a path of activities approach. The float commodity approach is selected in this Thesis to find the daily trade-off cost for the lost float in noncritical activities in order to use it in the proposed optimization framework later.

To conclude, the review of related literature has pointed out the originality of the idea of incorporating the effect of float loss as a unique idea that wasn't approached before, and emphasized the need to include the effect of float loss in the time-cost tradeoff analysis to account for risks associated with the float loss in noncritical activities and improve the reliability and effectiveness of the time-cost optimization process in construction projects.

CHAPTER THREE: MANUAL-PROBABILISTIC OPTIMIZATION FRAMEWORK CONSIDERING FLOAT CONSUMPTION IMPACT

3.1 Chapter Overview

This chapter addresses the manual stochastic method of the proposed framework. The first part of the chapter states the assumptions and the proposed method, while the second part analyzes in detail a small project example; showing the detailed solution of this example via the deterministic approach and the new proposed framework. The framework is inspired by the fact that reducing the project duration reduces the float of non-critical activities; as well as, the probability of completing the project on time. It uses Monte Carlo simulation to analyze stochastically, at each crashing cycle, the float loss impact over the project total cost.

3.2 Proposed Framework

3.2.1 Assumptions

- Each activity is to be crashed by one day per cycle
- Activity's duration and cost are defined as a normal probability distribution with a mean and a standard deviation
- Duration of the activity equals its mean duration
- While crashing an activity per day, the activity duration will be decreased by one day; as well as the mean. The standard deviation is assumed to be equal in both cases
- 10000 iterations are used for each run by @risk

3.2.2 Manual-Stochastic Framework Steps

- Step One: Normal Schedule Analysis
  - Perform stochastic analysis over the Baseline Schedule or the network
  - Find the mean "M_{ij}" and the standard deviation "Std_{ij}" of the project duration, where the subscript i represents the number of crashing cycles, and the subscript j represents the activity being crashed.

- Determine the probability of completing the project on time "POF"; given the mean and the standard deviation found in the previous step.

- Step Two: Schedule Compression Analysis
  - If one critical path is available, the succeeding steps can be followed:
    a) Identify the critical activities. All critical activities are to be considered for the analysis.
    b) Perform stochastic analysis over the new crashed network
    c) Find the new project duration mean "M_{ij}" and standard deviation "Std_{ij}"
    d) Find the new probability of finishing the project on time associated with the new mean and standard deviation of the new crashed duration
    e) Using Equation 4, Calculate the difference between the new obtained probabilistic duration (found using the probability of finishing at the previous step and the M_{ij} and Std_{ij} found after the new simulation run at step two) and the new deterministic crashed duration at this step to find the float loss impact "FLD" in terms of days:

$$FLD_{ij} = D_{prob,ij} - D_{det,\,ij} \tag{4}$$

Where;

$D_{prob,ij}$ = Probabilistic duration for crashed activity j at cycle i

$D_{det,\,ij}$ = deterministic duration for crashed activity j at cycle i

FLD_{ij} = Float loss impact in terms of days for crashed activity j at cycle i

    f) Float loss Cost "FLC" = (the difference between the two durations in days) x (the savings per day). Equation 5 represents the float loss cost as a product of the duration difference and the savings per day.

$$FLC_{ij} = FLD_{ij} \times C_{SPD} \tag{5}$$

Where;

C_{SPD}= Savings per day (indirect cost, incentives, …etc)

FLC_{ij} = Float loss cost for crashed activity j at cycle i

    g) Add the float cost to the extra direct cost to find the new total extra cost
    h) Steps b through g will be repeated for all critical activities identified. The activity with the least total extra cost will be crashed at this point

i) Float impact in days (at the following crashing cycle) = duration associated with probability of finishing the project on time of the previous cycle – new deterministic duration of the new crashed schedule at the cycle in point

j) Steps a – i will be repeated progressively until reaching the optimum solution

- If two or more critical paths are available:

  a) Check the available activities to be crashed (either a common activity or two or more activities that correspond to the lowest crashing slope)

  b) Perform the same steps that were performed when one critical path has occurred while considering all possible cases occurring, then compare the accepted cases to select the activity/activities with the lowest crashing impact

Figure 5 illustrates the framework steps via a flowchart.

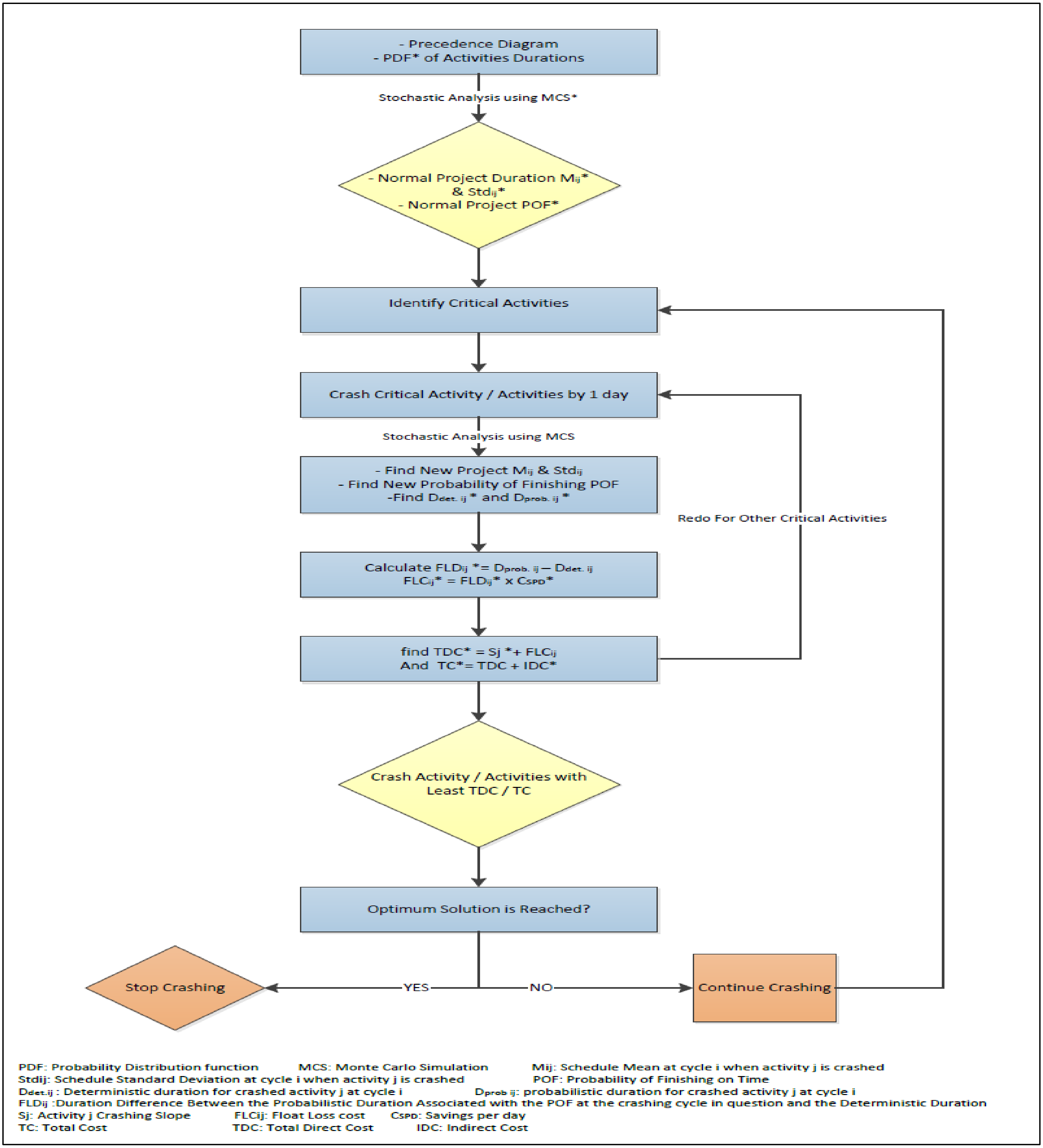


Figure 5: Framework flowchart

3.3 Application Example

Example one (adopted from Hinze [10]) explains first the manual-deterministic steps carried out to compute the optimum project duration and total cost and the minimum project duration and its associated total cost. Four cycles are needed to reach the optimum project duration, while three extra cycles are needed to reach the least project duration. The optimum project duration is 23 days with an associated cost of $12,490, while the minimum project duration and its associated cost are 20 days, $12,600, respectively.

Table 1 presents the project data in terms of durations and costs. The indirect cost is assumed to be $ 280 per day. The last two columns; duration mean and duration standard deviation, are added to Hinze [10] example in order to use them in the stochastic analysis.

Table 1: Project normal & crashed costs and durations

Activity	Normal Duration	Normal Cost	Crashed Duration	Crashing Cost	Potential Days Saved	Cost per Day	Duration Mean	Duration Standard Deviation
A	1	800	1	800	0	-	1	1.2
B	7	1,000	4	1,600	3	200	7	2
C	6	300	4	500	2	100	6	1.5
D	3	400	2	800	1	400	3	1.35
E	3	100	1	200	2	50	3	1.88
F	7	500	5	800	2	150	7	2.12
G	8	200	4	1,400	4	300	8	3
H	7	350	6	600	1	250	7	1.25
I	5	700	3	850	2	75	5	2.5
J	3	500	2	1,000	1	500	3	1.5
K	5	450	4	800	1	350	5	1.6
		Total= 5300						

3.3.1 Normal Project Compression (Deterministic Approach)

This section presents the solution cycles using normal deterministic project compression without considering the effect of float loss:

Cycle Zero: Normal Schedule: Based on the baseline schedule network represented in Figure 6, the project duration is found to be 27 days, with an associated total project cost equal to $12,860 that consists of a direct cost of $5,300 and indirect cost of $7,650. The critical path is A, B, F, H, K.

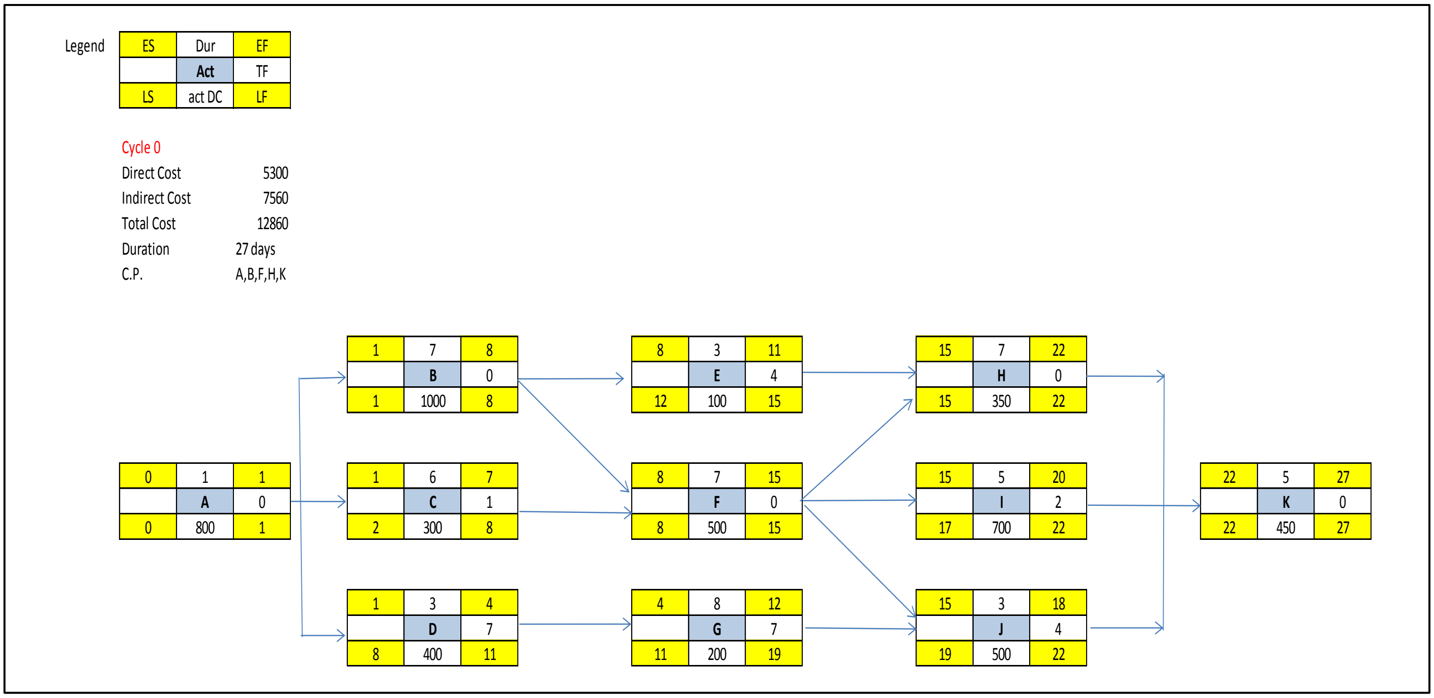


Figure 6: Cycle zero: normal schedule

Table 2 presents the total float available for noncritical activities at cycle zero.

Table 2: Activities total float at cycle zero

Activity	Activity Total Float (Days)
C	1
D	7
E	4
G	7
I	2
J	4

Cycle One: Crashed Schedule 1: The least expensive activity to expedite at this cycle is F; therefore, the decision is to expedite F by 2 days. The new Project Duration is 25 days and the new C.P. is A, B, F, H, K. The new updated compression calculations are as follows:

The new Direct Cost = Direct Cost + Crashing Cost = 5,300 + (2*150) = $ 5,600

The new Indirect Cost = Duration * indirect cost per day= 25 * 280 = $ 7,000

The new Total Project Cost = Direct Cost + Indirect Cost = 5,600 + 7,000 = $ 12,600

The updated schedule at step two is represented in Figure 7.

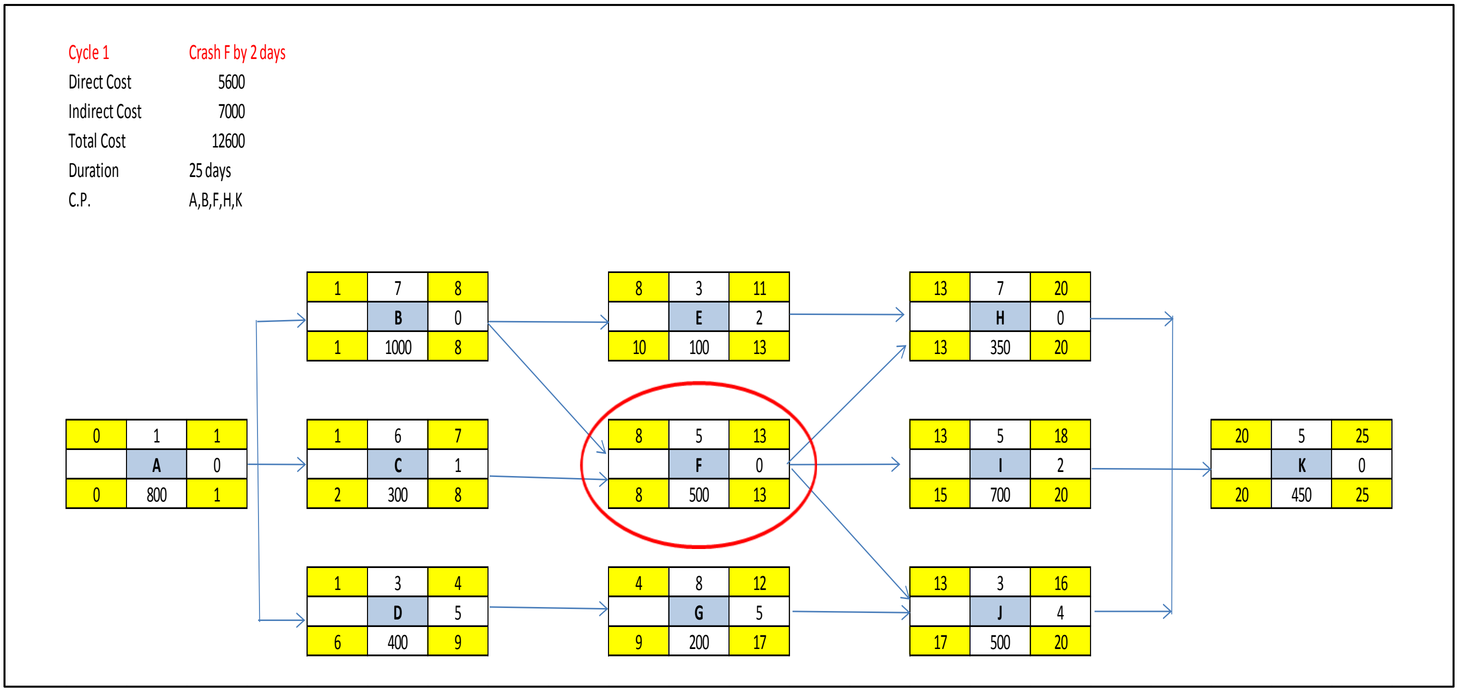


Figure 7: Cycle one: crashed schedule 1

Table 3 presents the remaining total float days for the noncritical activities after the first crashing cycle.

Table 3: Activities total float at cycle one

Activity	Activity Total Float (Days)
C	1
D	5
E	2
G	5
I	2
J	4

Cycle Two: Crashed Schedule 2: Least Expensive Activity to Expedite at this cycle is B, so the decision is to expedite activity B by 1 day. The new project duration is 24 days and the new C.P. now is A, B, F, H, K and A, C, F, H, K. The new updated calculations are as follows:

The new Direct Cost = 5,600 + (1*200) = $ 5,800

The new Indirect Cost = 24 * 280 = $ 6,720

The new Total Project Cost = 5,800 + 6,720 = $ 12,520

The updated schedule at step three is represented in Figure 8.

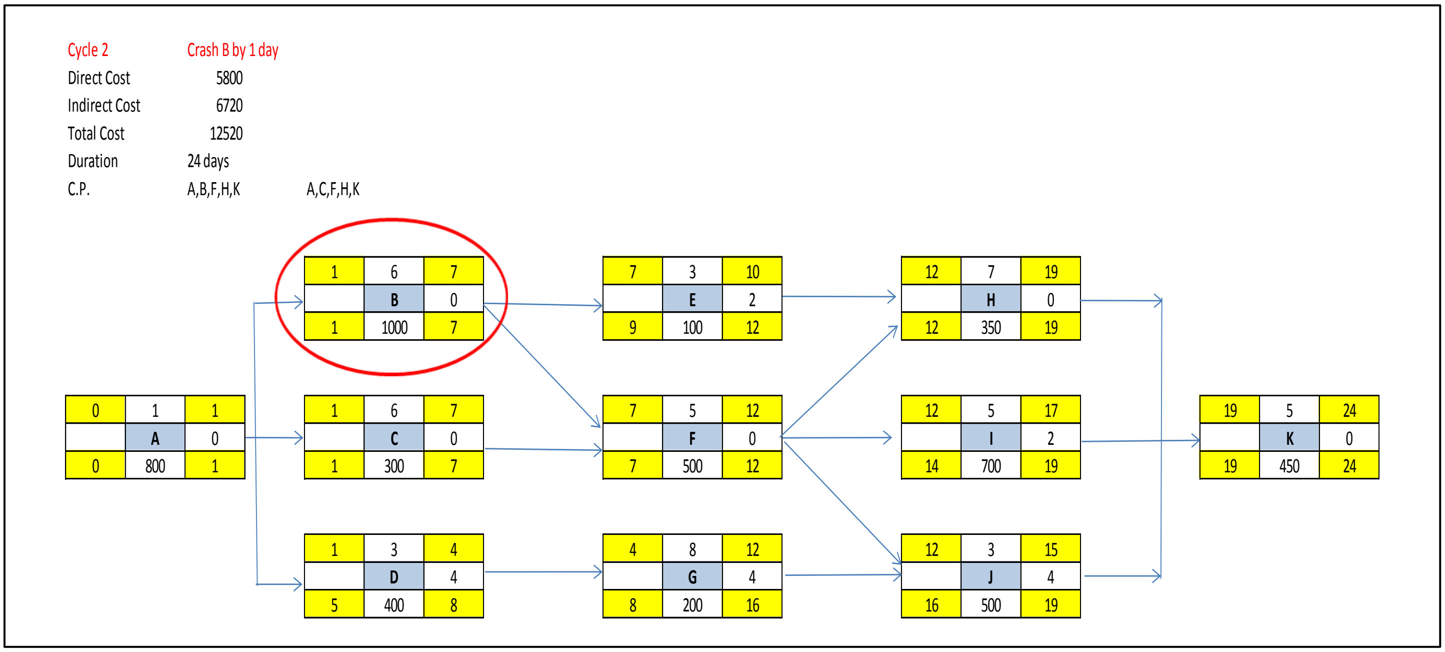


Figure 8: Cycle two: crashed schedule 2

Table 4 presents the remaining total float days for the noncritical activities after the second crashing cycle.

Table 4: Activities total float at cycle two

Activity	Activity Total Float (Days)
C	0
D	4
E	2
G	4
I	2
J	4

Cycle Three: Crashed Schedule 3: The activities available for crashing at cycle three: either B&C or H. Activity H exhibits the least expensive crashing slope. Therefore, the decision is to crash activity H by 1 day. The new project duration now is 23 days and the new C.P. = A,B,F,H,K and A,C,F,H,K. The new updated calculations are as follows:

The new Direct Cost = 5,800 + (1*250) = $ 6,050

The new Indirect Cost = 23 * 280 = $ 6,440

Therefore, the new Total Project Cost = 6,050 + 6,440 = $ 12,490

The updated schedule at step three is represented in Figure 9.

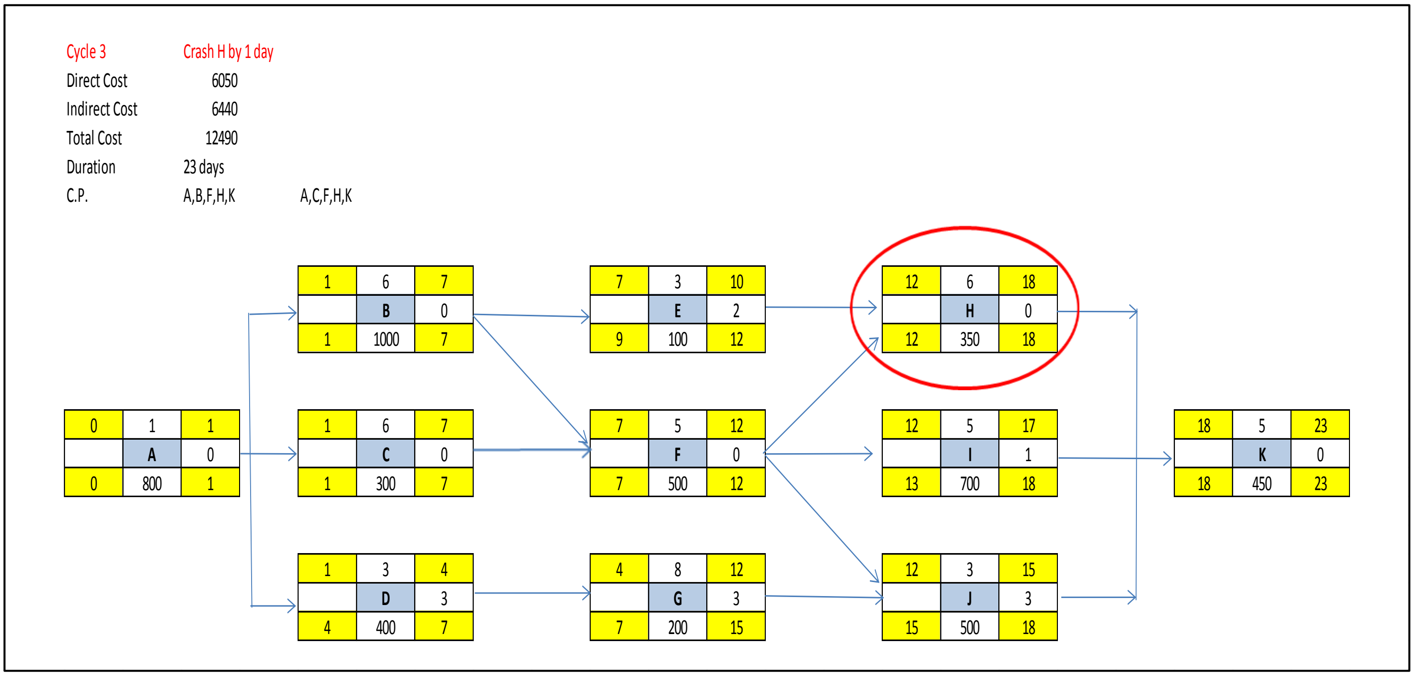


Figure 9: Cycle three: crashed schedule 3

Table 5 presents the remaining total float days for the noncritical activities after the third crashing cycle.

Table 5: Activities total float at cycle three

Activity	Activity Total Float (Days)
C	0
D	3
E	2
G	3
I	1
J	3

Cycle Four: Crashed Schedule 4: Available activities to be crashed at this cycle are either B&C or K. Crashing activities B&C will give the lowest crashing cost. Activities B&C can be crashed effectively 2 days. The new project duration after crashing is 21 days, and the new C.P.'s are A, B, F, H, K and A, C, F, H, K. The new updated calculations are as follows:

Direct Cost = 6,050 + (2*(200+100)) = $ 6,650

Indirect Cost = 21 * 280 = $ 5,880

Total Project Cost = 5,880 + 6,650 = $ 12,530. Since the total cost started to increase, the optimum project duration is 23 days. The updated schedule at step three is represented in Figure 10.

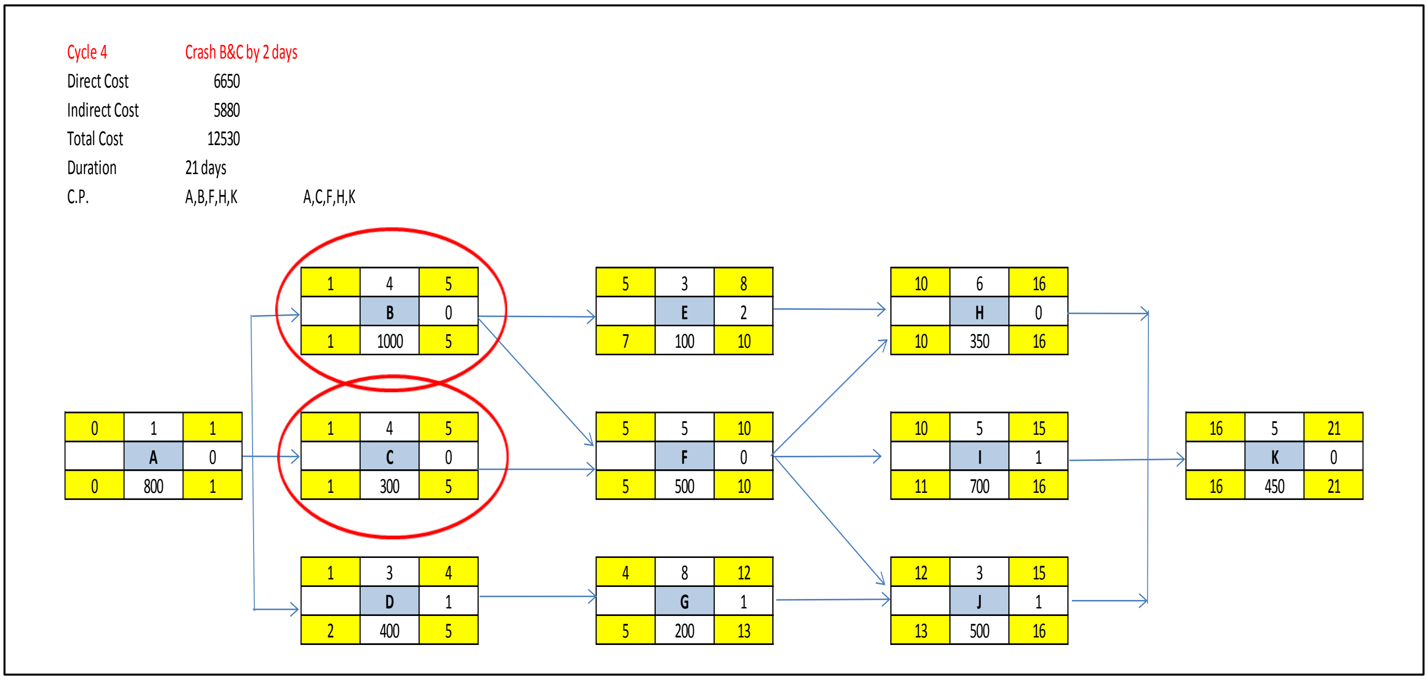


Figure 10: Cycle four: crashed schedule 4

Table 6 presents the remaining total float days for the noncritical activities after the fourth crashing cycle.

Table 6: Activities total float at cycle four

Activity	Activity Total Float (Days)
C	0
D	1
E	2
G	1
I	1
J	1

Cycle Five: Crashed Schedule 5: The last activity available for crashing is activity K; therefore, activity K is expedited by 1 day. The new Project Duration now is 20 days and the New C.P. is A, B, F, H, K and A, C, F, H, K.

The new Direct Cost at this step= 6,650 + (1*350) = $ 7,000

The new Indirect Cost at this step= 20 * 280 = $ 5,600

Total Project Cost at this step= 7,000 + 5,600 = $ 12,600. Since no further activities are available for crashing, the minimum project duration is reached and it is 20 days.

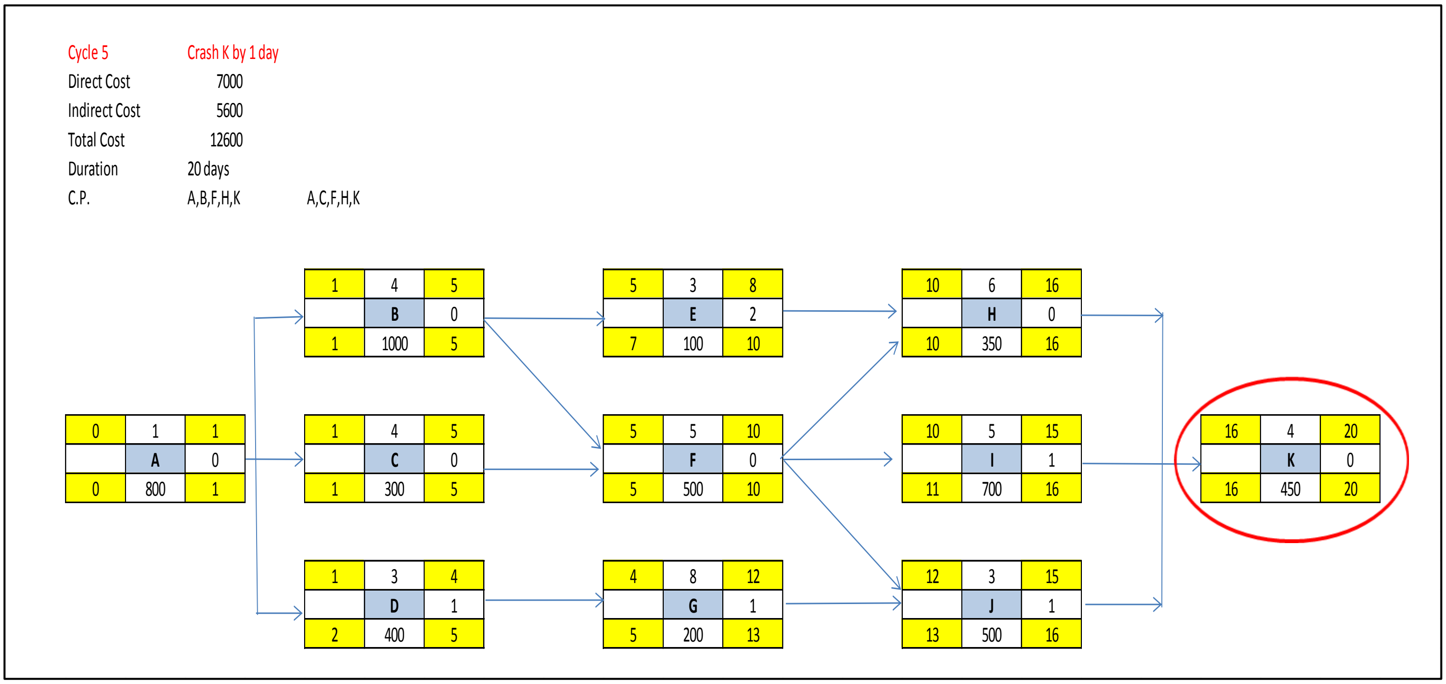


Figure 11: Cycle five: crashed schedule 5

Table 7 presents the remaining total float days for the noncritical activities after the fifth crashing cycle.

Table 7: Activities total float at cycle five

Activity	Activity Total Float (Days)
C	0
D	1
E	2
G	1
I	1
J	1

Table 8 illustrates the crashing results performed over the cycles zero to five:

Table 8: Project crashing results

Cycle	Project Duration	Direct Cost	Indirect Cost	Total Cost
0	27	5,300	7,560	12,860
1	25	5,600	7,000	12,600
2	24	5,800	6,720	12,520
3	23	6,050	6,440	12,490
4	21	6,650	5,880	12,530
5	20	7,000	5,600	12,600

Figure 12 shows the time-cost tradeoff. During crashing process the direct cost starts to increase while the indirect cost decreases as it is a function of time.

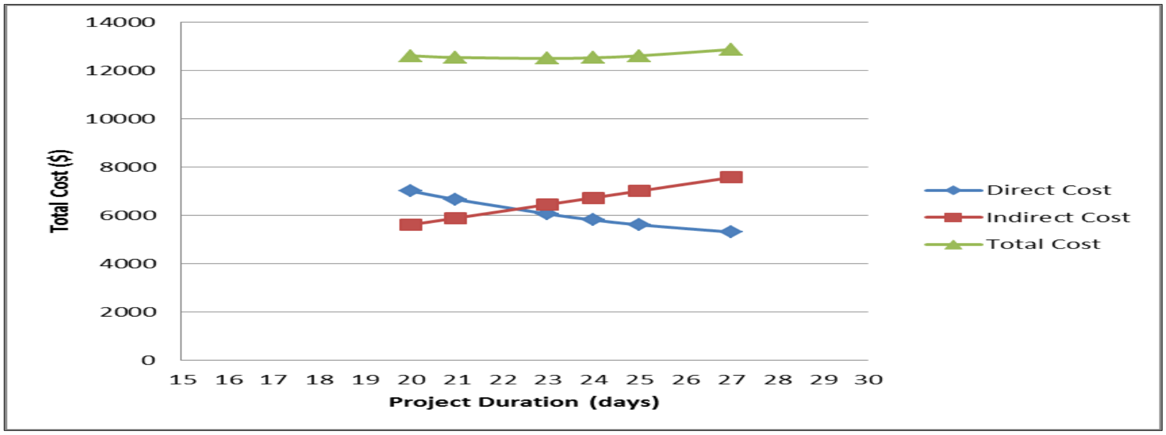


Figure 12: Project time-cost tradeoff

Figure 13 illustrates the total project cost vs. duration curve. It can be noticed that the optimum project duration and total cost are 23 days, $12,490 respectively, and afterward the total cost starts to increase until reaching the cost associated with minimum project duration of 20 days.

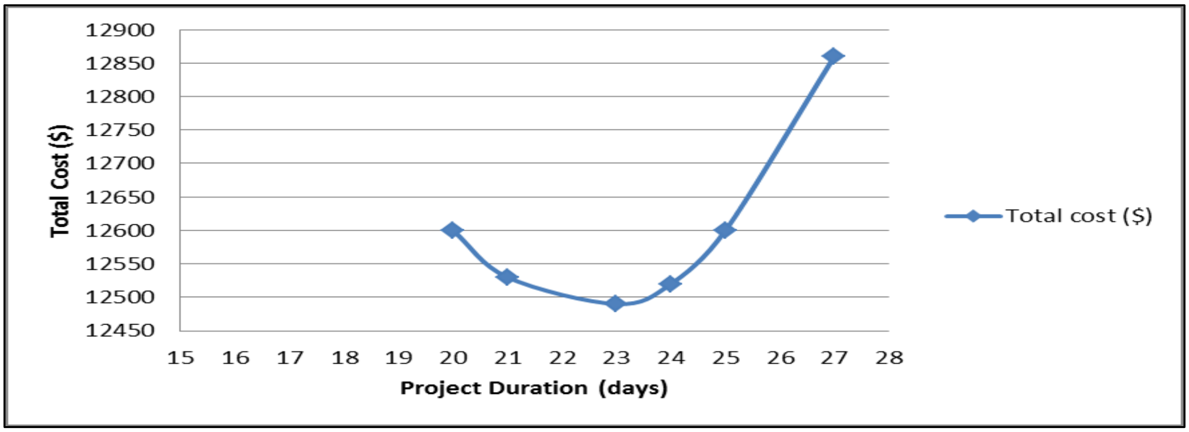


Figure 13: Project total cost vs. duration curve

3.3.2 Project Compression Considering Float Consumption Impact

This section presents the solution cycles using the new proposed probabilistic compression framework considering that considers the impact of float loss within noncritical activities during the crashing process:

Cycle Zero: Normal Schedule Analysis: The normal schedule analysis is performed as in section 3.3.1 and the schedule is shown in Figure 6. The project duration of the baseline schedule is 27 days, with a direct cost of $5,300, indirect cost of $7,560, and total cost $12,860. The critical path is A, B, F, H, and K.

Table 9 illustrates the simulation run for the baseline schedule and the resulting mean and standard deviation.

Table 9: Simulation results for the baseline schedule at cycle zero

Mean "M_0"	28.1082 days
Standard Deviation "Std_0"	3.4705 days
Probability of Finishing the Project within 27 days Given the Mean and Std. Found	37.4742 %

Cycle One: Crashed Schedule 1: Activities available to be expedited are: B, F, H, K (activity A can't be expedited). Therefore, four crashing scenarios need to be checked to find the best activity to crash.

The first scenario is for crashing activity B by 1 day. After crashing activity B by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 10 illustrates the simulation run results after crashing activity B by 1 day.

Table 10: Simulation results after crashing activity B at cycle one

Mean "M_{1B}"	27.5028 days
Standard Deviation "Std_{1B}"	3.401 days
POF within 26 Deterministic Days Given the Mean and Std. Found	32.9291627 %

New deterministic duration $D_{det,\ 1B}$ = 26 days

Duration at 37.74742% given the mean and standard deviation in Table 10 ($D_{prob,1B}$) = 26.41679274 days

According to Equation 4, the difference between the deterministic and the new duration: $FLD_{1B} = D_{prob,1B} - D_{det,\ 1B}$ =26.41679274 – 26 = 0.41679274 days

Float cost FLC_{1B} is calculated according to Equation 5:

$FLC_{1B} = FLD_{1B}$ x C_{SPD} = 0.41679274 (days) x 280 $ / day = $116.70

Extra direct cost (slope) = $ 200

Total extra cost = 200 + 116.70 = $316.70

Direct Cost Now = total extra + direct cost = $5,616.70

Indirect Cost = $7,280, and the Total Cost = $12,896.70

The second scenario is for crashing activity F by 1 day. After crashing activity F by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 11 illustrates simulation run results after crashing activity F by 1 day.

Table 11: Simulation results after crashing activity F at cycle one

Mean "M_{1F}"	27.16 days
Standard Deviation "Std_{1F}"	3.3939 days
POF within 26 Deterministic Days Given the Mean and Std. Found	36.6254588 %

New $D_{det,\ 1F}$ = 26 days

$D_{prob,1F}$ at 37.74742% given M_{1F} and Std_{1F} in Table 11 = 26.07625991 days

$FLD_{1F} = D_{prob,1F} - D_{det,\ 1F}$ = 26.07625991 – 26 = 0.07625991 days

FLC_{1F} = 0.07625991 (days) x 280 $ / day = $21.35

Extra direct cost (slope) = $ 150

Total extra cost = 150 + 21.35 = $171.35

Direct Cost Now = total extra + direct cost = $5,471.35

Indirect Cost = $7,280, and the Total Cost = $12,751.35

The third scenario is for crashing activity H by 1 day. After crashing activity H by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 12 illustrates simulation run results after crashing activity H by 1 day.

Table 12: Simulation results after crashing activity H at cycle one

Mean "M_{1H}"	27.4942 days
Standard Deviation "Std_{1H}"	3.6883 days
POF within 26 Deterministic Days Given the Mean and Std. Found	34.2695061 %

New $D_{det,\ 1H}$= 26 days

$D_{prob,1H}$ at 37.74742% given M_{1H} and Std_{1H} in Table 12 = 26.31645211 days

FLD_{1H}= $D_{prob,1H}$ – $D_{det,\ 1H}$ = 26.31645211 – 26 = 0.31645211 days

FLC_{1H} = 0.31645211 (days) x 280 $ / day = $88.61

Extra direct cost (slope) = $ 250

Total extra cost = 250 + 88.61 = $338.61

Direct Cost Now = total extra + direct cost = $5,638.61

Indirect Cost = $7,280, and the Total Cost = $12,918.61

The fourth scenario is for crashing activity K by 1 day. After crashing activity K by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 13 illustrates simulation run results after crashing activity K by 1 day.

Table 13: Simulation results after crashing activity K at cycle one

Mean "M_{1k}"	27.1397 days
Standard Deviation "Std_{1K}"	3.4861 days
POF within 26 Deterministic Days Given the Mean and Std. Found	37.1861573 %

New $D_{det,\ 1K}$ = 26 days

$D_{prob,1K}$ at 37.74742% given M_{1K} and Std_{1K} in Table 13 = 26.02651861 days

FLD_{1K}= $D_{prob,1K}$ – $D_{det,\ 1K}$ = 26.02651861 – 26 = 0.02651861 days

FLC_{1K} = 0.02651861 (days) x 280 $ / day = $7.43

Extra direct cost (slope) = $ 350

Total extra cost = 350 + 7.43 = $357.43

Direct Cost Now = total extra + direct cost = $5,657.43

Indirect Cost = $7,280, and the Total Cost = $12,937.43

According to the total extra cost and total cost, activity F exhibited the least total extra cost and total cost; therefore, the decision is to expedite activity F by 1 day.

Cycle Two: Crashed Schedule 2: Activities available to be expedited: B, F, H, and K. Based on that, again four available scenarios need to be checked to find the best activity to crash

The first scenario is for crashing activity B by 1 day. After crashing activity B by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 14 illustrates simulation run results after crashing activity B by 1 day in cycle two.

Table 14: Simulation results after crashing activity B at cycle two

Mean "M_{2B}"	26.591 days
Standard Deviation "Std_{2B}"	3.3549 days
POF within 25 Deterministic Days Given the Mean and Std. Found	31.7667338 %

New $D_{det,\,2B}$ = 25 days

$D_{prob,2B}$ at 36.6254588% given M_{2B} and Std_{2B} in Table 14 = 25.4443298days

FLD_{2B}= $D_{prob,2B}$ – $D_{det,\,2B}$ = 25.4443298– 25 = 0.4443298 days

FLC_{2B} = 0.4443298 (days) x 280 $ / day = $124.41

Extra direct cost (slope) = $ 200

Total extra cost = 200 + $124.41 = $324.41

Direct Cost Now = total extra + direct cost of last crashing cycle = $5,795.77

Indirect Cost = $7,000, and the Total Cost = $12,795.77

The second scenario is for crashing activity F by 1 day. After crashing activity F by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 15 illustrates simulation run results after crashing activity F by 1 day in cycle two.

Table 15: Simulation results after crashing activity F at cycle two

Mean "M_{2F}"	26.445 days
Standard Deviation "Std_{2F}"	3.3059 days
POF within 25 Deterministic Days Given the Mean and Std. Found	33.1020389 %

New $D_{det,\,2F}$ = 25 days

$D_{prob,2F}$ at 36.6254588% given M_{2F} and Std_{2F} in Table 15 = 25.31507749 days

$FLD_{2F} = D_{prob,2F} – D_{det,\,2F}$ = 25.31507749 – 25 = 0.31507749 days

FLC_{2F} = 0.31507749 (days) x 280 $ / day = $88.22

Extra direct cost (slope) = $ 150

Total extra cost = 150 + 88.22 = $238.22

Direct Cost Now = total extra + direct cost of last crashing cycle = $5,709.57

Indirect Cost = $7,000, and the Total Cost = $12,709.57

The third scenario is for crashing activity H by 1 day. After crashing activity H by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 16 illustrates simulation run results after crashing activity H by 1 day in cycle two.

Table 16: Simulation results after crashing activity H at cycle two

Mean "M_{2H}"	26.5491 days
Standard Deviation "Std_{2H}"	3.3519 days
POF within 25 Deterministic Days Given the Mean and Std. Found	32.1984801 %

New $D_{det,\,2H}$ = 25 days

$D_{prob,2H}$ at 36.6254588% given M_{2H} and Std_{2H} in Table 16 = 25.40345517 days

$FLD_{2H} = D_{prob,2H} – D_{det,\,2H}$ = 25.40345517 – 25 = 0.40345517days

FLC_{2H} = 0.40345517 (days) x 280 $ / day = $112.97

Extra direct cost (slope) = $ 250

Total extra cost = 250 + 112.97 = $362.97

Direct Cost Now = total extra + direct cost of last crashing cycle = $5,834.32

Indirect Cost = $7,000, and the Total Cost = $12,834.32

The fourth scenario is for crashing activity K by 1 day. After crashing activity K by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 17 illustrates simulation run results after crashing activity K by 1 day in cycle two:

Table 17: Simulation results after crashing activity K at cycle two

Mean "M_{2K}"	26.2251 days
Standard Deviation "Std_{2K}"	3.4652 days
POF within 25 Deterministic Days Given the Mean and Std. Found	36.1840397 %

New $D_{det,\ 2K}$ = 25 days

$D_{prob,2K}$ at 36.6254588% given M_{2K} and Std_{2K} in Table 17 = 25.0407304 days

FLD_{2K}= $D_{prob,2K}$ – $D_{det,\ 2K}$ = 25.0407304 – 25 = 0.0407304 days

FLC_{2K} = 0.0407304 (days) x 280 $ / day = $11.40

Extra direct cost (slope) = $ 350

Total extra cost = 350 + 11.40 = $361.40

Direct Cost Now = total extra + direct cost of last crashing cycle = $5,832.76

Indirect Cost = $7,000, and the Total Cost = $12,832.76

According to the total extra cost and total project cost, activity F exhibited the least total extra cost and total project cost. Based on that, the decision in this cycle is to expedite F by 1 day.

Cycle Three: Crashed Schedule 3: three available scenarios have to be checked to find the best activity to crash at cycle three. The three scenarios include either crashing activity B or H or K.

The first scenario is for crashing activity B by 1 day. After crashing activity B by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 18 illustrates simulation run results after crashing activity B by 1 day in cycle three.

Table 18: Simulation results after crashing activity B at cycle three

Mean "M_{3B}"	25.9106 days
Standard Deviation "Std_{3B}"	3.2834 days
POF within 24 Deterministic Days Given the Mean and Std. Found	28.0318093 %

New $D_{det,\ 3B}$ = 24 days

$D_{prob,3B}$ at 33.1020389% given M_{3B} and Std_{3B} in Table 18 = 24.47543469 days

FLD_{3B}= $D_{prob,3B}$ – $D_{det,\ 3B}$ = 24.47543469 – 24 = 0.47543469 days

FLC_{3B} = 0.47543469 (days) x 280 $ / day = $133.12

Extra direct cost (slope) = $ 200

Total extra cost = 200 + 133.12 = $333.12

Direct Cost Now = total extra + direct cost of last crashing cycle = $6,042.70

Indirect Cost = $6,720, and the Total Cost = $12,762.70

The second scenario is for crashing activity H by 1 day. After crashing activity H by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 19 illustrates simulation run results after crashing activity H by 1 day in cycle three.

Table 19: Simulation results after crashing activity H at cycle three

Mean "M_{3H}"	25.8742 days
Standard Deviation "Std_{3H}"	3.3498 days
POF within 24 Deterministic Days Given the Mean and Std. found	28.79116 %

New $D_{det, 3H}$ = 24 days

$D_{prob,3H}$ at 33.1020389% given M_{3H} and Std_{3H} in Table 19 = 24.41001143 days

FLD_{3H}= $D_{prob,3H}$ – $D_{det, 3H}$ = 24.41001143 – 24 = 0.41001143 days

FLC_{3H} = 0.41001143 (days) x 280 $ / day = $114.80

Extra direct cost (slope) = $ 250

Total extra cost = 250 + 114.80= $ $364.80

Direct Cost Now = total extra + direct cost of last crashing cycle = $6,074.38

Indirect Cost = $6,720, and the Total Cost = $12,794.38

The third scenario is for crashing activity K by 1 day. After crashing activity K by 1 day, Monte Carlo Simulation run is performed on the crashed schedule. Table 20 illustrates simulation run results after crashing activity K by 1 day in cycle three.

Table 20: Simulation results after crashing activity K at cycle three

Mean "M_{3K}"	25.6759 days
Standard Deviation "Std_{3K}"	3.5006 days
POF within 24 Deterministic Days Given the Mean and Std. Found	31.605949 %

New $D_{det, 3K}$ = 24 days

$D_{prob,3K}$ at 33.1020389% given M_{3K} and Std_{3K} in Table 20 = 24.14579715 days

FLD_{3K}= $D_{prob,3K}$ – $D_{det, 3K}$ = 24.14579715 – 24 = 0.14579715 days

FLC_{3K} = 0.14579715 (days) x 280 $ / day = $40.82

Extra direct cost (slope) = $ 350

Total extra cost = 350 + 40.82 = $390.82

Direct Cost Now = total extra + direct cost of last crashing cycle = $6,100.40

Indirect Cost = $6,720, and the Total Cost = $12,820.40

Total Cost started to increase at this cycle; therefore, the optimum project duration and total cost are 25 days and $12,709; respectively.

3.4 Analysis and Discussion of the Results

As per the example presented and solved earlier in this chapter, the optimum duration considering float consumption impact stochastically is 25 days, while the associated optimum total cost considering float consumption impact stochastically is found to be $12,709.

Table 21 compares the remaining total float for the noncritical activities between the deterministic compression method and the new proposed compression framework.

Table 21: Comparison of remaining TF between deterministic compression method & new compression framework

Noncritical Activity	Activity Total Float in Days @ 23 Days Duration (Deterministic)	Activity Total Float in Days @ 25 Days Duration (New Proposed Framework)
C	0	1
D	3	5
E	2	2
G	3	5
I	1	2
J	3	4

It can be noticed that the new proposed compression framework is better in terms of remaining float as it finds an optimum solution that can save some total float for future use with a less risky cost. In terms of the probability of finishing the project on time, the probability of finishing the project within 27 days is found to be 0.374741941. The probability of finishing the project within 25 days when float loss impact is considered stochastically is 0.331020389, while the probability of finishing the project within 23 days when float loss impact is not considered is 0.236667746. From the previous probabilities found, it can be seen that when float loss impact is considered, the probability of finishing the project is considerably higher than that when float loss impact is not considered. The optimum solution found using deterministic approach (without float loss effect), in comparison with the optimum solution found considering the float loss effect, is also better in terms of activities' criticality indices that are found using Monte Carlo Simulation and presented in Table 22.

Table 22: Activities critical index at 25 and 23 days durations

Activity	Activity critical index @23 days duration	Activity critical index @ 25 days duration
A	1	1
B	0.62	0.68
C	0.55	0.39
D	0.21	0.12
E	0.18	0.3
F	0.82	0.84
G	0.21	0.12
H	0.79	0.82
I	0.45	0.31
J	0.24	0.13
K	1	1

From the results in Table 22, one can understand that the new optimum duration with float can provide better results in terms of activities' criticality indices; given that the criticality index represents the percentage of the number of times the activity was found to be on the critical path. Therefore, the new optimum duration provides better project flexibility by preserving more float for future use when unforeseen events occur.

The criticality ratio is calculated as a ratio between the number of critical activities to the total number of activities. The criticality ratio of the schedule of the optimum solution found using deterministic approach (without float loss effect) is calculated to be 0.545. On the other hand, the criticality ratio of the schedule of the optimum solution found considering the float loss effect in terms of critical ratio is also calculated to be 0.545. In both cases, the critical ratio happened to be the same since the critical path didn't change at 23 and 25 days duration.

Figure 14 compares the results between the optimum solution found using deterministic approach (without float loss effect) and the optimum solution found considering the float loss effect in terms of total cost curves.

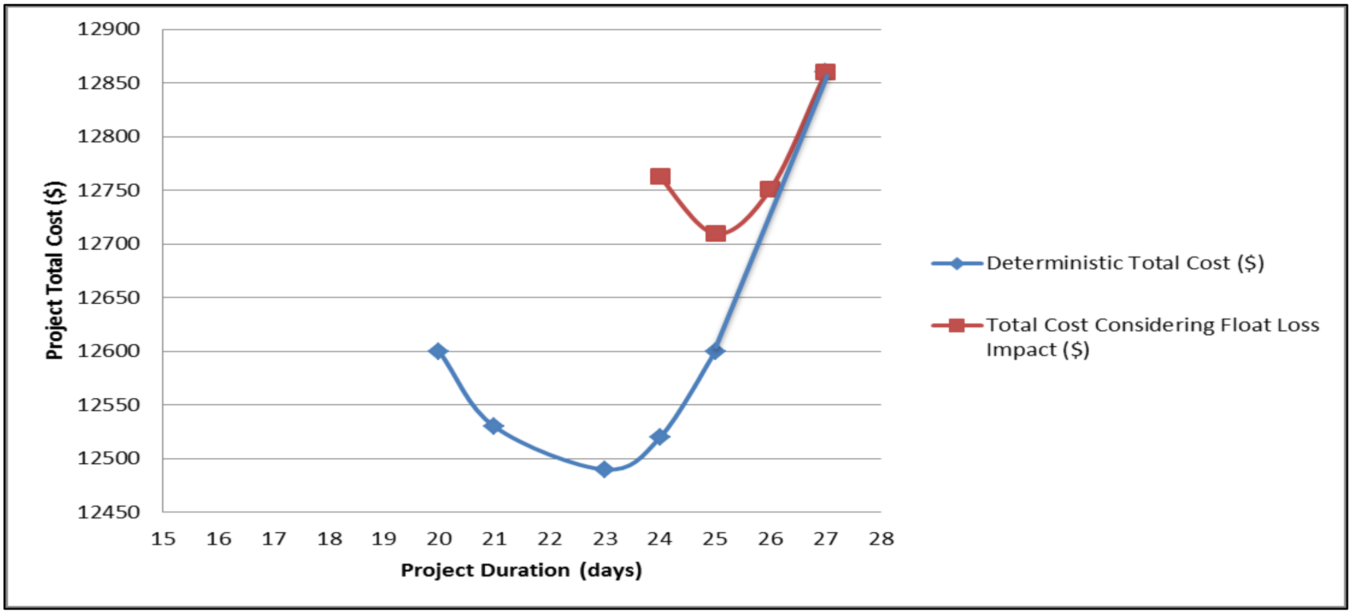


Figure 14: Total cost curves comparison

From Figure 14, it can be shown that the optimum project total cost considering float consumption impact is higher than the optimum normal cost. Optimum project duration as well is higher than that when float loss cost isn't considered. This result is predicted earlier hypothetically, and the increase in the project total cost of the curve considering float consumption impact is related to the increase in the direct cost that accounts for the float loss cost in noncritical activities. Although the framework presents a curve with a higher cost (the difference between the optimum total cost when float is considered and the deterministic normal optimum total cost is equal to $ 219 in this example). This higher cost if paid accounts and quantifies the float cost impact and can save dollars associated with risks appearing from project flexibility loss. Decision makers or project managers; depending on the nature of their projects, are free to choose between the two curves; whether to stick to the normal compression method and bear the risk associated with losing total float, or use the new curve and be at the safe side while maintaining a compressed schedule.

3.5 Chapter Conclusion

This chapter presented a new approach for optimizing project time and cost by introducing the concept of float loss impact into the tradeoff analysis. The float loss impact is quantified using @risk simulation by determining the probability of finishing the project at each crashing cycle for each activity and comparing the total extra direct cost (activity slope + float loss cost) to select the activity that best exhibits the least total cost. The idea is inspired by the fact that whenever the project schedule is crashed, total float of activities is consumed, and the probability of finishing the project on time is reduced as well. The framework by that measures the difference between the probabilistic duration and the deterministic duration and translates this loss of time into a cost to be added to the project direct cost. In light of the results obtained in this chapter, one can say that incorporating the float loss impact into the optimization process can produce a more realistic optimum cost and account for risks arising from future potential delays.

CHAPTER FOUR: PROJECT COMPRESSION CONSIDERING FLOAT CONSUMPTION IMPACT VIA NONLINEAR-INTEGER PROGRAMMING

4.1 Chapter Overview

This chapter presents the proposed optimization framework for project compression considering the float consumption impact of noncritical activities. The first section highlights the main assumptions and justifications that are assumed to calculate the float cost using the trade-in value of total float method developed by Garza et al. [4]. This method is selected from a variety of methods used for total float preallocation discussed earlier in Chapter Two as it is one of few methods that considers the effect of float loss on cost and it can provide an estimate of the risk associated with total float loss per day, which makes it suitable for time-cost tradeoff analysis. Moreover, it assumes a linear distribution of the daily TF trade-in for each activity; which provides more simplicity for the framework user. The second section demonstrates the proposed framework of the nonlinear-integer programming process including the stating of the objective functions, decision variables and constraints. The third section solves example one explained in Chapter Three via the nonlinear-integer optimization model and the deterministic approach considering the float consumption impact. The last section presents the analysis and the comparison of the results.

4.2 Proposed Optimization Model

The float cost per day is calculated according to Equation 3 established by Garza et al. [4]:

$$\frac{\text{LFC}-\text{EFC}}{\text{TF}} \tag{3}$$

Where;

EFC: Early finish cost

LFC: Late finish cost

TF: Total float

The early finish cost (EFC) represents the best cost estimate assuming normal conditions for the noncritical activity. In this case flexibility and resources are assumed to be available as needed. The late finish cost (LFC) represents the cost at abnormal conditions as the early finish date moves into the late finish dates due to unforeseen events. In this case flexibility is assumed to be consumed and efficiency is accomplished at a late finish times with an increase in the cost. And the total float (TF) represents the total float available for the noncritical activity

4.2.1 Assumptions

The following assumptions are made in the proposed NLIP framework:

- A linear relationship or a uniform float cost distribution per day throughout the entire activities duration is assumed for simplicity.
- The total cost for the activity increases proportionally with the amount of total float consumed
- EFC is assumed to be equal to the normal cost (mean) as the normal cost represents the best estimate assuming normal conditions for the activity including starting and finishing at their earliest dates. LFC is assumed in the example as the value representing abnormal conditions where the activity finishes at its latest finish dates.
- Activities' durations are assumed to be in integer numbers to make it as real as possible.

4.2.2 Model Formulation

A nonlinear-integer programming formulation is used to find the minimum total project cost associated with the optimum duration. The nonlinearity arises from the constraints over activities' relations and precedence. Two binary (one-zero) parameters; B_{ik} and D_{ik}, are defined for each activity to denote whether this activity will be crashed or not and whether the activity in question is noncritical and has a float loss cost or not at each crashing cycle.

Framework Parameters:

Table 23 illustrates the model parameters of the framework along with their descriptions.

Table 23: NLIP framework parameters

Parameter Symbol	Description	Parameter Symbol	Description
I	Project activities are denoted by the symbol *i* ϵ I where I is a set that comprises all project activities, and *i* = 1,2,3,…,n	F_{ai}	Noncritical activity *i* original float
K(*i*)	A set of all possible time-cost combinations of activities in I	F_{bi}	Noncritical activity *i* current float
B*ik*	A parameter equal to 1 if activity *i* is selected for the time-cost combination k or equal to 0 otherwise	FUC*if*	Float unit cost of noncritical activity *i*
F(i)	A set of all noncritical activities in I	LFC*if*	Late finish cost of noncritical activity *i*
D*i*	A parameter = 1 if activity *i* is noncritical and has a float cost at the crashing cycle under consideration, or equal to 0 otherwise	EFC*if*	Early finish cost of noncritical activity *i*
TC	Project total cost	TF*if*	Total float of noncritical activity *i*
IDC	Project indirect cost	CUCi	Activity *i* crashing unit cost / crashing slope
C_{OH}	Project overhead cost per day	D_{ai}	Original duration of activity *i*
DC*i*	Direct cost of activity *i*	D_{bi}	Current duration of activity *i*
CC*i*	Expenditure cost of activity *i*	D_{Ci}	Activity *i* crashed duration
FC*if*	Float consumption cost of noncritical activity *i*	NCi	Activity *i* normal cost
X_T	Project targeted completion date	$x_{i\ min}$	Activity *i* minimum possible duration
$LT_{i,j}$	Lag time between activity *i* and the succeeding activity *j*	$x_{i\ max}$	Activity *i* maximum possible duration

Framework Decision Variables:

Table 24 illustrates the decision variables of the framework along with their descriptions.

Table 24: NLIP framework decision variables

Decision Variable Symbol	Description
D_T	Project duration
a_i	Start time of activity *i*
a_j	Start time of succeeding activity *j*
x_i	Activity *i* duration
x_j	Succeeding activity *j* duration

Objective Function:

The objective function is to minimize the total project cost according to Equation 6:

$$\text{Minimize TC} = \mathbf{IDC} + \sum_{n=1}^{i} \sum_{k \in K(i)} ((\boldsymbol{DC}_{ik} + \boldsymbol{CC}_{ik}) * \mathbf{B}_{ik}) + \sum_{f \in F(i)} (\boldsymbol{FC}_{if} * D_{ik}) \qquad (6)$$

Rewriting the objective function in an expanded manner:

$$\text{Minimize TC} = (C_{OH} * D_T) + \sum_{n=1}^{i} \sum_{k \in K(i)} (DC_{ik} + (((D_{aik} - D_{bik}) * CUC_{ik})) * B_{ik}) + \sum_{f \in F(i)} ((F_{ai} - F_{bi}) * FUC_{if}) * D_i)$$

Where;

$$FC_{if} \geq 0$$

$$FUC_{if} = \frac{LFC_{if} - EFC_{if}}{TF_{if}} \qquad (3)$$

$$CUC_i = \frac{CC_i - NC_i}{D_{ai} - D_{ci}} \qquad (7)$$

Equation 3 illustrates the float unit cost (float cost) for noncritical activity *i*, while Equation 7 illustrates how the crashing unit cost (or crashing slope) is calculated for activity *i*.

Subject to the following constraints:

- Activities Duration:

  $x_{i\ min} \leq x_i \leq x_{i\ max}$

  where $x_i \geq 0$ for all i

  $x_{i\ min} \geq 0,\ x_{i\ min} \leq x_{i\ max}$

  $x_{i\ max} \geq 0$

- Activities Relations:

  Equations 8 to 11 explain the relations between the activities; FS, SS, SF, and FF, respectively:

  Finish – to- start relationship (FS):

$$a_i + x_i + LT_{i,j} \leq a_j \tag{8}$$

  Start - to - start relationship(SS):

$$a_i + LT_{i,j} \leq a_j \tag{9}$$

  Start – to- finish relationship(SF):

$$a_i + LT_{i,j} \leq a_j + x_j \tag{10}$$

  Finish – to - finish relationship (FF):

$$a_i + x_i + LT_{i,j} \leq a_j + x_j \tag{11}$$

  Equations 12 and 13 explain the forward pass rules while scheduling:

$$ES_j = \text{Max}\ [ES_i + FS_{i,j}\ ;\ ES_i + SS_{i,j}] \tag{12}$$

$$EF_j = \text{Max}\ [ES_j + x_j\ ;\ EF_i + FF_{i,j}\ ;\ ES_i + SF_{i,j}] \tag{13}$$

  Equations 14 and 15 explain the backward pass rules while scheduling:

$$LF_j = \text{Min}\ [LS_i - FS_{j,i}\ ;\ LF_i - FF_{j,i}] \tag{14}$$

$$LS_j = \text{Min}\ [LF_j - x_j\ ;\ LS_i - SS_{j,i}\ ;\ LF_i - SF_{j,i}] \tag{15}$$

- Project Completion Date:

  $D_T \leq X_T$

Figure 15 illustrates the general framework steps via a flowchart.

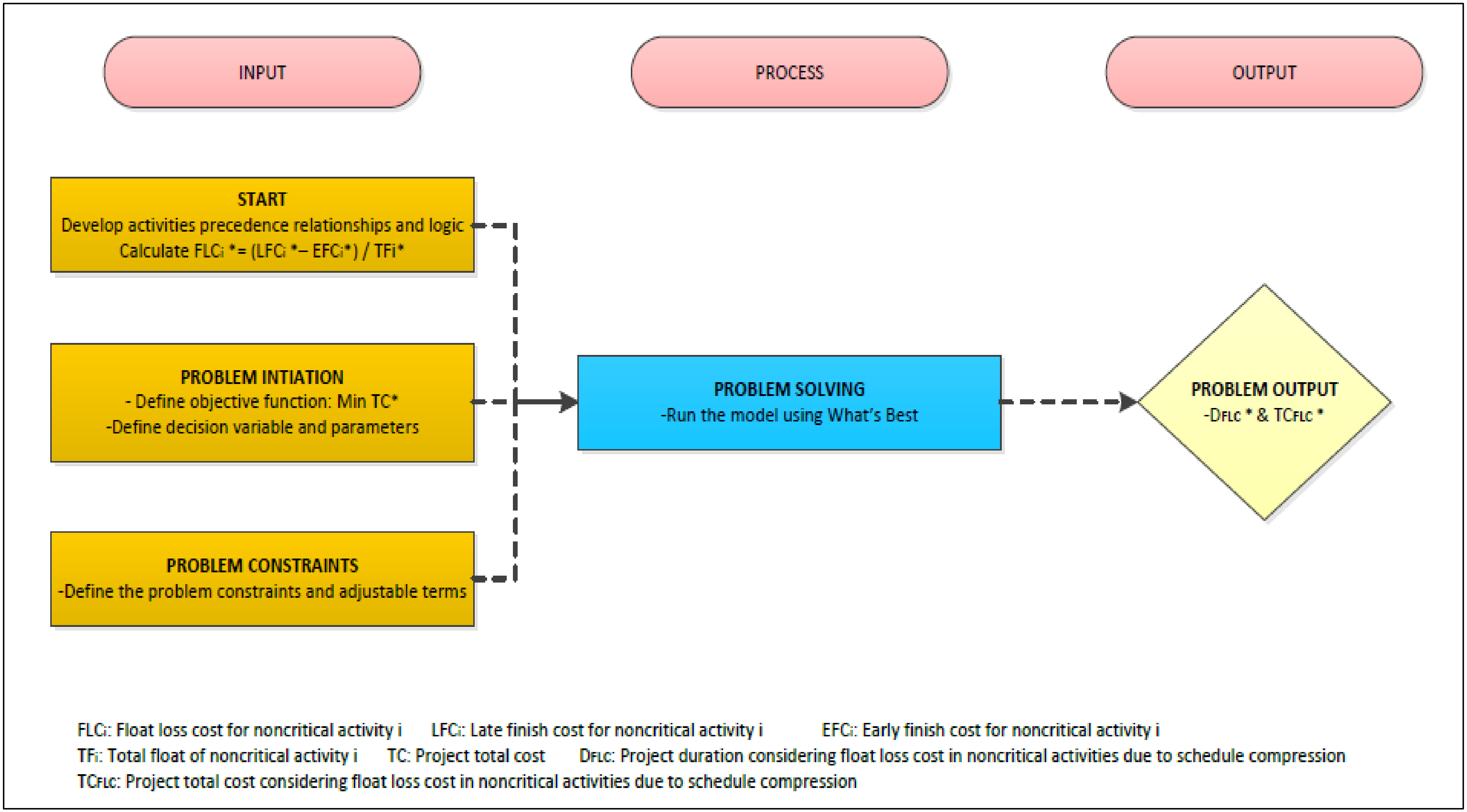


Figure 15: NLIP framework flowchart

4.3 Application Example

4.3.1 Project Compression Considering Float Consumption Impact (Manual Approach)

Table 25 lists the activities durations and costs in normal and crashed cases, along with the early and late finish costs and float unit costs for example one explained in Chapter Three. The early finish cost and the late finish cost are assumed and added to the example. The float unit cost is calculated using Equation 3 presented earlier.

As an example, the float unit cost for activity D is calculated as follows:

FUC_D = (505 – 400) / 7 = $15 / day

Table 25: Activities costs and durations

Activity	Normal Duration	Normal Cost	Crashed Duration	Crashing Cost	Potential Days Saved	Cost per Day	Duration Mean	Duration Standard Deviation	Total Float	Early Finish Cost	Late Finish cost	Float Unit Cost
A	1	800	1	800	0	-	1	1.2	0	-	-	-
B	7	1,000	4	1,600	3	200	7	2	0	-	-	-
C	6	300	4	500	2	100	6	1.5	1	300	310	10
D	3	400	2	800	1	400	3	1.35	7	400	505	15
E	3	100	1	200	2	50	3	1.88	4	100	148	12
F	7	500	5	800	2	150	7	2.12	0	-	-	-
G	8	200	4	1,400	4	300	8	3	7	200	340	20
H	7	350	6	600	1	250	7	1.25	0	-	-	-
I	5	700	3	850	2	75	5	2.5	2	700	720	10
J	3	500	2	1,000	1	500	3	1.5	4	500	532	8
K	5	450	4	800	1	350	5	1.6	0	-	-	-
	Total = 5300											

For the example above, and without considering the float effect, four crashing cycles are needed to reach the optimum project duration, while extra three cycles are needed to reach the minimum project duration. The deterministic optimum project duration is 23 days with an associated optimum total cost of $ 12,490, while the minimum project duration is 20 days with an associated project total cost of $ 12,600.

Cycle Zero: Normal Schedule: Based on the baseline schedule network represented in Figure 16, the project duration is found to be 27 days, with an associated total project cost equal to $12,860 that consists of a direct cost of $5,300 and indirect cost of $7,650. The critical path was A, B, F, H, K.

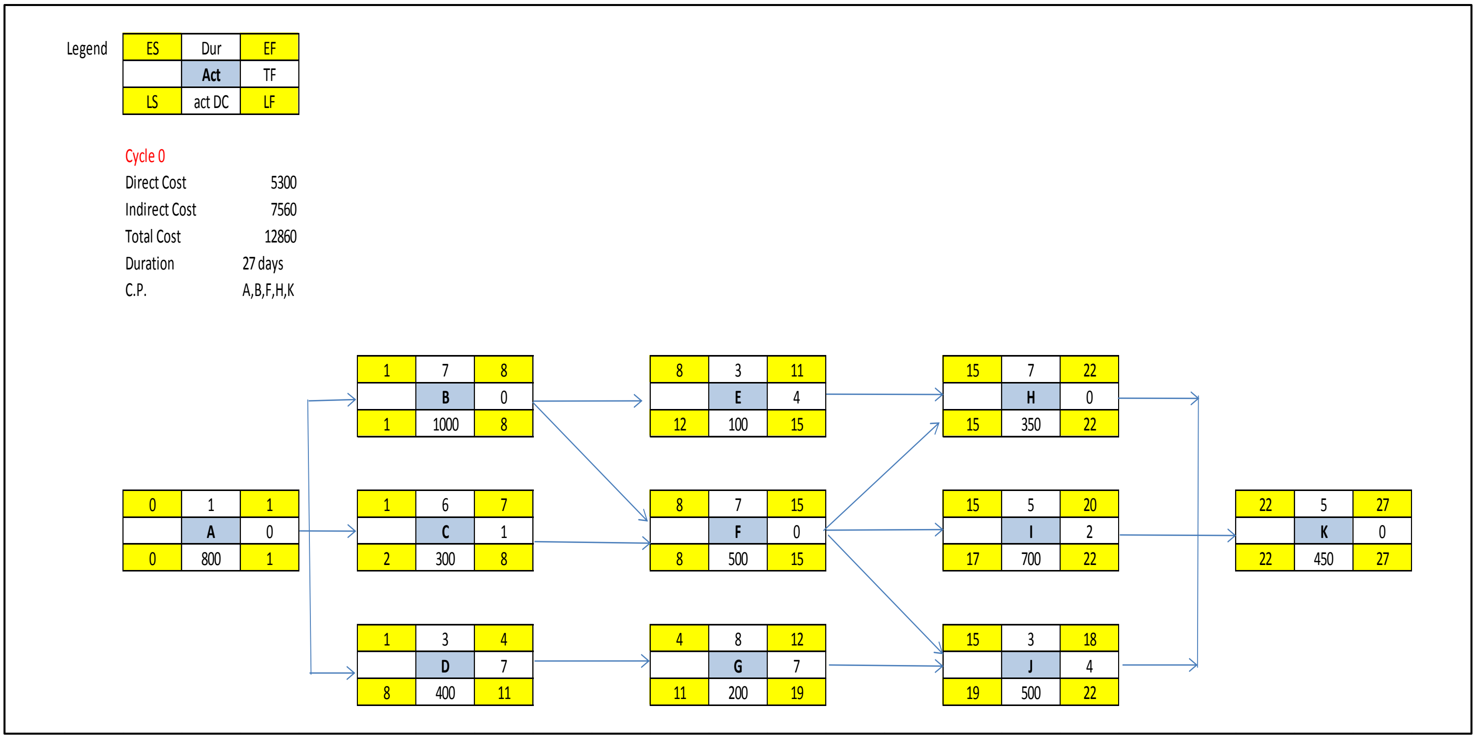


Figure 16: Cycle zero: Project normal schedule

Table 26 presents the remaining total float days for the noncritical activities at cycle zero.

Table 26: Noncritical activities total float at cycle zero

Activity	Activity Total Float
C	1
D	7
E	4
G	7
I	2
J	4

Cycle One: Crashed Schedule 1: The available activities to expedite are B, F, H, and K. The least expensive activity to expedite is F. Activity F can be crashed effectively by 2 days. The new project duration now is 25 days, and the new C.P. is A, B, F, H, K.

Total Float Cost= 30+24+40 = $94 (Table 28 illustrates the float cost per day for the non-critical activities that lost float)

The New Direct Cost = 5300 + (2*150) + 94= $5,694

The New Indirect Cost = 25 * 280 = $7,000

And the New Total Project Cost = 5,600 + 7,000 = $12,694

Figure 17 shows the updated schedule at cycle one.

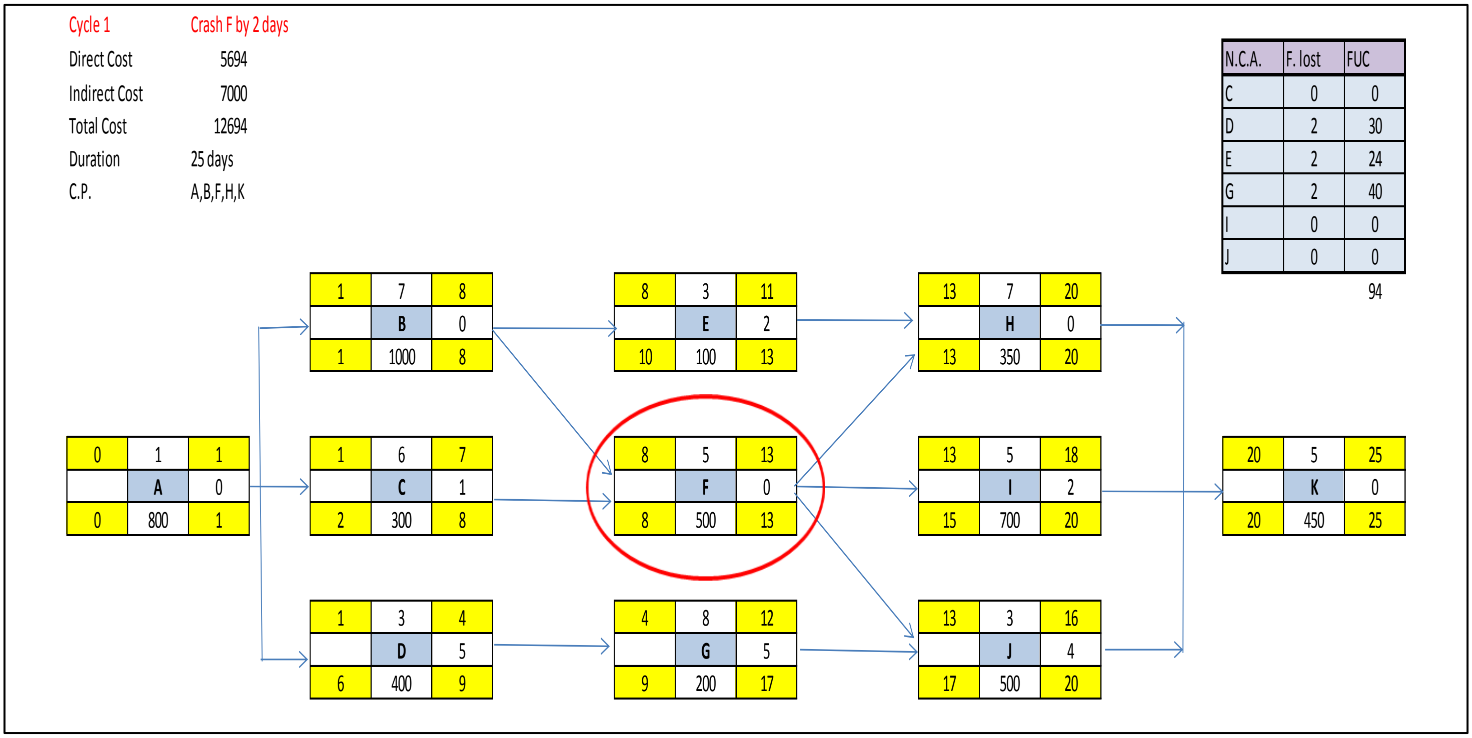


Figure 17: Cycle one: crashed schedule 1

Table 27 illustrates the remaining total float days for the noncritical activities after the first crashing cycle:

Table 27: Noncritical activities total float at cycle one

Activity	Activity Total Float
C	1
D	5
E	2
G	5
I	2
J	4

Table 28: Float loss cost at cycle one

Non-Critical Activity	Float lost (Days)	Float Cost (Per Day)	Total Float Cost
C	0	10	0
D	2	15	30
E	2	12	24
G	2	20	40
I	0	10	0
J	0	8	0
			Total = 94

To make sure that activity F is the best activity to be crashed at this step, one can try the other crashing scenarios available; crashing B, H, and K, regardless of the slope cost, and then compare the results. The first scenario is to crash activity B by 1 day. Although activity B can be crashed by 3 days, it can compress the schedule effectively only by 1 day. So the total float cost associated with crashing this activity = 10 + 15 + 20 = $45

Direct cost= 5,300 + (1*200) + 45 = $5,545

Indirect cost= 26* 280 = $7,280, and the Total cost= 7,280 + 5,545 = $12,825 > $ 12,694

The second scenario is to crash activity H by 1 day. The total float cost associated with crashing this activity = 15 + 20 +10 +8 = $53

Direct cost= 5,300 + (1*250) + 53 =$5,603

Indirect cost= 26 * 280 = $7,280, and the Total cost= 7,280 + 5,603 = $12,883 > $ 12,694

The third scenario is to crash activity K by 1 day. The total float cost = 0 (No float loss associated with crashing activity K)

Direct cost= 5,300 + (1*350) = $5,650

Indirect cost= 26 * 280 = $7,280, and the Total cost= $12,930 > $ 12,694

Since Activity F exhibited the least total cost at this step, the decision is to proceed with expediting activity F by 2 days.

Cycle Two: Crashed Schedule 2: Least Expensive Activity to Expedite is B and it can compress the schedule effectively by 1 day, so the decision at this step is to expedite activity B by 1 day. The new resulting project duration is 24 days and the new C.P. is A, B, F, H, K and A, C, F, H, K.

Total Float Cost= 10+15+20 = $45 (Table 30 illustrates the float cost per day for the non-critical activities that lost float). So the new direct cost now = 5,694 + (1*200) + 45= $5,939

Indirect Cost = 24 * 280 = $6,720; and therefore, the total project cost = 5,600 + 7,000 = $12,659

Figure 18 shows the updated schedule at cycle two.

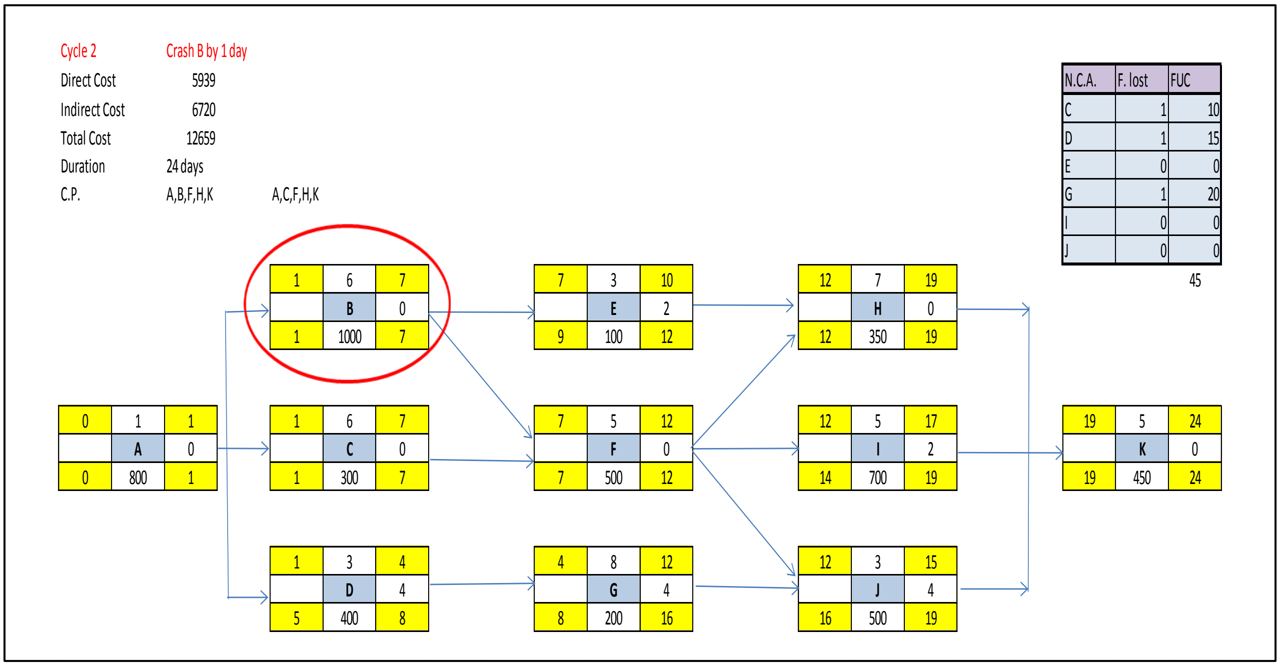


Figure 18: Cycle two: crashed schedule 2

Table 29 illustrates the remaining total float days for the noncritical activities after the second crashing cycle:

Table 29: Noncritical activities total float at cycle 2

Activity	Activity Total Float
C	0
D	4
E	2
G	4
I	2
J	4

Table 30: Float loss cost at cycle 2

Non-Critical Activity	Float lost (Days)	Float Cost (Per Day)	Total Float Cost
C	1	10	10
D	1	15	15
E	0	12	0
G	1	20	20
I	0	10	0
J	0	8	0
			Total = 45

In order to make sure that activity B is the best activity to be crashed at this step, other two available crashing scenarios are tested. The other crashing scenarios available are crashing H, and K each by 1 day. If activity H is crashed by 1 day in the first scenario:

Total float cost associated with crashing this activity = 15 + 20 +10 +8 = $53

Direct cost= $5,694 + (1*250) + 53 =$5,997

Indirect cost= 24 * 280 = $6,720, and the Total cost= 6,720 + 5,997 = $12,717 > $12,659

The second crashing scenario to try is crashing activity K by 1 day. The total float cost= 0 (No float loss associated with crashing activity K)

Direct cost= $5,694 + (1*350) = $6,044

Indirect cost= 24 * 280 = $6,720, and the Total cost= $12,764 > $12,659

Since Activity B exhibited the least total cost at this step, the decision is to proceed with expediting activity B by 1 day.

Cycle Three: Crashed Schedule 3: the least expensive activity to expedite at this step is H. the decision is to crash activity H by 1 day. The new resulting project duration equals 23 days, while the new C.P. is A, B, F, H, K and A, C, F, H, K. The detailed compression calculations are explained below:

Total Float Cost= 15+20+10+8 = $53 (Table 32 illustrates the float cost per day for the non-critical activities that lost float). So the new direct cost now = 5939 + (1*250) + 53= $6,242

Indirect Cost = 23 * 280 = $ 6,440

Total Project Cost = 6,242 + 6,440 = $12,682

Since the total cost started to increase, the optimum project duration considering float loss effect is 24 days.

Figure 19 shows the updated schedule at cycle three.

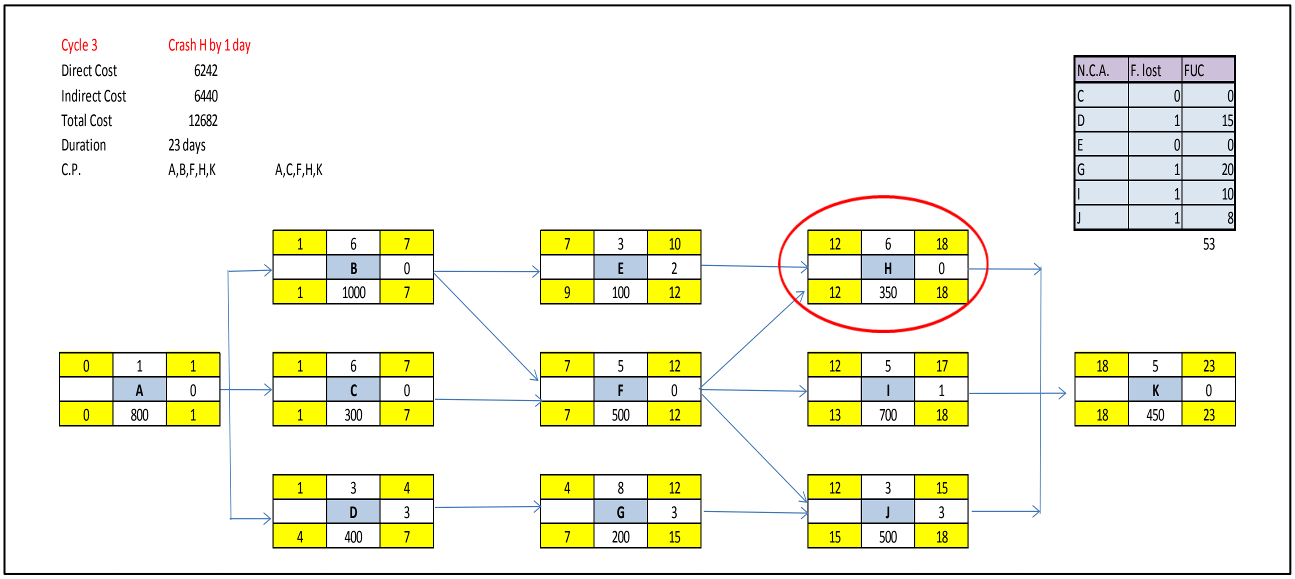


Figure 19: Cycle three: crashed schedule 3

Table 31 demonstrates the remaining total float days for the noncritical activities after the third crashing cycle:

Table 31: Noncritical activities total float at cycle 3

Activity	Activity Total Float
C	0
D	3
E	2
G	3
I	1
J	3

Table 32: Float loss cost at cycle 3

Non-Critical Activity	Float lost (Days)	Float Cost (Per Day)	Total Float Cost
C	0	10	0
D	1	15	15
E	0	12	0
G	1	20	20
I	1	10	10
J	1	8	8
			Total = 53

To make sure that activity H is the best activity to be crashed at this cycle, one can try the other crashing scenarios available; crashing activity K by 1 day, and then compare the results. If activity K is crashed by 1 day, the calculations are as follows:

Total float cost= 0 (No float loss associated with crashing activity K)

Direct cost= 5,939+ (1*350) = $6,289

Indirect cost= 23 * 280 = $6,440, and the Total cost= $12,729 > $ 12,694

Since Activity H exhibited the least total cost at this step, the decision is to proceed with expediting activity H by 1 day, but since the total project cost started to increase at this cycle, the optimum solution is $12,659 with 24 days duration.

Table 33 tabulates the crashing results from cycle zero to cycle three

Table 33: Project crashing results (crashing considering the float consumption impact)

Cycle	Duration	Direct Cost	Indirect Cost	Total Cost
0	27	5,300	7,560	12,860
1	25	5,694	7,000	12,694
2	24	5,939	6,720	12,659
3	23	6,242	6,440	12,682

Figure 20 shows the time-cost tradeoff. During crashing process the direct cost starts to increase while the indirect cost decreases since it’s a function of time.

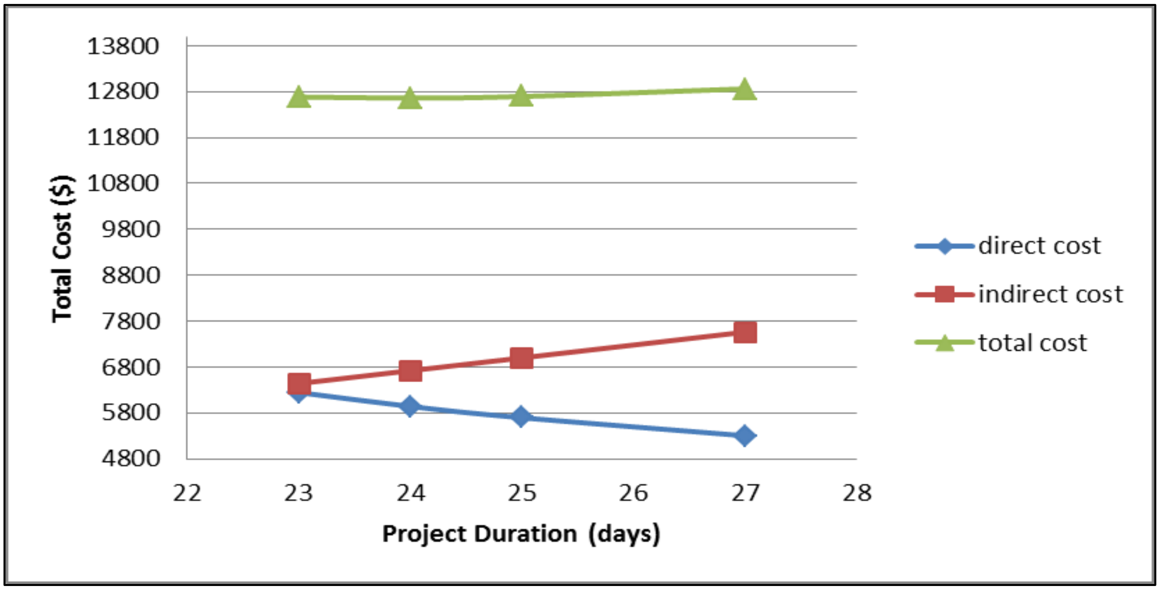


Figure 20: Project time-cost tradeoff considering float consumption impact

Figure 21 illustrates the total project cost vs. duration curve. It can be noticed that the optimum project duration and total cost are 24 days, $12,659 respectively, and afterward the total cost starts to increase until reaching the cost associated with minimum project duration.

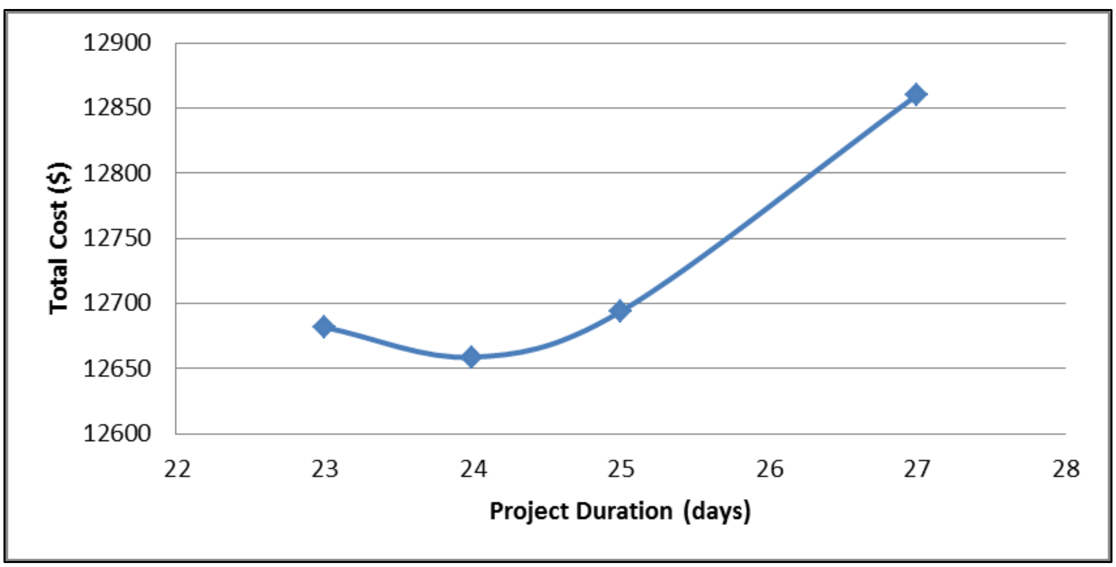


Figure 21: Project total cost vs. duration curve considering float consumption impact

4.3.2 Project Compression via Nonlinear-Integer Programming Considering Float Consumption Impact

The developed framework is a nonlinear-integer programing framework that can find efficient solutions for the two cases with and without float loss effect.

To develop the framework, What's Best Solver 11.0 is used. What's Best basically is an add-in to Excel that can support building a variety of optimization models such as linear, nonlinear, quadratic and integer models within an excel spreadsheet.

Example one is solved using What's Best framework via the following steps:

- Step One: Develop the network on Excel and define the activities' relations and logic. Define the duration, early start, early finish, late start, late finish, and total float for each activity.
- Step Two: Define the adjustable terms. The activities durations are the adjustable terms in the model
- Step Three: Establish the model table that includes; for each activity, the following:
  - Activity Normal Duration
  - Activity Original Float
  - Activity Crashed Duration
  - Activity Current Float
  - Activity Crashing Unit Cost "CUC": Crashing Slope
  - Activity Float Unit Cost "FUC"
  - Activity Crashing Cost = (Normal Duration – Adjustable Duration) * CUC
  - Activity Float Loss Cost = (Original Float – Current Float) * FUC
- Step Four: Define the constraints. The constraints for the application example presented earlier are the minimum and maximum durations of the activities, and the activities logic and relations (if the project has to be finished before a targeted duration, then this constraint has to be added as well). For activity H for example, the duration is 7 days and it has a FS relationship with activities E and F. Applying the precedence constraint for the FS relation represented via Equation 8:

$$a_i + x_i + LT_{i,j} \leq a_j$$

From activity B: 8+3+ 0 (no time lag)= 11 < 15

From activity C: 8+7+0 (no time lag)= 15 ≤ 15

Activity H early start = 15

Applying the forward pass constraint using Equations 12 and 13 to find the early start and the early finish; respectively, of activity H:

$$ES_j = \text{Max}\ [ES_i + FS_{i,j}\ ;\ ES_i + SS_{i,j}] = 15$$

$$EF_j = \text{Max}\ [ES_j + x_j\ ;\ EF_i + FF_{i,j}\ ;\ ES_i + SF_{i,j}] = 22$$

Applying the backward pass constraint using Equations 14 and 15 to find the late finish and the late start; respectively, of activity H:

$$LF_j = \text{Min}\ [LS_i - FS_{j,i}\ ;\ LF_i - FF_{j,i}] = 22$$

$$LS_j = \text{Min}\ [LF_j - x_j\ ;\ LS_i - SS_{j,i}\ ;\ LF_i - SF_{j,i}] = 15$$

Applying the min and max duration constraint: $x_{i\ min} \leq x_i \leq x_{i\ max}$

$6 \leq 7 \leq 7$

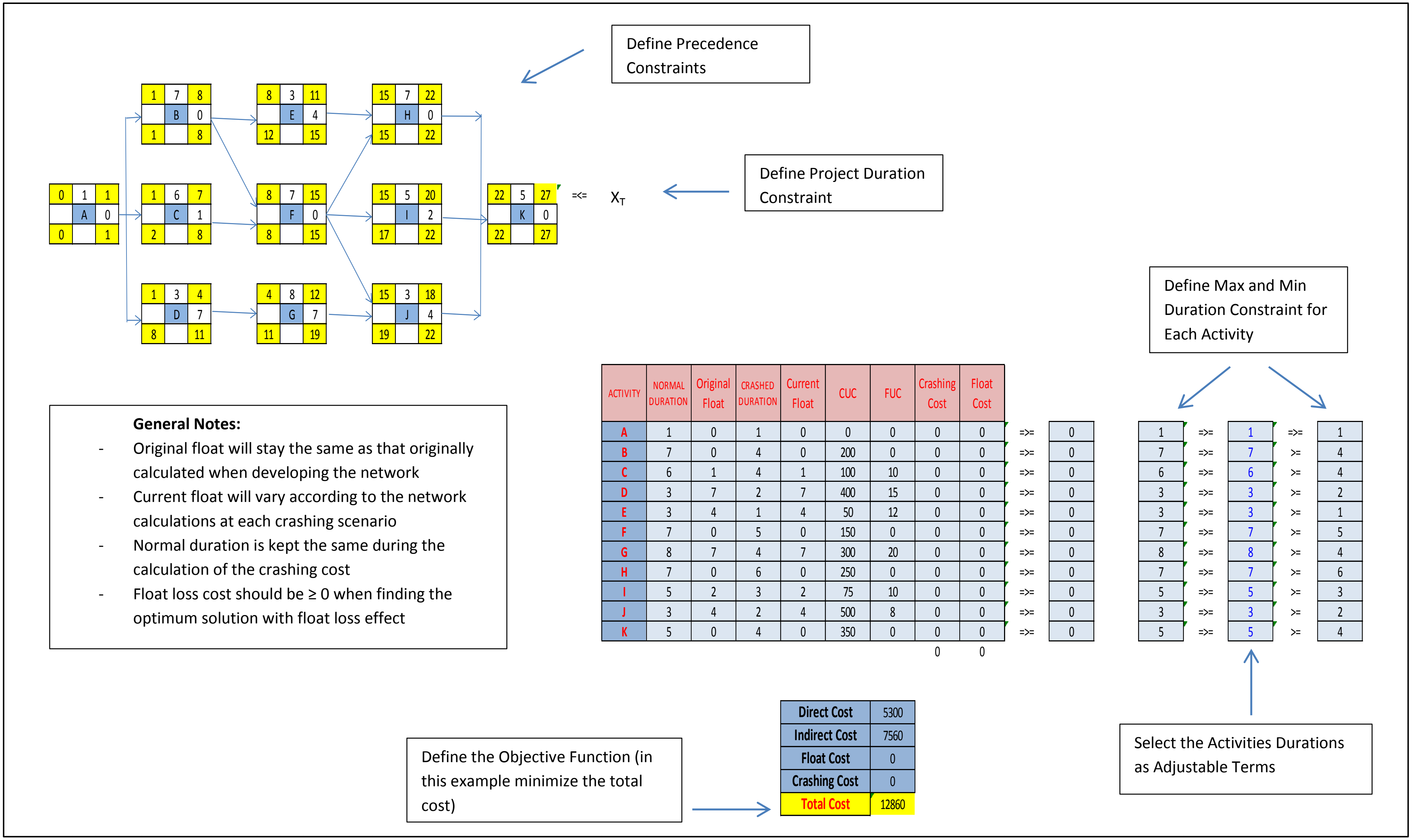
where $7 \geq 0$, $6 \geq 0$ and $6 \leq 7$

Step Five: Define the objective function in each scenario:

- Scenario One: to find the optimum solution without considering float loss cost: minimize the total project cost= direct cost + extra direct cost due to crashing + indirect cost.
- Scenario Two: to find the minimum duration: minimize the project duration. To find the total cost associated with minimum duration: constrain the project duration to be equal to the minimum duration found, and then minimize the total project cost.
- Scenario Three: to find the optimum solution considering the float loss cost: minimize the total project duration = direct cost + extra direct cost due to crashing + extra cost due to float loss in noncritical activities + indirect cost. In this scenario the float loss cost should be constrained to be ≥ 0 for each activity.

- Step Six: Run the model and check the What's Best report for the results.

Figure 22 shows an example of the model spreadsheet with the adjustable terms, objective function, and constraints.



ACTIVITY	NORMAL DURATION	Original Float	CRASHED DURATION	Current Float	CUC	FUC	Crashing Cost	Float Cost							
A	1	0	1	0	0	0	0	0	=>=	0	1	=>=	1	=>=	1
B	7	0	4	0	200	0	0	0	=>=	0	7	=>=	7	>=	4
C	6	1	4	1	100	10	0	0	=>=	0	6	=>=	6	>=	4
D	3	7	2	7	400	15	0	0	=>=	0	3	=>=	3	>=	2
E	3	4	1	4	50	12	0	0	=>=	0	3	=>=	3	>=	1
F	7	0	5	0	150	0	0	0	=>=	0	7	=>=	7	>=	5
G	8	7	4	7	300	20	0	0	=>=	0	8	=>=	8	>=	4
H	7	0	6	0	250	0	0	0	=>=	0	7	=>=	7	>=	6
I	5	2	3	2	75	10	0	0	=>=	0	5	=>=	5	>=	3
J	3	4	2	4	500	8	0	0	=>=	0	3	=>=	3	>=	2
K	5	0	4	0	350	0	0	0	=>=	0	5	=>=	5	>=	4
							0	0							

Direct Cost	5300
Indirect Cost	7560
Float Cost	0
Crashing Cost	0
Total Cost	12860

Figure 22: Example of model building using Excel and What's Best

4.4 Results and Discussion

After developing example one on Excel, and developing the model accordingly, the framework is run and the optimum duration is found to be 23 days with an associated total cost of $ 12,490. The minimum project duration is 20 days and the total cost associated with the minimum duration is $12600. The optimum duration considering float consumption impact found using the model is 24 days with a total cost considering float consumption impact equal to $12,659. The results obtained are consistent with the deterministic approaches (with and without float loss effect), which proves the validity of the model in reaching an optimum/efficient solution.

Table 34 compares the remaining total float for the noncritical activities between the deterministic compression method and the new proposed NLIP compression framework.

Table 34: Comparison of TF between deterministic compression method & new proposed NLIP framework

Noncritical Activity	Activity Total Float in Days at 23 Days Duration (Deterministic)	Activity Total Float in Days at 24 Days Duration (New Proposed Framework)
C	0	0
D	3	4
E	2	2
G	3	4
I	1	2
J	3	4

It can be noticed that the new proposed compression model is better in terms of remaining float as it finds an efficient solution that can save some total float for future use with a less risky cost.

The Probability of finishing the project within 27 days using Monte Carlo Simulation is found to be 0.374741941. Comparing the results between the optimum solution found using deterministic approach (without float loss effect) and the optimum solution found considering the float loss effect in terms of probability of finishing the project on time, the probability of finishing on time when float loss impact is considered is considerably higher than that when float loss impact is not considered; as the probability of finishing the project within 24 days when float loss impact is considered is 0.281465113, and the probability of finishing the project within 23 days when float loss impact is not considered is 0.236667746.

Table 35 tabulates the activities' criticality indices found after running the Monte Carlo Simulation over the two networks at 23 and 24 days duration.

Table 35: Activities' critical indices at 23 and 24 days duration

Activity	Activity critical index @23 days duration	Activity critical index @ 24 days duration
A	1	1
B	0.62	0.6
C	0.55	0.47
D	0.21	0.18
E	0.18	0.15
F	0.82	0.81
G	0.21	0.18
H	0.79	0.69
I	0.45	0.3
J	0.24	0.23
K	1	1

Comparing the results between the optimum solution found using deterministic approach (without float loss effect) and the optimum solution found considering the float loss effect in terms of activities' criticality indices presented in Table 33, it can be noticed that the new optimum duration with float can provide better results in terms of activities criticality index, given that the criticality index represents the percentage of the number of times the activity was found to be on the critical path. Therefore, the new optimum duration provides better project flexibility by preserving more float for future use when unforeseen events occur.

The criticality ratio is calculated as a ratio between the number of critical activities to the total number of activities. The criticality ratio of the schedule of the optimum solution found using deterministic approach (without float loss effect) is calculated to be 0.545. On the other hand, the criticality ratio of the schedule of the optimum solution found considering the float loss effect in terms of critical ratio is also calculated to be 0.545. In both cases, the critical ratio happened to be the same since the critical path didn't change at 23 and 24 days duration.

Figure 23 compares the total cost curves resulting from the optimum solution found using deterministic approach (without float loss effect) and the optimum solution found considering the float loss effect.

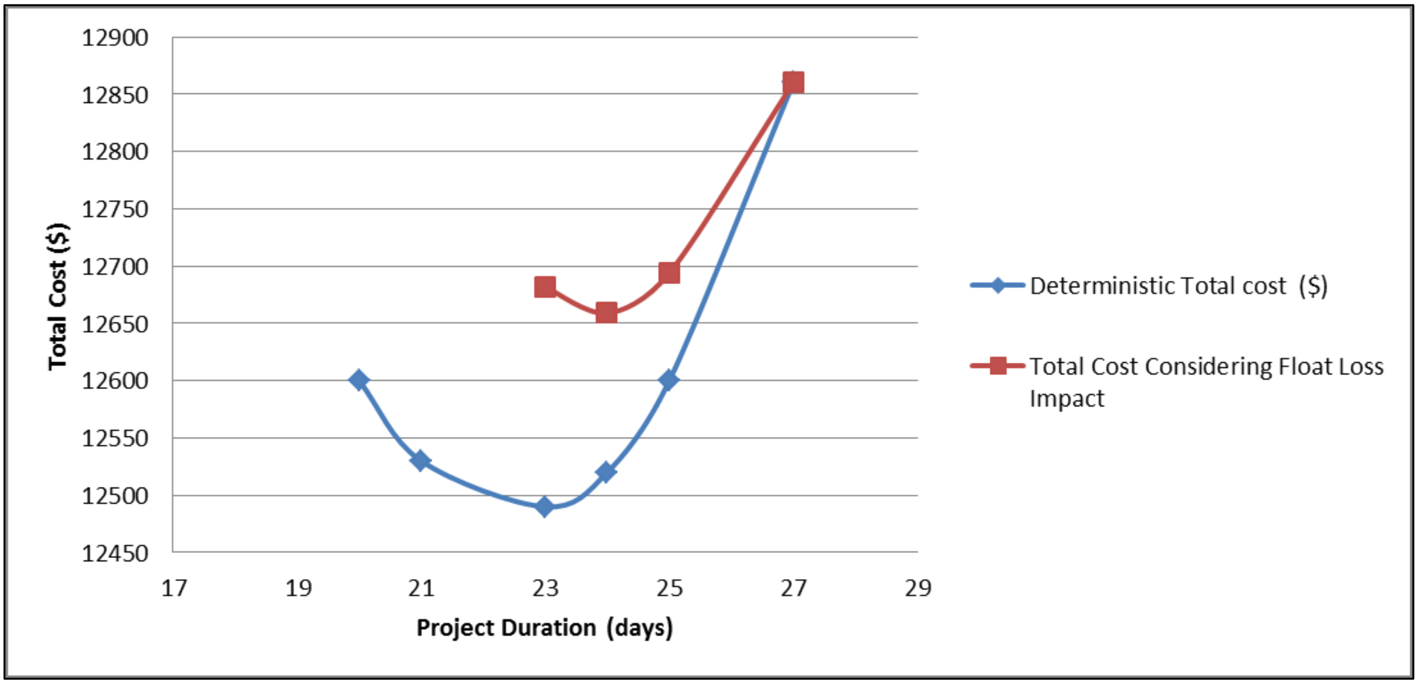


Figure 23: Comparison of total cost curves

From Figure 23, it can be shown that the optimum project total cost considering float consumption impact is higher than the optimum normal cost. Optimum project duration, as well, is higher than that when float loss cost wasn't considered. This result is predicted earlier hypothetically, and the increase in the project total cost of the curve considering float consumption impact is related to the increase in the direct cost that accounts for the float loss cost in non-critical activities. Although the framework presents a curve with a higher cost (the difference between the optimum total cost when float is considered and the deterministic normal optimum total cost is equal to $ 169 in this example), this higher cost accounts and quantifies the float cost impact and accounts for the risks associated with project flexibility loss in terms of money. Project managers can choose, depending on their projects and circumstances, between the two curves; either to go for the normal compression method and accept the risk associated with loosing total float, or to use the new curve and compress the schedule in a less risky manner.

4.5 Chapter Conclusion

This chapter presented the formulation and the assembly of the new proposed framework to find an efficient solution to the optimization problem considering a new criterion; float loss impact within noncritical activities.

As addressed before, float consumption within noncritical consumption can impact the project schedule and cost. The developed framework using What's Best solver proved this impact by pointing out the effect of float loss over the schedule flexibility, criticality index, and probability of finishing the project on time. The solution obtained considering the float loss cost compared to the normal deterministic solution provided a higher probability of completion (0.28 compared to 0.23), a better criticality index for some activities, and a small total cost difference (this small extra money can help in avoiding the risks associated with project flexibility loss). The total cost curves in both cases matched the hypothetical curves predicted earlier; validating the earlier concept predicted about the new framework.

CHAPTER FIVE: FRAMEWORKS VALIDATION

5.1 Chapter Overview

This chapter evaluates the results of five different examples solved using the two proposed frameworks and compares between the deterministic solution and the frameworks solution in terms of probability of finishing on time, criticality index, remaining float, criticality ratio, and total cost graphs to present the effect of considering float consumption in noncritical activities over project flexibility, cost, and duration.

5.2 Frameworks Validation: Results and Analysis

5.2.1 Example One

Example one is adopted from Isidore & Back [7], and reproduced in Table 36. Columns 6 to 8 are added in order to use them in finding the float unit cost, while columns 9 and 10 are added to perform the stochastic analysis.

Table 36: Isidore & Back project data

Activity	Normal Duration	Crashed Duration	CUC	TF	EFC	LFC	FUC	Duration Mean	Duration Standard Deviation
A	3	2	30	6	400	460	10	3	1.2
B	4	2	40	0	-	-	-	4	3.14
C	2	2	-	2	350	386	18	2	1.8
D	7	5	25	0	-	-	-	7	1.25
E	5	3	35	0	-	-	-	5	1
F	6	3	30	6	610	742	22	6	2.3
G	4	2	30	0	-	-	-	4	2.5
H	5	2	45	2	525	555	15	5	1.5
I	3	2	20	2	390	440	25	3	0.5
J	6	3	50	2	615	655	20	6	1.5

Figure 24 illustrates the project schedule and precedence relationships. According to Isidore & Back [7], the normal project duration is 20 days, and the normal total cost is $8,215. The optimum project duration is 12 days, while the total project cost associated with this duration is $7,940.

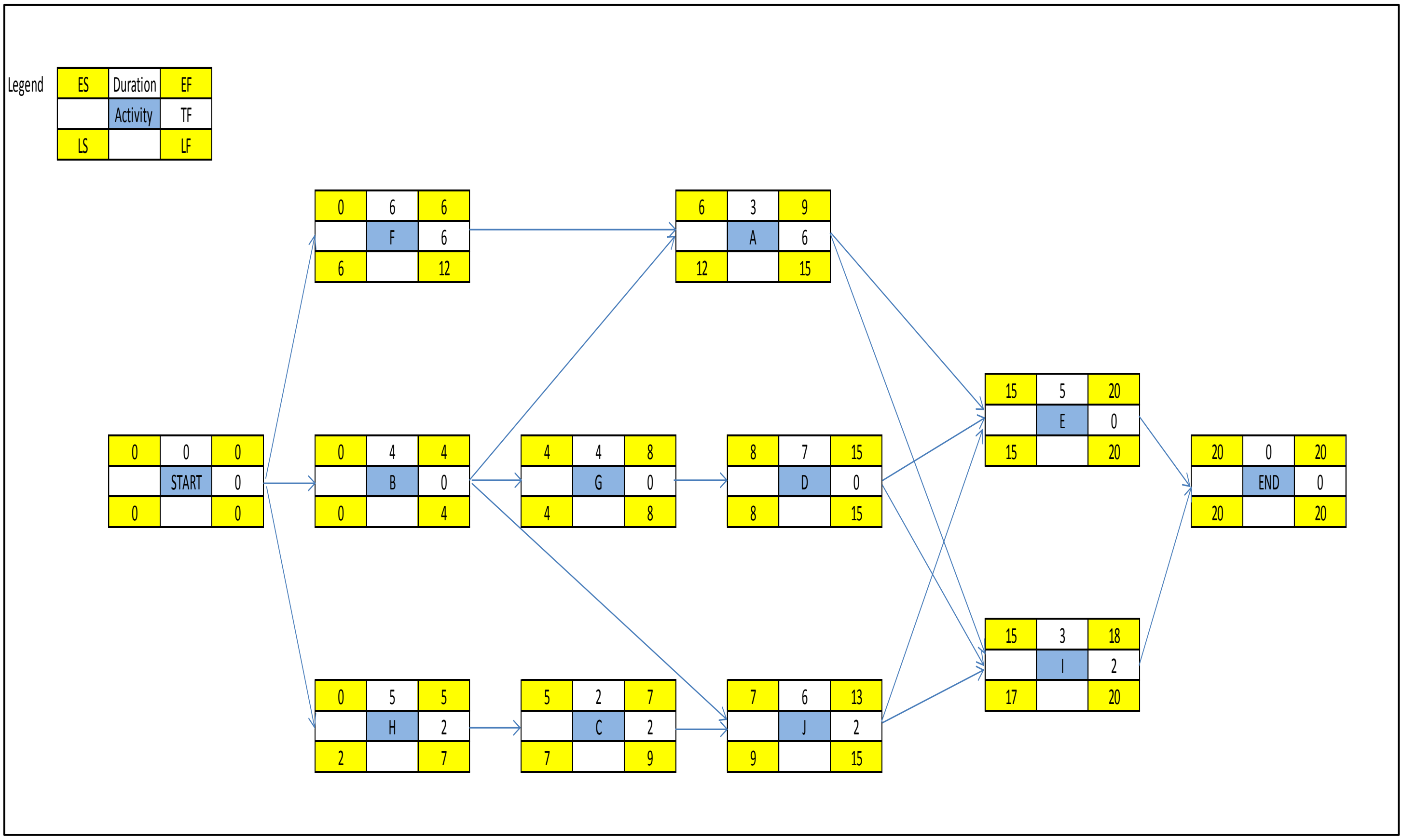


Figure 24: Schedule network and activities relations in Example One

After developing the framework and running it using What's Best, the deterministic duration without the float loss cost is found to be 12 days, and the associated total cost is $7,940. The results are consistent with one found by Isidore & Back [7]. The optimum duration considering float loss cost is 18 days with an associated cost of $8,155. The stochastic analysis resulted in 17 days project duration with an associated total cost of $8,146.5.

In light of the results obtained, a comparison is done between the deterministic results, the nonlinear-integer model results, and the stochastic results. In terms of criticality ratio, the deterministic scenario is 1 while the probabilistic model and the nonlinear integer model resulted in a criticality ratio of 0.5. in terms of probability of finishing on time, the probability of finishing the project within 20 days is 0.355124, and within 12 days (deterministic scenario) is 0.109315488, and within 17 days (probabilistic framework) is 0.252702066, while the probability of finishing the project within 18 days (NLIP framework) is 0.279670521. In terms of schedule flexibility, the nonlinear-integer framework and the stochastic framework performed better than the deterministic approach. Table 37 illustrates the remaining schedule total float in the three cases. Moreover, the activities criticality indices when using the nonlinear-integer framework and the stochastic framework are better than that when applying the deterministic approach to find the optimization solution. Table 38 shows a comparison between the three cases in terms of criticality index of each activity.

Table 37: Comparison between all cases in terms of remaining total float in Example One

Activity	Total Float in Normal Case	Total Float in Deterministic Case	Total Float in Nonlinear-Integer Framework Case	Total Float in Stochastic Framework Case
A	6	0	6	5
B	0	0	0	0
C	2	0	2	1
D	0	0	0	0
E	0	0	0	0
F	6	0	6	5
G	0	0	0	0
H	2	0	2	1
I	2	0	0	0
J	2	0	2	1

Table 38: Activities critical indices in Example One

Activity	Activity Critical Index		
	Deterministic Approach	Probabilistic Framework	Nonlinear-Integer Framework
A	1	0.07	0.05
B	1	0.7	0.69
C	1	0.48	0.43
D	1	0.68	0.69
E	1	0.77	0.84
F	1	0.07	0.05
G	1	0.68	0.69
H	1	0.48	0.43
I	1	0.8	0.82
J	1	0.55	0.45

Figure 25 shows the project total cost vs. the project duration for the results when using the deterministic approach, the nonlinear-integer framework, and the stochastic framework. From Figure 25, one can notice the optimum duration-cost for each case, and it can be realized that the higher cost in the nonlinear-integer model results and the stochastic model results quantifies the impact of project flexibility loss in money means.

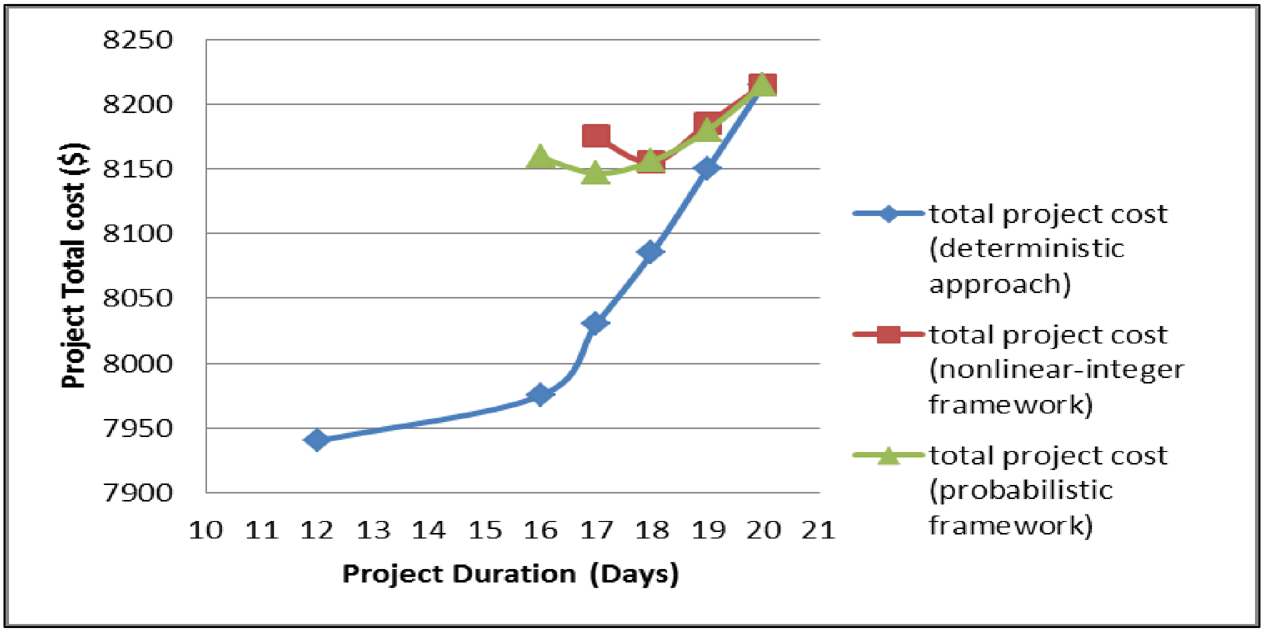


Figure 25: Total project cost vs project duration in all cases in Example One

5.2.2 Example Two

Example two is adopted from Oxley & Poskitt [61] and reproduced in Table 39. Columns 6 to 8 in the table are added in order to use them in finding the float unit cost, while columns 9 and 10 are added to perform the stochastic analysis. Figure 26 illustrates the project schedule and precedence relationships. According to Oxley & Poskitt [61], the normal project duration is 24 dyas, while the normal total cost is $30,520. The deterministic project optimum duration is 17 days, while the total project cost associated with this duration is $28,870.

After developing the framework and running it using What's Best, the deterministic duration without the float loss cost is found to be 17 days, and the associated total cost is $28,870. The results are consistent with one found by Oxley & Poskitt [61]. The duration considering float loss cost is 21 days with an associated cost of $30,252. The stochastic analysis resulted in 18 days project duration with an associated total cost of $29,742.5.

Table 39: Oxley & Poskitt project data

Activity	Normal Duration	Crashed Duration	CUC	TF	EFC	LFC	FUC	Duration Mean	Duration Standard Deviation
C	4	3	300	11	3,000	3,330	30	4	2.5
B	10	5	150	0	-	-	-	10	1
A	1	1	0	4	100	200	25	1	1.6
D	1	1	0	11	120	560	40	1	2.25
E	1	1	0	0	-	-	-	1	0.25
F	2	2	0	4	1,500	1,532	8	2	1.5
G	3	2	100	9	500	815	35	3	0.85
H	1	1	0	4	500	612	28	1	3.2
I	3	3	0	9	350	548	22	3	1.25
J	3	2	75	4	450	530	20	3	0.3
L	4	3	50	9	550	775	25	4	2.1
K	5	3	500	0	-	-	-	5	0.5
M	4	3	200	0	-	-	-	4	1.5
O	3	2	125	0	-	-	-	3	0.3
N	3	2	300	9	800	980	20	3	2.5
P	1	1	0	0	-	-	-	1	0.1

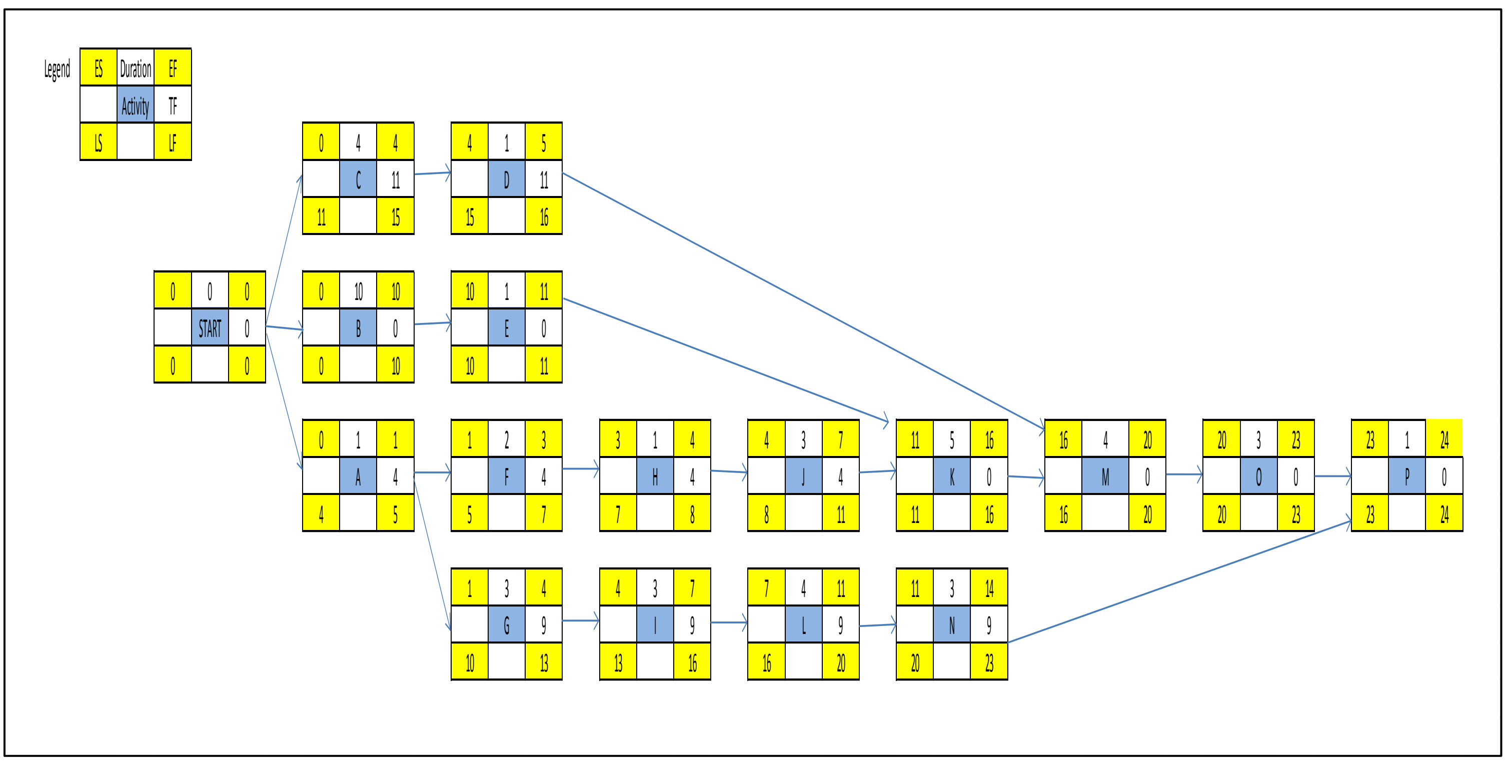


Figure 26: Schedule network and activities relations in Example Two

According to the results obtained, the deterministic scenario had a criticality index of 0.625 while the nonlinear integer framework and the probabilistic framework resulted in a criticality ratio of 0.375 and 0.625, respectively (the criticality ratio is equal in the deterministic and probabilistic model as the number of critical activities was the same and critical path didn't change in both cases). Moreover, the probabilistic framework and the NLIP framework performed better in terms of the probability of finishing on time. The probability of finishing the project normally within 24 days is 0.425895392, and within 17 days (deterministic case) is 0.208809028, and within 18 days (probabilistic framework) is 0.250002956, while the probability of finishing within 21 days (NLIP framework) is 0.385687061. In terms of schedule flexibility, the nonlinear-integer framework and the stochastic framework performed better than the deterministic approach.

Table 40 indicates the schedule flexibility in terms of the remaining schedule total float in the three cases. Furthermore, the activities criticality indices when using the nonlinear-integer framework and the stochastic framework are noticeably better than that when applying the deterministic approach to solve the optimization problem.

Table 40: Comparison between all cases in terms of remaining total float in Example Two

Activity	Total Float in Normal Case	Total Float in Deterministic Case	Total Float in Nonlinear-Integer Framework Case	Total Float in Stochastic Framework Case
C	11	6	10	6
B	0	0	0	0
A	4	0	4	0
D	11	6	10	6
E	0	0	0	0
F	4	0	4	0
G	9	2	8	3
H	4	0	4	0
I	9	2	8	3
J	4	0	4	0
L	9	2	8	3
K	0	0	0	0
M	0	0	0	0
O	0	0	0	0
N	9	2	8	3
P	0	0	0	0

Table 41 shows a comparison between the three cases in terms of schedule-activities' criticality indices. Figure 27 shows the project total cost vs. the project duration for the results when using the deterministic approach, the nonlinear-integer framework, and the stochastic framework.

Table 41: Activities critical indices in Example Two

Activity	Activity Critical Index		
	Deterministic Approach	Probabilistic Framework	Nonlinear-Integer Framework
C	0.03	0.02	0
B	0.42	0.52	0.66
A	0.78	0.65	0.44
D	0.03	0.02	0
E	0.42	0.52	0.66
F	0.52	0.45	0.31
G	0.36	0.24	0.13
H	0.52	0.45	0.31
I	0.36	0.24	0.13
J	0.52	0.45	0.31
L	0.36	0.24	0.13
K	0.95	0.8	0.87
M	0.95	0.8	0.87
O	0.95	0.8	0.87
N	0.36	0.24	0.13
P	1	1	1

From Figure 27, one can notice the optimum duration-cost for each case, and it can be realized that the higher cost in the nonlinear-integer framework results and the stochastic framework results quantifies the impact of project flexibility loss in money means.

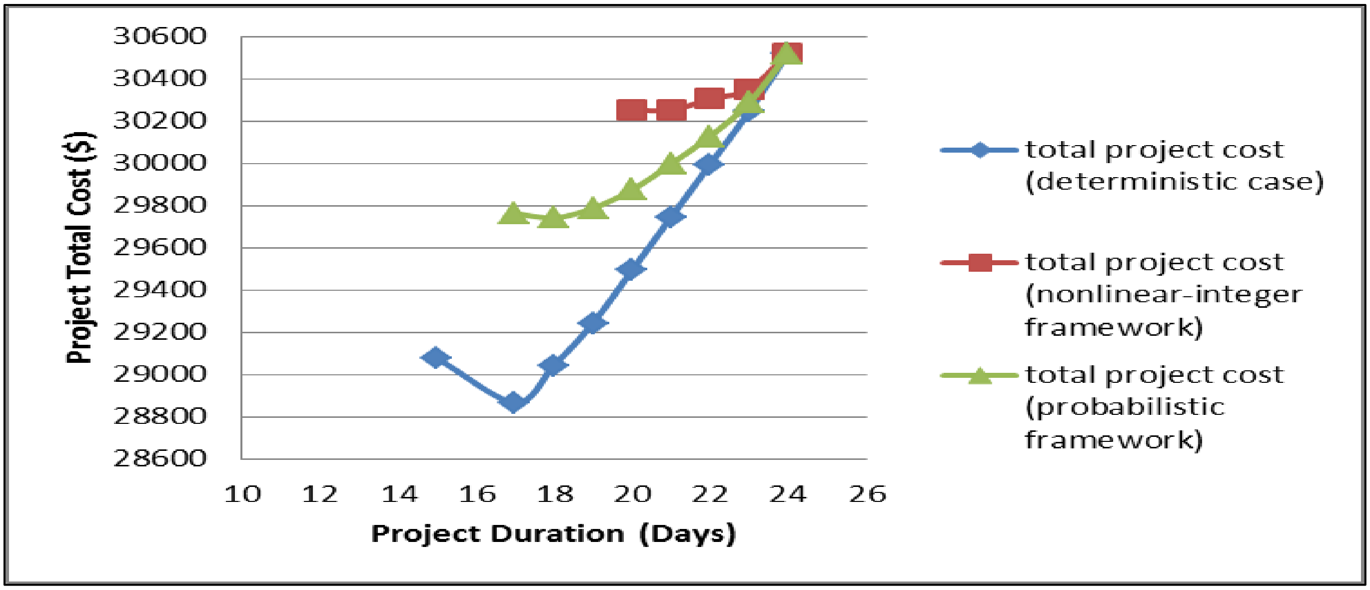


Figure 27: Total project cost vs project duration in all cases in Example Two

5.2.3 Example Three

Example three is adopted from Zeinalzadeh [62], and reproduced in Table 42. Columns 6 to 8 are added to use them in finding the float unit cost, while columns 9 and 10 are added to perform the stochastic analysis. Figure 28 illustrates the project schedule and precedence relationships. According to Zeinalzadeh [62], the normal project duration is 20 days, and the normal total project cost is $2,044,000. The project optimum duration is 16 days, while the total project cost associated with this duration is $1,990,000.

After developing the framework and running it using What's Best, the deterministic duration without the float loss cost is found to be 16 days, and the associated total cost is $1,990,000. The results are consistent with the ones found by Zeinalzadeh [62]. The duration considering float loss cost is 17 days with an associated cost of $2,005,795. The stochastic analysis resulted in 18 days project duration with an associated total cost of $2,020,269.525.

As per the results obtained in this example, the deterministic scenario scored 0.261 in terms of criticality ratio, while the probabilistic framework and the nonlinear integer framework resulted in a criticality ratio of 0.087 and 0.174, respectively.

Table 42: Zeinalzadeh project data

Activity	Normal Duration	Crashed Duration	CUC	TF	EFC	LFC	FUC	Duration Mean	Duration Standard Deviation
A	15	13	20,000	3	225,000	225,300	100	15	2.5
B	2	1	2,000	3	26,000	26,750	250	2	4
C	14	10	20,000	4	210,000	210,360	90	14	1.6
D	2	1	2,000	4	26,000	26,600	150	2	2.25
E	2	1	20,000	12	30,000	32,160	180	2	0.25
F	2	1	1,000	12	16,000	20,200	350	2	1.5
G	4	3	2,000	12	52,000	53,200	100	4	0.85
H	3	1	20,000	10	45,000	48,500	350	3	3.2
I	2	1	1,000	10	16,000	17,500	150	2	1.25
J	5	3	2,000	10	65,000	66,200	120	5	3
K	3	2	20,000	10	45,000	48,600	360	3	2.1
L	4	2	1,000	10	32,000	34,100	210	4	0.5
M	3	1	2,000	10	39,000	40,300	130	3	1.5
N	5	2	20,000	9	75,000	76,260	140	5	0.5
O	6	4	2,000	9	78,000	78,810	90	6	2.5
P	5	3	15,000	8	75,000	76,760	220	5	1.8
Q	7	5	2,000	8	91,000	92,280	160	7	1.2
R	2	1	15,000	18	20,000	29,000	500	2	0.5
S	5	3	15,000	15	50,000	56,300	420	5	0.35
T	3	1	1,000	15	24,000	25,650	110	3	1.5
U	2	1	15,000	15	20,000	21,275	85	2	1.8
V	12	9	20,000	0	-	-	-	12	2.2
W	8	6	2,000	0	-	-	-	8	2.8

Moreover, the probabilistic framework and the NLIP framework performed better in terms of the probability of finishing on time. The probability of finishing the project normally within 20 days is 0.341538455, and within 16 days (deterministic case) is 0.121625058, and within 18 days (probabilistic framework) is 0.203947032, while the probability of finishing within 17 days (NLIP framework) is 0.160543228. In terms of schedule flexibility, the nonlinear-integer framework and the stochastic framework performed better than the deterministic approach. Table 43 shows the remaining schedule total float in the three cases. Also, the activities criticality indices using the nonlinear-integer framework and the stochastic framework are better than the deterministic case.

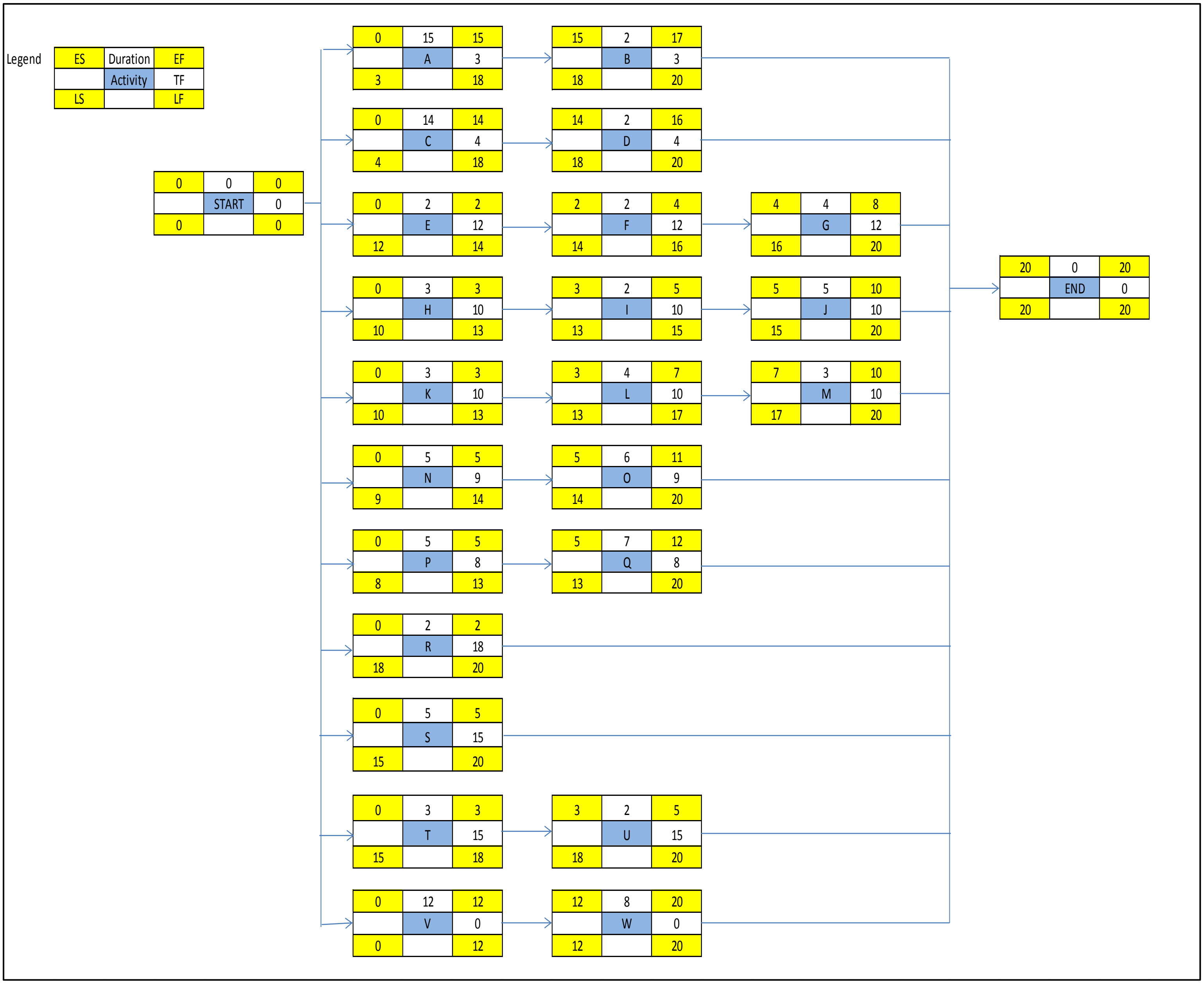


Figure 28: Schedule network and activities relations in Example Three

Table 44 shows a comparison between the three cases in terms of schedule-activities' criticality indices. Figure 29 shows the project total cost vs. the project duration for the results when using the deterministic approach, the nonlinear-integer framework, and the stochastic framework. From Figure 29, one can notice the optimum duration-cost for each case, and it can be realized that the higher cost in the nonlinear-integer model results and the stochastic model results quantifies the impact of project flexibility loss in money means.

Table 43: Comparison between all cases in terms of remaining total float in Example Three

Activity	Total Float in Normal Case	Total Float in Deterministic Case	Total Float in Nonlinear-Integer Framework Case	Total Float in Stochastic Framework Case
A	3	0	0	1
B	3	0	0	1
C	4	0	1	2
D	4	0	1	2
E	12	8	9	10
F	12	8	9	10
G	12	8	9	10
H	10	6	7	8
I	10	6	7	8
J	10	6	7	8
K	10	6	7	8
L	10	6	7	8
M	10	6	7	8
N	9	5	6	7
O	9	5	6	7
P	8	4	5	6
Q	8	4	5	6
R	18	14	15	16
S	15	11	12	13
T	15	11	12	13
U	15	11	12	13
V	0	0	0	0
W	0	0	0	0

Table 44: Activities critical indices in Example Three

Activity	Activity Critical Index		
	Deterministic Approach	Probabilistic Framework	Nonlinear-Integer Framework
A	0.5	0.42	0.42
B	0.5	0.42	0.42
C	0.31	0.23	0.31
D	0.31	0.23	0.31
E	0	0	0
F	0	0	0
G	0	0	0
H	0.08	0.03	0.05
I	0.08	0.03	0.05
J	0.08	0.03	0.05
K	0.01	0.01	0.01
L	0.01	0.01	0.01
M	0.01	0.01	0.01
N	0.01	0	0
O	0.01	0	0
P	0.05	0	0
Q	0.05	0	0
R	0	0	0
S	0	0	0
T	0	0	0
U	0	0	0
V	0.56	0.42	0.37
W	0.56	0.42	0.37

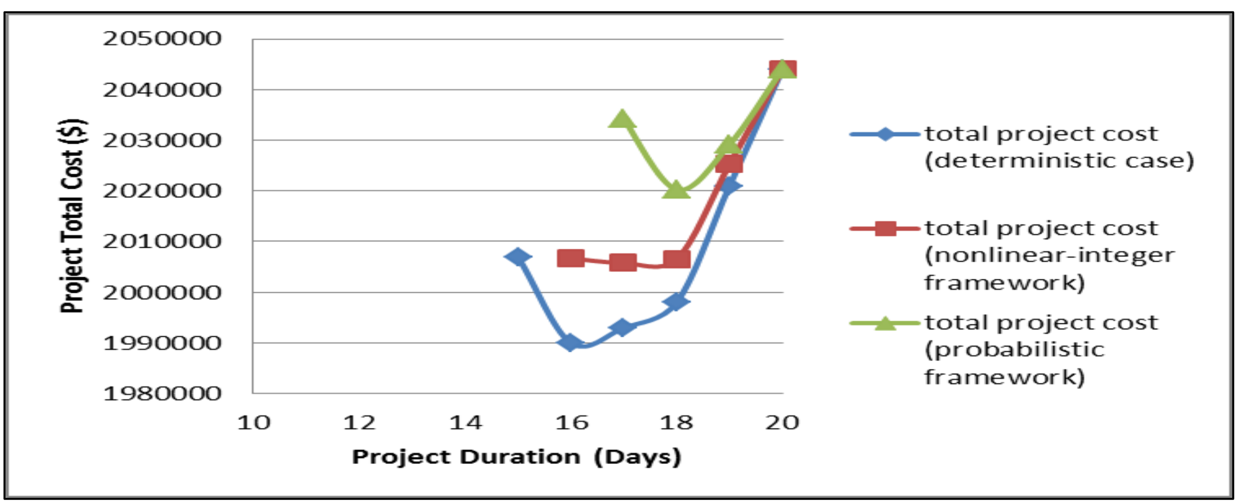


Figure 29: Total project cost vs project duration in all cases in Example Three

5.2.4 Example Four

Example four is adopted from Elbeltagi [63], and reproduced in Table 45. Columns 6 to 8 are added to use them in finding the float unit cost, while columns 9 and 10 are added to perform the stochastic analysis. Figure 30 illustrates the project schedule and precedence relationships. According to Elbeltagi [63], the normal project duration is 59 days, and the normal total cost is $43,875. The project optimum duration is 51 days, while the total project cost associated with this duration is $43,545.

Table 45:Elbeltagi project data

Activity	Normal Duration	Crashed Duration	CUC	TF	EFC	LFC	FUC	Duration Mean	Duration Standard Deviation
A	12	10	100	0	7000	_	_	12	1.5
B	8	6	150	2	5000	5036	18	8	2.25
C	15	12	200	0	4000	_	_	15	0.5
D	23	23	0	4	5000	5088	22	23	2.5
E	5	4	50	2	1000	1030	15	5	0.25
F	5	4	300	2	3000	3024	12	5	1.5
G	20	15	60	0	6000	_	_	20	0.65
H	13	11	40	2	2500	2548	24	13	1
I	12	10	75	0	3000	_	_	12	1.2

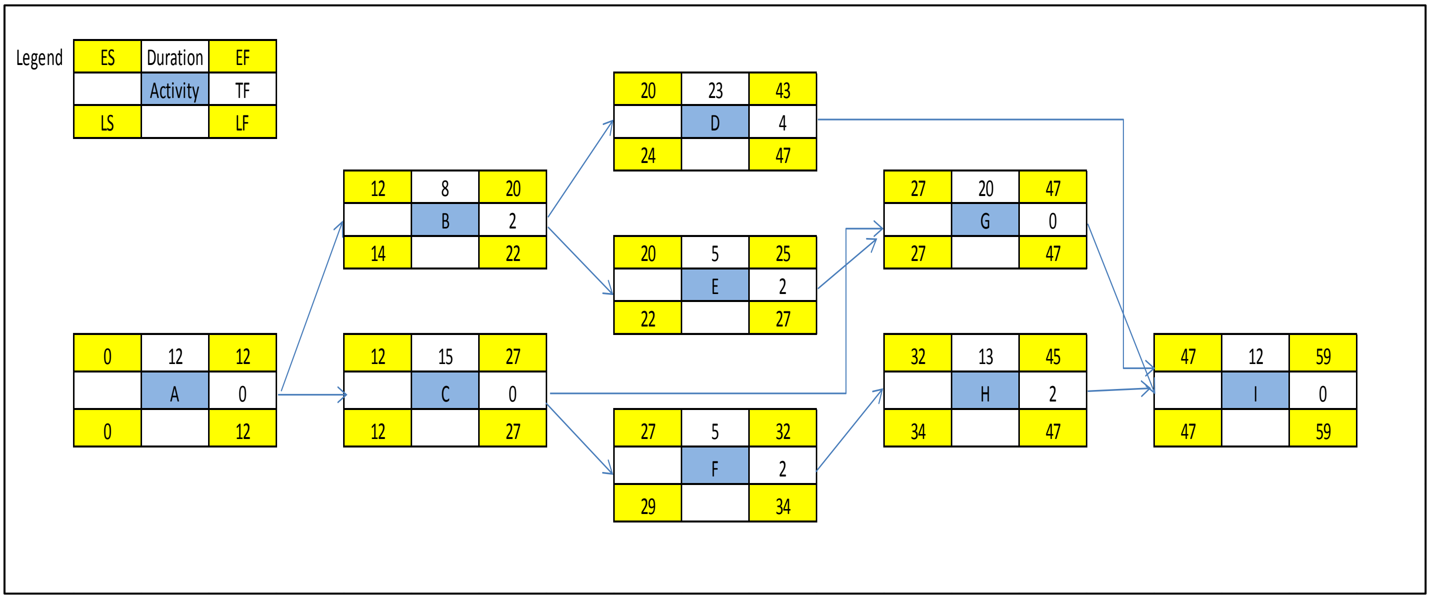


Figure 30: Schedule network & activities relations in Example Four

After developing the framework and running it using What's Best, the deterministic duration without the float loss cost is found to be 51 days, and the associated total cost is $43,545. The results are consistent with the ones found by Elbeltagi [63]. The duration considering float loss cost is 53 days with an associated cost of $43,711. The stochastic analysis resulted in 54 days project duration with an associated total cost of $ 43,792.73.

In terms of criticality ratio, the deterministic scenario resulted in a critical ratio of 0.667 while the probabilistic framework and the nonlinear integer framework resulted in a criticality ratio of 0.556 and 0.667, respectively (the criticality ratio is equal in the deterministic and the nonlinear framework as the number of critical activities is the same and critical path didn't change in both cases). Moreover, the probabilistic framework and the NLIP framework performed better in terms of the probability of finishing on time. The probability of finishing the project normally within 59 days is 0.393415562, and within 51 days (deterministic case) is 0.225519109, and within 54 days (probabilistic framework) is 0.356437597, while the probability of finishing within 53 days (NLIP framework) is 0.313370215. In terms of schedule flexibility, the nonlinear-integer framework and the stochastic framework performed better than the deterministic approach.

Table 46 illustrates the remaining schedule total float in the three cases. Moreover, the activities criticality indices when using the nonlinear-integer framework and the stochastic framework are better than that when applying the deterministic approach to find the optimization solution.

Table 46: Comparison between all cases in terms of remaining total float in Example Four

Activity	Total Float in Normal Case	Total Float in Deterministic Case	Total Float in Nonlinear-Integer Framework Case	Total Float in Stochastic Framework Case
A	0	0	0	0
B	2	0	2	2
C	0	0	0	0
D	4	0	2	2
E	2	2	2	3
F	2	0	0	0
G	0	0	0	1
H	2	0	0	0
I	0	0	0	0

Table 47 shows a comparison between the three cases in terms of schedule-activities' criticality indices.

Table 47: Activities critical indices in Example Four

Activity	Activity Critical Index		
	Deterministic Approach	Probabilistic Framework	Nonlinear-Integer Framework
A	1	1	1
B	0.54	0.39	0.4
C	0.86	0.79	0.76
D	0.51	0.36	0.26
E	0.07	0.08	0.2
F	0.72	0.49	0.6
G	0.42	0.43	0.6
H	0.72	0.49	0.6
I	1	1	1

Figure 31 shows the project total cost vs. the project duration for the results, and the optimum duration-cost when using the deterministic approach, the nonlinear-integer framework, and the stochastic framework. Figure 31 also indicates the difference between the three curves as an illustration of the impact of project flexibility loss over the total cost.

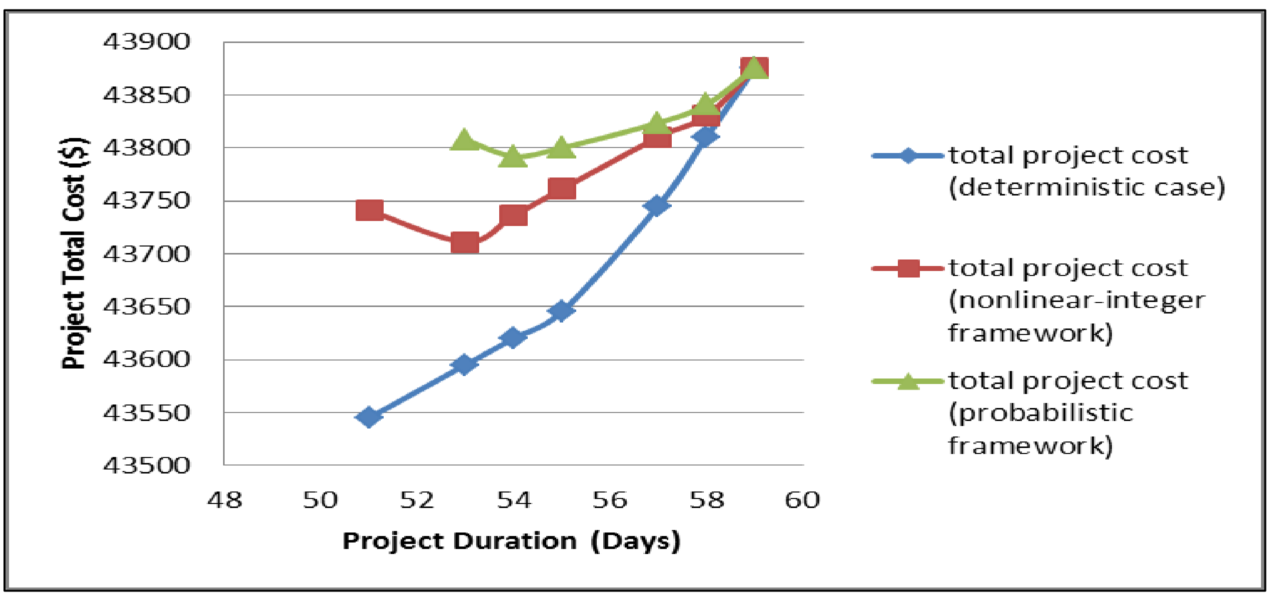


Figure 31: Total project cost vs project duration in all cases in Example Four

5.2.5 Example Five

Example five is adopted from Gould [64] and reproduced in Table 48. Columns 6 to 8 are added to use them in finding the float unit cost, while columns 9 and 10 are added to perform the stochastic analysis.

Figure 32 shows the project schedule and precedence relationships. According to Gould [64], the normal project duration is 37 days, and the normal project total cost is $119,745. The optimum project duration is 30 days, while the total project cost associated with this duration is $116,465.

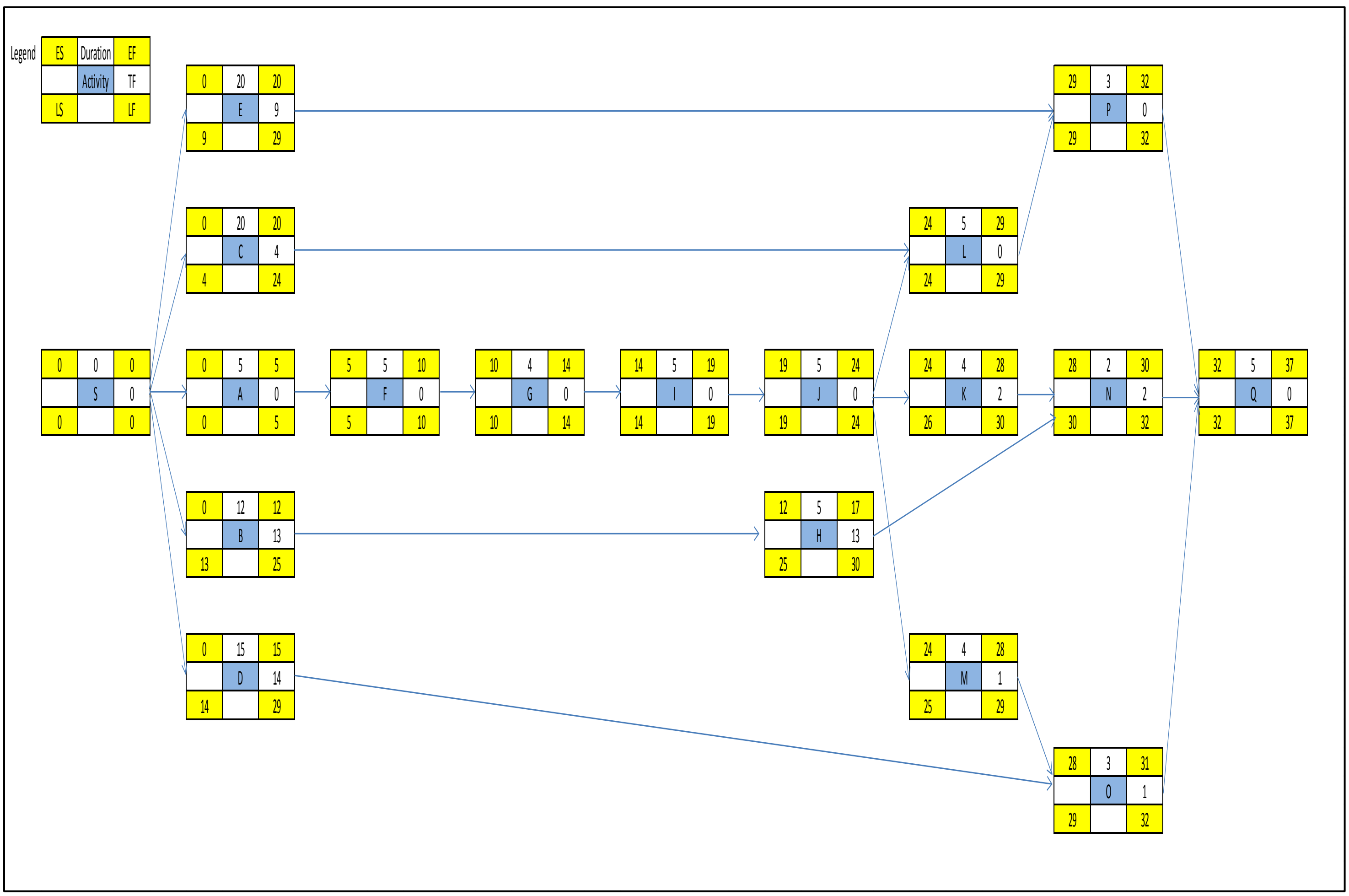


Figure 32: Schedule network & activities relations in Example Five

After developing the framework and running it using What's Best, the deterministic duration without the float loss cost is found to be 30 days, and the associated total cost is $116,465. The results are consistent with one found by Gould [64]. The duration considering float loss cost is 32 days with an associated cost of $118,035. The stochastic analysis resulted in 35 days project duration with an associated total cost of $116,604.992.

Table 48: F. Gould project data

Activity	Normal Duration	Crashed Duration	CUC	TF	EFC	LFC	FUC	Duration Mean	Duration Standard Deviation
A	5	3	500	0	3,000	-	-	5	0.35
B	12	-	0	13	6,200	7,500	100	12	0.5
C	20	-	0	4	8,400	8,620	55	20	1
D	15	-	0	14	8,550	9,390	60	15	3.25
E	20	-	0	9	5,900	6,530	70	20	3.85
F	5	3	1,175	0	9,450	-	-	5	0.2
G	4	3	1,100	0	7,500	-	-	4	1.5
H	5	3	800	13	6,000	6,325	25	5	1.325
I	5	3	850	0	10,200	-	-	5	0.25
J	5	3	700	0	6,300	-	-	5	1.2
K	4	3	750	2	8,250	8,340	45	4	2
L	5	3	660	0	2,550	-	-	5	0.75
M	4	3	750	1	3,360	3,420	60	4	2.62
N	2	1	1,050	2	6,450	6,520	35	2	1.22
O	3	2	900	1	3,375	3,400	25	3	1.34
P	3	2	480	0	1,560	-	-	3	0.3
Q	5	3	420	0	4,200	-	-	5	0.65

Based on the results obtained in this example, a comparison is done between the deterministic results, the nonlinear-integer framework results, and the stochastic framework results. In terms of criticality ratio, the deterministic scenario is 0.647 while the probabilistic framework and the nonlinear integer framework resulted in a criticality ratio of 0.471 and 0.588, respectively. Moreover, the probabilistic framework and the NLIP framework performed better in terms of the probability of finishing on time. The probability of finishing the project normally within 37 days is 0.341980523, and within 30 days (deterministic case) is 0.165335684, and within 35 days (probabilistic framework) is 0.338124405, while the probability of finishing within 32 days (NLIP framework) is 0.2545436. In terms of schedule flexibility, the nonlinear-integer framework and the stochastic framework performed better than the deterministic approach. Table 49 illustrates the remaining schedule total float in the three cases. Moreover, the activities' criticality indices when using the nonlinear-integer framework and the stochastic framework are better than that when applying the deterministic approach to find the optimization solution. Table 50 shows a comparison between the three cases in terms of criticality index for each activity.

Figure 33 shows the project total cost vs. the project duration for the results when using the deterministic approach, the nonlinear-integer model, and the stochastic model. Figure 33, shows the optimum duration-cost for each case, and the difference in cost between the nonlinear-integer model results and the stochastic model results from the deterministic results. This difference measures the impact of project flexibility loss in terms of money.

Table 49: Comparison between all cases in terms of remaining total float in Example Five

Activity	Total Float in Normal Case	Total Float in Deterministic Case	Total Float in Nonlinear-Integer Framework Case	Total Float in Stochastic Framework Case
A	0	0	0	0
B	13	8	10	13
C	4	0	2	4
D	14	9	11	14
E	9	5	7	9
F	0	0	0	0
G	0	0	0	0
H	13	8	10	13
I	0	0	0	0
J	0	0	0	0
K	2	1	1	2
L	0	0	0	0
M	1	0	0	1
N	2	1	1	2
O	1	0	0	1
P	0	0	0	0
Q	0	0	0	0

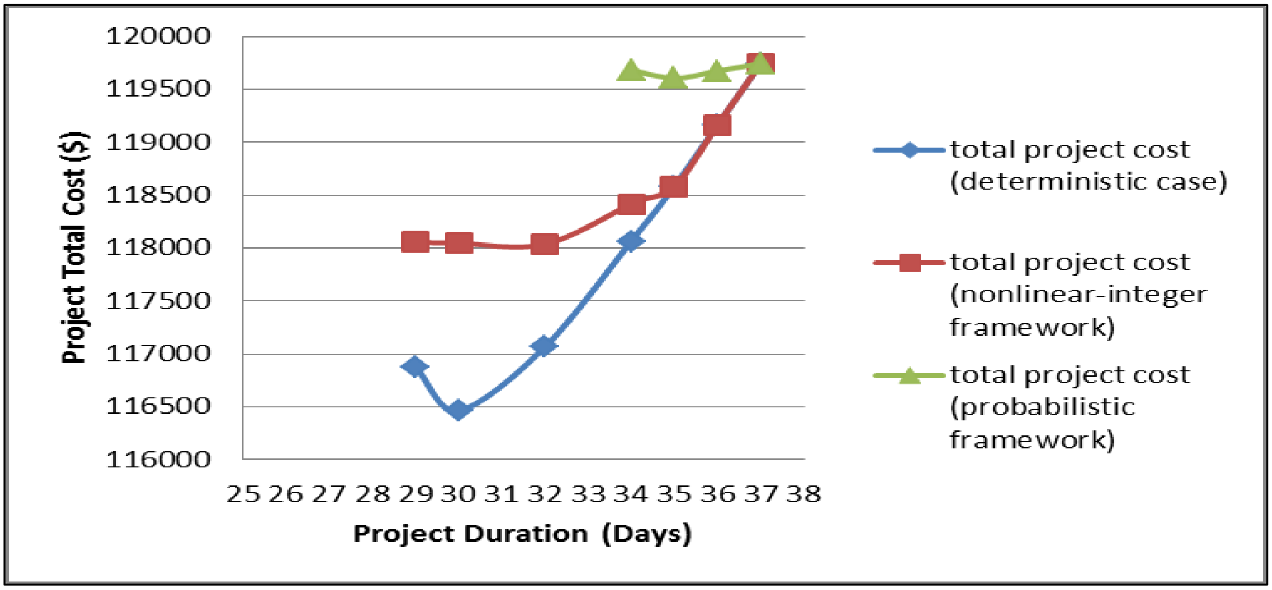


Figure 33: Total project cost vs project duration in all cases in Example Five

Table 50: Activities critical indices in Example Five

Activity	Activity Critical Index		
	Deterministic Approach	Probabilistic Framework	Nonlinear-Integer Framework
A	0.79	0.69	0.69
B	0	0	0
C	0.39	0.08	0.18
D	0.01	0	0
E	0.1	0	0.04
F	0.79	0.69	0.69
G	0.79	0.69	0.69
H	0	0	0
I	0.79	0.69	0.69
J	0.79	0.69	0.69
K	0.25	0.24	0.35
L	0.61	0.68	0.52
M	0.59	0.44	0.48
N	0.25	0.24	0.35
O	0.59	0.44	0.48
P	0.67	0.60	0.53
Q	1	1	1

Table 51 summarizes the results of the five examples found via the classical deterministic approach, manual-probabilistic framework, and the NLIP framework along with their probability of finishing the project on time.

Table 51: Summary of the five examples results

Example	Normal Case			Deterministic Solution			Manual- Probabilistic Framework Solution			NLIP Framework Solution		
	Project Duration	Total Project Cost	POF	Project Duration	Total Project Cost	POF	Project Duration	Total Project Cost	POF	Project Duration	Total Project Cost	POF
Example One (Isidore & Back [7])	20 days	$8,215	0.355	12 days	$7,940	0.109	17 days	$8,146.5	0.253	18 days	$8,155	0.279
Example Two (Oxley & Poskitt [61])	24 days	$30,520	0.426	17 days	$28,870	0.209	18 days	$29,742.5	0.250	21 days	$30,252	0.386
Example Three Zeinalzadeh [62])	20 days	$2,044,000	0.342	16 days	$1,990,000	0.122	18 days	$2,020,269.525	0.204	17 days	$2,005,795	0.161
Example Four (Elbeltagi [63])	59 days	$43,875	0.393	51 days	$43,545	0.226	54 days	$ 43,792.73	0.356	53 days	$43,711	0.313
Example Five (Gould [64])	37 days	$119,745	0.342	30 days	$116,465	0.165	35 days	$ 116,604.992	0.338	32 days	$118,035	0.255

In light of the results presented in Table 51, one can notice the improved probability of finishing the project on time if the manual-probabilistic framework or the NLIP framework is used to find an optimum/efficient solution to the optimization problem. The two developed frameworks indeed allows the decision makers to experience a new tradeoff between time, cost, and project flexibility while improving the chances of meeting the targeted project duration within the planned budgeted cost.

5.3 Chapter Conclusion

This chapter presented the results of five examples that are solved using the two frameworks developed; the stochastic framework and the nonlinear-integer framework. The five examples are adopted from various books and journal papers and reproduced with some required added data. The five examples are analyzed using the two frameworks in order to validate the two developed frameworks and compare between the deterministic solution and the frameworks solution in terms of probability of finishing on time, criticality index, remaining float, criticality ratio, and total cost graphs. The comparison is done to present the effect of considering float consumption in noncritical activities over project flexibility, cost, and duration. In all the cases, the frameworks' results are better than the deterministic results; which highlights the frameworks advantage over the deterministic approach.

CHAPTER SIX: SUMMARY, CONCLUSION & RECOMMENDATIONS

6.1 Summary and Conclusion

The Thesis was able to deliver a new time-cost optimization framework that considers a new criterion; float loss impact within noncritical activities. Based on the literature review performed, it was found that this criterion was never approached before and there is a need to incorporate the float loss impact to the compression problem. Adding the float loss impact to the time-cost tradeoff analysis will provide a modified logical approach for decision making and will improve the reliability and effectiveness of the crashing decision.

Two frameworks are developed in this Thesis; a stochastic framework and a nonlinear-integer framework. The stochastic framework makes use of @risk simulation to generate probabilities and find means and standard deviations for each activity being crashed at each crashing cycle, then analyzes the crashing results and compares between the activities; the activity with the lowest total extra cost/total project cost will be selected and crashed at that cycle. The initiation of the stochastic framework is based on the fact that the project's probability of finishing reduces when the project's duration is reduced; and consequently, the total float available for noncritical activities will be reduced as well, reducing the flexibility of the schedule. This reduction in the flexibility is associated with an impact. This impact is quantified first as a time then as a cost to be added to the crashing mechanism. The other nonlinear-integer framework uses What's Best solver to solve the optimization problem considering the float loss impact. The float loss cost is calculated in this framework using the commodity equation presented by Garza et al. [4] that finds the float cost for each noncritical activity as relation between the early finish cost, the late finish cost and the amount of total float available for the noncritical activity. The objective function, the decision variables and problem parameters, and the constraints are all formulated and presented in Chapter Four. The Thesis also presented a manual approach for schedule compression considering float loss impact in noncritical activities by incorporating the float loss cost found using Garza et al. [4] equation into the crashing problem and analyzing all the available activities for crashing then selecting the best activity with the least total extra cost/ total project cost at each cycle. The results of the two frameworks over six examples were consistent and met the hypothetical expectations while providing better results over the deterministic compression approach in terms of schedule flexibility, criticality index, and probability of finishing the project on time.

To conclude, the two frameworks developed in this Thesis could be beneficial, if implemented, as they can help management in providing an optimal/efficient solution to the time-cost tradeoff problem while maintaining a flexible schedule that meets the project needs with less inhabited risk.

6.2 Recommendation for Future Research

To best improve the speed or the computational efficiency of the framework developed, it is recommended to use the meta-heuristic techniques such as neural networks or particle swarm optimization techniques to reduce the calculation time as possible.

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