ROBUST CONTROL OF ELASTIC DRIVE SYSTEMS USING THE
IMMERSION AND INVARIANCE METHODOLOGY

by

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Thesis Title: Robust Control of Elastic Drive Systems Using the Immersion and Invariance Methodology

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DEDICATION

It’s an honor for me to dedicate this thesis to my loving and caring parents, Hakim Badshah and Mishal Zadi. Their role in my life is very important.
Abstract

This thesis is divided into three main parts. The first part presents a new robust control methodology to suppress the torsional vibrations in flexible drive systems. The position control of a nonlinear Two-Mass-Model system (2MM) is designed using the immersion and invariance approach. First, appropriate mapping functions are derived to convert the total nonlinear 2MM system into an equivalent reduced order system with rigid dynamics. This reduced order system is used as a target system to design a position controller, which is based on the standard PI type control technique. Next through immersion and invariance, the reduced order controller is applied to the nonlinear 2MM system to suppress the torsional vibrations and yield an over-damped response similar to the target system. The control law derivation and stability analysis of the target system are described and discussed. Simulation and experiments using a 2MM drive system are used to validate the proposed control methodology. The second part of the thesis presents the design of a nonlinear reduced order observer to estimate the load side position and load side speed of an elastic drive system. A reduced order observer is designed using the notion of invariant manifold. First, a manifold is defined in terms of the error dynamics, and then mapping functions are chosen in such a way that the error dynamics become asymptotically stable at the origin. Asymptotic stability of the error dynamics at the origin is proved. Simulation and experimental results are shown to validate the proposed methodology. Then the observer is combined with an I&I position controller and the results are validated through simulations and real time experiments. In the third part of this thesis, the input/output feedback linearization technique is explained and implemented on the same system. External and internal dynamics are derived for the 2MM. Feedback linearization gains are designed for external linear dynamics. The stability of the zero dynamics is also derived. Then, the I&I methodology, input/output feedback linearization, and PID position controllers are compared on the basis of their transient step response and robustness to external disturbance and parameter variations.

Keywords: Elastic drive system, immersion & invariance, invariant manifold, two mass model, nonlinear observer, vibration suppression
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Chapter 1: Introduction

In most industries, rolling mills, robotics arms, CNC machines, and semiconductor manufacturing machines are considered as multi-mass systems or two-mass systems. In all these machines, the drive end is connected to the load end through an elastic shaft and couplings. Figure 1-1 shows the rotating rolls that are used to reduce the thickness of the steel material and produce long steel strips that can be further used to produce different products. In order to get the correct weight, thickness, and width of the steel sheet at the end of the process, high precision speed and position control is required.

The easiest way to model the above system is as a one mass system but it is not practicable because of elasticity in the shaft and couplings. Most of these systems are modeled as two mass model elastic systems. If the shaft length is small, stiffer shafts can be used; however, for cost minimization, flexible shafts are mostly used. When the shaft length increases, rigid shafts cannot be used.

Position and speed control are widely used in applications such as robotic arms, rolling machines, and surface mounting machines. Most of these systems use rotatory actuators. The selection of the drive system is directed by factors such as the position and speed accuracy, type of load, load capacity, length of shaft, and maximum speed.
In applications like robotic arm welding or PCB manufacturing, high precision in position control and repeatability is required. For that purpose, rigid elements are used for power transmission because high performance cannot be obtained with elastic transmission elements. This increases the cost of manufacturing and also puts a constraint on the length of the shaft. To get the results close to rigid system with elastic shafts, direct feedback from the load side is also used by adding an additional encoder. Different techniques for better performance with elastic shafts and couplings exist, but the main obstacles in getting the performance close to rigid systems are the nonlinearities caused by elasticity, friction, and backlash. To cater for all the nonlinearities and to get the desired performance close to a rigid system, more extravagant control techniques are required.

High performance position and speed control is also required in hydraulic rolling machines and sheet molding machines. One of the hydraulic rolling machines and sheet molding machines is shown below.

![Figure 1-2 Hydraulic Four Roller Plate Rolling Machine [2]](image)

![Figure 1-3 Sheet Molding Compound Machine [3]](image)
Robotics manipulators can also be modeled as two mass systems. High performance position control is required in many applications using robotics arms (e.g., in the car manufacturing industry and high temperature welding or cutting).

![Figure 1-4 Robotic Arm [4]](image)

1.1 Background

Position control of elastic drive systems with high performance requires an accurate dynamic model. Elasticity in multi mass systems is unavoidable because of the elastic shaft and couplings. Elasticity in drive systems, in addition to nonlinearities also produces torsional vibration. It is necessary to dampen the torsional vibrations. For all the control schemes, the desired performance is to get a faster over-damped response. Achieving a faster response with damped torsional vibration is restricted by the elastic behavior of the system, friction, external disturbances, and unmodeled dynamics.

Torsional vibrations are compensated by using different control techniques including model-based predictive control, dynamics, and static models. Most of the techniques depend on the system modeling. In order to get the desired response, the system’s mathematical model should be close to the original system. System modeling also has
certain limitations due to nonlinearities in the system because of friction and
elasticity, backlash, dead-band, etc. Some nonlinearities can be modeled but some
hard nonlinearities are not possible to model. Due to these constraints, the model of
the system is not exactly the same as that of the real system. Thus, the control
techniques that depend on the model of the system cannot get the desired response
and dampen the torsional vibrations. Due to these issues, different nonlinear control
methodologies have gained importance. Soft computing control techniques have also
gained popularity because there is no need for mathematical modeling in such control
techniques.

In industries, torsional vibration in mechanical systems is a very critical issue. As the
length of the shaft increases, the damping of mechanical vibrations becomes more
difficult. These vibrations mostly occur at a low frequency, between 10-20 Hz. Vibrations in such systems are very dangerous and can cause damage to the whole
mechanical system. Such mechanical vibrations also affect the quality of the product.

Coupling in elastic drive systems is used to allow for any misalignments in the
system. Couplings are more elastic compared to shafts and are the main reason for
torsional vibrations. Torsional vibrations have different frequency components. In
most of industrial systems, feedback from the drive side is taken to the controller and
there is no feedback device at the load side. In a system with vibrations, it is not
possible to measure all states with a single feedback device. Because of these reasons,
vibration suppression in elastic drive systems have gained a lot of importance.
Industrial elastic drive systems like rolling mills and robotics arms need to operate
with a precise position and speed control and there is no room for torsional vibrations
in such systems. If the mechanical system is not modeled correctly or the controller
doesn’t take nonlinearities into effect, sudden changes in the reference signal may
also cause torsional vibrations in the system.

Some of the differences between rigid and elastic systems are mentioned below.

- Properties of an elastic system changes with time; on the other hand,
  properties of rigid systems do not change with time.
- Elastic systems are highly nonlinear because of the shaft and coupling elasticity, whereas in most of the cases rigid systems can be interpreted as linear systems.
- In elastic drive systems, the power transmission elements stretch due to elasticity before the motion and this stretching effect produces dead band. Dead band is a hard nonlinearity that is difficult to model mathematically. In rigid systems, there is no such nonlinearity.
- In elastic drive systems there are low frequency vibrations modes, which affect the performance of the controller. These vibrations modes add additional nonlinearities to the system. Rigid systems on the other hand have no such low frequency oscillations modes.
- Elastic drive systems have less load carrying capacity as compared to rigid systems.
- Rigid drive systems cannot be used in applications where high speed is required, whereas elastic drive systems can be used for high-speed applications. As the length of the shaft increases, the speed limit on a rigid system increases. That’s why in industries where the length of the shaft is longer, elastic drive systems are more attractive as compared to rigid drive systems.

If the performance of elastic drive systems can be enhanced and made closer to that of rigid systems, then it can be used in applications where position precision on the nano scale is required. In this way, constraints imposed by rigid systems on the shaft length and speed can be overcome and the system will also be cost effective. In order to get the desired performance from elastic drive systems, the controller should be chosen very carefully. Because of the nonlinearities in the system, it is not possible to get the desired performance using linear control techniques like PID. Even the nonlinear control techniques that linearize the system at certain operating points and ignore the nonlinearities are not good enough. There is a need for a nonlinear controller that takes into account all the nonlinearities in the system and operates on a wider range.

In this thesis the immersion and invariance control approach will be used to design a robust position controller for a single-axis flexible joint manipulator. The controller
will suppress the torsional vibrations and yield an over-damped response. The system is modeled as a two-mass-model system driven by a DC motor with a hysteresis current/torque control inner loop. A prototype experimental system is built to validate the developed control algorithm. Then, a nonlinear reduced order observer is designed for the same system using the notion of invariant manifold. Observer design is validated with simulations and experiments. The observer is then embedded with the position controller using I&I and tested again in simulations and real time experiments. At the end, in order to make a comparison with other control techniques, a position controller using input/output feedback linearization and PID is designed for two mass models and then the three controllers are compared on the basis of transient response and robustness to external disturbances and parameter variations.

1.2 Problem Statement

This thesis is divided into three parts:

- In the first part, a robust position control scheme to suppress the torsional vibrations in flexible drive systems using immersion and invariance methodology is presented. First, appropriate mapping functions are derived to convert the total nonlinear 2MM system into an equivalent reduced order system with rigid dynamics. This reduced order system is used as a target system to design a position controller, which is based on the standard PI type control technique. Next, through immersion and invariance, the reduced order controller is applied to the nonlinear 2MM system to suppress the torsional vibrations and yield an over-damped response similar to the target system. The control law derivation and stability analysis of the target system are described and discussed. Simulations and experiments using a 2MM drive system are used to validate the proposed control methodology. This method is well-suited for applications where the controller for the target system (reduced-order system) is known and need to be robustified with respect to the system under consideration. The control law which will be derived at the end will immerse the given system (higher order system) into the target system (lower order system).

- In the second part, the design of a nonlinear reduced-order observer to
estimate the load side position and load side speed of an elastic drive system is presented. The reduced-order observer is designed using the notion of invariant manifold. First, a manifold is defined in terms of the error dynamics, and then mapping functions are chosen in such a way that the error dynamics become asymptotically stable at the origin. Asymptotic stability of the error dynamics at the origin is proved. Simulation and experimental results are shown to validate the proposed methodology. Then the observer is combined with the I&I position controller and the results are shown in simulations and real time experiments.

- In the third part, input/output feedback linearization techniques are explained. Normal form is derived for 2MM system using I/O feedback linearization. Normal form splits the system into external and internal dynamics. External dynamics are the linear dynamics and the feedback controller is designed for the external dynamics. The zero dynamics are derived from the internal dynamics and then at the end, stability of zero dynamics is proved. Then a PID controller is designed for the 2MM system. At the end, the above three controllers are compared, using simulation, on the basis of transient response and robustness to external disturbances and parameter variation.

1.3 Thesis Contribution

The main contributions of this thesis can be summarized as follows

- Designing a robust position controller with vibration suppression for a 2MM system using immersion and invariance methodology.
- Implementing the designed controller in simulation.
- Designing and manufacturing a 2MM system prototype for real-time implementation.
- Implementing the designed controller in real time using dSPACE 1103.
- Designing a nonlinear reduced-order observer for a 2MM system using the notion of invariant manifold.
- Implementing the reduced-order observer in simulations and real time experiments.
Designing a position controller for a 2MM system using input/output feedback linearization and implementing it in simulation.

Comparing the performance of I&I controller, I/O feedback linearization controller, and PID controller.

1.4 Thesis Outline

In Chapter 1, an introduction of elastic drive systems is presented and a brief background describing the problems with elastic drive systems is given.

In Chapter 2, a detailed literature review is presented, related to different control techniques for elastic drive systems. The main area of review is focused on immersion and invariance methodology used to control different systems, nonlinear observer design using I&I, and feedback linearization techniques used for mechanical systems.

In Chapter 3 immersion and invariance methodology is explained in detail. A mathematical model for 2MM system is derived. A position controller with vibration suppression is designed for 2MM system using the I&I technique. Rigid body dynamics are chosen as the target system and a PI position controller is designed for the target system. Then, using I&I, 2MM system is immersed through mapping to behave like the target system. Simulation and experimental results are shown and discussed.

In Chapter 4, the input/output feedback linearization technique is explained in detail. 2MM system dynamics are split into external and internal dynamics using I/O feedback linearization. Then, feedback gains are designed for the external dynamics and stability for internal/zero dynamics is derived. Simulation results are shown and discussed.

In Chapter 5, the experimental setup is discussed in detail.

In Chapter 6, a nonlinear reduced-order observer is designed for 2MM system using the notion of invariant manifold. Simulation and experimental results are shown and discussed.

In Chapter 7, a comparison is made between position controllers designed via immersion and invariance methodology, input/output feedback linearization, and PID controllers for 2MM system based on transient response and robustness to
external disturbances and parameter variations. At the end, the conclusion, problems faced during the research, and future work is discussed.
Chapter 2: Literature Review

In this chapter, a literature review is presented. First, general linear and nonlinear control schemes are discussed. Then, controller design via immersion and invariance for different systems is studied in detail. After that, a literature review about input/output feedback linearization and PID controllers is presented. At the end, nonlinear observers are discussed.

2.1 Linear and Nonlinear Control Theory

Linear control theory has developed remarkably in the last few decades [5]-[6]. If the mathematical model of a given dynamical system is available, linearity can be determined by looking at the mathematical equations. If a mathematical model is not available then linearity can be checked by the input/output response of the dynamic system. Almost all practical systems are nonlinear but linear control methods can be still applied to practical systems [7]-[9]. Feedback linearization [10]-[11] and gain scheduling [12]-[13] are widely-used linear control techniques for nonlinear systems. However, if we want to operate the system over a wide range, linear control theory cannot be applied. In linear control theory, nonlinear systems are approximated as linear systems at certain operating points. A system behaves linearly only at these equilibrium points, and control laws are applicable only at these points. But this doesn’t mean nonlinear control is useless. Linear controllers have the following shortcomings:

- Linear control methods are only applicable on a small range because the system is approximated linearly only in that range.
- Linear control assumes that a system is linearizable but not all systems are linearizable (e.g., systems with hard nonlinearities).
- Linear controllers are designed with the assumption that all the parameters of the system are known.

In recent years, nonlinear control theory has gained a lot of importance because it takes the nonlinearities into consideration and allows the system to operate at a wider range [14]-[16]. Nonlinear disturbances in systems create many problems in
controlling the systems. Some of the disturbances can be modeled, but some nonlinearities are not possible to model. This greatly affects the performance of the controller.

Some properties of non-linear dynamic systems are:

- Real time systems have uncertainties in their parameters and some of the nonlinearities cannot be modeled. Nonlinear controllers are designed in order to account for all these uncertainties.
- Nonlinearities of a non-smooth nature are very common, such as friction and backlash. Most practical systems have these nonlinearities.
- Nonlinear systems do not obey the superposition principle.
- There is more than one equilibrium point in nonlinear systems.

Nonlinear control techniques are divided into two main categories:

1. Analytically-oriented techniques which are based on numerical models. The controller is designed using a systematic process.
2. Computationally-oriented techniques which do not rely on a numerical model of the system. These techniques are applied when the system is highly nonlinear and mathematical modeling is not possible. In most cases, system behavior is predicted by collecting input/output data of the system.

Analytically-oriented techniques are more popular than computationally-oriented techniques in most cases because the answers to the question of how the system behaves in certain conditions are well-answered by analytically-oriented techniques. Analytically-oriented techniques also helps in understanding a nonlinear system and how the controller is dealing with the nonlinearities, unknown parameters, and disturbances. The technique that is used in this thesis to design the controller is based on numerical computation and comes under the heading of an analytically-oriented technique.

The design of a high-performance motion control system requires accurate knowledge of the electromechanical dynamics, including linear and nonlinear transmission attributes of the system. Flexibility in the mechanical transmission elements is
inevitable in mechanical systems and is found to be undesirable by control system designers because it is often responsible for causing mechanical vibrations, which deteriorate the performance and restrict the closed-loop bandwidth. Combining friction with compliance makes it difficult to achieve a high-precision position and speed control in pointing and tracking servomechanisms.

Mechanical vibration suppression has been treated from many different points of view, including static and dynamic compensation models with feedback and feedforward compensation [17]-[21]. Most of the techniques used rely on a two-mass-model (2MM) system, i.e., a model in which the actuator and load are coupled by a flexible shaft, and friction depends only on the relative velocity of the motor and load. These models may be sufficiently accurate for systems in which relative velocity is almost always extremely small, such as in rolling mill drive systems or multi-axes flexible joint manipulators. The parameters of the 2MM system are thus required. Nonlinear and soft computing control techniques also have gained attention in vibration suppression [22]-[23] and speed control [24] of 2MM drive systems. Fuzzy control in [22] increases the robustness to parameters variation. In industry, soft computing control techniques are still not very popular.

2.2 Immersion and Invariance Methodology

In this thesis a new controller design methodology for two mass model is introduced. The method depends on the concept of immersion and invariant manifold. The new method, immersion and invariance (I&I) basic methodology is to accomplish the desired control objective by immersing the higher-order plant dynamics into a lower-order target system. It is suitable in conditions where a controller for a reduced order system is known and needs to be robustified with respect to the original system [25]. A graphical representation of the immersion and invariance (I&I) approach is shown in Figure 2-1. \( \pi(\cdot) \) maps the trajectory on the \( x \) space, restricted to the manifold as shown below.
In [26], a survey is made on immersion and invariance and its applicability is discussed via some examples. The method is also extended to feedback linearization where the design of an observer is typically required. In [27], a controller is designed based on immersion and invariance for an active suspension system and the results are compared with the back-stepping control law. Simulation results show that an immersion and invariance controller can stabilize the full-order system as well as the back-stepping controller in the nominal case, but is more robust to some parameter changes in the system. Moreover, when there is an unknown parameter, the adaptive immersion and invariance controller gives closer response to the known parameter case than the adaptive back-stepping controller. In [28], immersion and invariance is used to deal with the stabilization problem of magnetic suspension. This methodology has proven to be a very effective tool for the stabilization of nonlinear systems and its main advantage is making the closed loop system behave asymptotically like a target system. Experimental results illustrate the effectiveness of immersion and invariance in comparison with different additional studied controllers. In [29], immersion and invariance are used to design a controller for an antagonistic joint with nonlinear stiffness and then the immersion and invariance controller was compared with a simple PD controller. The immersion and invariance controller performed very well even in the presence of parameter uncertainties. In [30], a more detailed survey is made on the immersion and invariance method with more examples from real life systems. In [31], immersion and invariance is used for transient stability and voltage regulation of a power system with unknown mechanical power.
The authors of publications [26]-[33], however, do not elaborate on the derivation of the immersion and invariance control methodology, particularly the mapping between the original nonlinear system and the reduced-order target system. Some authors show the immersion and invariance control performance using computer simulation analysis, but they do not address the practical aspects of implementation.

2.3 Input / Output Feedback Linearization

In [34], an input-output robust tracking control scheme is designed for a robot manipulator with flexible joints. The controller works robustly to compensate for uncertainties including unknown flexibility, disturbances, parameters, and load variation. In [35], multi-variable discontinuous feedback linearization methodology is presented for the position control of a robot manipulator. The proposed controller can be applied to control different electrically-driven manipulators. The proposed control methodology can guarantee stability and satisfactory tracking performance. In [36], it is shown that dynamic feedback linearization is an efficient design scheme for trajectory tracking and set point regulation. The implementation is shown on the experimental setup. To check the performance of the proposed controller, it is compared with several existing control techniques. In [37], a controller for a nonlinear non-minimum-phase system is proposed, where input-output feedback linearization leads to unstable internal dynamics.

In this thesis the problem is approached by using the observability normal form in conjunction with input-output linearization. It is shown that the system is feedback-linearized upon neglecting a part of the system dynamics, with the neglected part being considered as a perturbation. A linear controller is designed for the perturbation resulting from the approximation. Stability analysis is provided.

2.4 PID Controller

In [38], the stability of manipulators with flexible joints under a class of PID is studied. It is shown that asymptotic stability of the position control system can be attained, even in the presence of unknown forces. It is shown that a PD action on the joint error and an integral action on the link error assure the asymptotic stability. [39] explains the implementation of a nonlinear digital PID controller for position control.
of electric drive systems. The non-aperiodic response of the PD controller is discussed and to overcome it, the nonlinear PD controller is redesigned so as to produce an aperiodic response. The nonlinear PD controller is then compared with the linear mode, and from the obtained results, its limitations and counter measures are discussed. [40] compares three controllers, PI, a PI-based state space control, and a model-based predictive control for the speed control of a drive system with resonant load. For the comparison, all the controllers are designed with equal bandwidth and the same setup is used to test all controllers. In [41], a fractional order PID controller is explained for the position control of a servomechanism. Actuator saturation and shaft torsional flexibility is taken into consideration. It is shown that if fractional order PID is properly designed and implemented, it gives better result than the conventional PID controller. In [42], a simple iterative and model independent control scheme is presented, starting initially with a PD control law and updating at discrete instants. In [43], it is shown that the PID controller does not create a suitable step response for a flexible link manipulator. Intense transient oscillation and high overshoot are the shortcomings of this controller.

2.5 Nonlinear Observers

An observer is designed for a system when some states of a system are not known or it is not practically possible to measure all of them. In control system applications, information about all the states at all times is important. Especially for feedback control techniques, all the states are required. State estimation has gained a lot of importance in both linear and nonlinear control systems. For linear control systems, observer design methodologies have been extensively developed, but for nonlinear systems, observer design methodologies are still under development and many research activities are going on in this area. Applying a linear observer design procedure to nonlinear systems has succeeded up to certain levels but nonlinearities in the system, impose significant constraints.

The observer theory started back in 1964 with the work of Luenberger. For state estimation, a model of the system called observer is constructed and in a way connected to the original system. If the observer order is the same as the order of the system under consideration and estimated are all the states, that observer is called a
“full-order observer.” If the observer only estimates the unknown states, it is called a “reduced-order observer.” When there is no random noise in the system and all the system parameters are known, the Luenberger observer and its extensions are usually used. When there are unknown parameters in the system, adaptive observers are used because with the state estimation, it also estimates unknown parameters. When there is noise in the system with known or unknown parameters, stochastic observers (Kalman filters) are used.

In [44] a speed observer is constructed for an n-degree of freedom mechanical system using immersion and invariance methodology. The observer design procedure is explained and implemented on a two-link planar robotic manipulator and robustness of the observer is shown in the presence of uncertainties. In [45], a nonlinear observer design for speed and load torque using the immersion and invariance technique is presented for general rotating machines. Convergence conditions are established using Lyapunov theory. The observer design is tested in simulations and the results are satisfactory. The robustness of the observer to parameter variations and disturbances is shown in simulations. In [46], partial state feedback for trajectory tracking of unicycle mobile robots with an exponentially stable angular attitude estimator is designed using the immersion and invariance technique. Simulations are done to show that it yields a locally asymptotically stable closed loop system. In [47] position feedback control for mechanical systems with speed estimators is discussed. Systems that can be rendered linear via a change of coordinates are considered. A reduced-order observer is constructed using immersion and invariance methodology. The observer design is explained in detail via several practical examples. At the end it is shown that the proposed observer can be used with state feedback and passivity-based controllers.

In [48], a vision-based range estimator is designed via immersion and invariance methodology. The proposed observer is combined with an input-state feedback controller and the stability of the closed loop system is proved using Lyapunov theory. In [49], a reduced-order observer design using invariant manifold theory is proposed. The method is explained in detail with the help of two practical examples. The proposed method is applicable to systems that might not be linear in unmeasured states. The validity of the method is shown in simulations. In [50], a general class of
nonlinear observers is considered and the observer design via immersion and invariance and circle criterion are explained in detail. In [51], an observer design for the unknown velocities of teleoperators using immersion and invariance is proposed. The closed loop stability of an I&I observer with a PD controller is shown. In [52], a globally-convergent reduced-order observer using invariant manifold theory for nonlinear systems is proposed. To illustrate the method, it is applied on practical systems like vision systems and two-degrees-of-freedom mechanical systems. The benefit of introducing a manifold in terms of the error dynamics is shown, which gives extra control for stabilizing the error dynamics.

In [53]-[56] an observer design using neural networks is proposed for different systems with high nonlinearities and for which mathematical modeling is very difficult or not feasible. Neural network techniques have shown good results. In [57], a high-gain observer with an output feedback controller is designed for minimum-phase nonlinear systems with unknown disturbance. In [58], three different high-gain observer design algorithms: classic high-gain Luenberger, high-gain extended Kalman filter, and adaptive-gain extended Kalman filter are presented. All of them are implemented in simulation and real time. Then, a comparison is made between them. In [59], a nonlinear observer design for mechanical systems is investigated. Nonlinear pendulums and Lienard systems are used to illustrate Sundarapan’s theorem for nonlinear observer design. In [60], an observer design survey is conducted. Different observers are examined on the basis of the dynamics structure of the plant, the information required by the observer, and the implementation of the observer. This review gives an idea about when to use which observer depending on the conditions. Detailed explanations of immersion and invariance methodology for controller and observer design with practical examples can be found in [61], [25], and [32]. Observer design for nonlinear systems using other methods are presented in [62]-[71].
Chapter 3: Controller Design Using Immersion and Invariance Methodology

In the I&I approach, a target dynamical system is introduced to capture the desired behavior of the system to be controlled. The control problem is then reduced to the design of a control law which guarantees that the controlled system asymptotically behaves like the target system. I&I methodology relies on finding a manifold in state-space that can be rendered invariant and attractive. It also relies on designing a control law that robustly steers the state of the system sufficiently close to this manifold.

3.1 I&I Theorem

This theorem will explain the set of sufficient conditions for the construction of a globally asymptotically stabilizing, static, state feedback control law for a general nonlinear system.

Consider the system,

\[ \dot{x} = f(x) + g(x)u \]  
(3-1)

with state \( x \in \mathbb{R}^n \) and control \( u \in \mathbb{R}^m \), with an equilibrium point \( x^* \in \mathbb{R}^n \) to be stabilized. Let \( p < n \) and assume we can find mapping \( x = \pi(\xi) \), such that the following holds:

(H1) (Target System) The System

\[ \dot{\xi} = a(\xi) \]  
(3-2)

with state \( \xi \in \mathbb{R}^p \), has an asymptotically stable equilibrium at \( \xi^* \in \mathbb{R}^p \) and \( x^* \in \pi(\xi^*) \).

(H2) (Immersion Condition) For all \( \xi \in \mathbb{R}^p \)

\[ f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi}{\partial \xi} a(\xi) \]  
(3-3)
where \( u = c \left( \pi \left( \xi \right) \right) \) that renders the manifold attractive.

(H3) (Implicit Manifold) The set identity
\[
\{ x \in \mathbb{R}^n \mid \phi(x) = 0 \} \{ x \in \mathbb{R}^n \mid x = \pi(\xi) \text{ for some } \xi \in \mathbb{R}^p \}\]
holds.

(H4) (Manifold attractively and trajectory boundedness) All trajectories of the system
\[
\dot{x} = f(x) + g(x) \psi(x, z)
\]
are bounded and satisfy \( \lim_{t \to \infty} z(t) = 0 \). \( z(t) \) is the distance between off the manifold dynamics and the manifold.

3.2 Mathematical Model of the System Under Consideration

The dynamic equations of the 2MM system are
\[
\begin{align*}
J_m \frac{d^2 \theta_m}{dt^2} &= \tau_m - K(\theta_m - \theta_l) - C(\dot{\theta}_m - \dot{\theta}_l) \\
J_l \frac{d^2 \theta_l}{dt^2} &= K(\theta_m - \theta_l) + C(\dot{\theta}_m - \dot{\theta}_l) - \tau_l
\end{align*}
\]
where \( J_m = J_{motor} + J_{Mdisk} \), \( J_l = J_{Ldisk} + J_{link} \), \( \theta_l \) is the load position, \( \theta_m \) is the motor position, \( J_m \) is the motor side inertia, \( J_l \) is the load side inertia, \( K \) is the shaft stiffness, \( C \) is the damping coefficient, \( \tau_m \) is the motor torque, and \( \tau_l \) is the load torque.

![Figure 3-1 Two Mass Model System](image)
The state variables selected are as follows:

\[
\begin{align*}
x_1 &= \theta_i \\
\dot{x}_1 &= \dot{\theta}_i \\
x_2 &= \theta_i \\
\dot{x}_2 &= \dot{\theta}_i \\
x_3 &= \theta_m \\
\dot{x}_3 &= \dot{\theta}_m \\
x_4 &= \dot{\theta}_m \\
\dot{x}_4 &= \ddot{\theta}_m
\end{align*}
\]

The 2MM system dynamics in terms of the state variables are given below:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{J_i} [K(x_3 - x_1) + C(x_4 - x_2) - Mg\frac{d}{2}\sin x_1] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{1}{J_m} [u - K(x_3 - x_1) - C(x_4 - x_2)]
\end{align*}
\]

(3-7)

where, \( \tau_m = u \), \( \tau_i = Mg\frac{d}{2}\sin x_1 \)

\[
\dot{x} = \begin{bmatrix}
x_2 \\
x_3 \\
\dot{x}_4 \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{J_i} [K(x_3 - x_1) + C(x_4 - x_2) - Mg\frac{d}{2}\sin x_1] \\
\frac{1}{J_m} [-K(x_3 - x_1) - C(x_4 - x_2)]
\end{bmatrix} u + \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

(3-8)

A rigid body system is shown in Figure 3-2.
3.3 I&I Implementation

Immersion and invariance methodology was implemented on the system under consideration, as explained below.

3.3.1 Defining the Target System

The target system is defined in terms of the rigid body dynamics of the 2MM system under consideration.

\[ \dot{\xi}_1 = \xi_2 \]

\[ \dot{\xi}_2 = \frac{1}{J_l} [\omega(\xi_1, \xi_2) - Mg \frac{d}{2} \sin \xi_1] \quad (3-9) \]

where \( \xi_1 \) is load position, \( \dot{\xi}_2 \) is load speed and \( \omega(\xi_1, \xi_2) = u + f(\xi_1, \xi_2) \). In this case \( u = 0 \).

3.3.2 Mapping

The mapping \( \pi(\xi) \) for the above defined target system needed to be determined.

\[ \pi(\xi) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \pi_3(\xi_1, \xi_2) \\ \pi_4(\xi_1, \xi_2) \end{bmatrix} \]

where \( \pi_3(\xi_1, \xi_2) \) and \( \pi_4(\xi_1, \xi_2) \) are the functions that need to be defined. Target system is defined as rigid body dynamics of 2MM, so \( x_1 = \xi_1, x_2 = \xi_2 \) and \( u = 0 \)

\[ \dot{x}_2 = \dot{\xi}_2 \quad (3-10) \]

Using Eq. 3-8

\[ \frac{1}{J_l} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1] = \frac{1}{J_l} [\omega(\xi_1, \xi_2) - Mg \frac{d}{2} \sin x_1] \]

We obtain

\[ [K(\pi - \xi_1) + C(\pi - \xi_2)] = \omega(\xi_1, \xi_2) \quad (3-11) \]
As

\[
\dot{\pi}_3(\xi_1, \xi_2) = \frac{\partial \pi_3}{\partial \xi_1} \dot{\xi}_1 + \frac{\partial \pi_3}{\partial \xi_2} \dot{\xi}_2
\]

\[
\dot{\pi}_3(\xi_1, \xi_2) = \frac{\partial \pi_3}{\partial \xi_1} \xi_2 + \frac{\partial \pi_3}{\partial \xi_2} \left[ -\frac{1}{J_l} [-\omega(\xi_1, \xi_2) + Mg \frac{d}{2} \sin \xi_1] \right]
\]

(3-12)

\[
\left[ K(\pi_3 - \xi_1) + C \left( \frac{\partial \pi_3}{\partial \xi_1} \xi_2 + \frac{\partial \pi_3}{\partial \xi_2} \left[ -\frac{1}{J_l} [-\omega(\xi_1, \xi_2) + Mg \frac{d}{2} \sin \xi_1] \right] - \xi_2 \right) \right] = \omega(\xi_1, \xi_2)
\]

(3-13)

A particular solution \( \pi_3 \) is defined below. In order to get a target system with an overdamped response, a damping term \( D > 0 \) is added.

\[
\pi_3 = -\frac{Mgd}{2K} \sin \xi_1 + \xi_1 (1 - D)
\]

(3-14)

\[
\frac{\partial \pi_3}{\partial \xi_1} = -\frac{Mgd}{2K} \cos \xi_1 + 1 - D
\]

As

\[
\dot{\pi}_3 = \pi_4
\]

(3-15)

\[
\pi_4 = \frac{\partial \pi_3}{\partial \xi_1} \dot{\xi}_1 + \frac{\partial \pi_3}{\partial \xi_2} \dot{\xi}_2
\]

\[
\pi_4 = -\xi_2 \frac{Mgd}{2K} \cos \xi_1 + \xi_2 (1 - D)
\]

(3-16)

Eq. 3-13 becomes

\[
\omega(\xi_1, \xi_2) = \left[ K(\pi_3 - \xi_1) + C \left( \frac{\partial \pi_3}{\partial \xi_1} \xi_2 - \xi_2 \right) \right]
\]

\[
\omega(\xi_1, \xi_2) = \left[ K \left( -\frac{Mgd}{2K} \sin \xi_1 + \xi_1 - D \xi_1 \right) - \xi_1 \right] + C \left( \left[ -\frac{Mgd}{2K} \cos \xi_1 + 1 - D \right] \xi_2 - \xi_2 \right)
\]

37
Substituting $\omega(\xi_1, \xi_2)$ in Eq. 3-9, the target system becomes

\[
\dot{\xi}_1 = \xi_2 \\
\dot{\xi}_2 = -\xi_2 \frac{CMgd}{2KJ_l} \cos\xi_1 - \frac{Mgd}{J_l} \sin\xi_1 - \frac{DK}{J_l} \dot{\xi}_1 - \frac{CD}{J_l} \dot{\xi}_2
\]  (3-18)

Comparing the rigid body dynamics with the dynamics of single pendulum given below,

\[
\dot{\xi}_1 = \xi_2 \\
\dot{\xi}_2 = -E'(\xi_1) - \xi_2 R(\xi_1)
\]  (3-19)

where $E(\xi_1)$ is the potential energy, $E'(\xi_1) = \frac{\partial E}{\partial \xi_1}$ and $R(\xi_1)$ is the damping function.

\[
E'(\xi_1) = \frac{Mgd}{J_l} \sin\xi_1 + \frac{DK}{J_l} \dot{\xi}_1
\]  (3-20)

\[
E(\xi_1) = \frac{Mgd}{J_l} (1 - \cos\xi_1) + \frac{DK}{2J_l} \dot{\xi}_1^2
\]

\[
R(\xi_1) = \frac{CMgd}{2KJ_l} \cos\xi_1 + \frac{CD}{J_l}
\]  (3-21)

To prove the asymptotic stability of the target system at the origin the Lyapunov function is defined below,

\[
V(\xi_1, \xi_2) = \frac{1}{2} \dot{\xi}_2^2 + E(\xi_1)
\]  (3-22)

The target dynamics will have an asymptotically stable equilibrium at the origin if the following conditions are satisfied,
\[ V(0,0) = 0 \]

\[ V(\xi_1, \xi_2) > 0, \quad \text{in } D_0 - \{0\}, \quad D_0 \to R^p, \quad -\frac{\pi}{2} \leq D_0 \leq \frac{\pi}{2} \]

\[ \dot{V}(\xi_1, \xi_2) < 0, \quad \text{in } D_0 - \{0\} \]

(3-23)

Eq. 3-19 becomes

\[ V(\xi_1, \xi_2) = \frac{1}{2} \xi_2^2 + \frac{Mgd}{J_l} (1 - \cos \xi_1) + \frac{DK}{2J_l} \xi_1^2 \quad (3-24) \]

\[ \dot{V}(\xi_1, \xi_2) = \xi_2 \dot{\xi}_2 + \frac{Mgd}{J_l} \sin \xi_1 \dot{\xi}_1 + \frac{DK}{J_l} \xi_1 \dot{\xi}_1 \]

\[ \dot{V}(\xi_1, \xi_2) = \xi_2 \dot{\xi}_2 + E'(\xi_1) \xi_2 \]

\[ \dot{V}(\xi_1, \xi_2) = \xi_2 \left( \dot{\xi}_2 + E'(\xi_1) \right) \]

\[ \dot{V}(\xi_1, \xi_2) = -\xi_2^2 R(\xi_1) \quad (3-25) \]

As it can be clearly seen that the first two conditions of stability are satisfied. For the third condition, \( R(\xi_1) \) should be greater than zero in the specified domain. The Jacobian method is also used to prove the stability of the target system.

The Jacobian matrix of the target system is given below

\[ J = \begin{bmatrix} 0 & 1 \\ \frac{CMgd}{2KJ_l} \sin \xi_1 - \frac{Mgd}{J_l} \cos \xi_1 - \frac{DK}{J_l} & -\frac{CMgd}{2KJ_l} \cos \xi_1 - \frac{CD}{J_l} \end{bmatrix} \quad (3-26) \]
The Jacobian matrix at the origin becomes

\[
J_{(0,0)} = \begin{bmatrix}
0 & 1 \\
\frac{-Mgd}{J_l} & \frac{-CMgd}{2KJ_l} \\
\frac{-DK}{J_l} & \frac{-CD}{2KJ_l}
\end{bmatrix}
\]  

(3-27)

The eigenvalues are

\[
\xi_1 = -\frac{2 \left(C^2D^2 - C^2DJ_l^2KMdg + \frac{C^2J_l^2K^2M^2d^2g^2}{4} + 4DJ_lK - 4J_lMd g \right)^{\frac{1}{2}} - 2CD + CJ_l^2KMdg}{4J_l}
\]

\[
\xi_2 = -\frac{2 \left(C^2D^2 - C^2DJ_l^2KMdg + \frac{C^2J_l^2K^2M^2d^2g^2}{4} + 4DJ_lK - 4J_lMd g \right)^{\frac{1}{2}} + 2CD - CJ_l^2KMdg}{4J_l}
\]

After substitution, the eigen values become

\[
\xi_1 = -2.9821 - 57.1269i
\]

\[
\xi_2 = -2.9821 + 57.1269i
\]

As the eigenvalues are in the left half plane, the target system has an asymptotically stable origin.

3.3.3 Manifold

The manifold can be implicitly defined as

\[
\mathcal{M} = \{x \in \mathbb{R}^4, \phi(x) = 0\}
\]  

(3-28)

where

\[
\phi_1(x) = x_3 - \pi_3(x_1, x_2)
\]

\[
\phi_2(x) = x_4 - \pi_4(x_1, x_2)
\]  

(3-29)
3.3.4 Manifold Attractivity and Trajectory Boundedness

In the last step, a controller is designed to ensure the manifold attractivity and the trajectory boundedness.

\[ z_1 = \phi_1(x) \quad \& \quad z_2 = \phi_2(x) \]  
\[ (3-30) \]

\( z_1 \) and \( z_2 \) represents the distance of off-the-manifold dynamics from the manifold.

\[ \dot{z}_1 = z_2 \]  
\[ (3-31) \]

\[ \dot{z}_2 = v \]

The controller for off the manifold dynamics is given by \( v \)

\[ v = -k_1 z_1 - k_2 z_2 \]  
\[ (3-32) \]

\( k_1 \) and \( k_2 \) are chosen in such a way that \( s^2 + k_2 s + k_1 \) is Hurwitz.

\[ \dot{z}_2 = \dot{x}_4 - \dot{\pi}_4(x_4, x_2) \]  
\[ (3-33) \]

\[ \dot{z}_2 = \frac{1}{J_m} [u - K(x_3 - x_1) - C(x_4 - x_2)] - \frac{\partial \pi_4}{\partial x_1} \dot{x}_1 - \frac{\partial \pi_4}{\partial x_2} \dot{x}_2 \]

\[ -k_1 z_1 - k_2 z_2 = \frac{1}{J_m} [\psi(x, z) - K(x_3 - x_1) - C(x_4 - x_2)] - \frac{\partial \pi_4}{\partial x_1} x_2 
- \frac{\partial \pi_4}{\partial x_2} \frac{1}{J_i} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1] \]

\[ \psi(x, z) = J_m \left[ -k_1 z_1 - k_2 z_2 + \frac{\partial \pi_4}{\partial x_1} x_2 + \frac{\partial \pi_4}{\partial x_2} \frac{1}{J_i} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1] \right] 
+ K(x_3 - x_1) + C(x_4 - x_2) \]

Using Eq. 3-16 the controller equation becomes

\[ \psi(x, z) = J_m \left[ -k_1 z_1 - k_2 z_2 + \left[ x_2 \frac{mgd}{2k} \sin x_1 \right] x_2 + \left[ -\frac{mgd}{2k} \cos x_1 + 1 - D \right] \frac{1}{J_i} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1] \right] 
+ K(x_3 - x_1) + C(x_4 - x_2) \]  
\[ (3-34) \]
As it has been proven, that target system has an asymptotically stable equilibrium at the origin and every asymptotic stable equilibrium point is attractive. Once the error dynamics goes to zero, the system will behave similar to the target system. Next the PI controller comes into effect and will keep all the dynamics bounded to the manifold.

3.3.5 Simulink Models and Simulation Results

The Simulink models and simulation results are shown below.

![Simulink Model of Complete System](image)
The Simulink model for the 2MM is shown in Figure 3-4.

![Figure 3-4 Simulink Model for 2MM](image)

The Simulink model for the target system is shown in Figure 3-5.

![Figure 3-5 Simulink Model for Target System](image)
The Simulink model for the mapping functions is shown in Figure 3-6.

The Simulink model for computing the control signal $u = \psi(x, z)$ is shown in Figure 3-7.
The simulation results for the actual system and target system are shown in Figures 3-8 and 3-9, respectively. $k_1$ and $k_2$ for the error dynamics are chosen in such a way that $s^2 + k_2 s + k_1$ is Hurwitz. As $u = 0$, so initial conditions $(\xi_1(0) = x_1(0) = 0.01 \text{ rad}, \xi_2(0) = x_2(0) = -0.01 \text{ rad/s})$ are given to both the system. It can be seen clearly that the actual system behaves similar to the target system once it is on the manifold. A step disturbance is applied at $t = 1$ second and the response of both systems is the same.

Figure 3-8 Plot of Load Position & Load Speed for Actual System

Figure 3-9 Plot of Load Position & Load Speed for Target System
3.4 PI Controller Design

From the above results it has been shown that the original system behaves similar to the target system once it comes to the manifold. But in that case there is no position controller involved. Initial conditions were given to both the actual and target systems, to check whether they behave in the same manner. After verifying the results in simulation, the next step is to introduce a PI position controller.

The controller gains for the PI were designed using an IP configuration. At first, an IP controller was implemented on the target system and the response was underdamped. Because mapping for this controller was not possible, we moved to PI implementation, for which mapping was developed.

![Figure 3-10 Inner IP loop for Speed Control and outer P loop for Position Control](image)

In order to find the general equations for the controller gains, a closed-loop transfer function was derived analytically, as shown in Eq. 3-35 below. Then, tuning was done to get the over-damped response.

\[
G_{cl} = \frac{\frac{K_p K_i}{J_t s^3}}{1 + \frac{K_p K_i}{J_t s^3} + \frac{K_i}{J_t s^2} + \frac{K_v}{J_t s}}
\]  \hspace{1cm} (3-35)

\[
G_{cl} = \frac{\frac{K_p K_i}{J_t}}{s^3 + \frac{K_p K_i}{J_t} + \frac{K_i}{J_t} s + \frac{K_v}{J_t} s^2}
\]  \hspace{1cm} (3-36)
We rewrite the denominator in the following factor form

\[ \Delta = (s^2 + 2\zeta w_n s + w_n^2)(s + a) \]  \hspace{1cm} (3-37)

\[ \Delta = s^3 + (a + 2\zeta w_n)s^2 + (2\zeta w_n a + w_n^2)s + aw_n^2 \]  \hspace{1cm} (3-38)

To find the controller gains, the denominator of Eq. 3-36 is compared with Eq. 3-38.

\[ a + 2\zeta w_n = \frac{K_p}{J_t} \]  \hspace{1cm} (3-39)

\[ 2\zeta w_n a + w_n^2 = \frac{K_i}{J_t} \]  \hspace{1cm} (3-40)

\[ aw_n^2 = \frac{K_p K_i}{J_t} \]  \hspace{1cm} (3-41)

The equations for the controller gains are

\[ K_p = \frac{aw_n}{2\zeta a + w_n} \]

\[ K_i = J_t(2\zeta w_n a + w_n^2) \]  \hspace{1cm} (3-42)

\[ K_v = J_t(a + 2\zeta w_n) \]

Below table shows the PI controller design parameters and gains.

<table>
<thead>
<tr>
<th>( t_z ) ms</th>
<th>( \zeta ) (zeta)</th>
<th>( K_p )</th>
<th>( K_i )</th>
<th>( K_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.6</td>
<td>10.86</td>
<td>3.97</td>
<td>0.217</td>
</tr>
</tbody>
</table>

Table 3-1 PI Controller Design Parameters and Gains
A PI configuration was then implemented on the target system. The gains that were designed in the IP configuration were used and the response was over-damped.

![Figure 3-11 Inner PI loop for Speed Control and outer P loop for Position Control](image)

The output $u$ of the PI controller is derived below.

$$u = K_v e_v + K_i \int e_v dt$$  \hspace{1cm} (3-43)

where $e_v = e_p K_p - \xi_2$ is the velocity error and $e_p$ is the position error.

$$u = K_v K_p e_p + K_i K_p \int e_p dt - K_v \xi_2 - K_i \int \xi_2 dt$$

$$u = K_v K_p e_p + K_i K_p \int e_p dt - K_v \xi_2 - K_i \xi_1$$

$$u = K_i e_p + K_2 \int e_p dt - K_3 \xi_1 - K_4 \xi_2$$  \hspace{1cm} (3-44)

### 3.5 I&I Implementation with PI Controller

A PI controller was designed for the target system to get an over-damped response. The same controller should also control the original system to get the same response once it will come to the manifold. The same steps were followed as before in section 3.3, but the mapping is different because a controller is introduced in this case.
3.5.1 Mapping

The mapping $\pi(\xi)$ for the above defined target system need to be determined.

$$
\pi(\xi) = \begin{bmatrix}
\xi_1 \\
\xi_2 \\
\pi_3(\xi_1, \xi_2) \\
\pi_4(\xi_1, \xi_2)
\end{bmatrix}
$$

(3-45)

where $\pi_3(\xi_1, \xi_2)$ and $\pi_4(\xi_1, \xi_2)$ are the functions that need to be defined. The target system is defined as the rigid body dynamics of the 2MM, so $x_1 = \xi_1$ and $x_2 = \xi_2$.

$$
\dot{x}_2 = \dot{\xi}_2
$$

(3-46)

$$
\frac{1}{J_1} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1] = \frac{1}{J_t} \omega(\xi_1, \xi_2)
$$

$$
[K(\pi_3 - \xi_1) + C(\pi_4 - \xi_2) - Mg \frac{d}{2} \sin x_1] = \frac{J_1}{J_t} \omega(\xi_1, \xi_2)
$$

(3-47)

The equation for $\omega(\xi_1, \xi_2)$ is

$$
\omega(\xi_1, \xi_2) = b_1 \sin \xi_1 + b_2 \xi_2 \cos \xi_1 + K_1 e + K_2 \int e \, dt - K_3 \xi_2 - K_4 \xi_1
$$

(3-48)

Choosing the mapping $\pi_3$

$$
\pi_3 = -\frac{Mgd}{2K} \sin \xi_1 + \xi_1 + a_1 e + a_2 \int e \, dt + a_3 \xi_2 + a_4 \xi_1
$$

(3-49)

we have

$$
\dot{\pi}_3 = \pi_4
$$

$$
\pi_4 = \frac{\partial \pi_3}{\partial \xi_1} \dot{\xi}_1 + \frac{\partial \pi_3}{\partial \xi_2} \dot{\xi}_2
$$
\[ \pi_4 = -\xi_2 \frac{Mg d}{2K} \cos \xi_1 + \xi_2 - a_1 \xi_2 + a_2 e + a_3 \dot{\xi}_2 + a_4 \dot{\xi}_2 \]  \hspace{1cm} (3-50)

Substituting \( \omega(\xi_1, \xi_2) \), \( \pi_3 \), and \( \pi_4 \) in Eq. 3-47, we get

\[
\begin{align*}
\left[ K \left( -\frac{Mg d}{2K} \sin \xi_1 + a_1 e + a_2 \int e \, dt + a_3 \xi_2 + a_4 \xi_1 \right) + C \left( -\xi_2 \frac{Mg d}{2K} \cos \xi_1 - a_1 \xi_2 + a_2 e + a_3 \dot{\xi}_2 + a_4 \dot{\xi}_2 \right) - Mg \frac{d}{2} \sin x_1 \right] &= \frac{l_1}{l_2} \omega(\xi_1, \xi_2) \\
\left[ K \left( -\frac{Mg d}{2K} \sin \xi_1 + a_1 e + a_2 \int e \, dt + a_3 \xi_2 + a_4 \xi_1 \right) + C \left( -\xi_2 \frac{Mg d}{2K} \cos \xi_1 - a_1 \xi_2 + a_2 e + a_3 \frac{1}{l_1} \omega(\xi_1, \xi_2) + a_4 \dot{\xi}_2 \right) - Mg \frac{d}{2} \sin x_1 \right] &= \frac{l_1}{l_2} \omega(\xi_1, \xi_2) \\
\left[ K \left( -\frac{Mg d}{2K} \sin \xi_1 + a_1 e + a_2 \int e \, dt + a_3 \xi_2 + a_4 \xi_1 \right) + C \left( -\xi_2 \frac{Mg d}{2K} \cos \xi_1 - a_1 \xi_2 + a_2 e + a_4 \dot{\xi}_2 \right) - Mg \frac{d}{2} \sin x_1 \right] &= \frac{l_1}{l_2} \left( 1 - \frac{C a_3}{l_1} \right) \left[ a_1 \sin \xi_1 + a_2 \dot{\xi}_2 \cos \xi_1 + a_3 \frac{1}{l_1} \right] \\
\left[ K \left( -\frac{Mg d}{2K} \sin \xi_1 + a_1 e + a_2 \int e \, dt + a_3 \xi_2 + a_4 \xi_1 \right) + C \left( -\xi_2 \frac{Mg d}{2K} \cos \xi_1 - a_1 \xi_2 + a_2 e + a_4 \dot{\xi}_2 \right) - Mg \frac{d}{2} \sin x_1 \right] &= \frac{l_1}{l_2} \left( 1 - \frac{C a_3}{l_1} \right) \\
\end{align*}
\]  \hspace{1cm} (3-51)

Eq. 3-51 has six unknown constants. In order to find the unknowns, the coefficients are compared.

\[
\begin{align*}
-Mg d &= \frac{l_1}{l_2} b_1 \left( 1 - \frac{C a_3}{l_1} \right) \\
-Mg d C &= \frac{l_1}{l_2} b_2 \left( 1 - \frac{C a_3}{l_1} \right) \\
K a_1 + C a_2 &= \frac{l_1}{l_2} K_1 \left( 1 - \frac{C a_3}{l_1} \right) \\
K a_2 &= \frac{l_1}{l_2} K_2 \left( 1 - \frac{C a_3}{l_1} \right)
\end{align*}
\]  \hspace{1cm} (3-52) \hspace{1cm} (3-53) \hspace{1cm} (3-54) \hspace{1cm} (3-55)
\[ Ka_3 + C a_4 - C a_1 = -\frac{J_t}{J_t} K_3 \left( 1 - \frac{C a_3}{J_t} \right) \quad (3-56) \]

\[ Ka_4 = -\frac{J_t}{J_t} K_4 \left( 1 - \frac{C a_3}{J_t} \right) \quad (3-57) \]

Solving Eq. 3-56

\[ Ka_3 + C a_4 - C a_1 = -\frac{J_t}{J_t} K_3 \left( 1 - \frac{C a_3}{J_t} \right) \]

\[ Ka_3 = -\frac{J_t}{J_t} K_3 \left( 1 - \frac{C a_3}{J_t} \right) - C a_4 + C a_1 \quad (3-58) \]

From Eq. 3-57

\[ a_4 = -\frac{J_t}{J_t} K_4 \left( 1 - \frac{C a_3}{J_t} \right) \quad (3-59) \]

Substituting \( a_4 \) in Eq. 3-58

\[ Ka_3 = -\frac{J_t}{J_t} K_3 \left( 1 - \frac{C a_3}{J_t} \right) + \frac{J_t}{J_t} K_4 \left( 1 - \frac{C a_3}{J_t} \right) + C a_1 \]

\[ Ka_3 = -\frac{J_t}{J_t} \left( 1 - \frac{C a_3}{J_t} \right) \left[ -K_3 + \frac{K_4 C}{K} \right] + C a_1 \]

\[ Ka_3 + \frac{C a_3}{J_t} \left( -K_3 + \frac{K_4 C}{K} \right) = \frac{J_t}{J_t} \left( -K_3 + \frac{K_4 C}{K} \right) + C a_1 \]

\[ a_3 \left[ K + \frac{C}{J_t} \left( -K_3 + \frac{K_4 C}{K} \right) \right] = \frac{J_t}{J_t} \left[ -K_3 + \frac{K_4 C}{K} \right] + C a_1 \]

\[ a_3 \left[ \frac{K^2 J_t + C \left( -K_3 K + K_4 C \right)}{K J_t} \right] = \frac{J_t}{J_t} \left( -K_3 K + K_4 C \right) + K J_t C a_1 \]
\( a_3 \) in terms of \( a_1 \) is

\[
a_3 = \frac{J_t(-K_3 K + K_4 C) + KJ_t Ca_1}{K^2 J_t + C(-K_3 K + K_4 C)} \quad (3-60)
\]

Solving Eq. 3-54

\[
Ka_1 + Ca_2 = \frac{J_t}{J_t} K_1 \left( 1 - \frac{Ca_3}{J_t} \right)
\]

\[
Ka_1 = \frac{J_t}{J_t} K_1 \left( 1 - \frac{Ca_3}{J_t} \right) - Ca_2
\]

\[
a_1 = \frac{J_t K_1}{J_t K_1} \left( 1 - \frac{Ca_3}{J_t} \right) - \frac{J_t C K_2}{J_t K_2} \left( 1 - \frac{Ca_3}{J_t} \right)
\]

\[
a_1 = \left( 1 - \frac{C J_t (-K_3 K + K_4 C) + KJ_t C^2 a_1}{J_t K^2 J_t + J_t C(-K_3 K + K_4 C)} \right) \left( \frac{J_t K_1}{J_t K_2} - \frac{J_t C K_2}{J_t K_2} \right)
\]

\[
a_1 = \left( \frac{J_t K^2 J_t + J_t C(-K_3 K + K_4 C) - [C J_t (-K_3 K + K_4 C) + KJ_t C^2 a_1]}{J_t K^2 J_t + J_t C(-K_3 K + K_4 C)} \right) \left( \frac{J_t K_1}{J_t K_2} - \frac{J_t C K_2}{J_t K_2} \right)
\]

\[
a_1 = \left( \frac{J_t K^2 J_t - KJ_t C^2 a_1}{J_t K^2 J_t + J_t C(-K_3 K + K_4 C)} \right) \left( \frac{J_t K_1}{J_t K_2} - \frac{J_t C K_2}{J_t K_2} \right)
\]

\[
a_1 = \left( \frac{J_t K - C^2 a_1}{K^2 J_t + C(-K_3 K + K_4 C)} \right) \left( \frac{K_1 K - C K_2}{K} \right)
\]

52
\[
\begin{align*}
a_1 + \left( \frac{C^2a_1}{K^2J_t + C(-K_3K + K_4C)} \right) \left( \frac{K_1K - CK_2}{K} \right) &= \frac{J_1K(K_1K - CK_2)}{K^3J_t + CK(-K_3K + K_4C)} \\
a_1 \left( 1 + \frac{C^2(K_1K - CK_2)}{K^3J_t + CK(-K_3K + K_4C)} \right) &= \frac{J_1K(K_1K - CK_2)}{K^3J_t + CK(-K_3K + K_4C)} \\
a_1[K^3J_t + CK(-K_3K + K_4C) + C^2(K_1K - CK_2)] &= J_1K(K_1K - CK_2)
\end{align*}
\]

The equation for \(a_1\) is
\[
a_1 = \frac{J_1K(K_1K - CK_2)}{K^3J_t + CK(-K_3K + K_4C) + C^2(K_1K - CK_2)} \tag{3-61}
\]

Solving Eq. 3-55
\[
Ka_2 = \frac{J_1}{J_t}K_2 \left( 1 - \frac{Ca_3}{J_1} \right)
\]

\[
a_2 = \frac{J_1K_2}{J_tK} \left( \frac{J_1K^2J_t - KJ_tC^2a_1}{J_1K^2J_t + J_tC(-K_3K + K_4C)} \right)
\]

\[
a_2 = K_2 \left( \frac{J_1K - C^2a_1}{K^2J_t + C(-K_3K + K_4C)} \right)
\]

\[
a_2 = K_2 \left( \frac{J_1K - C^2a_1}{K^3J_t + CK(-K_3K + K_4C) + C^2(K_1K - CK_2)} \right)
\]
\[ a_2 = K_2 \left( \frac{J_1 K [K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)] - C^2 J_1 K (K_1 K - CK_2)}{[K^2 J_t + C (-K_3 K + K_4 C)][K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)]} \right) \]

\[ a_2 = K_2 \left( \frac{J_1 K [K^3 J_t + CK (-K_3 K + K_4 C)]}{[K^2 J_t + C (-K_3 K + K_4 C)][K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)]} \right) \]

The equation for \( a_2 \) is

\[ a_2 = \frac{J_1 K^2 K_2}{K^2 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)} \] (3-62)

Solving Eq. 3-60

\[ a_3 = \frac{J_1 (-K_3 K + K_4 C) + KJ_t Ca_1}{K^2 J_t + C (-K_3 K + K_4 C)} \]

\[ a_3 = \frac{J_1 (-K_3 K + K_4 C) + KJ_t C J_1 K (K_1 K - CK_2)}{K^2 J_t + C (-K_3 K + K_4 C)} \]

\[ a_3 = \frac{J_1 (-K_3 K + K_4 C)[K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)] + K^2 J_1 C J_1 K (K_1 K - CK_2)}{[K^2 J_t + C (-K_3 K + K_4 C)][K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)]} \]

\[ a_3 = \frac{J_1 (-K_3 K + K_4 C)[K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)] + K^2 J_1 C J_1 K (K_1 K - CK_2)}{[K^2 J_t + C (-K_3 K + K_4 C)][K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)]} \]

\[ a_3 = \frac{[K^2 J_t + C (-K_3 K + K_4 C)][J_1 K (-K_3 K + K_4 C) + C J_1 K (K_1 K - CK_2)]}{[K^2 J_t + C (-K_3 K + K_4 C)][K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)]} \]
The equation for $a_3$ is

$$a_3 = \frac{[J_t K(-K_3 K + K_4 C) + C J_t (K_1 K - C K_2)]}{K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - C K_2)}$$

(3-63)

Solving Eq. 3-57

$$K a_4 = -\frac{J_t}{J_t} \left( 1 - \frac{C a_3}{J_t} \right)$$

(3-64)

Solving in the same way as Eq. 3-55, $a_4$ becomes

$$a_4 = -K_4 \left( \frac{J_t K^3 J_t + CK (-K_3 K + K_4 C)}{[K^3 J_t + C (-K_3 K + K_4 C)] [K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - C K_2)]} \right)$$

The equation for $a_4$ is

$$a_4 = \frac{-J_t K^2 K_4}{K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - C K_2)}$$

(3-65)

Solving Eq. 3-52

$$-M gd = \frac{J_t}{J_t} b_1 \left( 1 - \frac{C a_3}{J_t} \right)$$

$$-M gd = b_1 \left( \frac{J_t K^3}{[K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - C K_2)]} \right)$$

The equation for $b_1$ is

$$b_1 = \frac{-M gd [K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - C K_2)]}{J_t K^3}$$

(3-66)
Solving Eq. 3-53

\[- \frac{MgdC}{2K} = \frac{J}{J_t} b_2 \left( 1 - \frac{Ca_3}{J_t} \right)\]

\[- \frac{MgdC}{2K} = b_2 \left( \frac{J_t K^3}{[K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)]} \right)\]

The equation for \( b_2 \) is

\[ b_2 = \frac{-MgdC[K^3 J_t + CK (-K_3 K + K_4 C) + C^2 (K_1 K - CK_2)]}{2J_t K^4} \tag{3-67} \]

The target system becomes

\[ \dot{\xi}_1 = \xi_2 \]

\[ \dot{\xi}_2 = \frac{1}{J_t} \left( b_1 \sin \xi_1 + b_2 \xi_2 \cos \xi_1 + K_1 e + K_2 \int e \, dt - K_3 \xi_2 - K_4 \xi_1 \right) \tag{3-68} \]

If \( u = 0 \), then the target system becomes

\[ \dot{\xi}_1 = \xi_2 \]

\[ \dot{\xi}_2 = \frac{1}{J_t} (b_1 \sin \xi_1 + b_2 \xi_2 \cos \xi_1) \tag{3-69} \]

Comparing the rigid body dynamics \((u = 0)\) with the dynamics of a single pendulum given below.

\[ \dot{\xi}_1 = \xi_2 \]

\[ \dot{\xi}_2 = -E'(\xi_1) - \xi_2 R(\xi_1) \tag{3-70} \]

where \( E'(\xi_1) = \frac{\partial E}{\partial \xi_1} \) and \( R(\xi_1) \) is the damping function.
To prove the asymptotic stability of the target system at the origin a Lyapunov function is defined below.

\[ V(\xi_1, \xi_2) = \frac{1}{2} \xi_2^2 + E(\xi_1) \]  

(3-73)

where \( E \) is the potential energy of the target system. The target dynamics will have an asymptotically stable equilibrium at the origin if the following conditions are satisfied.

\[ V(0,0) = 0 \]
\[ V(\xi_1, \xi_2) > 0, \text{ in } D_o - \{0\}, \quad D_o \rightarrow \mathbb{R}^p, \quad -\frac{\pi}{2} \leq D_o \leq \frac{\pi}{2} \]
\[ \dot{V}(\xi_1, \xi_2) < 0, \text{ in } D_o - \{0\} \]  

(3-74)

Eq. 3-73 becomes

\[ V(\xi_1, \xi_2) = \frac{1}{2} \xi_2^2 + \frac{b_1}{j_1} (1 - \cos \xi_1) \]  

(3-75)

\[ \dot{V}(\xi_1, \xi_2) = \xi_2 \dot{\xi}_2 + \frac{b_1}{j_1} \sin \xi_1 \dot{\xi}_1 \]
\[ \dot{V}(\xi_1, \xi_2) = \xi_2 \dot{\xi}_2 + E'(\xi_1) \xi_2 \]
\[ \dot{V}(\xi_1, \xi_2) = \xi_2 \left( \dot{\xi}_2 + E'(\xi_1) \right) \]
\[ \dot{V}(\xi_1, \xi_2) = -\xi_2^2 R(\xi_1) \]  

(3-76)
As it can be clearly seen that first condition of stability is satisfied. For the second
condition to be satisfied, \(b_1\) should be positive. For the third condition, \(R(\xi_1)\) should
be greater than zero and for that, \(b_2\) should be positive.

Eq 3-66 and Eq. 3-67 gives

\[
b_1 = 0.2544
\]

\[
b_2 = 0.00023868
\]

As both the values are positive, the target system has an asymptotically stable
equilibrium. In order to prove the close loop stability of the target system, the
Jacobian method is used.

The Jacobian matrix is given below

\[
J = \begin{bmatrix}
0 & 1 \\
\frac{b_1 \cos \xi_1 - b_2 \sin \xi_1 - K_1 - K_2 \xi_1 - K_4}{J_t} & \frac{b_2 \cos \xi_1 - K_3}{J_t}
\end{bmatrix}
\] (3-77)

The Jacobian matrix becomes at the origin becomes

\[
J_{(0,0)} = \begin{bmatrix}
0 & 1 \\
\frac{b_1 - K_1 - K_4}{J_t} & \frac{b_2 - K_3}{J_t}
\end{bmatrix}
\] (3-78)

The eigenvalues are

\[
\xi_1 = \frac{-K_3 + b_2 - (K_3^2 - 2K_3 b_2 + b_2^2 + 4J_t b_1 - 4J_t K_1 - 4J_t K_4)^{1/2}}{2J_t}
\]

\[
\xi_2 = \frac{-K_3 + b_2 + (K_3^2 - 2K_3 b_2 + b_2^2 + 4J_t b_1 - 4J_t K_1 - 4J_t K_4)^{1/2}}{2J_t}
\]

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Eigenvalues becomes

\[ \xi_1 = -35.0384 - 29.8807i \]
\[ \xi_2 = -35.0384 + 29.8807i \]

As the eigenvalues are in the left half of the plane, the target system has asymptotically stable origin.

3.5.2 Manifold

The manifold can be implicitly defined as

\[ \mathcal{M} = \{ x \in \mathbb{R}^4, \phi(x) = 0 \} \]  

where,

\[ \phi_1(x) = x_3 - \pi_3(x_1, x_2) \]
\[ \phi_2(x) = x_4 - \pi_4(x_1, x_2) \]  

3.5.3 Manifold Attractivity and Trajectory Boundedness

In this step, a controller is designed to ensure the manifold attractivity and trajectory boundedness.

\[ z_1 = \phi_1(x) \quad \& \quad z_2 = \phi_2(x) \]  

\[ z_1 \text{ and } z_2 \text{ represent the distance of off-the-manifold dynamics from the manifold.} \]

\[ \dot{z}_1 = z_2 \]
\[ \dot{z}_2 = v \]  

The controller for off the manifold dynamics is given by \( v \)

\[ v = -k_1 z_1 - k_2 z_2 \]  

\( k_1 \text{ and } k_2 \) are chosen in such a way that \( s^2 + k_2 s + k_1 \) is Hurwitz.

\[ \dot{z}_2 = \dot{x}_4 - \dot{\pi}_4(x_1, x_2) \]  

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The controller equation becomes

\[
\dot{z}_2 = \frac{1}{f_m} [u - K(x_3 - x_1) - C(x_4 - x_2)] - \frac{\partial \pi_4}{\partial x_1} \dot{x}_1 - \frac{\partial \pi_4}{\partial x_2} \dot{x}_2
\]

\[-k_1 z_1 - k_2 \dot{z}_2 = \frac{1}{f_m} [\psi(x, z) - K(x_3 - x_1) - C(x_4 - x_2)] - \frac{\partial \pi_4}{\partial x_1} x_2 - \frac{\partial \pi_4}{\partial x_2} \dot{x}_2
\]

\[
\psi(x, z) = f_m \left[ -k_1 z_1 - k_2 \dot{z}_2 + \frac{\partial \pi_4}{\partial x_1} x_2 + \frac{\partial \pi_4}{\partial x_2} \dot{x}_2 \right] + K(x_3 - x_1) + C(x_4 - x_2)
\]

As it has been proven, the target system has an asymptotically stable equilibrium at the origin and every asymptotic stable equilibrium point is attractive. Once the error dynamics goes to zero, the system will behave similar to the target system. Next the PI controller comes into effect and will keep all the dynamics bounded to the manifold.

3.6 Simulink Models and Simulation Results for the Target System

Figure 3-12 shows the block diagram for I&I implementation with PI controller.
The Simulink model of the target system with a PI controller is shown in Figure 3-13.
The Simulink model for the PI controller is shown in Figure 3-14.

![Figure 3-14 Simulink Model for PI Controller]

The Simulink model for the DC motor is shown in Figure 3-15.

![Figure 3-15 Simulink Model for DC Motor]

The Simulink model for the target system is shown in Figure 3-16.

![Figure 3-16 Simulink Model for Target System]
Simulation results for the load position and load speed are shown in Figures 3-17 and 3-18, respectively. It can be seen that the gains of the PI controller were tuned to get the overdamped response.

Figure 3-17 Actual and Reference Load Position Simulation Results for Target System

Figure 3-18 Actual Load Speed Simulation Results for Target System
Simulation results for reference and actual currents are shown in Figures 3-19 and 3-20.

Figure 3-19 Actual and Reference Current Simulation Results for Target System

Figure 3-20 Zoomed Version of Actual and Reference Current Simulation Results for Target System
3.7 Simulink Models and Simulation Results for 2MM

The Simulink model of the 2MM with an I&I and PI controller is shown in Figure 3-21.

Figure 3-21 2MM with I&I and PI Controller Simulation
The Simulink model for the 2MM is shown in Figure 3-22.

![Simulink Model for 2MM](image)

Figure 3-22 Simulink Model for 2MM

The Simulink model for mapping and computing u is shown in Figure 3-23.

![Simulink Model for Mapping and Computing u](image)

Figure 3-23 Simulink Model for Mapping and Computing u
The Simulink model for computing $u$ is shown in Figure 3-24.

![Simulink Model for Computing $u$](image)

**Figure 3-24 Simulink Model for Computing $u$**

The Simulink model for mapping is shown in Figure 3-25.

![Simulink Model for Mapping](image)

**Figure 3-25 Simulink Model for Mapping**
Simulation results for the load position and load speed are shown in Figures 3-26 and 3-27, respectively. It can be seen that the response is the same as that of the target system.

Figure 3-26 Actual and Reference Load Position Simulation Results for 2MM

Figure 3-27 Actual Load Speed Simulation Results for 2MM
Simulation results for the motor position and motor speed are shown in Figures 3-28 and 3-29, respectively.

Figure 3-28 Actual Motor Position Simulation Results for 2MM

Figure 3-29 Actual Motor Speed Simulation Results for 2MM
Simulation results for actual and reference currents are shown in Figures 3-30 and 3-31.

Figure 3-30 Actual and Reference Current Simulation Results for 2MM

Figure 3-31 Zoomed Version of Actual and Reference Current Simulation Results for 2MM
Simulation results for error dynamics $z_1$ and $z_2$ are shown in Figures 3-32 and 3-33, respectively.

Figure 3-32 Error Dynamics $z_1$ Simulation Results

Figure 3-33 Error Dynamics $z_2$ Simulation Results
3.8 Simulink Models and Experimental Results for Target System

The Simulink model of the target system with a PI controller for real-time implementation is shown in Figure 3-34.

![Figure 3-34 Target System with PI Controller Experimental](image1)

The subsystem of the above target system experimental block is shown in Figure 3-35.

![Figure 3-35 Subsystem of Target System Experimental Block](image2)
The Simulink model for the hysteresis current controller block for real time implementation is shown in Figure 3-36.

![Figure 3-36 Subsystem of Hysteresis Current Controller Block](image)

The Simulink model for the DC motor block for real-time implementation is shown in Figure 3-37.

![Figure 3-37 Subsystem of DC Motor Block](image)

The Simulink model for the encoder block is shown in Figure 3-38.

![Figure 3-38 Subsystem of Encoders Block](image)

The Simulink model for the averaging filter block is shown in Figure 3-39.

![Figure 3-39 Subsystem of Averaging Filter Block](image)
The experimental results for the load position and load speed are shown in Figures 3-40 and 3-41, respectively.

Figure 3-40 Actual Load Position Experimental Results for Target System

Figure 3-41 Actual Motor Position Experimental Results for Target System
Experimental results for the actual and reference currents are shown in Figures 3-42 and 3-43. In the simulation results for the target system, the actual and reference current does not have the oscillations because the system is acting exactly as rigid. In real time to make the elastic system rigid, the shaft diameter is increased by 3 times, which increases the shaft stiffness by 81 times. The mechanical system put a constraint on the shaft diameter. The system is not exactly rigid and that’s why oscillations appear in the current.

Figure 3-42 Actual and Reference Current Experimental Results for Target System

Figure 3-43 Zoomed Version of Actual and Reference Current Experimental Results for Target System
3.9 Simulink Models and Experimental Results for 2MM

The Simulink model of the 2MM with an I&I and PI controller for real-time implementation is shown in Figure 3-44.

![Figure 3-44 2MM with I&I and PI Controller Experimental](image.png)

The subsystem of the above 2MM with the experimental block is shown in Figure 3-45.

![Figure 3-45 Subsystem of 2MM with I&I Experimental Block](image.png)
Experimental results for the load position and load speed are shown in Figures 3-46 and 3-47, respectively.

Figure 3-46 Actual Load Position Experimental Results for 2MM

Figure 3-47 Actual Motor Position Experimental Results for 2MM
Experimental results for the actual and reference currents are shown in Figures 3-48 and 3-49.

Figure 3-48 Actual and Reference Current Experimental Results for 2MM

Figure 3-49 Zoomed Version of Actual and Reference Current Experimental Results for 2MM
Experimental results for error dynamics $z_1$ and $z_2$ are shown in Figures 3-50 and 3-51, respectively.

Figure 3-50 Error Dynamics $z_1$ Experimental Results

Figure 3-51 Error Dynamics $z_2$ Experimental Results

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3.10 Comparison of Experimental Results

Comparison of the experimental results for the target system, 2MM with I&I and PI controllers, and 2MM with only a PI controller are shown below.

Figure 3-52 Load Position Experimental Results

Figure 3-53 Motor Position Experimental Results
Figure 3-54 Reference Current Experimental Results

Figure 3-55 Actual Current Experimental Results
3.11 Robustness of I&I Position Controller

To check the robustness of the controller to disturbances and parameter uncertainty, the load inertia is increased by 30% and an additional 25% of load torque is introduced. The controller was also tested with a 50% load torque increase. The results are shown below. Controller shows very robust response to disturbances and parameter uncertainty.

![Graph 1: Comparison of Experimental Load Position with and without disturbances for I&I Controller](image1)

Figure 3-56 Comparison of Experimental Load Position with and without disturbances for I&I Controller

![Graph 2: Comparison of Experimental Motor Position with and without disturbances for I&I Controller](image2)

Figure 3-57 Comparison of Experimental Motor Position with and without disturbances for I&I Controller
New Experimental Results for 2MM with I&I Position Controller

Lecroy Oscilloscope is used to plot the reference position, load position, motor position and actual current at different rise time in real time. In Table 3-2 three different settling time for load positions are given with the PI controller gains and rise time. The minimum settling time achieved is 153 ms. Further minimizing the settling time make the system unstable. In Table 3-3 the zeta value is changed, which changes the $K_p$, $K_i$ and $K_v$ value is unaffected. The minimum settling time achieved is 50 ms in this case.

<table>
<thead>
<tr>
<th>$t_s$ (Ref) ms</th>
<th>Zeta</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_v$</th>
<th>$t_r$ ms</th>
<th>$t_s$ (2%) ms</th>
<th>Error (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>0.6</td>
<td>28.9855</td>
<td>28.2288</td>
<td>0.5799</td>
<td>67.20</td>
<td>153</td>
<td>3</td>
</tr>
<tr>
<td>200</td>
<td>0.6</td>
<td>21.7391</td>
<td>15.8787</td>
<td>0.4349</td>
<td>101.14</td>
<td>210</td>
<td>10</td>
</tr>
<tr>
<td>250</td>
<td>0.6</td>
<td>17.3913</td>
<td>10.3624</td>
<td>0.3480</td>
<td>131.90</td>
<td>274</td>
<td>24</td>
</tr>
</tbody>
</table>

Table 3-2 PI Controller Gains for Different Settling Times

<table>
<thead>
<tr>
<th>$t_s$ (Ref) ms</th>
<th>Zeta</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_v$</th>
<th>$t_r$ ms</th>
<th>$t_s$ (2%) ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>0.475</td>
<td>47.2464</td>
<td>42.4486</td>
<td>0.6691</td>
<td>30.95</td>
<td>50</td>
</tr>
<tr>
<td>130</td>
<td>0.425</td>
<td>54.8227</td>
<td>45.6963</td>
<td>0.6691</td>
<td>34.55</td>
<td>55</td>
</tr>
<tr>
<td>130</td>
<td>0.4</td>
<td>59.1716</td>
<td>47.7954</td>
<td>0.6691</td>
<td>43.59</td>
<td>77</td>
</tr>
</tbody>
</table>

Table 3-3 PI Controller Gains for Different Zeta Values

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Figure 3-58 Motor Position & Load Position Experimental Results for 2MM with Rise Time 67.20 ms

Figure 3-59 Actual Current & Load Position Experimental Results for 2MM with Rise Time 67.20 ms
Figure 3-60 Motor Position & Load Position Experimental Results for 2MM with Rise Time 101.14 ms

Figure 3-61 Actual Current & Load Position Experimental Results for 2MM with Rise Time 101.14 ms
Figure 3-62 Motor Position & Load Position Experimental Results for 2MM with Rise Time 131.90 ms

Figure 3-63 Actual Current & Load Position Experimental Results for 2MM with Rise Time 131.90 ms
Figure 3-64 Motor Position & Load Position Experimental Results for 2MM with Rise Time 30.95 ms

Figure 3-65 Actual Current & Load Position Experimental Results for 2MM with Rise Time 30.95 ms
Figure 3-66 Motor Position & Load Position Experimental Results for 2MM with Rise Time 34.55 ms

Figure 3-67 Actual Current & Load Position Experimental Results for 2MM with Rise Time 34.55 ms
Figure 3-68 Motor Position & Load Position Experimental Results for 2MM with Rise Time 43.59 ms

Figure 3-69 Actual Current & Load Position Experimental Results for 2MM with Rise Time 43.59 ms
Chapter 4: Feedback Linearization

The idea behind feedback linearization techniques is to find the transformation that will be used to get the normal form. From the normal form, the linear dynamics are separated from the nonlinear dynamics. Then controller can be designed for the linear dynamics and the stability of the nonlinear dynamics should also be satisfied.

There are two main techniques for feedback linearization:

- Input / Output Feedback Linearization
- Input / State Feedback Linearization

In general, feedback linearization techniques are based on two operations:

- Change of Coordinates (Transformation)
- State Feedback Control

The flow chart below shows different feedback control techniques.

![Flow Chart]

Figure 4-1 State Transformation & Feedback Control
4.1 Input-Output Feedback Linearization

Consider a nonlinear system given by the Eq. 4-1

\[
\dot{x} = f(x) + g(x)u
\]
\[
y = h(x)
\]  

(4-1)

In the input/state linearization, state equations are linearized, which does not necessarily confirm that the resulting map from input \( u \) to output \( y \) is linear. The reason is that when the coordinate transformation is derived to linearize the state equation, the nonlinearity in the output equation is not taken into account. In this section we will discuss input/output linearization and a coordinate transformation will be derived for the 2MM that will take into account the nonlinearity in the output and the problem of finding a control law that renders a linear differential equation relating the input \( u \) to the output \( y \). Nonlinear systems with \( r < n \) in \( R^n \), where \( n \) is the order of the system and \( r \) is the relative degree, are input-output feedback linearized into Byrnes-Isidori normal form according to the following steps [37].

1) The nonlinear control law that compensates the nonlinearities in the input-output behavior,

\[
u = v - L_f^rh(x)
\]
\[
L_gL_f^{r-1}h(x)
\]

(4-2)

where \( L_f \) is the Lie derivative with respect to \( f \), \( L_g \) is the Lie derivative with respect to \( g \), \( v \) is the state feedback controller, \( r \) is the relative degree.

2) Using the nonlinear transformation \( = T(x) \), \( z = [\xi^T \eta^T]^T \), to find the internal and external dynamics, where \( \xi \) (internal dynamics) = \( [y \dot{y} \ldots y^{r-1}]^T \) is of length \( r \) and \( \eta \) (external dynamics) is of length \( n - r \).

\[
y^r = v
\]

(4-3)

\[
\dot{\eta} = Q(y, \dot{y}, \ldots, y^{r-1}, \eta)
\]

where \( \eta \) is chosen so that \( T(x) \) is a diffeomorphism. \( Q \) is a nonlinear function defining the time derivative of \( \eta \), and \( y^i \) is the \( i \)th time derivative of \( y \).
The new form in Eq. 4-3 of system in Eq. 4-1 is called the Byrnes Isidori normal form. Input-output feedback linearization decouples the input-output behavior of a nonlinear system from the internal dynamics, i.e. \( \eta \) has no effect on \( y \). The system in Eq. 4-3 can be rewritten as a function of the new coordinates shown below:

\[
\begin{align*}
\dot{\xi}_i &= \xi_{i+1}, \quad i = 1, \ldots, r - 1 \\
\dot{\xi}_r &= v \\
\dot{\eta} &= Q(\eta, \xi)
\end{align*}
\]  

(4-4)

The nonlinear system in Eq. 4-1 is linearized in its input-output form, and a linear controller \( v \) can be used to control the system output. However, the stability of the internal dynamics using the zero dynamics needs to be proved.

### 4.1.1 Relative Degree

The nonlinear system shown in Eq. 4-8 with \( y^k(t) = L_f^k h(x(t)) \) for all \( k < r \) and all \( t \) near \( t^0 \)

\[
y^r(t^0) = L_f^r h(x^0) + L_g L_f^{r-1} h(x^0) u(t^0)
\]  

(4-5)

is said to have relative degree \( r \) at a point \( x^0 \)

(i) \( L_g L_f^k h(x) = 0 \) for all \( x \) in a neighborhood of \( x^0 \) and all \( k < r - 1 \)

(ii) \( L_g L_f^{r-1} h(x^0) \neq 0 \)

From the above definition, relative degree is equal to the number of the derivative of the defined output, until input appears and there is a direct relation between input and output. For a linear system, relative degree is equal to the difference between the number of zeros and poles of the system. Relative degree tells us about the unobservable dynamics of the system from the output.

Figure 4-2 shows the feedback linearization techniques applicability, based on the relative degree.
4.1.2 Normal Form

The normal form splits the system into internal and external dynamics. In feedback linearization techniques, using transformation nonlinear systems are written in normal form. In normal form linear dynamics, the part for which feedback gains are designed is called external dynamics and the nonlinear dynamics part is called internal dynamics. To make sure that the controller will work properly, the stability of internal dynamics using zero dynamics needs to be proved. Eq. 4-6 shows the normal form for the system shown in Eq. 4-1.

\[
\begin{align*}
\dot{\eta} &= f_0(\eta, \xi) \\
\dot{\xi} &= A_c + B_c v \\
y &= C_c \xi
\end{align*}
\]

- $\xi$ represents the external dynamics
- $\eta$ represents the internal dynamics
- $\dot{\eta} = f_0(\eta, 0)$ represents the zero dynamics

For the 2MM, the input/output feedback linearization technique was implemented. The procedure followed for I/O feedback linearization is shown in the diagram below.
Two cases were considered for the 2MM. In the first case, load position was considered as an output and then the system was linearized using input/output feedback linearization. At the end, a controller was designed for the linear part and stability was derived for the zero dynamics. In the second case, the same procedure was followed with motor position as an output.

4.2 Case One

In the first case, load position is considered as an output.

\[ y = x_1 \]
\[ \dot{y} = \dot{x}_1 = x_2 \]
\[ \ddot{y} = \dot{x}_2 = \frac{1}{l_1} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1] \]

\[ \ddot{y} = \frac{1}{l_1} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \cos x_1] \quad (4-7) \]

\[ \ddot{y} = v \]

The input appears at the third derivative, so the relative degree \( r \) is 3. The order of the system is 4. Unobservable states from output after I/O linearization are \(-r\), so there is one unobservable state or internal dynamics.

In order to find the normal form, first the transformation matrix was defined as

\[ z = T(x) = \begin{bmatrix} T_1(x) \\ T_2(x) \\ T_3(x) \\ T_4(x) \end{bmatrix} = \begin{bmatrix} z_1(x) \\ z_2(x) \\ z_3(x) \\ z_4(x) \end{bmatrix} \quad (4-8) \]

where

\{z_1(x)\} are the internal dynamics

\{z_2(x)\}

\{z_3(x)\} are the external dynamics

\{z_4(x)\}

The external dynamics are defined as

\[ z_2(x) = y = x_1 \]

\[ z_3(x) = \dot{y} = x_2 \quad (4-9) \]

\[ z_4(x) = \ddot{y} = \frac{1}{l_1} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1] \]

For the internal dynamics state, \( z_1(x) \) is chosen such that \( T(x) \) satisfies the properties of a diffeomorphism. For that, the following condition needs to be satisfied.
Choosing \( z_1(x) \) to satisfy the above condition.

\[
z_1(x_1, x_2, x_3) = x_3 \tag{4-10}
\]

The transformation matrix becomes

\[
z = T(x) = \begin{bmatrix}
x_3 \\
x_1 \\
x_2 \\
\frac{1}{J_1} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1]
\end{bmatrix} \tag{4-11}
\]

The internal dynamics are given by Eq. 4-12

\[
\dot{z}_1 = \dot{x}_3 = x_4
\]

From Eq. 4-11

\[
z_4 = \frac{1}{J_1} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1]
\]

\[
z_4 = \frac{1}{J_1} [K(z_1 - z_2) + C(x_4 - z_3) - Mg \frac{d}{2} \sin z_2]
\]
The external dynamics are given by Eq. 4-13

\[
\dot{z}_2 = \dot{x}_1 = x_2 = z_3
\]

\[
\dot{z}_3 = \dot{x}_2 = \frac{1}{I_1} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1]
\]

\[
\dot{z}_3 = \frac{1}{I_1} [K(z_1 - z_2) + C(x_4 - z_3) - Mg \frac{d}{2} \sin z_2]
\]

\[
\dot{z}_4 = \frac{1}{I_1} [K(\dot{x}_3 - \dot{x}_1) + C(\dot{x}_4 - \dot{x}_2) - Mg \frac{d}{2} \cos x_1]
\]

\[
\dot{z}_2 = z_3
\]

\[
\dot{z}_3 = z_4
\]

\[
\dot{z}_4 = v
\]  

(4-13)

To find the zero dynamics, make the external dynamics states \( z_2 = z_3 = z_4 = 0 \) in internal dynamics Eq. 4-12. The zero dynamics are given by Eq. 4-14.

\[
\dot{z}_1 = -\frac{K}{C} z_1
\]  

(4-14)

As it can be seen, the zero dynamics are asymptotically stable with a pole \( \frac{K}{C} \) on the left hand side of s-plane. Therefore the control law will locally stabilize the whole system.

In order to satisfy the diffeomorphism condition, we must find the Jacobian of \( z = T(x) \)
The inverse of the Jacobian is given by Eq. 4-16

\[
\begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-\frac{1}{J_1} [K + Mg \frac{d}{2} \cos x_1] & -\frac{1}{J_1} & -\frac{1}{J_1} \\
\end{pmatrix}
\]  

(4-16)

The Jacobian is nonsingular for any value of \( x \) so the condition of diffeomorphism is satisfied and the inverse transformation can be found out.

The feedback controller \( v \) for the external dynamics is given by Eq. 4-17

\[
v = -k_0y - k_1\dot{y} - k_2\ddot{y}
\]  

(4-17)

\( k_i \) are selected to make the characteristic equation \( s^3 + k_2s^2 + k_1s + k_0 \) Hurwitz.

The final control input is given by Eq. 4-18

From Eq. 4-7

\[
v = \frac{1}{J_1} [K(x_3 - \dot{x}_1) + C(x_4 - \dot{x}_2) - Mg \frac{d}{2} \cos x_1]
\]

\[-k_0y - k_1\dot{y} - k_2\ddot{y} = \frac{1}{J_1} [K(x_4 - x_2) + C \left( \frac{1}{J_m} [u - K(x_3 - x_1) - \\
C(x_4 - x_2)] - \frac{1}{J_1} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1] \right) - Mg \frac{d}{2} \cos x_1]
\]

\[-k_0z_2 - k_1z_3 - k_2z_4 = \frac{1}{J_1} [K \left( -\frac{K}{C} (z_1 - z_2) + Mg \frac{d}{2C} \sin z_2 + z_3 + \frac{J_1}{C} z_4 - z_3 \right) + C \left( \frac{1}{J_m} [u - K(z_1 - z_2) - C \left( -\frac{K}{C} (z_1 - z_2) + Mg \frac{d}{2C} \sin z_2 + z_3 + \frac{J_1}{C} z_4 - z_3 \right) - \frac{1}{J_1} [K(z_1 - z_2) + C \left( -\frac{K}{C} (z_1 - z_2) + Mg \frac{d}{2C} \sin z_2 + z_3 + \frac{J_1}{C} z_4 - z_3 \right) - Mg \frac{d}{2} \sin z_2] \right) - Mg \frac{d}{2} \cos z_2]
\]

98
\[ u = \frac{J_l m}{C} \left[ -k_0 z_2 - k_1 z_3 - k_2 z_4 + \frac{M g d}{2J_l} \cos z_2 + \frac{M g d}{2J_l} \sin z_2 \left( \frac{C}{J_m} - \frac{K}{C} \right) \\
+ z_4 \left( \frac{C}{J_m} - \frac{K}{C} - \frac{K^2}{J_l C} \right) \right] \] (4-18)

As the zero dynamics are asymptotically stable, the control law will locally stabilize the whole system.

4.3 Simulink Models and Simulation Results for Case One

Figure 4-4 shows the block diagram implementation of I/O feedback linearization.

![Figure 4-4 Block Diagram Implementation of I/O Feedback Linearization](image)
Figure 4-5 shows the simulink model for the first case.

Figure 4-5 Simulink Diagram for I/O Feedback Linearization for Case 1
The Simulink model for computing the controlled input for the first case is shown in Figure 4-6.

Figure 4-6 Simulink Block for v to u for Case 1

The Simulink model for computing the transformation for the first case is shown in Figure 4-7.

Figure 4-7 Simulink Block for Transformation for Case 1

The subsystem for the feedback controller for the first case is shown in Figure 4-8.

Figure 4-8 Subsystem for Feedback Controller Block for Case 1
Simulation results for the load position and load speed for the first case are shown in Figures 4-9 and 4-10, respectively.

Figure 4-9 Actual and Reference Load Position Simulation Results for 2MM

Figure 4-10 Actual Load Speed Simulation Results for 2MM
Simulation results for the motor position and motor speed for the first case are shown in Figures 4-11 and 4-12, respectively.

Figure 4-11 Actual Motor Position Simulation Results for 2MM

Figure 4-12 Actual Motor Speed Simulation Results for 2MM
4.4 Case Two

In this case, the motor position is considered as an output.

\[ y = x_3 \]  
\[ \dot{y} = \dot{x}_3 = x_4 \] (4-19)

\[ \ddot{y} = \ddot{x}_4 = \frac{1}{J_m} \left[ u - K(x_3 - x_1) - C(x_4 - x_2) \right] \] (4-20)

\[ \ddot{y} = v \]

The input appears at the second derivative, so the relative degree \((r)\) is 2. The order of the system is 4. Unobservable states from output after I/O linearization are \(-r\), so there are two unobservable state or internal dynamics.

In order to find the normal form, first the transformation matrix is defined as

\[ z = T(x) = \begin{bmatrix} T_1(x) \\ T_2(x) \\ T_3(x) \\ T_4(x) \end{bmatrix} = \begin{bmatrix} z_1(x) \\ z_2(x) \\ z_3(x) \\ z_4(x) \end{bmatrix} \] (4-21)

\( \{z_1(x)\} \) are the internal dynamics \( \{z_2(x)\} \) are the external dynamics

\( \{z_3(x)\} \) are the external dynamics \( \{z_4(x)\} \) are the external dynamics

The external dynamics are defined as

\[ z_3(x) = y = x_3 \]
\[ z_4(x) = \dot{y} = x_4 \] (4-22)

For the internal dynamics states, \( z_1(x) \) & \( z_2(x) \) are chosen such that \( T(x) \) satisfies the properties of a diffeomorphism. For that, the following conditions need to be satisfied.
\[
\frac{\partial z_{1,2}(x)}{\partial x} - g(x) = 0
\]
\[
\begin{bmatrix}
\frac{\partial z_1(x)}{\partial x_1} & \frac{\partial z_1(x)}{\partial x_2} & \frac{\partial z_1(x)}{\partial x_3} & \frac{\partial z_1(x)}{\partial x_4} \\
\frac{\partial z_2(x)}{\partial x_1} & \frac{\partial z_2(x)}{\partial x_2} & \frac{\partial z_2(x)}{\partial x_3} & \frac{\partial z_2(x)}{\partial x_4}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
1 \\
\end{bmatrix}
= 0
\]

The transformation matrix becomes
\[
\frac{1}{J_m} \begin{bmatrix}
\frac{\partial z_1(x)}{\partial x_4} \\
\frac{\partial z_2(x)}{\partial x_4}
\end{bmatrix}
= 0
\]

\[
\frac{\partial z_1(x)}{\partial x_4} = 0
\]

\[
\frac{\partial z_2(x)}{\partial x_4} = 0
\]

Choosing \( z_1(x) \) and \( z_2(x) \) to satisfy the above the condition.

\[
z_1(x_1, x_2, x_3) = x_1
\]

\[
z_2(x_1, x_2, x_3) = x_2
\]

(4-23)

The transformation matrix becomes
\[
z = T(x) = \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
\]

(4-24)

The internal dynamics are given by Eq. 4-25

\[
\dot{z}_1 = \ddot{x}_1 = x_2 = z_2
\]

\[
\dot{z}_2 = \ddot{x}_2 = \frac{1}{J_1} [K(x_3 - x_1) + C(x_4 - x_2) - Mg\frac{d}{2}\sin x_1]
\]

\[
\dot{z}_2 = \frac{1}{J_1} [K(\dot{x}_3 - \dot{x}_1) + C(\dot{x}_4 - \dot{x}_2) - Mg\frac{d}{2}\sin \dot{x}_1]
\]
\[ \dot{z}_1 = z_2 \] (4-25)

\[ \dot{z}_2 = \frac{1}{l_1} [K(z_3 - z_1) + C(z_4 - z_2) - Mg \frac{d}{2} \sin z_1] \]

The external dynamics are given by Eq. 4-26

\[ \dot{z}_3 = \dot{x}_3 = x_4 = z_4 \]

\[ \dot{z}_4 = \dot{x}_4 = \frac{1}{l_m} [u - K(x_3 - x_1) - C(x_4 - x_2)] \]

\[ \dot{z}_4 = \dot{x}_4 = \frac{1}{l_m} [u - K(z_3 - z_1) - C(z_4 - z_2)] \]

\[ \dot{z}_3 = z_4 \] (4-26)

\[ \dot{z}_4 = v \]

To find the zero dynamics, make the external dynamics states \( z_3 = z_4 = 0 \) in the internal dynamics Eq. 4-25. Zero dynamics are given by Eq. 4-27

\[ \dot{z}_1 = z_2 \]

\[ \dot{z}_2 = -\frac{1}{l_1} [Kz_1 + Cz_2 + Mg \frac{d}{2} \sin z_1] \] (4-27)

In order to satisfy the diffeomorphism condition, we must find the Jacobian of \( z = T(x) \)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (4-28)
The inverse of Jacobian is

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (4-29)

The Jacobian is nonsingular for any value of \( x \) so the condition of diffeomorphism is satisfied and the inverse transformation can be found out.

The feedback controller \( v \) for external dynamics is given by Eq. 4-30

\[
v = -k_0 y - k_1 \dot{y} \hspace{1cm} (4-30)
\]

\( k_i \) are selected to make the characteristic equation \( s^3 + k_2 s^2 + k_1 s + k_0 \) Hurwitz.

The final control input is given by Eq. 4-31

From Eq. 4-20

\[
v = \frac{1}{J_m} [u - K(x_3 - x_1) - C(x_4 - x_2)]
\]

\[
-k_0 z_3 - k_1 z_4 = \frac{1}{J_m} [u - K(z_3 - z_1) - C(z_4 - z_2)]
\]

\[
u = J_m (-k_0 z_3 - k_1 z_4) + K(z_3 - z_1) + C(z_4 - z_2) \hspace{1cm} (4-31)
\]
4.5 Simulink Models and Simulation Results for Case Two

Figure 4-13 shows the simulink model for the second case.
The Simulink model for computing the controlled input for the second case is shown in Figure 4-14.

![Figure 4-14 Simulink Block for v to u for Case 2](image)

The Simulink model for computing the transformation for the second case is shown in Figure 4-15.

![Figure 4-15 Simulink Block for Transformation for Case 2](image)

The subsystem for the feedback controller for the second case is shown in Figure 4-16.

![Figure 4-16 Subsystem for Feedback Controller Block for Case 2](image)
The simulation results for the load position and load speed for the second case are shown in Figures 4-17 and 4-18, respectively.

Figure 4-17 Actual and Reference Motor Position Simulation Results for 2MM

Figure 4-18 Actual Motor Speed Simulation Results for 2MM
The simulation results for the motor position and motor speed for the second case are shown in Figures 4-19 and 4-20, respectively.

Figure 4-19 Actual Load Position Simulation Results for 2MM

Figure 4-20 Actual Load Speed Simulation Results for 2MM
Chapter 5: Experimental Setup and Hardware

To validate the simulation results in real time, the two mass model prototypes shown in Figure 5-1 were designed and developed. The system was designed to have a mechanical resonant frequency of $f_r = 18.57$ Hz because most of the industrial elastic drive systems have mechanical resonant frequencies in the same range. In this system, there are two inertias, one on the drive side and other on the load side. The drive side is connected to the load side through a flexible shaft. A Buhler DC motor is used as an actuator on the drive side, which is connected to the drive side inertia through flexible couplings and bearings. On the load side, inertia is connected to the link through flexible couplings and bearings. Baumer incremental encoders with 40,000 ppr are mounted on both the drive and load side to get the feedback signals.

Figure 5-1 Experimental Setup Top View

Figure 5-2 Experimental Setup Front View
To make the rigid system in real time, the shaft diameter of elastic system is increased 3 times. The shaft stiffness is increased by 81 times and the resonant frequency of the rigid system is increased 9 times as compared to the elastic system. Figure 5-3 shows the elastic and rigid systems.

Figure 5-3 Elastic to Rigid System

The below tables show the 2MM dimensions, inertias of motor side mass, load side mass and link. The parameters in Table 1 were selected in order to keep the load-to-motor inertia ratio \( R = J_l/J_m = 1.209 \) for maximum power transmission.

<table>
<thead>
<tr>
<th>Motor Side Cylindrical Mass</th>
<th>Radius</th>
<th>Width</th>
<th>( V = \pi r^2 h )</th>
<th>Density of steel</th>
<th>Mass = ( dV )</th>
<th>Measured Mass</th>
<th>( I_{min} = \frac{Mr^2}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
<td>m</td>
<td>m³</td>
<td>kg/m³</td>
<td>kg</td>
<td>kg</td>
<td>kg m²</td>
</tr>
<tr>
<td></td>
<td>0.0375</td>
<td>0.032</td>
<td>0.00023</td>
<td>7750</td>
<td>1.8</td>
<td>2.01</td>
<td>0.00125</td>
</tr>
</tbody>
</table>

Table 5-1 Motor Side Cylindrical Mass Dimensions

<table>
<thead>
<tr>
<th>Load Side Cylindrical Mass</th>
<th>Radius</th>
<th>Width</th>
<th>( V = \pi r^2 h )</th>
<th>Density of steel</th>
<th>Mass = ( dV )</th>
<th>Measured Mass</th>
<th>( I_{max} = \frac{Mr^2}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
<td>m</td>
<td>m³</td>
<td>kg/m³</td>
<td>kg</td>
<td>kg</td>
<td>kg m²</td>
</tr>
<tr>
<td></td>
<td>0.032</td>
<td>0.032</td>
<td>0.000167</td>
<td>7750</td>
<td>1.29</td>
<td>1.54</td>
<td>0.00067</td>
</tr>
</tbody>
</table>

Table 5-2 Load Side Cylindrical Mass Dimensions
### Table 5-3 Link Dimensions

<table>
<thead>
<tr>
<th>Link</th>
<th>Length (m)</th>
<th>Thickness (m)</th>
<th>Width (m)</th>
<th>Density of steel (kg/m²)</th>
<th>Mass ( m ) (kg)</th>
<th>Measured Mass (kg)</th>
<th>( I = Mg/2 ) (Nm)</th>
<th>% to Nominal Torque (2.6 Nm)</th>
<th>( J = \frac{M^2}{3} ) (kgm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.005</td>
<td>0.03</td>
<td>2700</td>
<td>0.061</td>
<td>0.08</td>
<td>0.0734</td>
<td>13.02</td>
<td>0.00306</td>
</tr>
</tbody>
</table>

### Table 5-4 Inertias of the 2MM

<table>
<thead>
<tr>
<th>Inertias</th>
<th>Motor ( I_m ) (kgm²)</th>
<th>Motor Side Mass ( I_{m \text{side}} ) (kgm²)</th>
<th>Load Side Mass ( I_{l \text{side}} ) (kgm²)</th>
<th>Bus ( I_{b} ) (kgm²)</th>
<th>Motor Side ( I_{m \text{ax}} + I_{m \text{ax}} ) (kgm²)</th>
<th>Load Side ( I_{l \text{ax}} + I_{l \text{ax}} ) (kgm²)</th>
<th>( R = \frac{I_L}{I_N} ) (kgm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00018</td>
<td>0.00125</td>
<td>0.00067</td>
<td>0.00106</td>
<td>0.00143</td>
<td>0.00173</td>
<td>1.299</td>
<td></td>
</tr>
</tbody>
</table>

5.1 **Workstation**

Figure 5-4 shows the workstation from side view.

![Figure 5-4 Workstation Side View](image)
5.2 dSpace 1103 Controller
The dSPACE 1103 controller board changes the PC to a powerful and advanced system for fast control prototyping. The real-time hardware based on PowerPC technology (from Motorola) and its set of I/O interfaces makes the board ideal for developing controllers in various fields. It comprises of a digital I/O, analog I/O, PWM channels, encoder channels, RS-232 I/O channels, and RS 422 I/O channels. dSPACE 1103 is more advanced than the 1104 version in terms of its inbuilt oscillator frequency range, number of digital and analog I/O, and RS 232 channels.

![Figure 5-5 dSPACE 1103](image)

5.3 Control Desk
Control Desk is software that is used as an interface between Simulink and dSPACE. Layouts are created using this software through which different parameters are controlled in real time. Different system parameters and feedback signals from the sensors can be monitored in real time. Once the real-time Simulink model is created, it is then built to transfer all the parameters to the control desk layout. After that, Simulink has no control over the real-time model, so then Control Desk interacts with the real system. Figure 5-6 shows the Control Desk layout.

![Figure 5-6 Control Desk Layout](image)
5.4 Hysteresis Current Controller

Hysteresis current control is a nonlinear control technique. In this technique, reference current and actual current are compared using a hysteresis comparator. The hysteresis comparator then generates switching signals as shown in Figure 5-7.

![Figure 5-7 Hysteresis Current Controller Block Diagram](image)

The actual current is controlled in a way that it swings between the upper and lower limits of the hysteresis band as shown in Figure 5-8. Because of its instantaneous response, the hysteresis current controller loop has a unity gain and this doesn’t add an extra degree to the system.

![Figure 5-8 Hysteresis Current Control](image)
Figure 5-9 shows the hysteresis current controller board. Current controllers are used to control the torque/current.

5.5 Testing of Hysteresis Current Controller in Real Time

To test the hysteresis current controller board in real time, the Simulink model shown in Figure 5-10 was designed. Figure 5-11 shows the subsystem.
Reference current signal is generated from the Simulink model and is given to the dSPACE DAC channel. The DAC channel of dSPACE is connected to the hysteresis current controller board. The current sensor on the board is continuously reading the feedback from the DC motor. The current controller board has three phases. In the first phase, an Op-amp is used as a buffer to isolate the board from the dSPACE. In the second phase, the Op-amp is used as a comparator. In this phase the reference signal from dSPACE and the actual current signal from the current sensor are compared and the resultant signal goes to the third phase. In this phase, the Op-amp is used in hysteresis mode. The hysteresis band value is set by the resistor values used in the Op-amp configuration. Then PWM signals are generated on the basis of the signal coming from the comparator. The PWM signal is then fed to the H-Bridge to generate the controlled input. Figure 5-12 shows the reference current, actual current, and hysteresis band.

![Figure 5-11 Simulink Model for Subsystem](image1)

![Figure 5-12 Hysteresis Current Controller Response in Real Time](image2)
Figures 5-13 and 5-14 show the reference and actual current in real time with sampling frequencies at 1KHz and 10 KHz, respectively.

Figure 5-13 Hysteresis Current Controller Response at 1 KHz Sampling Frequency

Figure 5-14 Hysteresis Current Controller Response at 1 KHz Sampling Frequency
5.6 Testing of Hysteresis Current Controller in Simulation

To test the hysteresis current controller in simulation, the Simulink model shown in Figure 5-15 was designed. The hysteresis band value in this case is 0.1 ampere. A very high sampling frequency is required in simulation to achieve $\Delta i = 0.1$ ampere. In simulation we can even go further below this value of $\Delta i$, but in real time implementation, these values are not possible to achieve because of the limitation made by the oscillator frequency of dSPACE. That’s why in real-time implementation, a hysteresis current controller board was designed.

Figure 5-15 Hysteresis Current Controller Testing in Simulation

Figure 5-16 Hysteresis Current Controller Response in Simulation
Chapter 6: Reduced Order Observer Design Using the Notion of Invariant Manifold

Consider a nonlinear, time-varying systems described by equations of the form

\[ \dot{\eta} = f(\eta, y, t) \]  
(6-1)

and

\[ \dot{y} = h(\eta, y, t) \]  
(6-2)

where \( \eta \in \mathbb{R}^n \) is the unmeasured state and \( y \in \mathbb{R}^m \) is the measurable output. It is assumed that the vector fields \( f(\cdot) \) and \( h(\cdot) \) are forward complete, (i.e., trajectories starting at time \( t_0 \) are defined for all times \( \geq t_0 \)).

Definition 1: The dynamical system

\[ \dot{\hat{\eta}} = \alpha(y, \hat{\eta}, t) \]  
(6-3)

where \( \hat{\eta} \in \mathbb{R}^p, p \geq n \), is called a (local) observer for the system in Eq. 6-1 & Eq. 6-2, if there exists mappings

\[ \beta(\cdot) : \mathbb{R}^m \times \mathbb{R}^p \times \mathbb{R} \to \mathbb{R}^p \text{ and } \phi(\cdot) : \mathbb{R}^n \to \mathbb{R}^p \]

with \( \phi(\cdot) \) (locally) left invertible, such that the manifold

\[ \mathcal{M}_t = \{ (\eta, y, t) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p : \beta(y, \hat{\eta}, t) = \phi(\eta) \} \]

has the following properties:

1) All trajectories of the extended system in Eq. 6-1, 6-2, and 6-3, that start on the manifold \( \mathcal{M}_t \) at time \( t \) remain there for all future times \( \tau > t \), i.e. \( \mathcal{M}_t \) is positively invariant.

2) All trajectories of the extended system in Eq. 6-1, 6-2, and 6-3, that start in a neighborhood of \( \mathcal{M}_t \) asymptotically converge to \( \mathcal{M}_t \).

Mapping functions should be chosen in such a way to satisfy the below-mentioned conditions:
(A1)  \( \det \left( \frac{\partial \beta}{\partial \eta} \right) \neq 0 \)

(A2)  The system

\[
\frac{\partial \beta}{\partial y} \left( h(\eta, y, t) - \text{\(h(\phi(\eta) + z), y, t)\)} - \frac{\partial \phi}{\partial \eta} f(\eta, y, t) + \frac{\partial \phi}{\partial \eta} \big|_{\eta=\phi(\eta)+z} \right) f(\phi(\eta) + z), y, t)
\]

has a (locally) asymptotically stable equilibrium at \(z = 0\), uniformly in \(\eta, y, t\).

Then there exists a function \(\alpha(\cdot)\) such that system in Eq. 6-3 is an observer for the
system in Eq. 6-1 and Eq. 6-2,

\[
\alpha(\xi, y, t) = -\left( \frac{\partial \beta}{\partial \xi} \right)^{-1} \left( \frac{\partial \beta}{\partial y} f_2(\hat{\eta}, y, t) + \frac{\partial \phi}{\partial \eta} \big|_{\eta=\hat{\eta}} f_2(\hat{\eta}, y, t) - \frac{\partial \phi}{\partial \eta} \big|_{\eta=\hat{\eta}} f_1(\hat{\eta}, y, t) \right)
\]

6.1 Nonlinear Reduced-Order Observer Design

Dynamics of the two mass models are given below

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{J_l} [K(x_3 - x_1) + C(x_4 - x_2) - Mg \frac{d}{2} \sin x_1] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{1}{J_m} [u - K(x_3 - x_1) - C(x_4 - x_2)]
\end{aligned}
\]

where

\(x_1=\) load position (unmeasured)

\(x_2=\) load speed (unmeasured)

\(x_3=\) Motor position (measured)

\(x_4=\) Motor speed (measured)
Defining a mapping $\phi(x_1, x_2)$ in terms of unmeasured variables, such that $\phi(x_1, x_2)$ is left invertible. If a matrix $A$ is $m$-by-$n$ and the rank of that matrix is equal to $n$, then it is left invertible.

$$\phi(x_1, x_2) = \begin{bmatrix} x_1 \\ x_2 \\ \sin x_1 \\ \cos x_1 \end{bmatrix}$$  \hspace{1cm} (6.4)

$\phi(x_1, x_2)$ satisfies the condition for left inevitability because the rank of the $\phi(x_1, x_2)$ matrix is one.

Consider a variable $z$ that defines the distance of system trajectories from the manifold

$$z = \beta(x_3, x_4, \hat{\eta}) - \phi(x_1, x_2)$$  \hspace{1cm} (6.5)

where $\hat{\eta}$ = observer states, and $\beta(x_3, x_4, \hat{\eta})$ is the mapping function that needs to be defined

$$\beta(x_3, x_4, \hat{\eta}) = \begin{bmatrix} \beta_1(x_3, x_4, \hat{\eta}) \\ \beta_2(x_3, x_4, \hat{\eta}) \\ \beta_3(x_3, x_4, \hat{\eta}) \\ \beta_4(x_3, x_4, \hat{\eta}) \end{bmatrix}$$  \hspace{1cm} (6.6)

$$z_1 = \beta_1(x_3, x_4, \hat{\eta}) - x_1$$

$$z_2 = \beta_2(x_3, x_4, \hat{\eta}) - x_2$$

$$z_3 = \beta_3(x_3, x_4, \hat{\eta}) - \sin x_1$$

$$z_4 = \beta_4(x_3, x_4, \hat{\eta}) - \cos x_1$$

Error dynamics are given by Eq. 6.5

$$\dot{z} = \dot{\beta}(x_3, x_4, \hat{\eta}) - \dot{\phi}(x_1, x_2)$$  \hspace{1cm} (6.8)
\[
\dot{z} = \frac{\partial \beta}{\partial \eta} \dot{\eta} + \frac{\partial \beta}{\partial x_3} \dot{x}_3 + \frac{\partial \beta}{\partial x_4} \dot{x}_4 - \phi(x_1, x_2)
\]

\[
\dot{z} = \frac{\partial \beta}{\partial \eta} \dot{\eta} + \frac{\partial \beta}{\partial x_3} x_3 + \frac{\partial \beta}{\partial x_4} \left\{ \frac{1}{m} [u - K(x_3 - x_1) - C(x_4 - x_2)] \right\} - \left[ \begin{array}{c} x_2 \\
\dot{x}_2 \\
\cos x_1 \\
\sin x_1 \end{array} \right]
\]

Provided that Jacobian matrix \( \frac{\partial \beta}{\partial \eta} \) is invertible and error dynamics have asymptotically stable origin, then the observer dynamics becomes

\[
\dot{z} = \frac{\partial \beta}{\partial \eta} \dot{\eta} + \frac{\partial \beta}{\partial x_3} x_3 + \frac{\partial \beta}{\partial x_4} \left\{ \frac{1}{m} [u - K(x_3 - x_1) - C(x_4 - x_2)] \right\} - \left[ \begin{array}{c} x_2 \\
\dot{x}_2 \\
\cos x_1 \\
\sin x_1 \end{array} \right]
\]

\[
\left[ \begin{array}{c} 1 \\
\frac{1}{J_1} K(x_3 + z_1 - \beta_1(x_3, x_4, \tilde{h})) + C(x_4 + z_2 - \beta_2(x_3, x_4, \tilde{h})) - M g \frac{d}{2} (\beta_3(x_3, x_4, \tilde{h}) - z_3) \\
\beta_4(x_3, x_4, \tilde{h}) - z_4 \\
-\beta_3(x_3, x_4, \tilde{h}) + z_3 \end{array} \right] (6-9)
\]

Provided that Jacobian matrix \( \frac{\partial \beta}{\partial \eta} \) is invertible and error dynamics have asymptotically stable origin, then the observer dynamics becomes

\[
0 = \frac{\partial \beta}{\partial \eta} \dot{\eta} + \frac{\partial \beta}{\partial x_3} x_3 + \frac{\partial \beta}{\partial x_4} \left\{ \frac{1}{m} [u - K(x_3 + 0 - \beta_1(x_3, x_4, \tilde{h})) - C(x_4 + 0 - \beta_2(x_3, x_4, \tilde{h}))] \right\} - \\
\left[ \begin{array}{c} \beta_2(x_3, x_4, \tilde{h}) - 0 \\
\frac{1}{J_1} K(x_3 + 0 - \beta_1(x_3, x_4, \tilde{h})) + C(x_4 + 0 - \beta_2(x_3, x_4, \tilde{h})) - M g \frac{d}{2} (\beta_3(x_3, x_4, \tilde{h}) - 0) \\
\beta_4(x_3, x_4, \tilde{h}) - 0 \\
-\beta_3(x_3, x_4, \tilde{h}) + 0 \end{array} \right]
\]

\[
\ddot{\eta} = \left( \frac{\partial \beta}{\partial \eta} \right)^{-1} \left[ - \frac{\partial \beta}{\partial x_3} x_3 - \frac{\partial \beta}{\partial x_4} \left\{ \frac{1}{m} [u - K(x_3 - \beta_1(x_3, x_4, \tilde{h})) - C(x_4 - \beta_2(x_3, x_4, \tilde{h}))] \right\} + \\
\left[ \begin{array}{c} \beta_2(x_3, x_4, \tilde{h}) \\
\frac{1}{J_1} K(x_3 - \beta_1(x_3, x_4, \tilde{h})) + C(x_4 - \beta_2(x_3, x_4, \tilde{h})) - M g \frac{d}{2} (\beta_3(x_3, x_4, \tilde{h})) \\
\beta_4(x_3, x_4, \tilde{h}) \\
-\beta_3(x_3, x_4, \tilde{h}) \end{array} \right] \right] (6-10)
\]
\[ \dot{\beta}_1 = \left( \frac{\partial \beta_1}{\partial \eta_1} \right)^{-1} \left[ \frac{\partial \beta_1}{\partial x_3} x_4 - \frac{\partial \beta_1}{\partial x_4} f_m \left[ u - K(x_3 - \beta_1(x_3, x_4, \dot{\eta})) - C(x_4 - \beta_2(x_3, x_4, \dot{\eta})) \right] \right] \\
+ \left( \frac{\partial \beta_1}{\partial \eta_1} \right)^{-1} \beta_2(x_3, x_4, \dot{\eta}) \\
\]

\[ \dot{\beta}_2 = \left( \frac{\partial \beta_2}{\partial \eta_2} \right)^{-1} \left[ -\frac{\partial \beta_2}{\partial x_3} x_4 - \frac{\partial \beta_2}{\partial x_4} f_m \left[ u - K(x_3 - \beta_2(x_3, x_4, \dot{\eta})) - C(x_4 - \beta_3(x_3, x_4, \dot{\eta})) \right] \right] \\
+ \left( \frac{\partial \beta_2}{\partial \eta_2} \right)^{-1} \frac{1}{f_1} \left[ K(x_3 - \beta_1(x_3, x_4, \dot{\eta})) + C(x_4 - \beta_2(x_3, x_4, \dot{\eta})) \right] - \frac{M g}{\ell} \left[ \beta_3(x_3, x_4, \dot{\eta}) \right] \]  

(6-11)

\[ \dot{\beta}_3 = \left( \frac{\partial \beta_3}{\partial \eta_3} \right)^{-1} \left[ -\frac{\partial \beta_3}{\partial x_3} x_4 - \frac{\partial \beta_3}{\partial x_4} f_m \left[ u - K(x_3 - \beta_1(x_3, x_4, \dot{\eta})) - C(x_4 - \beta_2(x_3, x_4, \dot{\eta})) \right] \right] \\
+ \left( \frac{\partial \beta_3}{\partial \eta_3} \right)^{-1} \beta_4(x_3, x_4, \dot{\eta}) \\
\]

\[ \dot{\beta}_4 = \left( \frac{\partial \beta_4}{\partial \eta_4} \right)^{-1} \left[ -\frac{\partial \beta_4}{\partial x_3} x_4 - \frac{\partial \beta_4}{\partial x_4} f_m \left[ u - K(x_3 - \beta_1(x_3, x_4, \dot{\eta})) - C(x_4 - \beta_2(x_3, x_4, \dot{\eta})) \right] \right] \\
- \left( \frac{\partial \beta_4}{\partial \eta_4} \right)^{-1} \beta_3(x_3, x_4, \dot{\eta}) \]

Choosing the functions \( \beta(x_3, x_4, \dot{\eta}) \)

\[
\beta_1(x_3, x_4, \dot{\eta}) = \dot{\eta}_1 \\
\beta_2(x_3, x_4, \dot{\eta}) = \dot{\eta}_2 \\
\beta_3(x_3, x_4, \dot{\eta}) = \dot{\eta}_3 - f_m x_4 \\
\beta_4(x_3, x_4, \dot{\eta}) = \dot{\eta}_4
\]  

(6-12)

1. \( \beta(x_3, x_4, \dot{\eta}) \) functions are chosen in such a way to satisfy

\[ \det \left( \frac{\partial \beta}{\partial \dot{\eta}} \right) \neq 0 \]
Eq. 6-13 shows the Jacobian of $\frac{\partial \beta}{\partial \hat{\eta}}$

$$\frac{\partial \beta}{\partial \hat{\eta}} = \begin{bmatrix} \frac{\partial \beta_1}{\partial \hat{\eta}_1} & \frac{\partial \beta_1}{\partial \hat{\eta}_2} & \frac{\partial \beta_1}{\partial \hat{\eta}_3} & \frac{\partial \beta_1}{\partial \hat{\eta}_4} \\ \frac{\partial \beta_2}{\partial \hat{\eta}_1} & \frac{\partial \beta_2}{\partial \hat{\eta}_2} & \frac{\partial \beta_2}{\partial \hat{\eta}_3} & \frac{\partial \beta_2}{\partial \hat{\eta}_4} \\ \frac{\partial \beta_3}{\partial \hat{\eta}_1} & \frac{\partial \beta_3}{\partial \hat{\eta}_2} & \frac{\partial \beta_3}{\partial \hat{\eta}_3} & \frac{\partial \beta_3}{\partial \hat{\eta}_4} \\ \frac{\partial \beta_4}{\partial \hat{\eta}_1} & \frac{\partial \beta_4}{\partial \hat{\eta}_2} & \frac{\partial \beta_4}{\partial \hat{\eta}_3} & \frac{\partial \beta_4}{\partial \hat{\eta}_4} \end{bmatrix}$$

(6-13)

$$\frac{\partial \beta}{\partial \hat{\eta}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$det\left(\frac{\partial \beta}{\partial \hat{\eta}}\right) = 1$$

As the determinant of $\frac{\partial \beta}{\partial \hat{\eta}}$ is not equal to zero, its inverse exists and the first condition is satisfied.

2. The equilibrium point $z = 0$ should be asymptotically stable for the chosen values of $\beta(x_3, x_4, \hat{\eta})$.

The observer dynamics become

$$\hat{\eta}_1 = \hat{\eta}_2$$

$$\hat{\eta}_2 = \frac{1}{J_1} K(x_3 - \hat{\eta}_1) + C(x_4 - \hat{\eta}_2) - M g \frac{d}{L} (\hat{\eta}_3 - f_m x_4)$$

$$\hat{\eta}_3 = u - K(x_3 - \hat{\eta}_1) - C(x_4 - \hat{\eta}_2) + \hat{\eta}_4$$

$$\hat{\eta}_4 = -\hat{\eta}_3$$

(6-14)
Substituting observer dynamics in Eq. 6-9 gives

\[
\dot{z} = \begin{bmatrix}
\frac{1}{f_l} & \frac{\beta_2}{f_l} \\
K(x_3 - \beta_1) + C(x_4 - \beta_2) - Mg \frac{d}{2} (\beta_3) & u - K(x_3 - \beta_1) - C(x_4 - \beta_2) + \beta_4 - [u - K(x_3 + z_1 - \beta_1) - C(x_4 + z_2 - \beta_2)]
\end{bmatrix}
\]

To prove the asymptotical stability of error dynamics at the origin, the eigenvalues of the Jacobian matrix from the error dynamics should be in the left half of the plane.

The Jacobian matrix is given below

\[
J = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\frac{K}{f_l} & -\frac{C}{f_l} & -\frac{Mg d}{2f_l} & 0 \\
K & C & 0 & 1 \\
0 & 0 & -1 & 0
\end{bmatrix}
\]

The Jacobian matrix at the origin is same as above matrix. The eigenvalues of above matrix are given below

\[
z_1 = -5.9249 + 79.5150i
\]

\[
z_2 = -5.9249 - 79.5150i
\]
\[ z_3 = -0.0392 + 0.9992i \]
\[ z_4 = -0.0392 - 0.9992i \]

As all the eigenvalues are in the left half plane, hence error dynamics are asymptotically stable.

6.2 Simulation Results for Reduced-Order Observer

Figure 6-1 shows the block diagram for the observer implementation. Motor position and motor speed were feed to the reduced-order observer to compute load position and speed. Then load position and load speed from the 2MM and reduced-order observer were compared.

Figure 6-1 Block Diagram for Observer Implementation
To validate the reduced-order observer, the below model was built in Simulink (see Figure 6-2).

Figure 6-2 Simulink Model for Reduced Order Observer Design
A step signal was given as input to the 2MM to compare the actual and estimated load position and load speed shown in Figures 6-3 and 6-5, respectively.

Figure 6-3 Plot of Actual and Estimated Load Position Simulation Results

Figure 6-4 Zoomed Version of Actual and Estimated Load Position Simulation Results
Figure 6-5 Plot of Actual and Estimated Load Speed Simulation Results

Figure 6-6 Zoomed Version of Actual and Estimated Load Speed Simulation Results
Figure 6-7 and 6-8 shows the error dynamics $z_1$ and $z_2$ respectively. The error dynamics goes to zero as shown below.

![Figure 6-7 Error Dynamics $z_1$](image1)

![Figure 6-8 Error Dynamics $z_2$](image2)
6.3 Reduced-Order Observer with I&I Position Controller Simulation

After validating the reduced-order observer, the next step was to incorporate it in a complete simulation of position control of the 2MM using I&I. Figure 6-9 shows the block diagram of observer implementation with I&I controller.

![Figure 6-9 Block Diagram for Observer Implementation with I&I Controller](image-url)
The Simulink model in Figure 6-10 compares the actual and estimated load position and load speed in simulation.

Figure 6-10 Simulink Model for 2MM with I&I and Reduced Order Observer
Actual and estimated load position and load speed are shown in Figures 6-11 and 6-12, respectively.
6.4 Reduced-Order Observer with I&I Position Controller Experimental

Figure 6-13 shows the simulink model for real time implementation of the reduced order-observer incorporated in the 2MM with an I&I position controller.

---

Figure 6-13 Simulink Model of 2MM with I&I and Reduced-Order Observer for Real Time Implementation
The actual and estimated load position and load speed in real time implementation for the above model are shown in Figures 6-14 and 6-15, respectively.
Chapter 7: Comparison of Controllers and Concluding Remarks

In this chapter, a comparison is made between three different position controllers for 2MM and then concluding remarks are made. Two of the controllers were discussed in detail in Chapter 3 and Chapter 4. Before making the comparison, the third controller design is discussed briefly.

7.1 PID Position Controller Design

PID controllers are widely used in industrial processes and other applications. The PID controller output is the weighted sum of the input signal, integral of the input signal, and derivative of the input signal. The weights are proportional (P), integral (I), and derivative (D). A first-order pole filters the derivative action of the PID. The transfer function for the PID controller is given below

\[
C(s) = \left[ P + I \left( \frac{1}{s} \right) + D \left( \frac{Ns}{s + N} \right) \right]
\] (7-1)

There are different methods for the tuning of PID gains including the Ziegler-Nichols Method, Cohen-Coon Method, Tyreus-Luyben Method, and Software Tuning Tools. In this paper Matlab/Simulink software is used for PID tuning to get the desired response. The PID tuner in Simulink first computes the PI controller to achieve a tradeoff between the desired performance and robustness. The PID tuner first linearizes the plant and then checks the controller performance on the linear plant model. After that the controller can be tested on the nonlinear model. PID gains are given in the below table.

<table>
<thead>
<tr>
<th>(K_p)</th>
<th>(K_i)</th>
<th>(K_d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>3</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 7-1 PID Gains
7.2 Comparison of I&I, I/O FL and PID Position Controller

Figure 7-1 shows the Simulink model for comparison of the three controllers.

Figure 7-1 Controller Comparison Simulation

Figure 7-2 shows the step response of the three controllers. It can be seen that with PID it's not possible to get an over-damped response. The I&I controller shows the fastest over-damped response.

Figure 7-2 Transient Response of Controllers
Figure 7-3 shows the response of the controllers to step disturbance externally introduced at 2.5 sec, after a steady state was achieved by all the controllers. I&I shows the most robust response as compared to PID and I/O feedback linearization to external disturbances.

![Figure 7-3 Response of Controllers to External Disturbance](image)

Figures 7-4 show the response of the controllers to parameter variation. The load inertia is increased by 40%. Only I&I show the robust response in this case.

![Figure 7-4 Response of Controller to 20% Variation in Load Inertia](image)
7.3 Summary

This thesis presents a new robust control methodology to suppress the torsional vibrations in flexible drive systems. The position control of a nonlinear 2MM is designed using the immersion and invariance approach. First, appropriate mapping functions are derived to convert the total nonlinear 2MM system into an equivalent reduced-order system with rigid dynamics. This reduced-order system is used as a target system to design a position controller, which is based on the standard PI type control technique. Next through immersion and invariance, the reduced-order controller is applied to the nonlinear 2MM system to suppress the torsional vibrations and yield an over-damped response similar to that of the target system. The control law derivation and stability analysis of the target system are described and discussed. Simulation and experiments using a 2MM drive system are used to validate the proposed control methodology.

After the position controller design, the thesis presents the design of a nonlinear reduced-order observer to estimate the load side position and load side speed of an elastic drive system. The reduced-order observer is designed using the notion of invariant manifold. First, manifold is defined in terms of error dynamics, and then mapping functions are chosen in such a way that the error dynamics become asymptotically stable at the origin. Asymptotical stability of the error dynamics at the origin is proved. Simulation and experimental results are shown to validate the proposed methodology. Then the observer is combined with an I&I position controller and the results are shown in simulations and real time.

After observer design, an input/output feedback linearization technique is explained and implemented on the same system. External and internal dynamics are derived for the 2MM. Feedback linearization gains are designed for external linear dynamics. The stability of zero dynamics is also derived. Then I&I methodology, input/output feedback linearization, and PID position controllers are compared on the basis of transient step response and robustness to external disturbance and parameter variations.
7.4 Concluding Remarks

The aim of this thesis is to present a position controller for elastic drive systems that suppresses the torsional vibration. It also presents a nonlinear reduced-order observer for elastic drive systems to measure the load side position and speed. The following contributions were successfully made in this thesis:

- A robust position controller with vibration suppression for two mass model systems was designed using immersion and invariance methodology.
- The designed position controller using I&I was tested in simulation and the results were very satisfactory.
- Two mass model (2MM) prototypes for real-time implementation were designed and manufactured.
- The designed position controller using I&I was implemented in real-time using dSPACE 1103 and the results matched the simulation results.
- A nonlinear reduced-order observer for two mass models using the notion of invariant manifold were designed to measure the load side position and speed.
- A reduced-order observer was implemented in simulation and real time. The results in both cases were satisfactory. Then the nonlinear observer was incorporated with I&I position controller and tested in both simulation and real time.
- A position controller for two mass models were designed using input/output feedback linearization and implemented in simulation.

7.5 Future Work

The following work can be done as a future development to this thesis:

- In order to further enhance the results, the two mass model (2MM) can be manufactured with more sophisticated and advanced tools, to reduce the effect of misalignment and friction.
- The I&I technique can be further explored, especially adaptive I&I.
References


Appendix A

Numerical & Measured Values of System’s Parameters

Shaft material: Aluminum

Aluminum Density $= \rho_{\text{Al}} = 2700 \text{ kgm}^{-3}$

Length of Shaft $= L_{\text{sh}} = 0.49$ meters

Diameter of Shaft $= D_{\text{sh}} = 8 \text{ mm} = 0.008 = 8 \times 10^{-3}$ meters

Radius of Shaft $= R_{\text{sh}} = D_{\text{sh}}/2 = 0.004 = 4 \times 10^{-3}$ meters

Mass of Shaft $= M_{\text{sh}} = 6.65 \times 10^{-2}$ kg

Shaft Inertia $= J_{\text{sh}} = \frac{1}{2} M_{\text{sh}} \cdot R_{\text{sh}}^2$

$J_{\text{sh}} = (0.5) (6.65 \times 10^{-2}) (4 \times 10^{-3})^2 = 5.32 \times 10^{-7}$ kgm$^2$

Shaft Stiffness $= K_s = \frac{GJ}{L}$

Moment of Inertia $= I = (D_{\text{sh}}^4) \pi/64 = 2.01 \times 10^{-10}$ kgm$^2$

Shear Modulus of Aluminum $= G = 26$ GPa

$K_s = 10.66$ N.m

Disk material: Stainless Steel

Steel Density $= \rho_{\text{steel}} = 7750 \text{ kgm}^{-3}$

$K_t = 55$ mN-m/Amp $= 55 \times 10^{-3}$ Nm/A

$R_a = 0.4 \Omega$

$\tau_e = L/R$

where,

$\tau_e = \text{ Electrical time constant (sec)}$
L = Inductance

R = Terminal Resistance (ohm)

\[(2 \times 10^{-3}) \times (0.4) = L\]

\[L = 0.0008 = 8 \times 10^{-4} \text{ Henry}\]

Rotor Inertia = 1800 gcm\(^2\) = 18 \times 10^{-4} \text{ kg}^2

Resonant frequency

\[w_r = \frac{k_s}{\sqrt{\frac{J_s}{J_r}} (1 + \frac{J_r}{J_m})}\]

After substituting values,

\[w_r = 116.67 \text{ rad.s}^{-1}\]

\[w_r = 18.57 \text{ Hertz}\]
Appendix B

Matlab Code

```matlab
% delete(gco)
clear all
clc

load tss500.mat
load 2mm500.mat
load ini500.mat

tss=tss500; % name of data file
mm=mm500;
ini=ini500;

tx1=3267-400; % data starting point check the reference current
mmx1=2370-400;
inix1=2089-400;

tx2=tx1+4000;
mmx2=mmx1+4000;
inix2=inix1+4000;

x=ini500.X.Data;
x1=x(1,1:4001)

% Load Position

a=tss.Y(1,9).Data; % for target system
a1=a(1,tx1:tx2);

b=mm.Y(1,11).Data; % for 2MM
b1=b(1,mmx1:mmx2);

c=ini.Y(1,15).Data; % for 2MM WITH I&I
c1=c(1,inix1:inix2);

figure(1)
whitebg([0 .1 .2])
p=plot(x1,a1,'--b',x1,b1,':y',x1,c1,'-.r','linewidth',1.5)

p = title('Plot of Load Position vs Time for Target System, 2MM & 2MM with I&I');
set(p, 'FontSize', 10);
p = xlabel('Time (sec)');
set(p, 'FontSize', 12);
p = ylabel('Load Position (degree)');
set(p, 'FontSize', 12);
e=legend('Target System','2MM','2MM with I&I');
set(e, 'Location', 'NorthWest')
grid
```
% Motor Position

a=tss.Y(1,11).Data; % for target system
a1=a(1,tx1:tx2);

b=mm.Y(1,13).Data; % for 2MM
b1=b(1,mmx1:mmx2);

c=ini.Y(1,17).Data; % for 2MM WITH I&I
c1=c(1,inix1:inix2);

figure(2)
whitebg([0 .1 .2])
p=plot(x1,a1,'--b',x1,b1,':y',x1,c1,'-.r','linewidth',1.5)

p = title('Plot of Motor Position vs Time for Target System, 2MM & 2MM with I&I');
set(p, 'FontSize', 10);
p = xlabel('Time (sec)');
set(p, 'FontSize', 12);
p = ylabel('Motor Position (degree)');
set(p, 'FontSize', 12);
e=legend('Target System','2MM','2MM with I&I');
set(e, 'Location', 'NorthWest')
grid

% Reference Current

a=tss.Y(1,3).Data; % for target system
a1=a(1,tx1:tx2);

b=mm.Y(1,4).Data; % for 2MM
b1=b(1,mmx1:mmx2);

c=ini.Y(1,2).Data; % for 2MM WITH I&I
c1=c(1,inix1:inix2);

figure(3)
whitebg([0 .1 .2])
p=plot(x1,c1,'-.r',x1,a1,'--b',x1,b1,':y','linewidth',1.5)

p = title('Plot of Reference Current vs Time for Target System, 2MM & 2MM with I&I');
set(p, 'FontSize', 10);
p = xlabel('Time (sec)');
set(p, 'FontSize', 12);
p = ylabel('Reference Current (A)');
set(p, 'FontSize', 12);
e=legend('2MM with I&I','Target System','2MM');
set(e, 'Location', 'NorthEast')
grid
% Actual Current

a=tss.Y(1,2).Data; % for target system
a1=a(1,tx1:tx2);

b=mm.Y(1,3).Data; % for 2MM
b1=b(1,mmx1:mmx2);

c=ini.Y(1,1).Data; % for 2MM WITH I&I
c1=c(1,inix1:inix2);

figure(4)
whitebg([0 .1 .2])
p=plot(x1,c1,'r',x1,a1,'--b',x1,b1,'y','linewidth',1.5)

p = title('Plot of Actual Current vs Time for Target System, 2MM & 2MM with I&I');
set(p, 'FontSize', 10);

p = xlabel('Time (sec)');
set(p, 'FontSize', 12);

p = ylabel('Actual Current (A)');
set(p, 'FontSize', 12);

grid

% z1 & z2

c=ini.Y(1,5).Data; % for 2MM WITH I&I
c1=c(1,inix1:inix2);

d=ini.Y(1,6).Data; % for 2MM WITH I&I
d1=d(1,inix1:inix2);

figure(5)
whitebg([0 .1 .2])
p=plot(x1,c1,'r','linewidth',1.5)
p = title('Plot of z1 vs Time');
set(p, 'FontSize', 10);

p = xlabel('Time (sec)');
set(p, 'FontSize', 12);

p = ylabel('z1 (Rad)');
set(p, 'FontSize', 12);

grid

figure(6)
whitebg([0 .1 .2])
p=plot(x1,d1,'r','linewidth',1.5)
p = title('Plot of z2 vs Time');
set(p, 'FontSize', 10);

p = xlabel('Time (sec)');
set(p, 'FontSize', 12);

p = ylabel('z1 (Rad)');
set(p, 'FontSize', 12);

grid
Appendix C

Buhler DC Motor

A Buhler DC motor was used as an actuator in the experimental setup. Figure C-1 shows some of the properties of the Buhler DC motor. Detailed characteristics and motor parameters are given in Table C-1. It’s a 24 V DC brushed motor. The maximum rated torque offered by this motor is 0.6 Nm.

![Buhler DC Motor](image)

Table C-1 DC Motor Specification
Baumer Incremental Encoder

Two Baumer incremental encoders with 40,000 ppr were used in the experimental setup on drive and load sides to capture the feedback signals. Encoders were connected to the dSPACE 1103 through a standard RS-422 cable. Figure C-2 shows the encoder. Technical details are shown in Table C-2. Figure C-3 shows the circuit of the complementary line drive of the encoder.

![Baumer Incremental Encoder](image)

**Figure C-2 Baumer Incremental Encoder**

<table>
<thead>
<tr>
<th>Technical data - electrical ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage supply: 5 VDC ±10 %</td>
</tr>
<tr>
<td>Consumption w/o load typ: 60 mA (5 VDC), 40 mA (24 VDC)</td>
</tr>
<tr>
<td>Pulses per revolution: 4096...320000</td>
</tr>
<tr>
<td>Reference signal: Zero pulse, programmable</td>
</tr>
<tr>
<td>Sensing method: Optical</td>
</tr>
<tr>
<td>Output frequency: ±1300 kHz</td>
</tr>
<tr>
<td>Output signals: A 90° B, N + inverted</td>
</tr>
<tr>
<td>Output circuit: Antivalent; Push-pull short-circuit proof</td>
</tr>
<tr>
<td>Interference immunity: DIN EN 61000-6-2</td>
</tr>
<tr>
<td>Emitted interference immunity: DIN EN 61000-6-3</td>
</tr>
<tr>
<td>Approval: UL approval / E21782</td>
</tr>
</tbody>
</table>

**Table C-2 Baumer Incremental Encoder Specifications**

![Complementary Line Drive of Encoder](image)

**Figure C-3 Complementary Line Drive of Encoder**
Simple H Bridge

The simple H-bridge shown in Figure C-4 was used in the hysteresis current controller board. The PWM signal generated from the current controller board, based on the difference between the reference and actual current, was given to the H-bridge to generate the controlled input that would drive the DC motor. It was operated at 24 V and the logic signal was 0-5 V. It could sustain up to 25 amperes of load current with a fan for cooling. A spike of 45 amperes of current for 5 seconds could also be tolerated by this board.

![Simple H-Bridge](image)

Figure C-4 Simple H-Bridge

Detailed specifications of the simple H-bridge are shown in Table C-3.

<table>
<thead>
<tr>
<th></th>
<th>Without Fan</th>
<th>With Fan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Range</td>
<td>6V – 24V (28 V absolute max)</td>
<td>Same</td>
</tr>
<tr>
<td>Current (H-bridge)</td>
<td>20A cont. at 100% duty cycle</td>
<td>25A cont. at 100%</td>
</tr>
<tr>
<td></td>
<td>17A cont. at 70%</td>
<td>20A at 70%</td>
</tr>
<tr>
<td></td>
<td>45A 5 second peak</td>
<td>45A 5 second peak</td>
</tr>
<tr>
<td>Current (each half-bridge)*</td>
<td>Same as above</td>
<td>Same as above</td>
</tr>
<tr>
<td>Current (ganged half-bridge)*</td>
<td>40A cont. at 100%</td>
<td>48A cont. at 100%</td>
</tr>
<tr>
<td></td>
<td>38A cont. at 70%</td>
<td>38A cont. at 70%</td>
</tr>
<tr>
<td></td>
<td>70A 5 second peak</td>
<td>70A 5 second peak</td>
</tr>
<tr>
<td>PWM frequency</td>
<td>DC – 20kHz</td>
<td>DC-20kHz</td>
</tr>
<tr>
<td>Current Sense Output</td>
<td>Vc = 1.0^ 0.075</td>
<td>Same</td>
</tr>
<tr>
<td></td>
<td>Vc = 0.75 at 10A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vc = 2.99V at 40A</td>
<td></td>
</tr>
<tr>
<td>Input voltage levels</td>
<td>2.5V – 5.5V = logic high</td>
<td>Same</td>
</tr>
<tr>
<td>PA,PB,EA,EB</td>
<td>4.5V – 28V for HV version</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;1.7V = logic low</td>
<td></td>
</tr>
<tr>
<td>Size</td>
<td>2.5&quot; x 2.25&quot; x 0.5&quot;</td>
<td>2.5&quot; x 2.25&quot; x 0.75&quot;</td>
</tr>
<tr>
<td>Weight</td>
<td>37g</td>
<td>61g</td>
</tr>
<tr>
<td>Mounting</td>
<td>4x - 4-40 or M2.5 bolts</td>
<td>Same</td>
</tr>
<tr>
<td>Fan</td>
<td>None</td>
<td>50mm x 10mm – 12V</td>
</tr>
</tbody>
</table>

Table C-3 Simple H-Bridge Specifications
VITA

Irfan Ullah Khan was born on January 22, 1986, in Rawalpindi, Pakistan. He received his BE degree in Mechatronics Engineering from National University of Sciences and Technology, Pakistan in May 2009 with a CGPA of 3.57. From 6th semester, he started working in a research center (NexGen RC) as a research student. He worked there on non-invasive medical sensor development. In 2010, he got a job in Dubai as technical support engineer. His work was related to PLC and HMI programming and providing technical support to the customers. In 2011, he joined American University of Sharjah to pursue his Master in Mechatronics Engineering. He was graduate teaching assistant at American University of Sharjah for two years. He got his Master of Science Degree in Mechatronics Engineering in October, 2013 with a CGPA of 3.92.

Irfan is now a PhD candidate at University of Sheffield, UK in Mechanical Engineering Department. He is offered a full scholarship. During his PhD, the research focus will be to analyze multi-degree-of-freedom nonlinear structural vibration using a Lie group approach. In particular the connection between Lie groups associated with particular polynomial nonlinearities, and perturbation methods such as the normal form analysis.