

INTEGRATED MATERIAL LOT SIZING AND MULTI-RESOURCE  
LEVELING MODELS WITH ACTIVITY SPLITTING

by

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## **Dedication**

To my greatest joy, my emotional support, and my endless love, my family.

To whom I shared with my dreams, my downfall nights, and my deepest beliefs, my one and only irreplaceable sister.

To my inspiration, my motivation, and my fuel for innovation, my Aisha.

To all who believed in me, who inspired me, and who picked me up every time I fell, this is dedicated to you.

## **Abstract**

In the project management literature, project scheduling and material procurement problems are often studied separately and are addressed following a sequential approach. The few papers in the literature that integrated both problems assumed that materials are ordered and received prior to their needed activities. This research will contribute to the project management literature by developing mathematical models that integrate project scheduling and material lot sizing decisions when dealing with renewable and consumable resources having constant and variable consumption rates. The formulated models minimize renewable resource leveling related costs and material ordering and inventory associated costs for consumable resources. The models allow activity splitting to smooth the utilization of renewable resources and to optimally schedule the ordering of consumable resources. Both models with time independent and dependent rates of consumption are formulated as mixed integer linear programs. Sensitivity analysis is performed to assess the effect of varying the problem's parameters on the total project cost savings realized after the integration of leveling and ordering decisions. The results show that under certain values of the problem parameters, the integrated decision approach achieved significant cost savings compared to the sequential decision approach. While low holding costs resulted in low cost savings, high holding costs significantly boosted cost savings. Furthermore, the numerical results show that the time-variation of the resource utilization rates has an effect on cost savings. The results of the sensitivity analysis also show that reductions in both resource leveling and material ordering related costs are realized when non-critical activities are allowed to split. Among all input parameters analyzed, the ordering and holding costs had the greatest effect on the total project costs. However, for large values of the ordering and holding costs, cost savings from splitting are not significant. Thus, every cost element in the objective function has an effect on either total holding costs or resource leveling and a project manager has to consider both leveling related costs and material related costs at the same time in order to find the optimal and minimum project cost.

**Search Terms:** Project Scheduling, Material Procurement, Project Management, Resource Leveling, Lot Sizing, Activity Splitting

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## Chapter 1: Introduction

Chapter 1 presents an overview of project management, resource leveling, and materials management. In addition, the chapter highlights the problem statement, the research objectives and significance, and the methodology undertaken in this research.

### 1.1 Overview

In the current competitive global economy, organizations, regardless of their line of business, find the need of embracing project management principles to achieve their business goals. Organizations realize the value of project management in terms of efficiency, resource utilization, and customer satisfaction. It has become clear that effective project management can grow to become a business competitive advantage.

Construction and industrial projects involve structured sequential activities with different types of consumable and renewable resources. While consumable (non-renewable) resources are defined as resources that are consumed once used, such as raw materials and money, renewable resources are defined as resources that have limited availability, such as machines and manpower. Project managers in such projects find themselves faced with an important decision regarding the timing and size of the orders for consumable resources. For a given resource, the manager must decide whether to order all required quantities prior to the project start time or prior to their needed activities. Both options have their own tradeoffs. While ordering prior to project start time ensures the availability of components when they are needed in the project, it raises the issue of storage and inventory holding cost. Further, the ordered materials may be subject to deterioration and damage while in storage for a longer period. On the contrary, ordering materials prior to their needed activities minimizes the inventory holding cost but does not ensure the availability of materials on the start of its project activity (i.e. the lead time might be longer than anticipated), thus increasing the probability of project delay and possible stakeholders' dissatisfaction. Therefore, determining the right time and right quantity for the procurement of non-renewable resources will definitely make the project more cost-efficient.

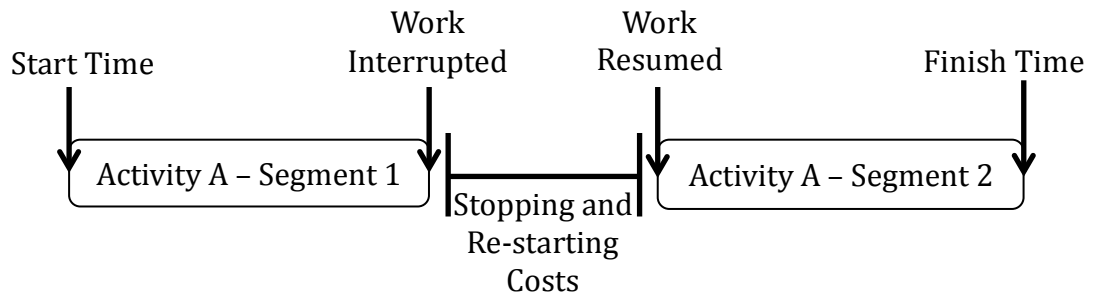
**1.1.1 Project scheduling.** Project management is defined as “the application of knowledge, skill, tools and techniques to manage project activities in order to meet

or exceed stakeholders' needs and expectations from the project" [1]. The process of project management involves four stages: defining, planning, executing, and closing [2]. The planning stage is very critical and important since effective planning can lead to smooth and agile execution of the project. Project planning involves defining the project scope, budget, resources, and schedule, along with project risks. In simple terms, it defines the project activities, their duration, their dependency relationships, and the means and resources for completing them.

The Critical Path Method (CPM) is a well-known and commonly used project scheduling technique. CPM is a method that calculates for each activity in the project the early and late start and finish times to find the longest path on the project network. The longest path found is called the 'critical path' since it consists of activities that are critical. Those critical activities have zero slack and any delay in them will lead to an overall delay in the project completion time. CPM is a simple and powerful method, yet it holds some critical assumptions: (1) activity durations are fixed, (2) activity can only start when all its predecessors are finished, and (3) resources are available in unlimited quantity. These assumptions in reality are not valid most, if not all, the time. Activity durations can be stochastic, and resource conflicts and over-allocation may arise. CPM focuses on the time aspect of the project and finds the minimum duration of a project while ignoring the resource utilization aspect.

In general, projects are usually classified as resource-constrained or time-constrained. A resource-constrained project assumes resources are fixed and time is flexible, while a time-constrained project assumes time is fixed and resources are flexible. In scheduling resource-constrained projects, the number of resources used should not exceed a specified level of quantity. Usually, in this type of scheduling, the project completion date set by CPM is exceeded because of the limitation on resource availability. The problem is then to minimize the project completion time subject to resource availability and activity precedence constraints. As for scheduling time-constrained projects, renewable resources are leveled using the noncritical activities' slacks to balance the resource usage and minimize the period-to-period variations. Despite the minimization of variations in resource demand in this type of scheduling, the project schedule loses flexibility as a result of slack reduction. Activity splitting can also help to reduce the fluctuation in resource utilization. It allows the work on

noncritical activities to be interrupted and their related resources to be allocated to other activities. However, activity splitting will introduce a cost associated with stopping and re-starting the activity again. Figure 1 illustrates the concept of activity splitting.



**Figure 1: Activity Splitting**

Resource allocation and utilization problems are solved using techniques that usually fall under one of the following categories: optimization, heuristics, or meta-heuristics. First, optimization techniques are those that provide an optimal solution (i.e. minimum cost or project duration) like linear and nonlinear programming techniques. These types of techniques optimize the objective function (e.g. a cost or project duration function) subject to a set of constraints (e.g. predecessor relationship, resource availability). Although optimization techniques give the best solution, their computation time increases as the project size increases. Second, heuristic techniques are those that depend on logic-based or experience-based iterative procedures that give a feasible, but not necessary, optimal solution for their corresponding problem. Compared to optimization techniques, heuristics are considered to be more time-efficient in solving large-scale problems. Last, meta-heuristics, unlike heuristics, are general-purpose techniques that can be used to solve any optimization problem but they do not necessarily give the optimal solution. Examples of meta-heuristic techniques are genetic algorithms and particle swarm optimization.

**1.1.2 Materials management.** Materials management is an approach to managing material flow from purchase to delivery in order to provide the right materials with the least cost, and the right quality and quantity at the right time and place. It involves material planning, procurement, inventory planning and quality

control. Four types of costs are incurred when ordering materials: purchasing cost, ordering cost, inventory holding cost, and material shortage cost. In case the unit material cost is unchanged during the project life, the purchasing cost is constant regardless of the procurement schedule followed. However, an ordering policy that orders materials more frequently with small quantities increases the ordering cost and reduces the holding cost. In this thesis, it is assumed that material shortages do not take place as this will result in project delay. Therefore, the best ordering policy is the one that trades-off the remaining three costs. Hereafter, such policy is referred to as the “material lot sizing policy” as it attempts to minimize the sum of purchasing, ordering and holding costs over a finite planning horizon determined by the length of the project.

In the inventory literature, the problem of determining the ordering quantities over a known and fixed planning horizon for a single product with a known and discrete time-varying pattern is termed the “dynamic lot sizing problem” (*DLSP*) [3]. Mathematically, *DLSP* is stated as follows:

$$\text{Minimize } \sum_{t=1}^T [A_t x_t + C_t Q_t + h_t I_t]$$

Subject to :

$$I_t = I_{t-1} + Q_t - D_t \quad \text{for } t = 1, 2, \dots, T$$

$$Q_t \leq M_t x_t \quad (M_t \text{ is a large positive number}) \quad \text{for } t = 1, 2, \dots, T$$

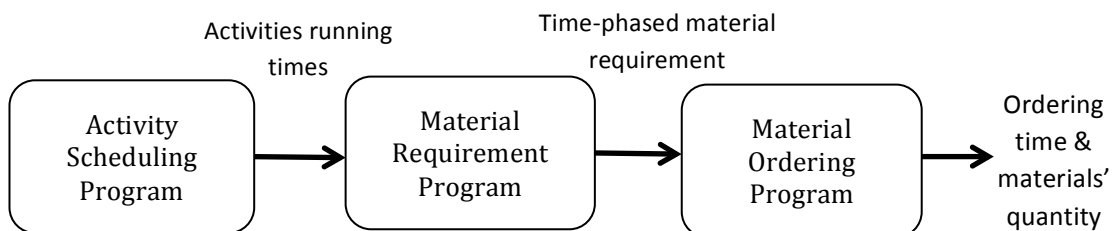
$$Q_t, I_t \geq 0, x_t \in \{0, 1\} \quad \text{for } t = 1, 2, \dots, T$$

$$I_0 = 0 \text{ and } I_T = 0$$

The decision variables in the above optimization problem are the ordering quantities  $Q_t$ , the ending inventory  $I_t$ , and the binary variable  $x_t$  for  $t = 1, 2, \dots, T$ . The binary variable is equal to one when an order is placed and zero otherwise. The first term in the objective function is the ordering cost whereas the second and third terms are the purchasing cost and holding cost, respectively. Note that when an order is not placed in period  $t$ , the ordering cost is zero. The first constraint is called the materials balance constraint as it makes the sum of the ordering quantity and initial inventory equal to the sum of the demand and ending inventory. The second constraint ensures that the ordering quantity is zero when an order is not placed. In spite of its simple

formulation, the *DLSP* optimization program is computationally difficult. Indeed, depending on its size, *DLSP* may not be solved efficiently using standard procedures such as the branch and bound technique or dynamic programming. In this thesis, the commercial software CPLEX, which is based on the branch and bound algorithm, will be used to develop optimal ordering quantities for a given project schedule.

**1.1.3 Project scheduling and material ordering decisions.** In general, there are two types of decision approaches when considering project scheduling and material ordering: the sequential approach and the integrated approach. The sequential approach, as shown in Figure 2, consists of three different independent decision programs which are the activity scheduling program, material requirement program, and material ordering program. The main outputs of the activity scheduling program are the project activities' running times where these are defined as the time periods during which the activity is active. In case the activities are scheduled according to their earliest times, then the running time of an activity is the difference between its earliest finishing time and earliest starting time. Based on the material quantity needed for each activity during each time unit of its duration, the materials requirement program computes a time-phased requirement for each material during the life of the project. In other words, it determines the quantity needed for each material per period, as some materials can be consumed by more than one activity. Using the discrete time varying requirement for each material, the material ordering program generates the optimal ordering lot sizes and times for each material using *DLSP*.



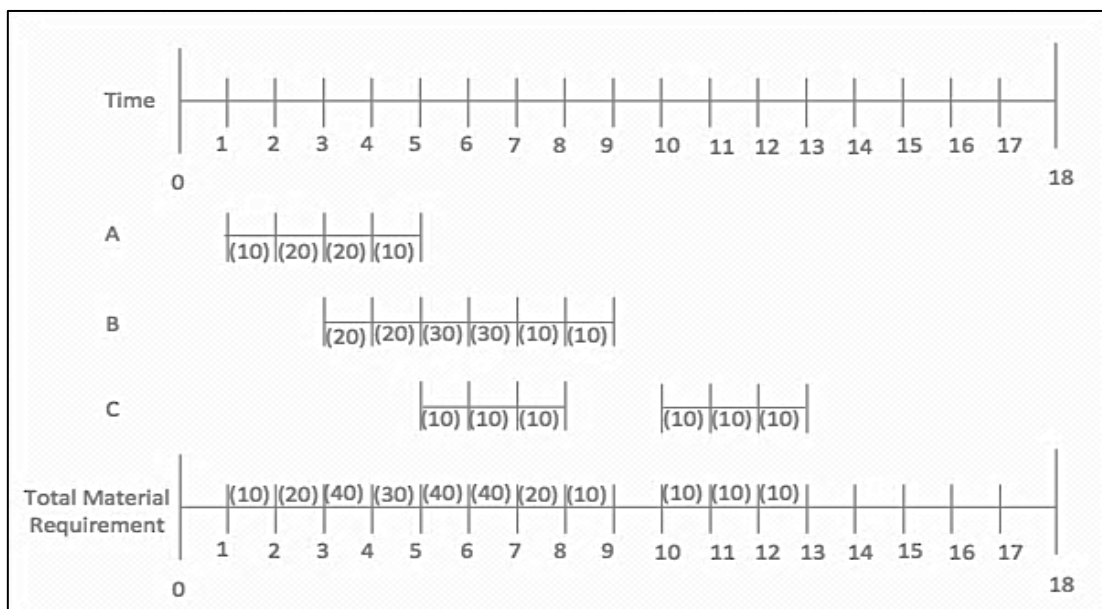
**Figure 2: Sequential Decisions Approach**

On the other hand, the integrated decision approach integrates the three separate programs into one model. The outputs of this model are the activity running



times, ordering times and ordering quantity for each material. The sequential approach will be used in this thesis to assess the economic benefit of following the integrated decision approach.

Figure 3 illustrates the calculation for the time-phased requirement of one material consumed by three activities (A, B, and C). The number between the parentheses are requirements for the material during each time period of the activity durations. According to the output of the scheduling problem, activity A will start at time 1 and finish at time 5, whereas activity B will start at time 3 and finish at time 9. Activity C was split. It will start at time 5, but will be stopped at time 8, and will resume at time 10 to finish at time 13.



**Figure 3: Calculation of Total Material Requirement**

## 1.2 Problem Statement

Project scheduling and material procurement decisions have traditionally been optimized independently. Project managers, normally, would start with project scheduling, often using CPM, and the resultant schedule would be taken as an input for material planning and ordering. The consumable and renewable resource-related costs are then summed to obtain the estimated total project cost. This procedure does not necessarily result in the global optimal project schedule. Therefore, it is important

to integrate project scheduling with material management to globally optimize the total cost of a project.

Further, it was mentioned before that CPM assumes unlimited availability of renewable and consumable resources; thus, the resulting schedule cannot be taken for granted to represent an optimized schedule in terms of time and cost. Although many procedures were developed to modify the CPM approach to account for limited resources, those approaches often refer to renewable resources only. Thus, there is a need to consider the consumable resources in project schedule along with the renewable resources. To this purpose, a mathematical model will be developed to minimize the material related costs (purchasing, ordering and holding costs) and to level the utilization of the renewable resources over a fixed duration of the project assuming that activity splitting is allowed. The model can be used to determine the material ordering schedule and a modified CPM schedule with preempted activities.

### **1.3 Research Objective**

The objective of this research is to develop mathematical models that integrate material procurement and project activity scheduling decisions. The models minimize the total cost of a project with project duration constraint being set by CPM or the customers. The total cost includes the resource-related costs such as acquiring and releasing costs, activity splitting cost, and material purchasing, holding, and ordering costs. The models allow activity splitting in order to smooth the utilization of renewable resources. Splitting an activity also alters the time-phased material requirements in a project and, in turn, may have an impact on those materials' ordering time and lot sizes. The problem will be formulated as mixed integer linear programs under the assumptions of unlimited resources, fixed activity time and slacks, and negligible lead-time. The lead-time is assumed to be negligible (shorter than the time period unit) in order to reduce the complexity of the developed mathematical model.

### **1.4 Research Significance**

The main contributions of this research include the following:

1. Supplement the project scheduling literature with new models that integrate project scheduling and material management decisions.

2. Formulate the integrated problem as mixed integer linear models.
3. Provide an optimal ordering schedule for the consumable resources in addition to the project's activities schedule that balances the utilization of renewable resources.
4. Assess the cost savings achieved through the integration of project scheduling and material ordering decisions.

### **1.5 Research Methodology**

The steps that will be followed to tackle the problem discussed in this research are as follows:

- Step 1: Review the literature related to project scheduling, material management, and resource leveling.
- Step 2: Formulate the optimization models by defining the assumptions, objective functions, constraints, decision variables, and problem parameters.
- Step 3: Code the formulated model using IBM Optimization CPLEX software.
- Step 4: Perform sensitivity analysis to test the effect of varying the models' input on the total project cost savings realized after the integration of project scheduling and material procurement decisions.

### **1.6 Thesis Organization**

Chapter 1 introduced the research problem, objective, significance, and methodology. In the following chapter, a review of the relevant literature is presented. Chapter 3 reviews the renewable resources leveling problem with activity splitting. Chapter 4 discusses the Integrated Resource Leveling and Lot Sizing model with constant consumption rate (IRLLS-C) and the results of a sensitivity analysis. Chapter 5 presents the Integrated Resource Leveling and Lot Sizing model with variable consumption rate (IRLLS-V) and the outputs of a sensitivity analysis. Finally, Chapter 6 provides a summary of the thesis along with conclusions and recommendations for future research areas.

## Chapter 2: Literature Review

Chapter 2 presents a review of the literature related to project scheduling, resource leveling, and integration of material planning and project scheduling.

### 2.1 Project Scheduling

Proper project scheduling plays an important role in the completion of the project on time and within budget. Optimality of project schedules can differ from one project to another, depending on the type of objective a project is focused on, whether it is, for example, minimizing project duration, minimizing project cost, or maximizing project profit. Many studies in the literature focused on defining optimal schedules by maximizing the net present value of a project since organizations mainly undertake projects to maximize their profits [4]. Elmaghraby and Herroelen [5] developed an algorithm that maximizes the net present value by building tree structures in an iterative fashion on the project network. Doersch and Patterson [6] proposed a model that also maximizes the discounted cash flow of a project but was formulated using zero-one integer programming subject to activity sequence, capital utilization, and job completion. The model's objective function included a penalty or bonus payments upon completion of the project after or prior to the required completion date, respectively. The considered problem was resource-constrained where resources in this context refer to consumable resources.

Although the above mentioned works focused on maximizing the net present value as an objective function, other studies focused on different types of objectives. Castro-Lacouture *et al.* [7] used a fuzzy mathematical model that optimized the schedule of a construction project subject to cost, time and unforeseen material shortage. Three methods were used to analyze the project network. They were the manual CPM, Primavera Project Management software (P5), and Optimization Programming Language software. In addition, a heuristic approach was used for sensitivity analysis and material allocation. They showed that the project duration is affected by the shortage of a material and also by the material allocation method to different activities, as materials should be assigned to critical or low-slack activities rather than arbitrary activities that are ready to start.

## 2.2 Resource Constrained Project Scheduling

Resource constrained project scheduling (RCPS) is a very active area of research since almost all projects in real life have limitations in terms of resources. Russel [8] addressed the problem of scheduling large-scaled projects that have resource restrictions while maximizing the net present value. He assumed while modeling the problem that activity durations are fixed and the resource level required for each time is determined. Further, activities could not start until their predecessors are completed. Six heuristics approaches were developed and tested on 80 problems, but none of them dominated in performance compared to others.

Easa [9] presented an optimization model for a single resource-leveling problem for small and medium sized projects in the construction field. The model utilized the CPM scheduling results to minimize variations in resource usage between the actual and the desired level using an integer-linear programming formulation. Although the model tackled the problem of leveling one resource, it could be extended to level multiple resources.

Neumann and Schwindt [10] introduced a single-mode project-planning problem under resource limitation with the objective of minimizing the project duration subject to temporal constraints defined by minimum and maximum time lags and resource constraints defined by minimum and maximum inventories. The inventories considered in this problem were limited cumulative resources that included renewable and consumable resources. The authors proposed to solve the problem with a branch-and-bound algorithm with a filtered beam search heuristic.

Son and Mattila [11] considered construction projects that allowed selective activity splitting (i.e. the work on an activity can be stopped and resumed later). They proposed a binary linear programming model that minimized the difference between the actual and desired daily resource requirement subject to daily resource levels, activity durations and predecessors. The authors found that integrating activity splitting in resource leveling models results in better solutions compared to models that did not allow activity splitting.

Nudrasomboon and Randhawa [12] developed an optimization model for resource-constrained projects that minimized the project total cost, the project duration, and the variations between periods' resource utilization level. The developed

model was a zero-one integer-programming model that took into account renewable and consumable resources, splittable and non-splittable activities, along with time-resource tradeoffs.

Hariga and El-Sayegh [13] developed a multi-resource leveling optimization model which allowed for activity splitting while minimizing an objective function composed of the startup and restarting costs of the preempted activities and the splitting costs. The model was formulated as a mixed binary and integer linear program and was optimized subject to resource balance constraints, network logic constraints, and activity duration constraints. Al-Sayegh and Hariga [14] proposed to solve the multi-resource leveling problem of Hariga and El-Sayegh [13] using a hybrid meta-heuristic method which combines particle swarm optimization with simulated annealing search procedures.

Ashuri and Tavakolan [15] designed a fuzzy-enabled hybrid genetic-particle swarm optimization algorithm to minimize the total project duration and cost, and total variations of resource utilization for complex time-cost-resource optimization problems in construction planning. The method succeeded in finding superior solutions compared to existing optimization algorithms in terms of project cost, project duration, total variation in resource allocation, and processing speed, but without allowing any activity splitting.

### **2.3 Integration of Material Planning and Project Scheduling**

Project scheduling techniques that account for resource constraints, like the ones mentioned above, were developed to overcome the shortcoming of scheduling techniques like CPM that were originally developed to account for projects with unconstrained resources [16]. However, none of the mentioned techniques integrates the project scheduling problem with the material planning problem. In fact, those two problems were dealt with separately in the literature.

For example, in material planning literature, Said and El-Rayes [17] presented an optimization model that integrated the material procurement decisions with material storage for construction logistics planning. The model minimizes the construction logistic costs (i.e. layout, financing, stock-out, and material ordering costs) using genetic algorithms. It aims to evaluate the impact of late material delivery that causes possible material shortages and delays in project completion.

Results showed that activities' criticality affects the material procurement decisions where the latter eventually affects the site layout decision along with storage space and layout constraints.

Georgy and Basily [18] proposed an optimization model for material ordering and delivery scheduling in the construction field. They developed a computer-based systematic procedure that minimized the total cost linked to material deliveries using genetic algorithms. The proposed system is linked with Primavera P3 software, a common commercial scheduling software, to ease the retrieval of project data and the calculations of the material delivery schedule. However, the authors assumed that the project schedule was predetermined before the materials requirement and planning process, and that the lead-time was constant.

To the best of our knowledge, Aquilano and Smith [16] were the first to integrate project scheduling and material planning decisions in a single model. They developed a hybrid model that integrates critical path method (CPM) and materials requirement planning (MRP) to schedule projects and resources. They integrated resources' requirements with resources' lead-time in an algorithm that they solved heuristically. This was possible through the integration of inventory records with the scheduling technique. The authors assumed in their model that all activities have constant duration. Their results showed that resources' lead-time affected the project schedule, and that there was a relationship between activity durations and release date of resource orders.

Smith and Aquilano [19] extended Aquilano and Smith's model by including resource constraints. Their model was developed under the assumption of deterministic and constant lead time. They used a heuristic procedure to schedule large projects subject to activity durations, precedence constraints, resources' acquisition lead-time and resources' availability. Resources in this model included materials, labor, facilities, equipment, and manufactured components. The developed algorithm achieved total cost reduction through better labor utilization and material cost reduction (ordering and holding costs).

Smith and Smith [20] attempted to maximize the net present worth subject to material and capital constraints. The maximized present worth included inflows resulting from progress payments paid as activities were completed. It also covered

cash outflows resulting from resource holding and ordering costs in addition to activities' expenses and incurred performance penalties if the project was completed late. The authors used mixed-integer programming formulation and found that the project schedule was affected by material management factors and material acquisition plans. However, a major assumption they made is that positive cash flows are not available for use until the start of its following period. This created a dependency between the project activities schedule and the amount of constrained capital.

In another paper, Smith and Smith [21] tackled the same problem of combining project scheduling along with materials ordering. They decomposed the problem into two sub-problems. The first one derives the project schedule whereas the second one determines the sizes of material lots. This decomposition allowed the problem to have optimal solution using mixed integer programming formulation subject to various costs related to material management. The authors found that activities are scheduled at their late start time and materials are ordered to arrive prior or on that time. These results are obtained under the assumptions that resources are unconstrained in availability, the flow of cash is negative during the project time, and that payments are only received when all the project activities are completed.

Shtub [22], on the other hand, targeted projects with expensive, long-lead inventory items ordered from external vendors, and attempted to minimize the cost of such projects. The problem was addressed by integrating materials management with CPM analysis along with assessing the CPM schedule feasibility. The proposed method dealt with materials management throughout the project's life cycle through the introduction of an inventory control module to the system's model base as well as inventory data to the system's database. The author used a heuristic solution procedure to solve the problem assuming that project activity durations are constant. Accordingly he found that integrating CPM and materials management in large construction projects results in project length reduction by about 20% compared to the original contract.

Caron *et al.* [23] introduced a stochastic model that protects against changes in materials delivery and completion rate by assessing the quantity of materials that should be available on site at a given time. Their approach focused on evaluating the



safety stock and the safety lead-time required for materials delivery in each planning period along with delivery date variability. The advantage of this method is that it can be applied during the early preliminary project planning phase (when detailed project activities are not yet clear).

Dodin and Elimam [24] considered the variability in activity duration along with the introduction of reward/penalty and materials discounts. The impact of such treatment was investigated on the activities schedule and materials plan. Their approach resulted in more flexible project scheduling and more cost reduction and savings in holding cost. They also found that activities always finish as late as possible within the completion time.

Sajadieh *et al.* [25] used the model in [24] to integrate the problem of materials ordering with project scheduling in order to minimize the total cost of a project. The authors defined the total cost in the project to include the material ordering cost, procurement cost, holding cost, completed activities holding cost, crashing cost and finally penalty cost if the project is delayed. The model was formulated using the mixed integer programming method and solved using the genetic algorithm approach. The output determined the project and materials ordering schedules assuming that lead time is deterministic and materials needed for each activity are ordered in a single batch.

Tserng *et al.* [26] formulated a model that aims to minimize the integrated inventory cost of a construction supply chain system. The authors focused on steel rebar production and studied its supply operations to develop a system that generates production and supply plans for steel rebar suppliers and buyers. The paper further studied the influence of transaction constraints on inventory cost and discussed cases of global optimization of the inventory cost for the entire supply chain.

Polat *et al.* [27] introduced a simulation-based decision support system that takes the project conditions and recommends to contractors the most economical rebar management system in terms of scheduling strategy, lot and buffer sizes. The system offers to the contractors, at the planning stage, cost of inventory for different rebar management systems that ranges from just in case (JIC) to just in time (JIT). In a case study directed in a developing country, it was found that JIC is the most economical rebar management system with savings of 4.8% over JIT.

Chen and Zhao [28] developed an optimization algorithm based on dynamic programming that integrates resource planning and scheduling with materials supply management in a project-driven supply chain model. The model achieved significant savings in cost compared to common practices that treat project scheduling and materials ordering independently. However, it is very simplified as it does not handle renewable resource constraints, economies of scale, material lead-time and activity duration tradeoff for recurrent projects.

Najafi *et al.* [29] proposed a model that integrates materials ordering and project scheduling that is based on minimizing the total ordering and holding costs of materials. The model is formulated using the mixed integer programming method and solved using a meta-heuristic algorithm.

Fu [30] considered a multi-mode resource-constrained project scheduling problem to minimize project total cost which includes materials purchasing cost, inventory ordering and holding costs, renewable resource cost, backorder cost in addition to delay/early penalty/reward costs. The model is formulated as a mixed integer program subject to activity predecessors' relationships, renewable resource requirements, and inventory level. A hybrid algorithm that included harmony search and a genetic algorithm was used to solve the problem provided that the activity lead times are constant and the materials consumption is uniform.

## **2.4 Chapter Summary**

As can be noticed from the preceding literature review, most papers addressed materials' procurement and project scheduling problems separately. In other words, materials ordering and activity scheduling decisions were addressed sequentially. However, the integrated decisions approach will be followed in this research. To the best of our knowledge, the few papers that integrated materials' procurement and project scheduling problems assumed that materials should be ordered and be available before their needed activity. Although this saves material ordering cost, it may result in excessive inventory holding costs. If more frequent orders are made, as suggested by this research, economic savings in holding costs could be achieved. Finally, while many papers in the literature considered resource leveling of renewable resources, this research will consider resource leveling for renewable resources while minimizing the cost of consumable resources.

## Chapter 3: Cost Optimization Model with Multi-resource Leveling and Allowed Activity Splitting

Chapter 3 presents an overview of the optimization model proposed by Hariga and El-Sayegh [13] that optimizes the cost of the multiresource leveling problem with allowed activity splitting. This chapter includes the mathematical formulation of Hariga and El-Sayegh's model and an illustrative example that will be used to benchmark the performance of the proposed model.

### 3.1 Optimization Model Overview

Hariga and El-Sayegh's [13] optimization model is aimed to solve multiresource leveling problems with activity splitting. The model balances the costs of acquiring and releasing renewable resources while considering splitting costs associated with the interruption of work on an activity (i.e. startup and restarting costs). The model uses mixed binary integer programming to optimize the costs subject to resource balance constraints, network logic constraints, and activity duration constraints.

The model takes into consideration a project with  $n$  activities. Each activity has a fixed duration of  $T_j$ ,  $j=1, 2, \dots, n$ . The model utilizes the CPM method to calculate the project duration, the earliest start time  $ES_j$ , the earliest finish time  $EF_j$ , the latest start time  $LS_j$ , the latest finish time  $LF_j$ , and the total float time  $TF_j$ . In addition, CPM identifies the critical and noncritical activities based on precedence relationships. Based on this information, the model minimizes the fluctuation of the usage of  $rp$  renewable resources over the project lifetime for noncritical activities using activity splitting while ensuring that the project completion time set by CPM is not exceeded.

**3.1.1 Model assumptions.** The formulation of Hariga and El-Sayegh's model is based on the following set of assumptions:

1. For running the project, there are  $rp$  available resource types.
2. Each activity has a fixed renewable resource requirement rate over its running duration.
3. There is a cost  $CS_j$  associated with splitting a noncritical activity  $j$ .

4. The setup time to restart an activity is relatively small to be carried out at the end of the split period.
5. After preemption of a noncritical activity, its renewable resource requirement remains the same and does not change.
6. For splittable activities, their precedence relationships must be met.

**3.1.2 Problem parameters.** The developed optimization model has the following problem parameters:

$nn$	Number of noncritical activities
$rp$	Number of renewable resources
$CI_p$	Cost of acquiring one unit of renewable resource type $p$ ( $p = 1, \dots, rp$ ).
$CD_p$	Cost of releasing one unit of renewable resource type $p$ ( $p = 1, \dots, rp$ ).
$CS_j$	Cost of splitting noncritical activity $j$ ( $j = 1, 2, \dots, nn$ ).
$rr_{ip}$	Number of units of renewable resource type $p$ ( $p = 1, 2, \dots, rp$ ) per time period needed to run activity $i=1, 2, \dots, n$ .
$z_{ti}$	Binary parameter equal to one when critical activity $i$ ( $i = 1, 2, \dots, nc$ ) is active from period $ES_i$ to period $EF_i$ and zero otherwise, where $t$ ( $t = 1, 2, \dots, T$ ).

**3.1.3 Problem decision variables.** The developed optimization model has the following decision variables:

$I_{tp}$	Number of units of renewable resource type $p$ ( $p = 1, \dots, rp$ ) acquired during period $t$ ( $t = 1, 2, \dots, T$ ).
$D_{tp}$	Number of units of renewable resource type $p$ ( $p = 1, \dots, rp$ ) released during $t$ ( $t = 1, 2, \dots, T$ ).
$RR_{tp}$	Requirement for renewable resource type $p$ ( $p = 1, \dots, rp$ ) during period $t$ ( $t = 1, 2, \dots, T$ ).
$y_{tj}$	Binary variable equal to one when noncritical activity $j$ is active (running) during period $t$ and zero otherwise, $t = ES_j, ES_{j+1}, \dots, LF_j$ and $j = 1, 2, \dots, nn$ .
$L_{tj}$	Non-negative variable to determine whether activity $j$ is split in period $t + 1$ .
$NL_j$	Number of times noncritical activity $j$ is split, $j = 1, 2, \dots, nn$ .
$S_j$	Start time of noncritical activity $j$ , $j = 1, 2, \dots, nn$ .
$F_j$	Finish time of noncritical activity $j$ , $j = 1, 2, \dots, nn$ .

**3.1.4 Model formulation.** The cost optimization model of Hariga and El-Sayegh for the resource leveling problem and allowed activity splitting is mathematically stated as follows:

$$\text{Minimize } \sum_{p=1}^{rp} \left[ CI_p \sum_{t=1}^T I_{tp} + CD_p \sum_{t=1}^T D_{tp} \right] + \sum_{j=1}^{nn} CS_j NL_j + \alpha \sum_{j=1}^{nn} (F_j - S_j) \quad (1)$$

Subject to:

- Resource Balance Constraints for Renewable Resources:

$$RR_{tp} - RR_{(t-1)p} + D_{tp} - I_{tp} = 0, \quad t=1, 2, \dots, T \text{ and } p = 1, 2, \dots, rp \quad (2)$$

$$RR_{tp} = \sum_{i=1}^{nc} [rr_{ip} z_{ti}] + \sum_{j=1}^{nn} [rr_{jp} y_{tj}], \quad t = 1, 2, \dots, T \text{ and } p = 1, 2, \dots, rp \quad (3)$$

- Activity Duration Constraints:

$$\sum_{t=ES_j}^{LF_j} y_{tj} = T_j, \quad j=1,2,\dots,nn \quad (4)$$

- Network Logic Constraints:

$$S_j = (T + 1) - \text{Max}\{(T + 1 - t)y_{tj} : t = ES_j, ES_j + TF_j\}, \quad j = 1, 2, \dots, nn \quad (5)$$

$$F_j = \text{Max}\{ty_{tj} : t = LF_j - TF_j, LF_j\}, \quad j = 1, 2, \dots, nn \quad (6)$$

$$S_k \geq F_j + 1, \quad j = 1, 2, \dots, nn \text{ and } k \in \text{Succ}(j) \quad (7)$$

- Splitting Cost Related Calculations:

$$L_{tj} = \text{Max}(y_{tj} - y_{(t+1)j}, 0), \quad j = 1, 2, \dots, nn \text{ and } t = ES_j, LF_j \quad (8)$$

$$NL_j = \sum_{t=ES_j}^{LF_j} L_{tj} - 1, \quad j = 1, 2, \dots, nn \quad (9)$$

- Non-negativity Constraints:

$$I_t, D_t \geq 0, \quad t = 1, 2, \dots, T-1$$

$$y_{tj} \in \{0,1\}, \quad j = 1, 2, \dots, nn \text{ and } t = ES_j, LF_j$$

$$y_{T+1,j} = 0, \quad j = 1, 2, \dots, nn$$

$$L_{tj} \geq 0, \quad j = 1, 2, \dots, nn \text{ and } t = ES_j, LF_j$$

The objective of Hariga and El-Sayegh's model, given by Equation 1, minimizes the total acquiring and releasing costs of the renewable resources used in addition to the total splitting costs of noncritical activities over the project duration. The fourth term in the objective function forces the starting times to take the largest

possible value and the finishing times to take the lowest possible value where  $\alpha$  is a very small constant value.

The objective function is minimized subject to resource balance constraints, activity duration constraints, and network logic constraints. The resource balance constraints, referred to as Equation 2 and 3, ensure that the renewable resource requirements for all activities active on period  $t+1$  are equal to the resource requirement for all activities active on period  $t$  plus the resources released on period  $t$  and minus the resources acquired on period  $t$ .

On the other hand, the activity duration constraint, referred to as Equation 4, ensures that the noncritical activities are active for the same amount of their duration.

As for the network logic constraints, Equations 5, 6 and 7 preserve the precedence relationships between noncritical activities and between critical and noncritical activities. This excludes the relationships between critical activities since they are performed on time in order to maintain the minimum project duration.

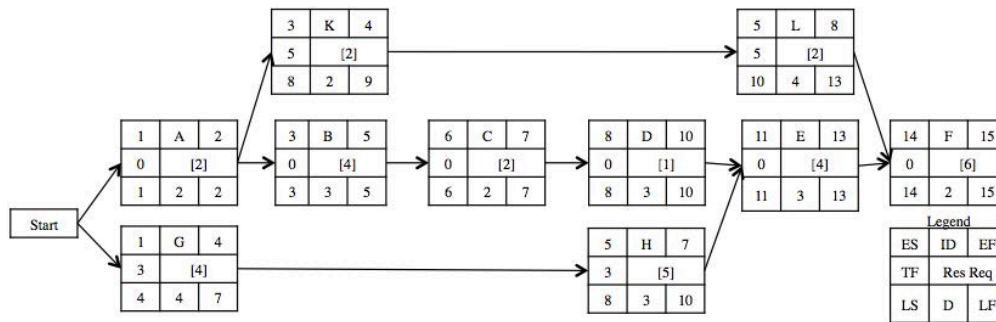
Overall, the model has  $\sum_{j=1}^{nn}(LF_j - ES_j + 1)$  binary variables and  $(2T * rp) + \sum_{j=1}^{nn}(LF_j - ES_j + 1) + 2nn$  continuous variables at most. In terms of constraints, it has  $(T - 1)rp + nn + \sum_{j=1}^{nn}(LF_j - ES_j + 1) + 2 \sum_{j=1}^{nn}(TF_j + 1) + \sum_{j=1}^{nn}(nn_j)$  at most where  $nn_j$  is the number of immediate successors of activity  $j$ .

### 3.2 Illustrative Example

In this section an example is used to illustrate Hariga and El-Sayegh's [13] optimization model and to evaluate its performance. The example defines 10 activities (A, B, C, D, E, F, G, H, K, and L) with constant activity durations and one resource type, R1. R1 is a renewable resource of undefined limit that is used across all activities with different resource requirements. Figure 4 shows the project network diagram along with each activity's duration and resource requirement.

Using CPM, the optimum schedule that results with the minimum project duration, the early start and finish times ( $ES$ ,  $EF$ ), late start and finish times ( $LS$ ,  $LF$ ), slack times ( $TF$ ), and critical and noncritical activities are defined. As shown in Figure 4, the optimum project duration is 15 while the critical activities are A, B, C, D, E, and F, the noncritical activities are G, H, K, and L. The project duration will remain unchanged throughout this example as well as the critical activities since changing them might increase the project duration which is an undesired outcome.

The only moveable activities are the noncritical ones since they have slack times which can be used without affecting the project duration. In Hariga and El-Sayegh's model, the slack times are utilized to smooth the R1 resource utilization diagram across time. This becomes more effective as splitting is allowed.



**Figure 4: Project Network for Base Model**

The CPM results are taken as an input to Hariga and El-Sayegh's model in addition to activity resource requirements, splitting cost (CS), acquiring costs, and releasing costs as Table 1 shows.

**Table 1: Data Input for Base Model**

Activity	Dur	Res1	ES	LF	TF	CS	CI	CD
A	2	2	1	2	0	1	20	20
B	3	4	3	5	0	1		
C	2	2	6	7	0	1		
D	3	1	8	10	0	1		
E	3	4	11	13	0	1		
F	2	6	14	15	0	1		
G	4	4	1	7	3	1		
H	3	5	5	10	3	1		
K	2	2	3	9	5	1		
L	4	2	5	13	5	1		

The resultant optimum schedule from Hariga and El-Sayegh's model is shown in Tables 2 and 3. Table 2 shows R1 utilization per period, whereas Table 3 shows the obtained values of  $y_{ij}$  for each activity with 1 referring to a period when the activity is active, and 0 when the activity is not active.

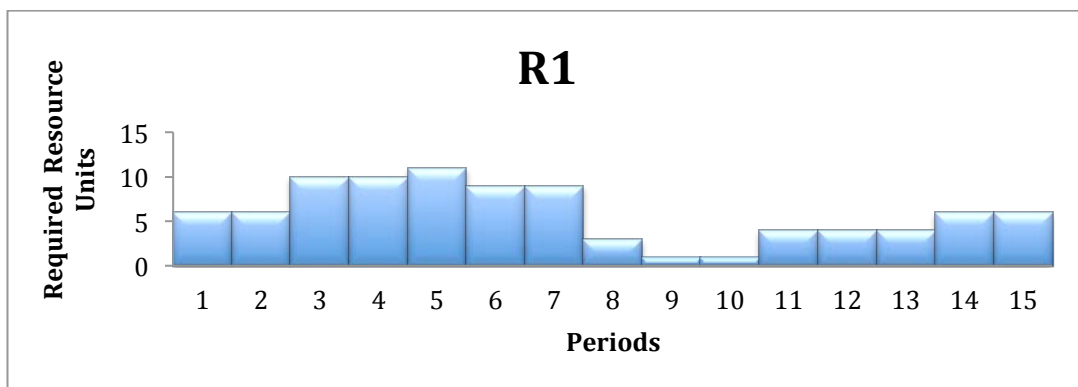
**Table 2: R1 Utilization per Period**

Activity	Periods														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R1	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
I	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

**Table 3: Gantt Chart for Base Model Result**

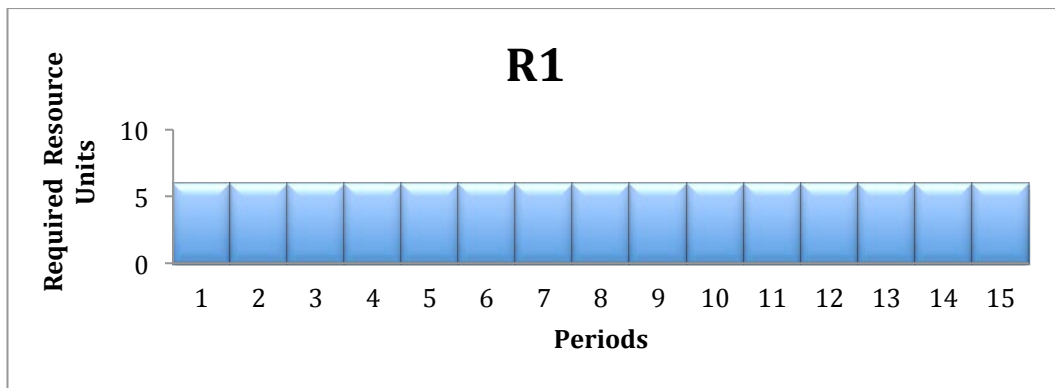
Activity	Periods														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1	1													
B			1	1	1										
C						1	1								
D								1	1	1					
E											1	1	1		
F														1	1
G	1	1	0	0	0	1	1								
H					0	0	0	1	1	1					
K			1	1	0	0	0	0	0						
L					1	0	0	0	0	0	1	1	1		

The results show that two, out of four, noncritical activities are split (G and L) to smooth the resource utilization profile. For example, activity G starts at time 1 and runs for two periods and then is stopped for three periods, and then it is resumed for additional two periods to complete. The stop and restart has a splitting cost associated with it, which in this particular example is 1. The resource utilization profile for R1 before and after Hariga and El-Sayegh’s model are shown in Figures 5 and 6, respectively.



**Figure 5: R1 Utilization Profile - Without Splitting**





**Figure 6: R1 Utilization Profile - With Splitting**

According to Figure 5, the total period-to-period deviation is equal to  $|6 - 10| + |10 - 11| + |11 - 9| + |9 - 3| + |3 - 1| + |1 - 4| + |4 - 6| = 4 + 1 + 2 + 7 + 2 + 3 + 2 = 21$ . In contrast, the total period-to-period deviation, as shown in Figure 6, is equal to 0. There is a significant impact that resource leveling has made to the resource utilization profile as the peak in Figure 5 is 11 while in Figure 6 is 6. This can be translated to cost reduction as well. The total cost for both cases when the splitting is allowed or not allowed is calculated below.

Total Cost (Without Splitting) = Acquiring Cost + Releasing Cost

$$= (16 * 20) + (10 * 20) = 520$$

Total Cost (With Splitting) = Splitting Cost + Acquiring Cost + Releasing Cost

$$= 2 + (6 * 20) + (0 * 20) = 122$$

In this example, the cost impact is substantial. Hariga and El-Sayegh's model balances between the splitting cost and the releasing and acquiring costs. In here the splitting cost is less than the releasing and acquiring costs, thus the expense is incomparable to the saving it generates.

### 3.3 Chapter Summary

Hariga and El-Sayegh's model levels renewable resource utilization using noncritical activities' float time and splitting when needed. Although the splitting cost is an added expense to the project, it helps smoothing the renewable resources utilization profile thus decreasing the acquiring and releasing costs. Whenever the decrease of the latter costs is more significant than the splitting costs, splitting occurs.

## Chapter 4: Integrated Resource Leveling and Lot Sizing Model with Constant Consumption Rate (IRLLS-C)

The model developed in this chapter jointly optimizes the cost of leveling the utilization of renewable resources and the material related costs of consumable resources with constant consumption rates. This chapter includes the mathematical formulation of the extended model and an illustrative example.

### 4.1 Optimization Model Overview

The formulated model in this section integrates materials procurement and project activity scheduling decisions. It minimizes the total cost of a project including its project scheduling related costs and material procurement related costs. While project scheduling related costs are acquiring costs, releasing costs, and activity splitting cost, material procurement related costs are material purchasing costs, material holding costs, and material ordering costs. The model is formulated as a mixed binary and integer linear program which optimizes the costs subject to resource balance constraints, network logic constraints, activity duration constraints, and material balance constraints.

As in Hariga and El-Sayegh's model, the extended model addresses a project involving  $n$  activities. Each activity has a fixed duration of  $T_j$ ,  $j=1, 2, \dots, n$ . It also utilizes the CPM method to calculate the project duration, the earliest start time  $ES_j$ , the earliest finish time  $EF_j$ , latest start time  $LS_j$ , latest finish time  $LF_j$ , and total float time  $TF_j$ .

**4.1.1 Model assumptions.** The formulation of the extended model holds the same assumptions mentioned in section 3.1.1 in addition to the following:

1. Each activity has a fixed consumable resource requirement rate over its running duration.
2. After preemption of a noncritical activity, its consumable resource requirement remains the same and does not change.

**4.1.2 Problem parameters.** The extended optimization model has the same problem parameters mentioned in section 3.1.2 in addition to the following:

- $cp$  Number of consumable resources.
- $cr_{ip}$  Number of units of consumable resource type  $p$  ( $p = 1, 2, \dots, cp$ ) per time period needed to run activity  $i=1, 2, \dots, n$ .
- $A_p$  Ordering cost of consumable resource type  $p$  ( $p = 1, \dots, cp$ ).
- $C_p$  Material cost of consumable resource type  $p$  ( $p = 1, \dots, cp$ ).
- $h_p$  Holding cost of consumable resource type  $p$  ( $p = 1, \dots, cp$ ).

**4.1.3 Problem decision variables.** The extended optimization model has the same problem decision variables mentioned in section 3.1.3 in addition to the following:

- $CR_{tp}$  Requirement for consumable resource type  $p$  ( $p = 1, \dots, cp$ ) in period  $t$  ( $t = 1, 2, \dots, T$ ).
- $O_{tp}$  Binary parameter equal to one when an order is placed for consumable resource type  $p$  ( $p = 1, \dots, cp$ ) at period  $t$ ,  $t = 1, 2, \dots, T$  and zero otherwise.
- $Q_{tp}$  Ordering quantity of consumable resource type  $p$  ( $p = 1, \dots, cp$ ) acquired during period  $t$ ,  $t = 1, 2, \dots, T$ .
- $Inv_{tp}$  Ending inventory of consumable resource type  $p$  ( $p = 1, \dots, cp$ ) at period  $t$ ,  $t = 1, 2, \dots, T$ .

**4.1.4 Model formulation.** The cost optimization model of the extended model of Hariga and El-Sayegh for the resource leveling problem for renewable and consumable resources of constant resource requirements and allowed activity splitting is mathematically stated as follows:

Minimize

$$\sum_{p=1}^{rp} \left[ CI_p \sum_{t=1}^T I_{tp} + CD_p \sum_{t=1}^T D_{tp} \right] + \sum_{j=1}^{nn} CS_j NL_j + \alpha \sum_{j=1}^{nn} (F_j - S_j) + \sum_{p=1}^{CP} \sum_{t=1}^T [A_p O_{tp} + C_p Q_{tp} + h_p Inv_{tp}] \quad (10)$$

Subject to:

- Resource Balance Constraints for Renewable Resources:

$$RR_{tp} - RR_{(t-1)p} + D_{tp} - I_{tp} = 0, \quad t=1, 2, \dots, T \text{ and } p = 1, 2, \dots, rp \quad (11)$$

$$RR_{tp} = \sum_{i=1}^{nc} [rr_{ip} z_{ti}] + \sum_{j=1}^{nn} [rr_{jp} y_{tj}], \quad t = 1, 2, \dots, T \text{ and } p = 1, 2, \dots, rp \quad (12)$$

- Activity Duration Constraints:

$$\sum_{t=ES_j}^{LF_j} y_{tj} = T_j, \quad j=1,2,\dots,nn \quad (13)$$

- Network Logic Constraints:

$$S_j = (T + 1) - \text{Max}\{(T + 1 - t)y_{tj}; t = ES_j, ES_j + TF_j\}, \quad j= 1, 2, \dots, nn \quad (14)$$

$$F_j = \text{Max}\{ty_{tj}; t = LF_j - TF_j, LF_j\}, \quad j= 1, 2, \dots, nn \quad (15)$$

$$S_k \geq F_j + 1, \quad j= 1, 2, \dots, nn \text{ and } k \in \text{Succ}(j) \quad (16)$$

- Splitting Cost Related Calculations:

$$L_{tj} = \text{Max}(y_{tj} - y_{(t+1)j}, 0), \quad j= 1, 2, \dots, nn \text{ and } t = ES_j, LF_j \quad (17)$$

$$NL_j = \sum_{t=ES_j}^{LF_j} L_{tj} - 1, \quad j= 1, 2, \dots, nn \quad (18)$$

- Material Balance Constraints for Consumable Resources:

$$Inv_{tp} = Inv_{(t-1)p} + Q_{tp} - CR_{tp} \quad t=1, 2, \dots, T \text{ and } p = 1, 2, \dots, cp \quad (19)$$

$$Q_{tp} \leq M O_{tp} \quad t=1, 2, \dots, T \text{ and } M \text{ is large number} \quad (20)$$

$$CR_{tp} = \sum_{i=1}^{nc} [cr_{ip}z_{ii}] + \sum_{j=1}^{mn} [cr_{jp}y_{ij}], \quad t = 1, 2, \dots, T \text{ and } p = 1, 2, \dots, CP \quad (21)$$

- Non-negativity Constraints:

$$I_t, D_t \geq 0, \quad t=1, 2, \dots, T-1$$

$$y_{tj} \in \{0,1\}, \quad j= 1, 2, \dots, nn \text{ and } t = ES_j, LF_j$$

$$y_{T+1,j} = 0, \quad j= 1, 2, \dots, nn$$

$$L_{tj} \geq 0, \quad j= 1, 2, \dots, nn \text{ and } t = ES_j, LF_j$$

$$Q_t, Inv_t \geq 0, \quad t=1, 2, \dots, T$$

$$Inv_0, Inv_T = 0$$

The extension of Hariga and El-Sayegh's model lies in integrating the project activity scheduling decisions with material procurement decisions. This translates to two main additions: (1) adding the material related costs to the objective function, and (2) adding material balance constraints for consumable resources.

The objective function, in Equation 10, minimizes six types of costs, three of which relate to project scheduling (acquiring, releasing and splitting costs), and the

other three are related to material procurement (ordering, purchase, and holding costs).

The materials balance constraint, in Equation 19, ensures that the sum of the ordering quantity and initial inventory equal the sum of the demand and ending inventory. Equation 20 ensures that the decision variable  $O_{tp}$  is 1 only when an order is placed and 0 otherwise.

## 4.2 Illustrative Example

This section uses the same example in Chapter 3 with multiple additions to assess the performance of the model developed in this chapter. The example in Chapter 3 only considers renewable resources while Chapter 4 incorporates both renewable and consumable resources, in addition to material related costs in the objective function. The example still defines 10 activities (A, B, C, D, E, F, G, H, K, and L) with constant activity durations and one renewable resource type, R1, and two consumable resource types, R2 and R3, of undefined limit that are used across all activities with different resource requirement levels. Figure 7 shows the project network diagram along with each activity's duration and resource requirement.

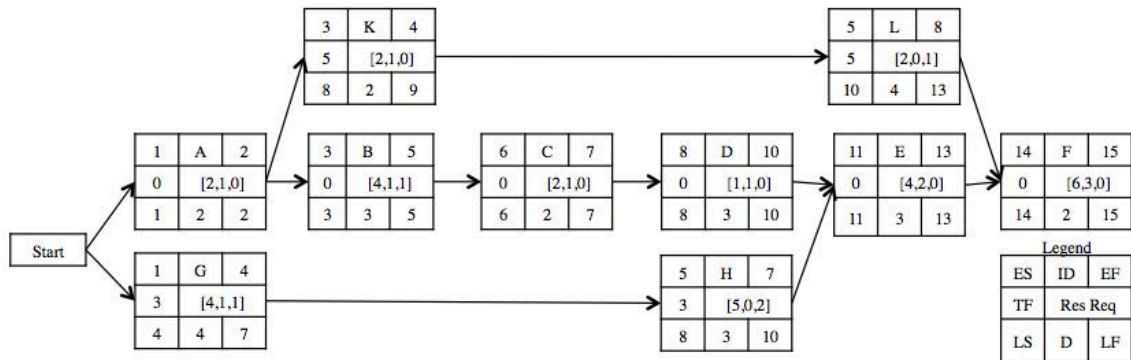


Figure 7: Project Network for IRLLS-C Model

Using CPM, the optimum schedule that results with the minimum project duration, the early start and finish times ( $ES$ ,  $EF$ ), late start and finish times ( $LS$ ,  $LF$ ), slack times ( $TF$ ), and critical and noncritical activities are defined. The CPM result remains unchanged compared to Chapter 3 since CPM does not take the resource requirements into consideration when scheduling the activities. As shown in Figure 7, the optimum project duration is 15 while the critical activities are A, B, C, D, E, and

F, and the noncritical activities are G, H, K, and L. This model makes use of the slack times and splitting to smooth the R1 resource utilization profile across time while also obtaining the optimum ordering scheduling and quantities of consumable resources, R2 and R3. The model assumes that material ordering does not have to occur only at the start of activities but at any time during the running period as well.

The CPM results are taken as an input to the integrated resource leveling and lot sizing model in addition to activities' resource requirements, splitting costs (CS), acquiring costs, releasing costs, ordering costs, purchasing costs, and holding costs as shown in Table 4.

**Table 4: Data Input for IRLLS-C Model**

Activity	Dur	Res1	Res2	Res3	ES	LF	TF	CS
A	2	2	1	0	1	2	0	1
B	3	4	1	1	3	5	0	1
C	2	2	1	0	6	7	0	1
D	3	1	1	0	8	10	0	1
E	3	4	2	0	11	13	0	1
F	2	6	3	0	14	15	0	1
G	4	4	1	1	1	7	3	1
H	3	5	0	2	5	10	3	1
K	2	2	1	0	3	9	5	1
L	4	2	0	1	5	13	5	1

CI	20
CD	20

A	[80,100]
C	[5, 3]
h	[1, 1]

The resultant optimum schedule from the IRLLS-C model is shown in Tables 5 and 6. Table 5 shows the obtained values of  $y_{ij}$  for each activity with 1 referring to the period when the activity is active, and 0 when the activity is not active, whereas Table 6 shows R1, R2, and R3 utilizations per period.

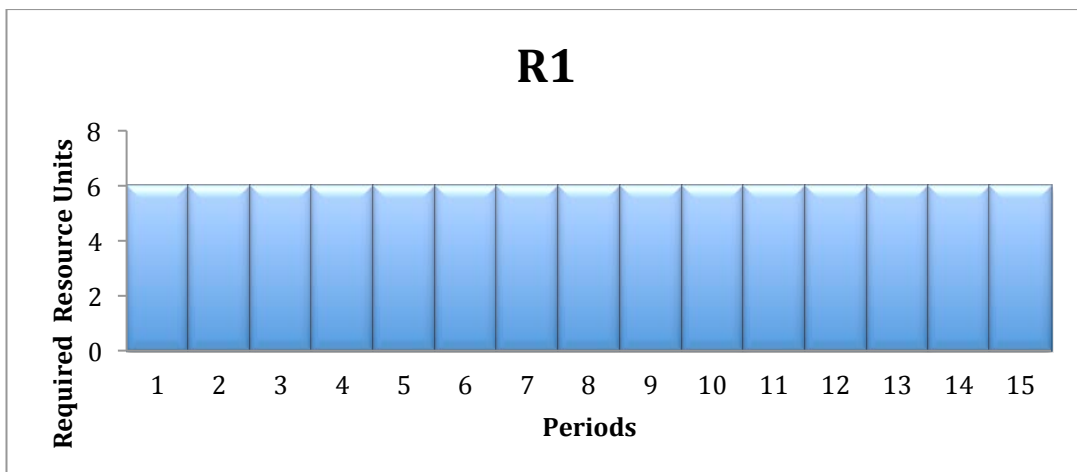
In this example, the project schedule happens to not change after adding the consumable resources and the material related costs. Two, out of four, noncritical activities are split (G and L) to smooth the resource utilization profile. The total number of orders is 3 and 1 whereas the inventory level is 77 and 106 for R2 and R3, respectively. The resource utilization profile for R1, R2, and R3 are shown in Figures 8-10.

**Table 5: Gantt Chart for IRLLS-C Model Result**

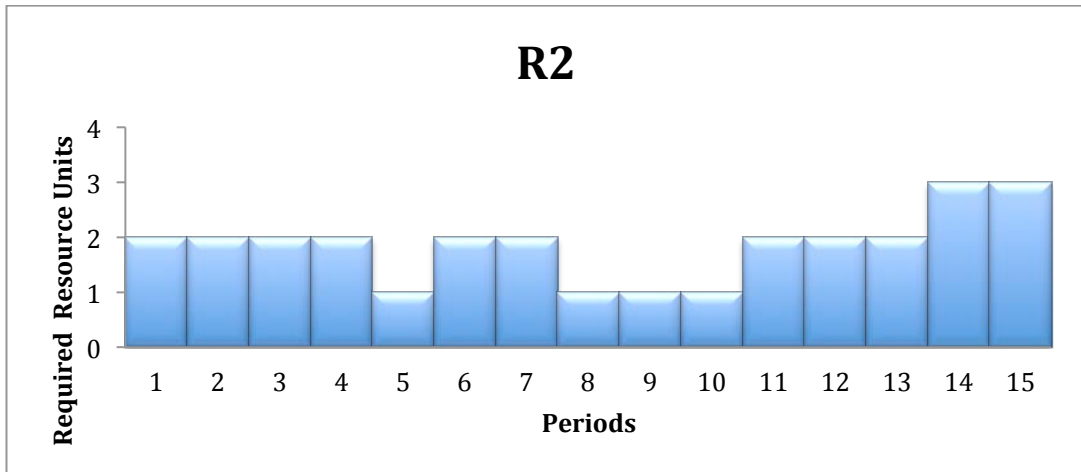
Activity	Periods														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1	1													
B			1	1	1										
C						1	1								
D								1	1	1					
E											1	1	1		
F													1	1	
G	1	1	0	0	0	1	1								
H					0	0	0	1	1	1					
K			1	1	0	0	0	0	0						
L					1	0	0	0	0	0	1	1	1		

**Table 6: R1, R2, and R3 Utilizations per Period**

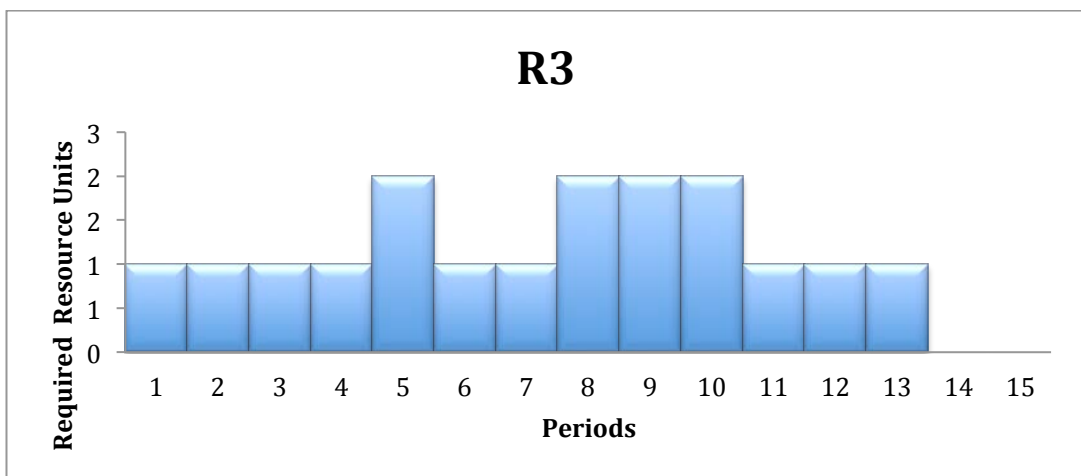
	Periods															Sum
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
R1	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
I	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6
D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R2	2	2	2	2	1	2	2	1	1	1	2	2	2	3	3	
O	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1	3
Q	15	0	0	0	0	0	0	0	0	10	0	0	0	0	3	28
Inv	13	11	9	7	6	4	2	1	0	9	7	5	3	0	0	77
R3	1	1	1	1	2	1	1	2	2	2	1	1	1	0	0	
O	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Q	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17
Inv	16	15	14	13	11	10	9	7	5	3	2	1	0	0	0	106



**Figure 8: R1 Utilization Profile –IRLLS-C**



**Figure 9: R2 Utilization Profile – IRLLS-C**



**Figure 10: R3 Utilization Profile – IRLLS-C**

The resource utilization profile for R1 is constant because of splitting of activity G and L. As R2 and R3 are consumable resources, they are depleted once they are used, and thus the utilization profile of them is not of concern. Consumable resources add purchasing cost, ordering cost, and holding cost to the objective function. The total cost is now defined as follows:

$$\begin{aligned}
 \text{Total Cost} &= \text{Splitting Cost} + \text{Acquiring Cost} + \text{Releasing Cost} + \text{Purchasing Cost} \\
 &\quad + \text{Ordering Cost} + \text{Holding Cost} \\
 &= 2 + (6 * 20) + (0 * 20) + (28 * 5 + 17 * 3) + (3 * 80 + 1 * 100) \\
 &\quad + (77 * 1 + 106 * 1) = 836
 \end{aligned}$$

This result is only valid for the defined parameters. If the parameters are changed, the schedule is affected along with the ordering quantities and inventory



levels per time period. The following subsections address the materials' related cost parameters (ordering and holding costs) on the project scheduling. The purchasing cost effect is not included, as it has no effect on the total project costs since it is kept constant over the project life.

**4.2.1 Ordering cost effect.** The ordering cost is an expense that is associated with the number of orders placed for consumable resources. In this example, the ordering cost per resource unit is 80 for R2 and 100 for R3. For instance, if two orders are made throughout the project duration for R2, regardless of the ordered quantity, the total ordering cost will be  $80 \times 2$  (160).

The IRLLS-C model was run for different ordering costs, keeping the other parameters unchanged to evaluate the effect of ordering cost on the project schedule. First, the ordering cost of R2 was changed to 10, 50, and 120, while the R3 ordering cost remained at 100. Second, the ordering cost of R3 was changed to 10, 50, and 120, while R2 ordering cost remained at 80. The results of this sensitivity analysis are summarized in the table below.

**Table 7: Ordering Cost Sensitivity Analysis - IRLLS-C**

Variables	Obj	NL	O [R2]	Inv [R2]	O [R3]	Inv [R3]
Base [80, 100]	836	[1 0 0 1]	3	77	1	106
Ordering Cost =[10, 100]	601	[1 0 0 1]	6	22	1	106
Ordering Cost =[50, 100]	746	[1 0 0 1]	3	77	1	106
Ordering Cost =[120, 100]	926	[1 0 0 1]	2	167	1	106
Ordering Cost =[80, 10]	686	[1 0 0 1]	3	77	3	26
Ordering Cost =[80, 50]	773	[1 0 0 1]	3	77	2	43
Ordering Cost =[80, 120]	936	[1 0 0 1]	3	77	1	106

Table 7 shows that as the ordering cost of R2 increases, the number of orders for R2 decreases while its inventory increases. There was no effect on other resources' schedules nor on splitting. The same was found for R3. This result is expected because as the ordering cost increases, it becomes more expensive to make frequent orders. Thus, the number of orders made is decreased as an attempt to

decrease the total ordering cost incurred in the project. Because the number of orders decreased, the quantity ordered per order is increased to satisfy the activities' demand for resources. Therefore, the summation of inventory level per period of time is increased. For example, when the ordering cost per unit of R2 is 120, there are only two orders made: 25 units at period 1 (start of the project), and 3 units at period 15 (end of the project). Referring to Table 8, only 2 units of R2 are required for period 1 however, the order is made to satisfy all the resource requirements of R2 for the next 13 periods. Thus, the inventory level starts at 25 units and decreases as time moves on. The summation of the inventory level in all periods of the project returns 167, which is higher than the base example 77.

**Table 8: R2 Demand, Ordered Quantity, and Inventory Level (IRLLS-C-Order Cost R2=120)**

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
R2	2	2	2	2	1	2	2	1	1	1	2	2	2	3	3	28
O	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2
Q	25	0	0	0	0	0	0	0	0	0	0	0	0	0	3	28
Inv	23	21	19	17	16	14	12	11	10	9	7	5	3	0	0	167

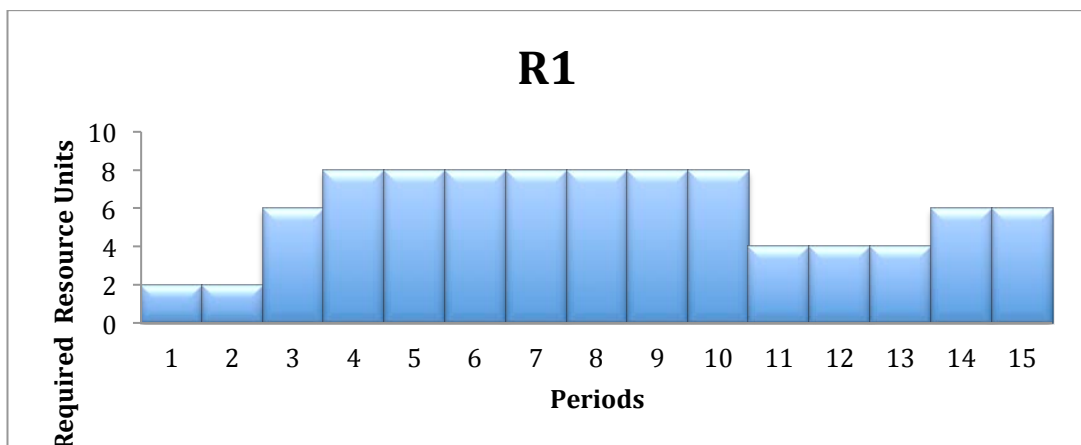
**4.2.2 Holding cost effect.** Like the ordering cost, the holding cost only occurs when consumable resources are involved in a project. It is an expense that is associated with the number of the consumable resources units kept in stock. In this example, the holding cost per resource unit is 1 for R2 and R3. For instance, if 5 resource units of R2 and 3 resource units of R3 are ordered in period  $t$  but not used during the same period, they are considered as inventory and their total holding cost is 8 ( $1*5+1*3$ ). If they are also not used in period  $t+1$ , then the same holding cost is added again. Holding cost is calculated on a per unit of time basis.

The IRLLS-C model was run for different holding costs, keeping the other parameters unchanged to evaluate the effect of holding cost on the project schedule. First, the holding cost of R2 was changed to 5, 50, and 80, while the R3 holding cost remained 1. Second, the holding cost of R3 was changed to 5, 20, and 50, while the R2 holding cost was unchanged. The results of this sensitivity analysis are summarized in the table below.

**Table 9: Holding Cost Sensitivity Analysis - IRLLS-C**

Variables	Obj	NL	O [R2]	Inv [R2]	O [R3]	Inv [R3]
Base [1, 1]	836	[1 0 0 1]	3	77	1	106
Holding Cost =[5, 1]	1049	[1 0 0 1]	3	42	1	106
Holding Cost =[50, 1]	1629	[1 0 0 1]	12	3	1	106
Holding Cost =[80, 1]	1729	[1 0 0 1]	15	0	1	106
Holding Cost =[1, 5]	1045	[1 0 0 1]	3	77	2	43
Holding Cost =[1, 20]	1370	[1 0 0 1]	3	77	4	17
Holding Cost =[1, 50]	1547	[0 0 1 0]	3	85	7	1

It can be noticed from Table 9 that as the holding cost of R2 increases, the number of orders for R2 increases while its inventory decreases. There was no effect on other resource schedules nor on splitting. This result is expected because as the holding cost increases, it becomes more expensive to keep units in storage as inventory. Thus, the number of orders made is increased to satisfy the requirement for shorter periods as an attempt to decrease the inventory level and accordingly the total holding cost. The same logic applies for R3. However, when the holding cost of R3 is 50, the activity splits were affected. In the base case, activities G and L were split both once, but when the R3 holding cost was 50, activities G and L were no longer split, and activity K which was not split before was now interrupted once. Accordingly, the resource utilization profile for R1 was changed as Figure 11 shows. The detailed results of this case are shown in Tables 10 and 11.



**Figure 11: R1 Utilization Profile – IRLLS-C (Holding Cost R3=50)**

**Table 10: Gantt Chart for IRLLS-C Model Result (Holding Cost R3=50)**

Activity	Periods														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1	1													
B			1	1	1										
C						1	1								
D								1	1	1					
E											1	1	1		
F														1	1
G	0	0	0	1	1	1	1								
H					0	0	0	1	1	1					
K			1	0	0	1	0	0	0						
L					0	0	1	1	1	1	0	0	0		

**Table 11: R1, R2, and R3 Utilizations per period (Holding Cost R3=50)**

	Periods															Sum
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
R1	2	2	6	8	8	8	8	8	8	8	4	4	4	6	6	
I	2	0	4	2	0	0	0	0	0	0	0	0	0	2	0	10
D	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	4
R2	1	1	2	2	2	3	2	1	1	1	2	2	2	3	3	
O	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	14
Q	1	1	2	2	2	3	3	0	1	1	2	2	2	3	3	28
Inv	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R3	0	0	1	2	2	1	2	3	3	3	0	0	0	0	0	
O	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	8
Q	0	0	1	2	2	1	2	3	3	3	0	0	0	0	0	17
Inv	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From the above results it can be noticed that when the holding cost of R3 is high, the resource leveling of R1 is affected. The total acquired units are 10 and the total released units are 4 as opposed to 6 and 0, respectively, in the base case. Because the acquiring and releasing costs are lower than the holding cost of R2, the IRLLS-C model found an optimal solution that sacrifices the smoothing of the R1 utilization profile in order to schedule the activities in such a way that activities which require resource R3 occur in parallel. Table 12 shows the demand profile of R3 in the base case and the case when the holding cost for R3 is 50.

**Table 12: R3 Utilization Profile (Base Case VS. High R3 Holding Cost)**

Base Case – Holding Cost R3 = 1															
Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R3	1	1	1	1	2	1	1	2	2	2	1	1	1	0	0
O	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Q	17	0	0	0	0	0	0	0	0	0	0	0	0	0	17
Inv	16	15	14	13	11	10	9	7	5	3	2	1	0	0	106
Holding Cost R3 = 50															
Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R3	0	0	1	2	2	1	2	3	3	3	0	0	0	0	0
O	0	0	1	1	1	1	1	1	1	1	0	0	0	0	8
Q	0	0	1	2	2	1	2	3	3	3	0	0	0	0	17
Inv	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The above table clearly shows that in the base case, R3 was used from period 1 to 13 while in the high holding cost case, the R3 utilization period is decreased from 13 periods to 8 periods. This decrease helped in minimizing the holding cost. If splitting did not occur, then 13 orders will be made (instead of 8) to satisfy the period-by-period demand of R3 to achieve zero inventories.

### 4.3 Chapter Summary

The IRLLS-C model integrates project scheduling and material procurement decisions to minimize resource leveling related costs and material ordering associated costs. Integrating material ordering decisions was found to have an effect on both project scheduling and splitting. The model balances between the different costs included in the objective function in order to find the minimum total costs. The ordering and holding costs were found to have an effect on the number of orders made and the inventory level. For high ordering costs, the number of orders is decreased while the inventory level is increased. When the holding cost is large, the number of orders is increased while the inventory level is decreased. If the holding cost is very high, the model might not be able to maintain the best resource leveling profile. The resource leveling profile is affected to minimize the total ordering and holding cost of consumable resources. This is the case especially when the acquiring and releasing

costs of renewable resources are less than the ordering and holding costs of the consumable resources. Although the IRLLS-C model integrates the project scheduling and material procurement decisions very well, it assumes constant consumption rates for both renewable and consumable resources.

## **Chapter 5: Integrated Resource Leveling and Lot Sizing Model with Variable Consumption Rate (IRLLS-V)**

Chapter 5 presents a mathematical model that optimizes the integrated renewable resource leveling and consumable resource lot sizing problem with time-varying consumption rate and allowed activity splitting. This chapter includes the mathematical formulation of the optimization model and an illustrative example.

### **5.1 Optimization Model Overview**

The majority of research work done in RCPS is dedicated to problems that deal with constant demand profiles. However, there are few papers in the literature that consider variable consumption rate for resource profiles. Li and Willis [31] considered an RCPS problem that included consumable resources with constant demand over activity duration, and renewable resources with variable demand over activity duration. Each activity is divided into sections and each section has a different demand profile and no delay should occur between successive sections. Poder *et al.* [32] mentioned that the application field has complex resources that do not follow constant demand profiles such as water, electricity and oil as their consumption profiles varies over activity duration. Willis [33] referred to a maintenance activity example where different types of resources (e.g. crane, rigger, plant helpers) are used throughout an activity with different quantities over time. If a constant demand profile is assumed, then the crane, for example, will be assigned to the whole maintenance activity even if it is demanded for only a short portion of the activity duration. This will not lead to efficient usage of resources. Road laying is another example of an activity that requires a variable demand profile [33].

As mentioned in Chapter 4, the IRLLS-C optimization model only considers activities with time-independent consumption rates for consumable resources. The extension of the model in this section integrates material procurement and project activity scheduling decisions with time variable consumption rates. It still minimizes the total cost of a project including its project scheduling related costs and material procurement associated costs. The IRLLS-V model is formulated as a mixed binary and integer linear program to optimize the total costs subject to resource balance

constraints, network logic constraints, activity duration constraints, and material balance constraints.

As is the case for IRLLS-C, the current model takes into consideration a project with  $n$  activities. Each activity has a fixed duration of  $T_j, j=1, 2, \dots, n$ . It also utilizes the CPM method to calculate the project duration, the earliest start time  $ES_j$ , the earliest finish time  $EF_j$ , the latest start time  $LS_j$ , the latest finish time  $LF_j$ , and the total float time  $TF_j$ .

**5.1.1 Model assumptions.** The formulation of the IRLLS-V model is based on the following set of assumptions:

1. For running the project, there are  $rp$  and  $cp$  types available of renewable and consumable resources, respectively.
2. There is a cost  $CS_j$  associated with splitting a noncritical activity  $j$ .
3. The setup time to restart an activity is relatively small and it is carried out at the end of the split period.
4. For splittable activities, their precedence relationships must be met.
5. The consumption rates for both renewable and consumable resources are time dependent.

**5.1.2 Problem parameters.** The extended optimization model has the same problem parameters mentioned in section 4.1.2 in addition to the following:

$rc_{ipu}$  Number of units of consumable resource type  $p$  ( $p = 1, \dots, cp$ ) consumed during  $u^{th}$  unit of the duration of activity  $i$  ( $i = 1, 2, \dots, n$ ).

$rr_{ipu}$  Number of units of renewable resource type  $p$  ( $p = 1, \dots, rp$ ) utilized during  $u^{th}$  unit of the duration of activity  $i$  ( $i = 1, 2, \dots, n$ ).

**5.1.3 Problem decision variables.** The extended optimization model has the same problem decision variables mentioned in section 4.1.3 in addition to the following:

$X_{iut}$  Binary variable equal to one when the  $u^{th}$  unit of the duration of activity  $i$  ( $i = 1, 2, \dots, n$ ) is performed during period  $t, t = 1, 2, \dots, T$  and zero otherwise.

**5.1.4 Model formulation.** The cost optimization model of the IRLLS-V for the integrated renewable resource leveling and consumable resource lot sizing



problem with variable resource requirements and allowed activity splitting is mathematically stated as follows:

Minimize

$$\sum_{p=1}^{rp} \left[ CI_p \sum_{t=1}^T I_{tp} + CD_p \sum_{t=1}^T D_{tp} \right] + \sum_{j=1}^m CS_j NL_j + \alpha \sum_{j=1}^m (F_j - S_j) + \sum_{p=1}^{CP} \sum_{t=1}^T [A_p O_{tp} + C_p Q_{tp} + h_p Inv_{tp}] \quad (22)$$

Subject to:

- Resource Balance Constraints for Renewable Resources:

$$RR_{tp} - RR_{(t-1)p} + D_{tp} - I_{tp} = 0, \quad t=1, 2, \dots, T \text{ and } p = 1, 2, \dots, rp \quad (23)$$

$$RR_{tp} = \sum_{i=1}^n \sum_{u=1}^{T_i} [rr_{ipu} X_{iut}], \quad t = 1, 2, \dots, T \text{ and } p = 1, 2, \dots, rp \quad (24)$$

- Activity Duration Constraints:

$$\sum_{t=ES_j}^{LF_j} y_{tj} = T_j, \quad j=1,2,\dots,nn \quad (25)$$

- Network Logic Constraints:

$$S_j = (T + 1) - \text{Max}\{(T + 1 - t)y_{tj} : t = ES_j, ES_j + TF_j\}, \quad j = 1, 2, \dots, nn \quad (26)$$

$$F_j = \text{Max}\{ty_{tj} : t = LF_j - TF_j, LF_j\}, \quad j = 1, 2, \dots, nn \quad (27)$$

$$S_k \geq F_j + 1, \quad j = 1, 2, \dots, nn \text{ and } k \in \text{Succ}(j) \quad (28)$$

- Splitting Cost Related Calculations:

$$L_{tj} = \text{Max}(y_{tj} - y_{(t+1)j}, 0), \quad j = 1, 2, \dots, nn \text{ and } t = ES_j, LF_j \quad (29)$$

$$NL_j = \sum_{t=ES_j}^{LF_j} L_{tj} - 1, \quad j = 1, 2, \dots, nn \quad (30)$$

- Material Balance Constraints for Consumable Resources:

$$Inv_{tp} = Inv_{(t-1)p} + Q_{tp} - CR_{tp} \quad t=1, 2, \dots, T \text{ and } p = 1, 2, \dots, cp \quad (31)$$

$$Q_{tp} \leq M O_{tp} \quad t=1, 2, \dots, T \text{ and } M \text{ is large number} \quad (32)$$

$$CR_{tp} = \sum_{i=1}^n \sum_{u=1}^{T_i} [cr_{ipu} X_{iut}], \quad t = 1, 2, \dots, T \text{ and } p = 1, 2, \dots, cp \quad (33)$$

- Variable Demand Additional Constraints:

$$\sum_{t=ES_i+u-1}^{LF_i-T_i+u} X_{iut} = 1, \quad u=1, 2, \dots, T_i, i=1,2,\dots, n \quad (34)$$

$$\sum_{u=1}^{T_i} X_{jut} = y_{tj}, \quad t = ES_j, LF_j, \text{ and } j= 1, 2, \dots, nn \quad (35)$$

$$\sum_{u=1}^{T_i} X_{iut} = z_{ti}, \quad t = ES_j, LF_j, \text{ and } i= 1, 2, \dots, nc \quad (36)$$

$$X_{iut} \leq \sum_{v=ES_i+u-1}^{t-1} X_{ivt}, \quad u= 1, 2, \dots, T_i-1, i=1,2,\dots, n, \text{ and } t = ES_i+1, \dots, LF_i \quad (37)$$

- Non-negativity Constraints:

$$I_t, D_t \geq 0, \quad t=1, 2, \dots, T-1$$

$$y_{tj} \in \{0,1\}, \quad j= 1, 2, \dots, nn \text{ and } t = ES_j, LF_j$$

$$y_{T+1,j} = 0, \quad j= 1, 2, \dots, nn$$

$$L_{tj} \geq 0, \quad j= 1, 2, \dots, nn \text{ and } t = ES_j, LF_j$$

$$Q_t, Inv_t \geq 0, \quad t=1, 2, \dots, T$$

$$Inv_0, Inv_T = 0$$

$$X_{iut} \in \{0, 1\} \quad j= 1, 2, \dots, nn \text{ and } t = ES_j, LF_j$$

The extension of the IRLLS-C model lies in the ability of the model to handle variable, not only constant, consumption rates for renewable and consumable resources. This translates into the addition of a three-dimension Boolean decision variable  $X_{iut}$  and four additional constraints (Eq. 34-37).

Equation 34 ensures that, for each activity, the summation of  $X_{iut}$  across the feasible active periods equals 1 for each  $u^{\text{th}}$  unit of its duration. In other words, each unit of the duration of an activity  $i$  must be carried out only once.

Equation 35 ensures that, at any time  $t$ , the summation of  $X_{iut}$  for noncritical activities over its duration equals to  $y_{tj}$  (which defines when the activity is active). Equation 36 ensures that, at any time  $t$ , the summation of  $X_{iut}$  for critical activities over its duration equals  $z_{ti}$ .

The last is Equation 37 which ensures that the  $u^{\text{th}}$  unit of the duration of activity  $i$  is performed in order with respect to time. In other words, the  $u^{\text{th}}$  unit cannot be carried out at time  $t$  if the previous units were not performed prior. For example, if activity A has a duration of 2, and its consumption rate is [3, 4] for a given resource, Equation 37 ensures that the second unit of the activity duration only occurs after the first unit has taken place.

The Boolean variable  $X_{iut}$  must be incorporated in the calculation of resource requirements. Thus, Equations 24 and 33 are here modified where  $X_{iut}$  now substitutes for the variables  $y_{tj}$  and  $z_{ti}$  to accommodate for the third dimension  $u$  defined in this model.

## 5.2 Illustrative Example

This section uses the same example from Section 2 of Chapter 4, but changes the constant consumption rates of resources R1, R2, and R3 to be time dependent. The example still defines 10 activities (A, B, C, D, E, F, G, H, K, and L) with one renewable resource type, R1, and two consumable resource types, R2 and R3, that are used across all activities with different resource requirement levels.

Using CPM, the optimum schedule that results with the minimum project duration, the early start and finish times ( $ES$ ,  $EF$ ), late start and finish times ( $LS$ ,  $LF$ ), slack times ( $TF$ ), and critical and noncritical activities are defined. The optimum project duration is 15 while the critical activities are A, B, C, D, E, and F, and the noncritical activities are G, H, K, and L. This model uses the slack times and splitting to smooth the R1 resource utilization profile across time while attaining the optimum ordering schedule and quantities of consumable resources, R2 and R3. The model assumes that material ordering does not have to occur only at the start of activities but at any time during the running period as well.

The CPM results are taken as an input to the integrated resource leveling and lot sizing model in addition to activities' resource requirements, splitting cost ( $CS$ ), acquiring costs, releasing costs, ordering costs, purchasing costs, and holding costs as shown in Table 13.

**Table 13: Data Input for IRLLS-V Model**

Activity	Dur	Res1	Res2	Res3	ES	LF	TF	CS
A	2	[3,1]	[1,1]	[0,0]	1	2	0	1
B	3	[5,4,3]	[1,1,1]	[1,1,1]	3	5	0	1
C	2	[2,2]	[1,1]	[0,0]	6	7	0	1
D	3	[1,1,1]	[1,1,1]	[0,0,0]	8	10	0	1
E	3	[4,4,4]	[3,2,1]	[0,0,0]	11	13	0	1
F	2	[4,8]	[4,2]	[0,0]	14	15	0	1
G	4	[3,5,5,3]	[1,1,1,1]	[1,1,1,1]	1	7	3	1
H	3	[5,5,5]	[0,0,0]	[2,1,3]	5	10	3	1
K	2	[2,2]	[1,1]	[0,0]	3	9	5	1
L	4	[2,2,2,2]	[0,0,0,0]	[1,1,1,1]	5	13	5	1

CI	20
CD	20

A	[80,100]
C	[5, 3]
h	[1, 1]

Although the consumption rates of resources are time dependent, the total consumption per activity is kept the same as the example in Chapter 3. For instance, in Chapter 3, the consumption rate for activity “A” during its running time for resource R1 is [2, 2]. Thus the total consumption rate for activity “A” is 4. In this chapter, the consumption rate for activity “A” is [3,1] and its total still remains at 4. This has been taken into consideration for all activities in the project.

The resultant optimum schedule from the IRLLS-V model is shown in Tables 14 and 15. Table 14 shows the optimum values of  $y_{ij}$  for each activity, whereas Table 15 shows the R1, R2, and R3 utilization profiles per period.

**Table 14: Gantt Chart for IRLLS-V Model Result**

Activity	Periods														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1	1													
B			1	1	1										
C						1	1								
D								1	1	1					
E											1	1	1		
F														1	1
G	0	1	0	0	1	1	1								
H					0	0	0	1	1	1					
K			0	1	0	0	1	0	0						
L					0	0	0	1	1	1	1	0	0		

**Table 15: R1, R2, and R3 Utilizations per Period**

	Periods															Sum
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
R1	3	4	5	6	8	7	7	8	8	8	6	4	4	4	8	
I	3	1	1	1	2	0	0	1	0	0	0	0	0	4	0	
D	0	0	0	0	0	1	0	0	0	0	2	2	0	0	0	
R2	1	2	1	2	2	2	3	1	1	1	3	2	1	4	2	
O	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	
Q	16	0	0	0	0	0	0	0	0	0	10	0	0	0	2	
Inv	15	13	12	10	8	6	3	2	1	0	7	5	4	0	0	
R3	0	1	1	1	2	1	1	3	2	4	1	0	0	0	0	
O	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	
Q	0	17	0	0	0	0	0	0	0	0	0	0	0	0	0	
Inv	0	16	15	14	12	11	10	7	5	1	0	0	0	0	0	

In this example, the project schedule is different from that found in Chapter 4 after changing the constant consumption rate to be time variable. Two, out of four, noncritical activities are split (G and K) in order to smooth the resource utilization profile. The total number of orders is 3 and 1 whereas the inventory level is 86 and 91 for R2 and R3 respectively. The resource utilization profiles for R1, R2, and R3 are shown in Figures 12-14.

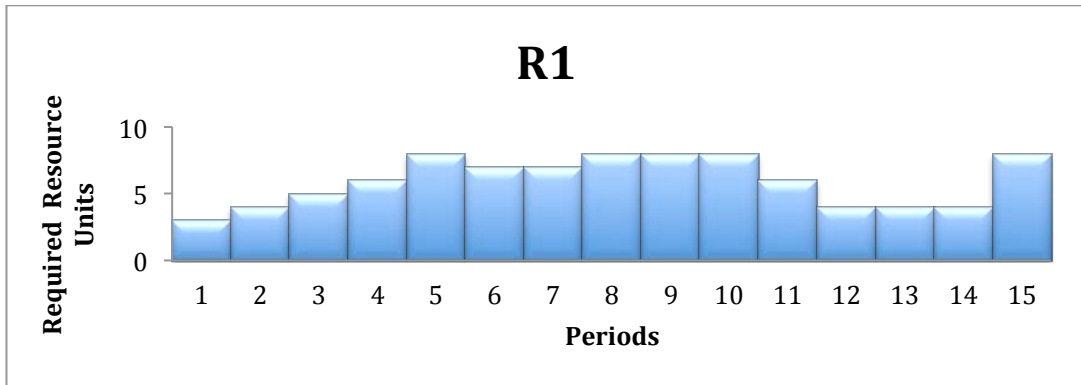


Figure 12: R1 Utilization Profile –IRLLS-V

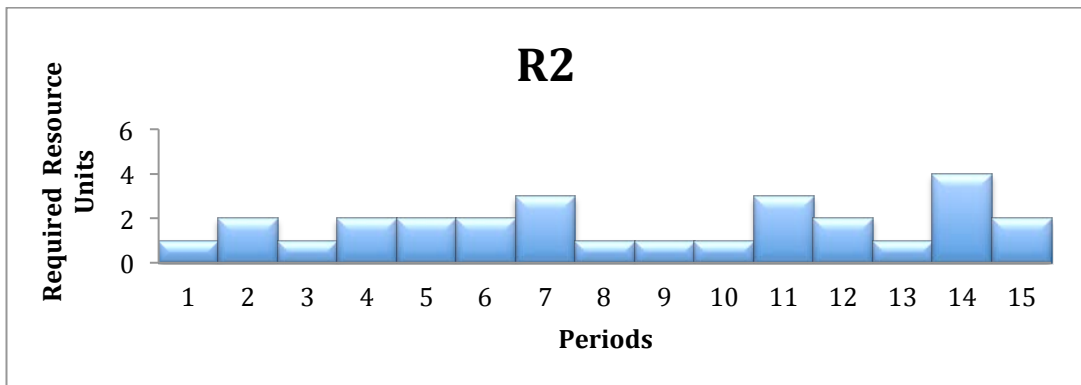


Figure 13: R2 Utilization Profile – IRLLS-V

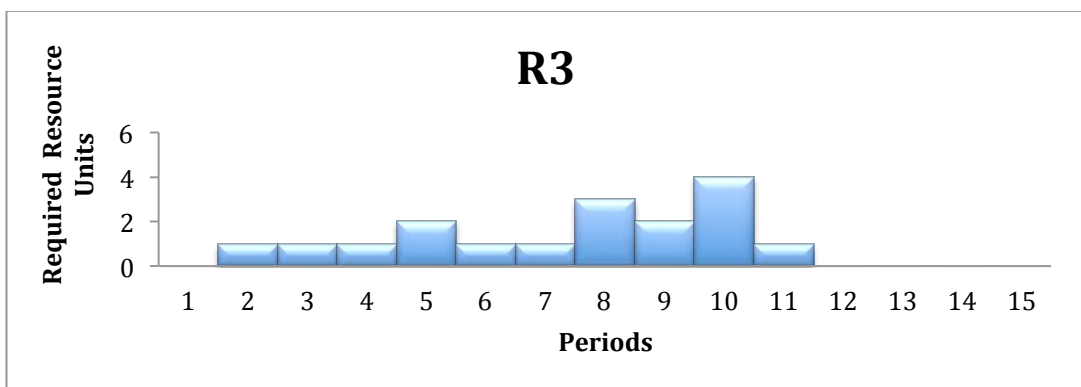


Figure 14: R3 Utilization Profile – IRLLS-C

The resource utilization profile for R1 is no longer constant, as the Chapter 3 example showed, because of the demand variability of resource R1's consumption rate. However, the profile is smoothed due to the splitting of activities G and K. The total cost is now defined as follows:

$$\begin{aligned}
 \text{Total Cost} &= \text{Splitting Cost} + \text{Acquiring Cost} + \text{Releasing Cost} \\
 &\quad + \text{Purchasing Cost} + \text{Ordering Cost} + \text{Holding Cost} \\
 &= 2 + (13 * 20) + (5 * 20) + (28 * 5 + 17 * 3) \\
 &\quad + (3 * 80 + 1 * 100) + (86 * 1 + 91 * 1) = 1070
 \end{aligned}$$

Obviously, this result is only valid for the defined parameters. If the values of some other parameters are changed, the schedule is affected along with the ordering quantity and inventory level per time period. The following subsections address the effect of (1) material related cost parameters (ordering and holding costs), (2) leveling related cost parameters, (3) splitting, (4) resource utilization rate, and (5) integrated decisions on the project schedule and total cost. The purchasing cost effect is not included and is removed from the objective function as it is kept constant over the project life.

**5.2.1 Ordering cost effect.** As mentioned before, the ordering cost is associated with the number of orders placed for consumable resources. In this example, the ordering cost per resource unit is 80 for R2 and 100 for R3.

The IRLLS-V model was run for 12 problems with different ordering costs, keeping the other parameters unchanged to assess the effect of ordering cost on the project schedule. First, the ordering cost of R2 was changed to 10, 30, 50, 100, and 120, while the R3 ordering cost remained at 100. The result of this study is shown in Table 16.

**Table 16: R2 Ordering Cost Sensitivity Analysis - IRLLS-V**

Ordering Cost [R2, R3]	Obj	NL	O [R2, R3]	Inv [R2, R3]
[10, 100]	633	[1 0 1 0]	[6, 1]	[20, 91]
[30, 100]	719	[1 0 1 0]	[4, 1]	[46, 91]
[50, 100]	789	[1 0 1 0]	[3, 1]	[86, 91]
Base = [80, 100]	879	[1 0 1 0]	[3, 1]	[86, 91]
[100, 100]	939	[1 0 1 0]	[2, 1]	[186, 91]
[120, 100]	979	[1 0 1 0]	[2, 1]	[186, 91]

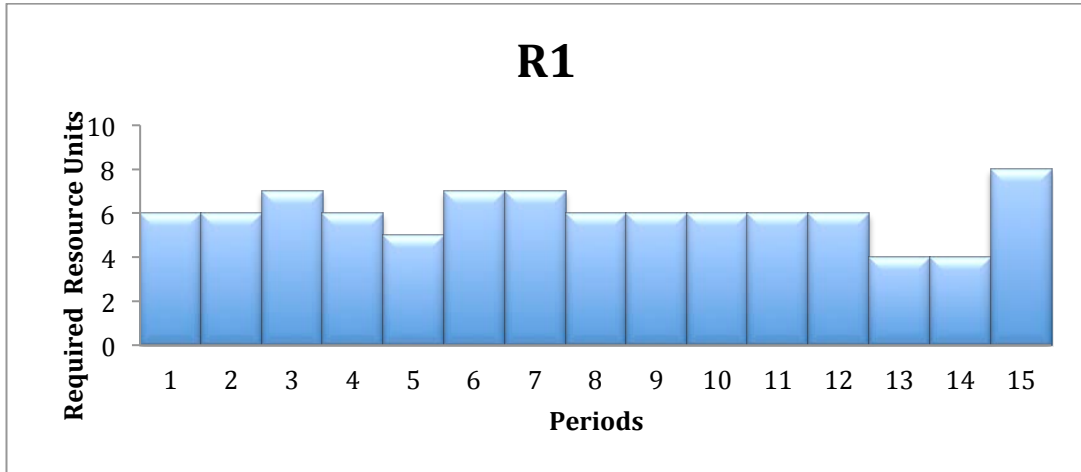
Table 16 shows that as the ordering cost of R2 increases, the number of orders for R2 decreases while its inventory increases. There was no effect on other resource schedules nor on splitting. This result is expected because as the ordering cost increases, it becomes more expensive to make frequent orders. Thus, the number of orders made is decreased as an attempt to decrease the total ordering cost incurred in the project. Because the number of orders decreased, the quantity ordered per order is increased to satisfy the activities' demand for resources. Therefore, the summation of inventory level per period of time is increased.

Second, the ordering cost of R3 was changed to 10, 25, 35, 50, 65, 85, and 120, while the R2 ordering cost was unchanged. The results of this sensitivity analysis are summarized in the table below.

**Table 17: R3 Ordering Cost Sensitivity Analysis - IRLLS-V**

Ordering Cost [R2, R3]	Obj	NL	O [R2, R3]	Inv [R2, R3]
[80, 10]	735	[1 0 0 2]	[3, 3]	[78, 24]
[80, 25]	769	[1 0 1 0]	[3, 2]	[86, 31]
[80, 35]	789	[1 0 1 0]	[3, 2]	[86, 31]
[80, 50]	819	[1 0 1 0]	[3, 2]	[86, 31]
[80, 65]	844	[1 0 1 0]	[3, 1]	[86, 91]
[80, 85]	864	[1 0 1 0]	[3, 1]	[86, 91]
Base = [80, 100]	879	[1 0 1 0]	[3, 1]	[86, 91]
[80, 120]	899	[1 0 1 0]	[3, 1]	[86, 91]

Table 17 shows that as the ordering cost of R3 increases, the number of orders for R3 decreases while its inventory increases. This is a similar behavior to the one noticed while changing the ordering cost of R2. However, when the ordering cost of R3 was 10, the activities to be split changed thus changing the project schedule. In the base case, activities G and K were both split once, but when the R3 ordering cost was 10, activity K was no longer split, and activity L which was not split before was now split twice. Accordingly, the resource utilization profile for R1 was changed as Figure 15 shows.



**Figure 15: R1 Utilization Profile – IRLLS-V (Ordering Cost R3=10)**

The first thing to notice in this case is the difference in the R1 utilization profile. According to Figure 15, the total period-to-period deviation is equal to  $|6 - 7| + |7 - 6| + |6 - 5| + |5 - 7| + |7 - 6| + |6 - 4| + |4 - 8| = 12$ . In the contrary, the total period-to-period deviation in the base example as shown in Figure 12 is equal to  $|3 - 4| + |4 - 5| + |5 - 6| + |6 - 8| + |8 - 7| + |7 - 8| + |8 - 6| + |6 - 4| + |4 - 8| = 15$ . Although both profiles have a peak of 8, the resource leveling of this case is enhanced compared to the base model. Accordingly, the number of released resources now increased by 1 unit but the inventory levels of R2 and R3 resources are reduced by 8 and 67 units, respectively. The decrease in inventory outweighs, in terms of benefit, the one unit increase in the released resources. The detailed results of this case are shown in Tables 18 and 19.

**Table 18: Gantt Chart for IRLLS-V Model Result (Ordering Cost R3=10)**

Activity	Periods														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1	1													
B			1	1	1										
C						1	1								
D								1	1	1					
E											1	1	1		
F														1	1
G	1	1	0	0	0	1	1								
H					0	0	0	1	1	1					
K			1	1	0	0	0	0	0						
L					1	0	1	0	0	0	1	1	0		



**Table 19: R1, R2, and R3 Utilizations per period (Ordering Cost R3=10)**

	Periods															Sum
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
R1	6	6	7	6	5	7	7	6	6	6	6	6	4	4	8	
I	6	0	1	0	0	2	0	0	0	0	0	0	0	0	4	13
D	0	0	0	1	1	1	0	0	1	0	0	0	2	0	0	6
R2	2	2	2	2	1	2	2	1	1	1	3	2	1	4	2	
O	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	3
Q	16	0	0	0	0	0	0	0	0	0	10	0	0	0	2	28
Inv	14	12	10	8	7	5	3	2	1	0	7	5	4	0	0	78
R3	1	1	1	1	2	1	2	2	1	3	1	1	0	0	0	
O	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	3
Q	4	0	0	0	8	0	0	0	0	5	0	0	0	0	0	17
Inv	3	2	1	0	6	5	3	1	0	2	1	0	0	0	0	24

**5.2.2 Holding cost effect.** Like the ordering cost, the holding cost only occurs when consumable resources are involved in a project and it is associated with the number of consumable resources kept in stock. In this example, the holding cost per resource unit is 1 for R2 and R3. The IRLLS-V model was run for 12 problems with different holding costs, keeping the other parameters unchanged to better evaluate the effect of holding cost on the project schedule. First, the holding cost of R2 was changed to 10, 30, 50, 80, and 100, while the R3 holding cost remained at 1. Second, the holding cost of R3 was changed to 10, 25, 35, 50, 65, 85, and 100, while the R2 holding cost was unchanged. The results of this sensitivity analysis are summarized in the following tables.

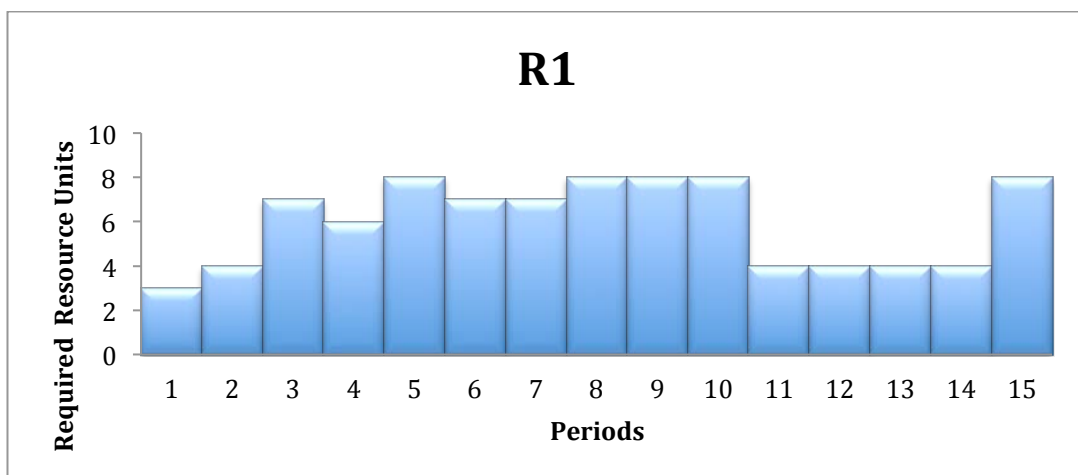
**Table 20: R2 Holding Cost Sensitivity Analysis - IRLLS-V**

Holding Cost [R2, R3]	Obj	NL	O [R2, R3]	Inv [R2, R3]
Base = [1, 1]	879	[1 0 1 0]	[3, 1]	[86, 91]
[10, 1]	1233	[1 0 1 0]	[6, 1]	[20, 91]
[30, 1]	1533	[1 0 1 0]	[10, 1]	[6, 91]
[50, 1]	1633	[1 0 1 0]	[11, 1]	[4, 91]
[80, 1]	1753	[1 0 1 0]	[15, 1]	[0, 91]
[100, 1]	1753	[1 0 1 0]	[15, 1]	[0, 91]

**Table 21: R3 Holding Cost Sensitivity Analysis - IRLLS-V**

Holding Cost [R2, R3]	Obj	NL	O [R2, R3]	Inv [R2, R3]
Base = [1, 1]	879	[1 0 1 0]	[3, 1]	[86, 91]
[1, 10]	1178	[1 0 1 0]	[3, 3]	[86, 9]
[1, 25]	1313	[1 0 1 0]	[3, 4]	[86, 9]
[1, 35]	1403	[1 0 0 1]	[3, 4]	[86, 9]
[1, 50]	1523	[1 0 0 0]	[3, 7]	[82, 4]
[1, 65]	1553	[1 0 0 0]	[3, 7]	[82, 2]
[1, 85]	1589	[0 0 0 0]	[3, 7]	[84, 1]
[1, 100]	1604	[0 0 0 0]	[3, 7]	[84, 1]

It can be noticed from Table 20 that as the holding cost of R2 increases, the number of orders for R2 increases while its inventory decreases. There was no effect on other resource schedules nor on splitting. This result is expected because as the holding cost increases, it becomes more expensive to keep units in storage as inventory. Thus, the number of orders made is increased to satisfy the requirement for shorter periods as an attempt to decrease the inventory level and, in turn, the total holding cost. The same logic applies to R3 as Table 21 shows. However, when the holding cost of R3 is 50, the activities split was affected. In the base case, activity G and K were split both once, but when the R3 holding cost was 50, activity K was no longer split. Accordingly, the resource utilization profile for R1 was changed as Figure 16 shows. The detailed results of this case are shown in Tables 22 and 23.



**Figure 16: R1 Utilization Profile – IRLLS-V (Holding Cost R3=50)**

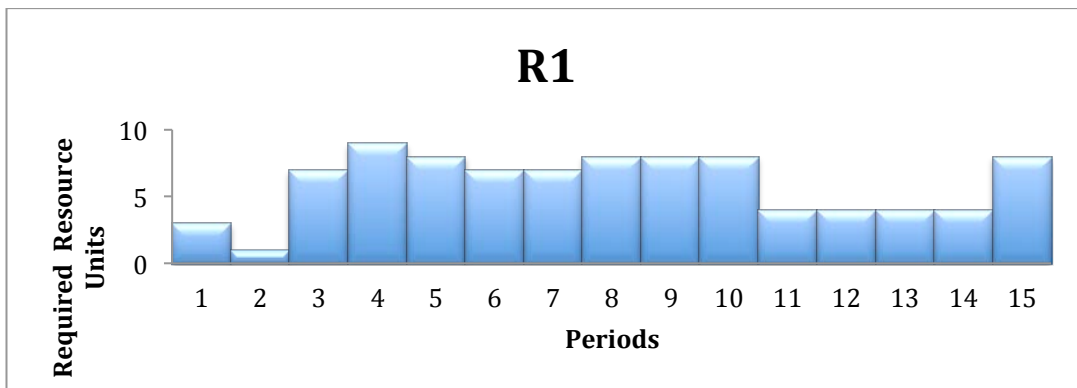
**Table 22: Gantt Chart for IRLLS-V Model Result (Holding Cost R3=50)**

Activity	Periods														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1	1													
B			1	1	1										
C						1	1								
D								1	1	1					
E											1	1	1		
F														1	1
G	0	1	0	0	1	1	1								
H					0	0	0	1	1	1					
K			1	1	0	0	0	0	0						
L					0	0	1	1	1	1	0	0	0		

**Table 23: R1, R2, and R3 Utilizations per period (Holding Cost R3=50)**

	Periods															Sum
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
R1	3	4	7	6	8	7	7	8	8	8	4	4	4	4	8	
I	3	1	3	0	2	0	0	1	0	0	0	0	0	0	4	14
D	0	0	0	1	0	1	0	0	0	0	4	0	0	0	0	6
R2	1	2	2	2	2	2	2	1	1	1	3	2	1	4	2	
O	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	3
Q	16	0	0	0	0	0	0	0	0	0	10	0	0	0	2	28
Inv	15	13	11	9	7	5	3	2	1	0	7	5	4	0	0	82
R3	0	1	1	1	2	1	2	3	2	4	0	0	0	0	0	
O	0	1	0	1	1	0	1	1	0	1	0	0	0	0	0	6
Q	0	2	0	1	3	0	2	5	0	4	0	0	0	0	0	17
Inv	0	1	0	0	1	0	0	2	0	0	0	0	0	0	0	4

In other cases, when the holding cost of R3 is 85 and 100, no activities are interrupted anymore. Thus, the resource utilization profile for R1 was changed as Figure 17 shows and the detailed results are shown in Tables 24 and 25.



**Figure 17: R1 Utilization Profile – IRLLS-V (Holding Cost R3=100)**

**Table 24: Gantt Chart for IRLLS-V Model Result (Holding Cost R3=100)**

Activity	Periods														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1	1													
B			1	1	1										
C						1	1								
D								1	1	1					
E											1	1	1		
F														1	1
G	0	0	0	1	1	1	1								
H					0	0	0	1	1	1					
K			1	1	0	0	0	0	0						
L					0	0	1	1	1	1	0	0	0		

**Table 25: R1, R2, and R3 Utilizations per period (Holding Cost R3=100)**

	Periods															Sum
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
R1	3	1	7	9	8	7	7	8	8	8	4	4	4	4	8	
I	3	0	6	2	0	0	0	1	0	0	0	0	0	0	4	16
D	0	2	0	0	1	1	0	0	0	0	4	0	0	0	0	8
R2	1	1	2	3	2	2	2	1	1	1	3	2	1	4	2	
O	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	3
Q	16	0	0	0	0	0	0	0	0	0	10	0	0	0	2	28
Inv	15	14	12	9	7	5	3	2	1	0	7	5	4	0	0	84
R3	0	0	1	2	2	1	2	3	2	4	0	0	0	0	0	
O	0	0	1	1	1	0	1	1	1	1	0	0	0	0	0	7
Q	0	0	1	2	3	0	2	3	2	4	0	0	0	0	0	17
Inv	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1

Analyzing Figure 16, the total period-to-period deviation is equal to  $|3 - 4| + |4 - 7| + |7 - 6| + |6 - 8| + |8 - 7| + |7 - 8| + |8 - 4| + |4 - 8| = 17$  compared to the base example which gave 15. In Figure 17, the total period-to-period deviation is equal to  $|3 - 1| + |1 - 7| + |7 - 9| + |9 - 8| + |8 - 7| + |7 - 8| + |8 - 4| + |4 - 8| = 20$ . In addition, the released and acquired units were [13, 5] in the base example, but in these two cases it is [14, 6] and [16, 8] respectively. It is very clear from these figures that when the holding cost of R3 gets higher, resource leveling is no longer the priority considering that the holding cost is higher than the releasing and acquiring costs. This case is similar to the one discussed in Chapter 4 in the sensitivity analysis of the high holding cost of R3. The IRLLS-V model found an optimal solution that sacrifices the smoothing of R1 utilization profile in order to schedule the activities in such a way that activities that require resource R3

occur in parallel. Table 26 shows the demand profile of R3 in the base case and the two cases of high holding cost of R3. The table clearly shows that in the base case, R3 was used over a total of 10 periods, while in the high holding cost cases, 50 and 100, the R3 utilization period decreased to 9 then 8 periods. This decrease helped in minimizing the holding cost.

**Table 26: R3 Utilization Profile (Base Case VS. High R3 Holding Cost)**

Base Case – Holding Cost R3 = 1															
Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R3	0	1	1	1	2	1	1	3	2	4	1	0	0	0	0
O	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
Q	0	17	0	0	0	0	0	0	0	0	0	0	0	0	0
Inv	0	16	15	14	12	11	10	7	5	1	0	0	0	0	91
Holding Cost R3 = 50															
Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R3	0	1	1	1	2	1	2	3	2	4	0	0	0	0	0
O	0	1	0	1	1	0	1	1	0	1	0	0	0	0	6
Q	0	2	0	1	3	0	2	5	0	4	0	0	0	0	0
Inv	0	1	0	0	1	0	0	2	0	0	0	0	0	0	4
Holding Cost R3 = 100															
Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
R3	0	0	1	2	2	1	2	3	2	4	0	0	0	0	0
O	0	0	1	1	1	0	1	1	1	1	0	0	0	0	8
Q	0	0	1	2	3	0	2	3	2	4	0	0	0	0	0
Inv	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0

It is very evident that the effect of increasing the holding cost of R2 and R3 is different. While the increase in R2 holding cost had an effect on only consumable resource related costs, the increase in the R3 holding cost had a further impact on both renewable and consumable related costs. This is because resource R2 is utilized by six critical activities and two noncritical activities. On the other hand, resource R3 is utilized by only 1 critical activity and 3 (out of four) noncritical activities. Therefore, when the holding cost of R3 increases, the noncritical activities utilizing R3 resource can be moved within their slack in an attempt to minimize the effect of the high holding cost. In the high holding cost of R2, the model does not have the freedom to

move the critical activities since they do not have a slack; thus the impact on schedule is not observed.

**5.2.3 Leveling related cost effect.** The objective function in this chapter includes both materials related costs and leveling related costs. The leveling related costs include acquiring, releasing, and splitting costs. To study these cost effects, the IRRLS-V model was run for 16 problems with different acquiring, releasing, and splitting costs, one at a time, keeping the other parameters unchanged. First, the acquiring cost was changed to 10, 30, 50, 80, and 100, while releasing and splitting costs remained at 20 and 1, respectively. Second, the releasing cost was changed to 10, 30, 50, 80, and 100, while the acquiring and splitting costs remained at 20 and 1, respectively. Last, the splitting cost was changed to 10, 30, 35, 40, 45, and 50, while the releasing and acquiring costs remained both at 20. The results of these sensitivity analyses are summarized in the following tables.

**Table 27: Acquiring Cost Sensitivity Analysis - IRRLS-V**

Acquiring Cost	Obj	NL	O [R2, R3]	Inv [R2, R3]
10	749	[1 0 1 0]	[3, 1]	[86, 91]
Base = 20	879	[1 0 1 0]	[3, 1]	[86, 91]
30	1009	[1 0 1 0]	[3, 1]	[86, 91]
50	1269	[1 0 1 0]	[3, 1]	[86, 91]
80	1659	[1 0 1 0]	[3, 1]	[86, 91]
100	1919	[1 0 1 0]	[3, 1]	[86, 91]

**Table 28: Releasing Cost Sensitivity Analysis - IRRLS-V**

Releasing Cost	Obj	NL	O [R2, R3]	Inv [R2, R3]
10	829	[1 0 1 0]	[3, 1]	[86, 91]
Base = 20	879	[1 0 1 0]	[3, 1]	[86, 91]
30	929	[1 0 1 0]	[3, 1]	[86, 91]
50	1029	[1 0 1 0]	[3, 1]	[86, 91]
80	1179	[1 0 1 0]	[3, 1]	[86, 91]
100	1279	[1 0 1 0]	[3, 1]	[86, 91]

**Table 29: Splitting Cost Sensitivity Analysis - IRLLS-V**

Splitting Cost	Obj	NL	O [R2, R3]	Inv [R2, R3]
Base = 1	879	[1 0 1 0]	[3, 1]	[86, 91]
10	897	[1 0 1 0]	[3, 1]	[86, 91]
30	937	[1 0 1 0]	[3, 1]	[86, 91]
35	944	[1 0 0 0]	[3, 1]	[82, 87]
40	949	[1 0 0 0]	[3, 1]	[82, 87]
45	950	[0 0 0 0]	[3, 1]	[90, 80]
50	950	[0 0 0 0]	[3, 1]	[90, 80]

Referring to Tables 27 and 28, the results show that changes in the values of acquiring and releasing costs did not have an effect on activity splitting nor number of orders. However, splitting cost had an effect, as expected, on the project schedule. As the splitting cost increases, it becomes more expensive to split noncritical activities. Although the model aims to smooth the resource utilization profile, the high splitting cost makes this objective hard to reach. The model was run for larger values of splitting costs to find at which threshold the noncritical activities G and K that were split are no longer split. It was found that when all noncritical activities had equal splitting cost, activities K and G did not split at cost 32 and 42, respectively. However, if activity K and others' splitting cost was kept unchanged (remains 1) and activity G's splitting cost was only increasing, activity G is no longer split at 66 (instead of 32). Similarly, if activity K is increased by 1 cost unit (from 1 to 2) while other splitting costs remain at 1, activity L is split instead of activity K.

So far, 41 problems have been solved in total by varying all the parameters involved in the objective function one at a time. These 41 problems will be used to assess the splitting effect and the sequential versus integrated decision approaches as the following sections explain.

**5.2.4 Splitting effect.** Activity splitting is a method that allows noncritical activities to be interrupted and their related resources to be allocated to other activities. This can help reduce the fluctuation in resource utilization, thus reducing

the acquiring and releasing total costs. However, activity splitting introduces a cost associated with stopping and re-starting the activity again, and this cost is referred to as splitting cost (CS). In the example used in this chapter, the splitting cost per interruption is 1 for all activities. To study the effect of splitting, the variable  $NL$ , that defines the number of times an activity is split, was forced to zero. The same 41 problems, excluding the ones used for the splitting cost analysis, which were solved earlier with different ordering, holding, releasing and acquiring costs, are now solved with the extra constraint of non-splitting to assess the impact of splitting on the total costs. The detailed results of this analysis can be found in Appendix A while the summary of the cost saving achieved when splitting is allowed is presented in Table 30.

**Table 30: Percent Cost Savings when Splitting is Allowed**

Cost Saving Type	Renewable Related Costs	Consumable Related Costs	Total
Minimum Cost Reduction	0.00%	-5.97%	0.00%
Maximum Costs Reduction	24.58%	1.59%	15.30%
Average Cost Reduction	17.05%	-1.45%	7.01%

Table 30 shows that splitting reduces renewable related costs (i.e. acquiring and releasing costs) but increases the consumable related costs due to changes in the time-phased requirement for consumable resources as a result of activity splitting. This finding is expected as the purpose of activity splitting is to smooth the utilization of renewable resources and, consequently, to minimize the leveling related costs. On the other hand, when an activity is not split, the time interval between its finishing and starting times will be reduced, which may affect the ordering schedule for the consumable resource needed for this activity. In other words, the total ordering and holding costs for the consumable resource may be reduced. Therefore, splitting is no longer only dependent on releasing, acquiring, and splitting costs, but is also related to other cost factors such as ordering and holding costs.

**5.2.5 Resource utilization rate effect.** All results above showed that resource requirements play an important factor in determining the total cost. This section will study the effect of changing the resource utilization for an activity per



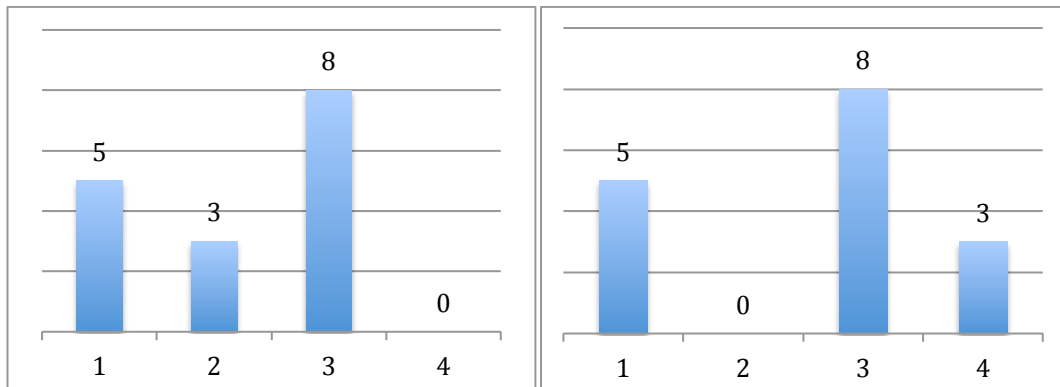
period on the total project cost for both sequential and integrated decision approaches. The sensitivity analysis is done on activity G since it has the longest duration and it utilizes all resources R1, R2, and R3. The distribution of utilization over the activity duration was varied for each resource at a time for different standard deviation (SD) values while keeping the total consumption rate constant. The IRLLS-V model was run for 31 problems (23 problems for variations in R1, 4 for variations in R2, and 4 for variations in R3 utilization rate distribution). Because the total consumption rate for R1 is 14 while for R2 and R3 it is 4, most problems were generated for R1 variations. The detailed analysis results are in Appendix B while a portion of them is shown in Table 31. All problems had an R3 holding cost of 100.

**Table 31: Cost Savings for Different Standard Deviations of R1 Utilization Rate**

Resources						Cost Saving
R1	SD	R2	SD	R3	SD	
[4, 4, 4, 4]	0.00	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	0.00%
[4,4,5,3]	0.82	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	14.15%
[3,5,5,3]	1.15	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	14.86%
[4,4,6,2]	1.63	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	12.69%
[5,3,6,2]	1.83	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	12.69%
[4,4,7,1]	2.45	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	13.50%
[5,3,7,1]	2.58	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	14.21%
[6,2,7,1]	2.94	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	7.83%
[4,4,8,0]	3.27	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	14.96%

The results show that the distribution of resource utilization over activity duration does have an effect on cost saving achieved following the integrated decision approach instead of the sequential one. However, there is no apparent relationship or trend between them.

To better understand the results, the resource consumption time profile was varied for a fixed standard deviation value. Figure 18 shows two different resource consumption profiles ([5, 3, 8, 0] and [5, 0, 8, 3]) that have the same standard deviation 3.37. A summary of the results is shown in Table 32.



**Figure 18: Example of Two Different Resource Consumption Profiles with Fixed Standard Deviation**

**Table 32: Results of Variation in Resource Consumption Time Profile with Fixed Standard Deviation**

Resources						Cost Saving
R1	SD	R2	SD	R3	SD	
[5,3,8,0]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	15.61%
[5,3,0,8]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	0.00%
[5,0,3,8]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	2.53%
[5,0,8,3]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	13.46%
[5,8,3,0]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	10.74%
[5,8,0,3]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	12.43%
[5,3,8,0]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	15.61%
[5,3,0,8]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	0.00%
[5,0,3,8]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	2.53%

Table 32 shows that the variations in the resource consumption time profile over the duration of an activity obviously has an effect on cost savings, though no clear trend was found. Therefore, cost savings are not only directly dependent on the standard deviation of the resource consumption rate, but also on the resultant resource time profile. Different time-phased resource requirement schedules produce different total ordering, holding, and leveling related costs, thus different cost savings. The following section will elaborate more on the effect of integrated decisions on the total cost and how different project parameters affect the overall cost savings.

**5.2.6 Integrated decision effect.** Project managers often follow a sequential approach when considering project scheduling and material ordering decisions. As mentioned in Chapter 1, the sequential approach consists of three independent decision programs which are the activity scheduling program, materials requirement program, and materials ordering program. While the scheduling program defines the project activities' running times, the materials requirement program computes the time-phased requirement, based on the output of the scheduling program, for each material during the life of the project. Using the output of the materials requirement program, the material ordering program generates the optimal ordering lot sizes and times for each material using *DLSP*. This procedure does not necessarily result in the global optimal project schedule. Therefore, it is important to integrate project scheduling with materials management to globally optimize the total cost of a project. The integrated decision approach integrates the three independent programs in the sequential approach into one model that generates the optimal project scheduling and material ordering solution.

In Chapters 4 and 5, the IRLLS-C and IRLLS-V models were based on the integrated decision approach. In this section, the sequential decision approach will be used on IRLLS-V to assess the economic benefits of following the integrated decision approach compared to the sequential approach. The 72 problems solved in the previous sections with different utilization rates, ordering costs, holding costs, releasing costs, acquiring costs, and splitting costs are solved again following the sequential approach to assess the impact of integrating the leveling and lot sizing decisions. The detailed results of this analysis can be found in Appendix B and C

while the summary of the cost savings achieved through adopting the integrated decision approach is presented in Table 33.

Examining Table 33, it can be found that the integrated decision approach, in general, increases the renewable related costs but decreases the consumable related costs. Overall, the decrease in the consumable related costs outweighs the increase in the renewable related costs, hence the model achieves cost savings and outperforms the model that depends on the sequential decision approach.

**Table 33: Percent Cost Savings of Integrated Decision Approach**

Cost Saving Type	Renewable Related Costs	Consumable Related Costs	Total
Minimum Cost Reduction	-63.72%	0.00%	0.00%
Maximum Costs Reduction	0.00%	33.33%	15.61%
Average Cost Reduction	-8.92%	7.69%	3.48%

Looking through the analysis results, it was found that as the acquiring and releasing costs are changed, no effect was observed in the project scheduling. The only difference between the sequential and integrated decision approaches results is the activities that are split. In the sequential approach, two activities are split which are G and L, while in the integrated approach, activities G and K are split. This difference in the time-phased resource requirements reduced the holding cost of the integrated approach by 1. Thus, the cost savings for all changes in releasing and acquiring cost is 1. The same result was obtained for different values of splitting cost. However, it was found that when the splitting cost is large, it becomes expensive to afford it, thus no splitting occurs in both the sequential and integrated decision approaches.

The ordering cost, on the other hand, was found to have an inverse relationship with the amount of economic savings the integrated approach achieves compared to the sequential approach. As the ordering cost of R2 or R3 increases, the total cost savings gained from applying the integrated approach decreases. This is because as the ordering cost increases, the ending inventory increases as fewer orders are made. The integrated decision approach takes into consideration this relationship and schedule activities and splits them accordingly to achieve the minimum possible

total cost. On the other hand, the sequential approach does not schedule the activities while considering materials' related costs. Thus, there was a difference in the scheduling of activities. In the sequential decision approach, activities G and L are split while in the integrated decision approach activities G and K are split. The integrated decision approach cases always performed better than the sequential decision approach cases, although the cost savings are not considerably significant.

As for changes in the holding cost, the analysis showed that as the holding costs of R3 increases, the cost savings increase. This reduction is due to activity splitting that affects the time-phased resource requirements for each resource type. As for increases of R2 holding costs, there was no clear cost saving trend. Changes in holding costs were found to have the greatest effect on the cost savings offered by the integrated decision approach. Figure 19 shows how the cost savings increases as the holding cost of resource R3 increases.

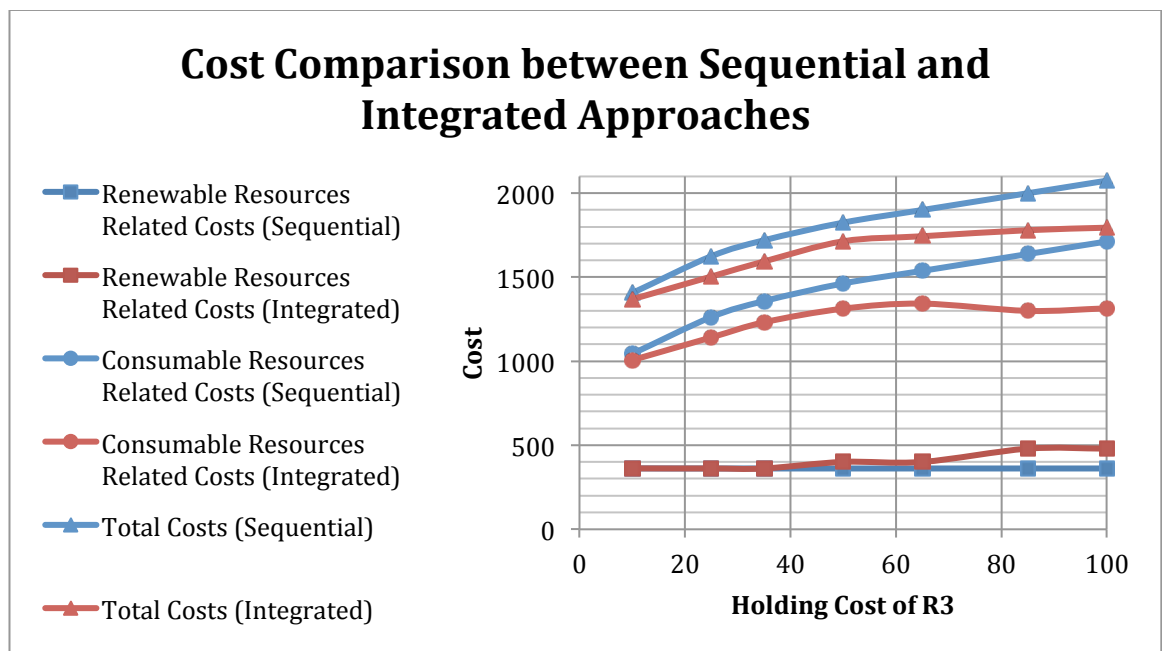


Figure 19: Cost Savings (Sequential Versus Integrated Approaches)

### 5.3 Chapter Summary

The IRLLS-V model integrates project scheduling and material procurement decisions to minimize resource leveling and material procurement related costs. Unlike the IRLLS-C model, IRLLS-V considers variable consumption rates for

consumable and renewable resources. The model balances between the different costs included in the objective function in order to find the minimum total costs. The ordering and holding costs were found to have an effect on the number of orders made and the inventory level. When the ordering cost is large, the number of orders is decreased while the inventory level is increased. On the other hand, when the holding cost is large, the number of orders is increased while the inventory level is decreased. If the holding and ordering costs are very high, the model might not be able to maintain the best resource leveling profile. This resource leveling profile is affected to minimize the total ordering and holding cost of consumable resources. Splitting was found to have an impact in terms of cost savings. It was found that when the ordering and holding costs are increasing, the cost savings from splitting decreases. In other words, for large ordering and holding costs, splitting is not recommended. This is due to the limited slack that activities have and there being a limit as to what splitting can salvage in terms of cost. Further, the distribution of resource utilization and the variations in the resource consumption time profile over activity duration were found to have an effect on cost savings achieved after following the integrated decision approach. However, there is no apparent relationship between them. Savings were not directly dependent on the standard deviation of the resources consumption rate, but rather on the resultant time-phased resource requirement schedule. In the last section, the sequential and integrated decision approaches were compared for different values of project parameters and it was found that the integrated decision approach always performed better or equal to the sequential decision approach, and the cost savings was more significant when the holding cost was large.

## Chapter 6: Conclusion and Future Research Directions

### 6.1 Conclusion

This research addressed the resource leveling problem where project scheduling and material procurement decisions were handled concurrently. Project managers would traditionally determine the project schedule and then make the material procurement decisions. This sequential approach does not necessarily result in the minimum total costs which can alternatively be found using the integrated decisions approach that is used in this research. After a comprehensive review of the literature, only a few papers were found to integrate material procurement and project scheduling problems. To the best of our knowledge, the papers that did so assumed that materials should be available before their needed activity. This saves material ordering costs, but may result in excessive inventory holding costs.

Thus, the objective of this research was to develop mathematical models that integrate material procurement and project activity scheduling decisions. The developed models minimize the material related costs (purchasing, ordering and holding costs) and project scheduling relating costs (splitting, releasing, and acquiring costs) and to the best extent level the utilization of the renewable resources over the fixed duration of the project assuming that activities can be split. The model can be used to determine the materials ordering schedule and a modified CPM schedule with preempted activities. Many papers in the literature consider resource leveling of renewable resources, but in this research resource leveling for renewable resources is considered while minimizing the cost of consumable resources. Finally, the models were formulated as mixed integer linear programs under the assumptions of unconstrained resources, fixed activities' time and slacks, and negligible lead-time.

The first step was to code the optimization model proposed by Hariga and El-Sayegh [13] which seeks to optimize the cost of the multi-resource leveling problem with allowed activity splitting. Hariga and El-Sayegh's model minimized the project scheduling related costs (splitting, releasing, and acquiring costs) while leveling renewable resource utilization by making use of noncritical activities' float time and splitting when needed. Although splitting helps smooth the time-phased resource requirements profile and reduce the total period-to-period deviation, it adds an

expense to the project called the splitting cost. However, it was found while analyzing the performance of the model that splitting helps decrease the acquiring and releasing costs. Thus, splitting is beneficial to the project as long as the reduction it makes in the releasing and acquiring costs is higher than the associated splitting cost.

Hariga and El-Sayegh's model did not integrate material procurement decisions with project scheduling decisions. Thus, the extended model IRLLS-C was developed to integrate the project scheduling and material procurement decisions. The objective function in this model consists of both material related costs and project scheduling related costs. Integrating materials ordering decisions was found to have an effect on both project scheduling and splitting. The model strives to find the best tradeoff between the different costs included in the objective function in order to find the minimum total costs. The ordering and holding costs were found to have an effect on the number of orders made and the inventory level. For high ordering costs, the number of orders is decreased while the inventory level is increased. When the holding cost is large, the number of orders is increased while the inventory level is decreased. If the holding cost is very high, the model might not be able to maintain the best resource leveling profile. In such situations, the resource leveling profile is altered in an attempt to minimize the total ordering and holding cost of consumable resources. This is the case especially when the acquiring and releasing costs of renewable resources are less than the ordering and holding costs.

It should be mentioned that the effect of the mentioned parameters was not exactly the same for both consumable resources R2 and R3. It was found that consumable resources that are utilized mostly by noncritical activities (like R3) have a greater impact on activity splitting and total cost compared to resources that are mostly utilized by critical activities (like R2). For example, when R2's holding cost is increased, only the number of orders and inventory level of R2 were affected. Conversely, when R3 is increased, the number of orders of R3 and the inventory levels of R2 and R3 along with activity splitting were affected.

Although the IRLLS-C model integrates the project scheduling and material procurement decisions very well, it assumes a constant consumption rate for both renewable and consumable resources. Thus, the IRLLS-V model was developed as an extension to IRLLS-C to handle variable demand requirements per activity duration



for renewable and consumable resources. A sensitivity analysis was performed to test the effect of varying the model's input parameters on the total project cost savings. The results came out similar to the IRLLS-C. When the holding cost is large, the model was found to sacrifice the smoothing of the renewable resource utilization profile so that the activities that utilize the consumable resources (with high holding cost) are overlapped or if possible scheduled in parallel.

Further analysis was carried out to assess the effect of splitting on project scheduling and total cost. Splitting was found to have an impact in terms of cost savings. However, when the ordering and holding costs are large, the cost savings are low, thus splitting is not recommended. This is because splitting decisions are now based on material related costs and leveling related costs, not the latter only.

In addition, the sequential and integrated decision approaches were compared for different values of project parameters and it was found that the integrated decision approach always performed better or, at worst, equal to the sequential decision approach, and the cost savings was more significant for large values of the holding cost. Although the distribution of resource utilization and the variations in the resource consumption time profile over activity duration were found to have an effect on cost savings, there was no apparent relationship between them. Savings were not directly dependent on the standard deviation of the resource consumption rate but rather on the resultant time-phased resource requirement schedule. The largest cost savings obtained was 15.30% while evaluating the model. The analysis showed that this percentage is not limited but rather related to the value of the holding cost. As the holding cost increases, the economic savings is expected to increase as well. The integrated model was able to provide an optimal ordering schedule for the consumable resources in addition to the optimal project's activities schedule that balances, to the greatest extent possible, the utilization of renewable resources.

The main insight from this research is that all types of costs were found to have an effect on either resource leveling or the total holding costs. In addition, leveling related costs were found to have an effect on materials' related costs and vice versa. For example, when splitting cost is high, activities do not split, and they run over shorter time periods that may result in lower number of orders depending on the

value of ordering cost. Therefore, decision makers have to consider all types of costs at the same time to level resources and minimize material related costs.

## **6.2 Future Research Directions**

The models presented in this research, IRLLS-C and IRLLS-V, were developed under the assumptions of unlimited resources, fixed activity time and slacks, and negligible lead-time. Renewable resources are usually constrained. For instance, project managers would typically have a limited and defined amount of manpower to utilize during the project duration. Adding resource constraints to the IRLLS-V model poses as a promising future research avenue. Further, the activities considered in this research are assumed to have fixed and known durations. This is true especially when projects are well known and have been done before and the work breakdown structure is developed accurately by an experienced team. However, when there is a lack of experience or in the common situation where unforeseen events take place, the project activities cannot be assumed to have fixed durations as they might take more or less time to be completed compared to the initial plan. An extended model to IRLLS-V can accommodate for the case where activity durations are random. The lead-time as well can be considered to be significant and not negligible as assumed in this research. For cases where the lead-time is large, the ordering time and quantity will differ from those cases where lead-time is negligible.

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## Appendix A

### Effect of Splitting Analysis Results

Analysis Type		Type	Renewable Resource					Consumable Resources				Obj	Cost Saving	
			NL	Split	CI*I	CD*D	CS*NL	Total	O	O*A	Inv*h			Total
Base	Variable	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 1]	340	170	510	950	7.47%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[3 1]	340	177	517	879	
Ordering Cost	R2=10	Without Splitting	[0 0 0 0]	0	300	140	0	440	[6 1]	160	100	260	700	9.57%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[6 1]	160	111	271	633	
	R2=30	Without Splitting	[0 0 0 0]	0	300	140	0	440	[4 1]	220	127	347	787	8.64%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[4 1]	220	137	357	719	
	R2=50	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 1]	250	170	420	860	8.26%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[3 1]	250	177	427	789	
	R2=100	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 1]	400	170	570	1010	7.03%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[2 1]	300	277	577	939	
	R2=120	Without Splitting	[0 0 0 0]	0	300	140	0	440	[2 1]	340	272	612	1052	6.94%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[2 1]	340	277	617	979	
	R3=10	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 3]	270	108	378	818	10.15%
		With Splitting	[1 0 0 2]	3	260	100	3	363	[3 3]	270	102	372	735	
	R3=25	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 2]	290	120	410	850	9.53%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[3 2]	290	117	407	769	
	R3=35	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 2]	310	120	430	870	9.31%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[3 2]	310	117	427	789	
	R3=50	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 1]	290	170	460	900	9.00%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[3 2]	340	117	457	819	
	R3=65	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 1]	305	170	475	915	7.76%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[3 1]	305	177	482	844	
R3=85	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 1]	325	170	495	935	7.59%	
	With Splitting	[1 0 1 0]	2	260	100	2	362	[3 1]	325	177	502	864		

### Effect of Splitting Analysis Results (continued)

Analysis Type		Type	Renewable Resource					Consumable Resources				Obj	Cost Saving	
			NL	Split	CI*I	CD*D	CS*NL	Total	O	O*A	Inv*h			Total
Ordering Cost	R3=120	Without Splitting	[0 0 0 0]	0	300	140	0	440	[3 1]	360	170	530	970	7.32%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[3 1]	360	177	537	899	
Holding Cost	R2=10	Without Splitting	[0 0 0 0]	0	300	140	0	440	[6 1]	580	280	860	1300	5.15%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[6 1]	580	291	871	1233	
	R2=30	Without Splitting	[0 0 0 0]	0	300	140	0	440	[9 1]	820	350	1170	1610	4.78%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[10 1]	900	271	1171	1533	
	R2=50	Without Splitting	[0 0 0 0]	0	300	140	0	440	[12 1]	1060	230	1290	1730	5.61%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[11 1]	980	291	1271	1633	
	R2=80	Without Splitting	[0 0 0 0]	0	300	140	0	440	[15 1]	1300	80	1380	1820	3.68%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[15 1]	1300	91	1391	1753	
	R2=100	Without Splitting	[0 0 0 0]	0	300	140	0	440	[15 1]	1300	80	1380	1820	3.68%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[15 1]	1300	91	1391	1753	
	R3=10	Without Splitting	[0 0 0 0]	0	320	160	0	480	[3 3]	540	230	770	1250	5.76%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[3 3]	540	276	816	1178	
	R3=25	Without Splitting	[0 0 0 0]	0	320	160	0	480	[3 4]	640	290	930	1410	6.88%
		With Splitting	[1 0 1 0]	2	260	100	2	362	[3 4]	640	311	951	1313	
	R3=35	Without Splitting	[0 0 0 0]	0	320	160	0	480	[3 5]	740	259	999	1479	5.14%
		With Splitting	[1 0 0 1]	2	260	100	2	362	[3 4]	640	401	1041	1403	
	R3=50	Without Splitting	[0 0 0 0]	0	320	160	0	480	[3 7]	940	134	1074	1554	1.99%
		With Splitting	[1 0 0 0]	1	280	120	1	401	[3 7]	940	182	1122	1523	
	R3=65	Without Splitting	[0 0 0 0]	0	320	160	0	480	[3 7]	940	149	1089	1569	1.02%
		With Splitting	[1 0 0 0]	1	280	120	1	401	[3 7]	940	212	1152	1553	
R3=85	Without Splitting	[0 0 0 0]	0	320	160	0	480	[3 7]	940	169	1109	1589	0.00%	
	With Splitting	[0 0 0 0]	0	320	160	0	480	[3 7]	940	169	1109	1589		
R3=100	Without Splitting	[0 0 0 0]	0	320	160	0	480	[3 7]	940	184	1124	1604	0.00%	
	With Splitting	[0 0 0 0]	0	320	160	0	480	[3 7]	940	184	1124	1604		



### Effect of Splitting Analysis Results (continued)

Analysis Type	Type	Renewable Resource						Consumable Resources				Obj	Cost Saving	
		NL	Split	CI*I	CD*D	CS*NL	Total	O	O*A	Inv*h	Total			
Releasing Cost	10	Without Splitting	[0 0 0 0]	0	300	70	0	370	[3 1]	340	170	510	880	5.80%
		With Splitting	[1 0 1 0]	2	260	50	2	312	[3 1]	340	177	517	829	
	30	Without Splitting	[0 0 0 0]	0	300	210	0	510	[3 1]	340	170	510	1020	8.92%
		With Splitting	[1 0 1 0]	2	260	150	2	412	[3 1]	340	177	517	929	
	50	Without Splitting	[0 0 0 0]	0	300	350	0	650	[3 1]	340	170	510	1160	11.29%
		With Splitting	[1 0 1 0]	2	260	250	2	512	[3 1]	340	177	517	1029	
	80	Without Splitting	[0 0 0 0]	0	300	560	0	860	[3 1]	340	170	510	1370	13.94%
		With Splitting	[1 0 1 0]	2	260	400	2	662	[3 1]	340	177	517	1179	
100	Without Splitting	[0 0 0 0]	0	300	700	0	1000	[3 1]	340	170	510	1510	15.30%	
	With Splitting	[1 0 1 0]	2	260	500	2	762	[3 1]	340	177	517	1279		
Acquiring Cost	10	Without Splitting	[0 0 0 0]	0	150	140	0	290	[3 1]	340	170	510	800	6.38%
		With Splitting	[1 0 1 0]	2	130	100	2	232	[3 1]	340	177	517	749	
	30	Without Splitting	[0 0 0 0]	0	450	140	0	510	[3 1]	340	170	510	1020	1.08%
		With Splitting	[1 0 1 0]	2	390	100	2	492	[3 1]	340	177	517	1009	
	50	Without Splitting	[0 0 0 0]	0	750	140	0	890	[3 1]	340	170	510	1400	9.36%
		With Splitting	[1 0 1 0]	2	650	100	2	752	[3 1]	340	177	517	1269	
	80	Without Splitting	[0 0 0 0]	0	1200	140	0	1340	[3 1]	340	170	510	1850	10.32%
		With Splitting	[1 0 1 0]	2	1040	100	2	1142	[3 1]	340	177	517	1659	
100	Without Splitting	[0 0 0 0]	0	1500	140	0	1640	[3 1]	340	170	510	2150	10.74%	
	With Splitting	[1 0 1 0]	2	1300	100	2	1402	[3 1]	340	177	517	1919		

## Appendix B

### Variations in Resource Utilization Analysis Results

Resources						Type	Renewable Resource		Consumable Resource			Obj	Cost Saving
R1	SD	R2	SD	R3	SD		NL	Total	O*A	Inv*h	Total		
[4, 4, 4, 4]	0.00	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,1,0]	322	1140	85	1225	1547	0.00%
						Integrated	[1,0,1,0]	322	1040	185	1225	1547	
[4,4,5,3]	0.82	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	322	1440	82	1522	1844	14.15%
						Integrated	[1,0,0,0]	361	1040	182	1222	1583	
[3,5,5,3]	1.15	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	362	1440	82	1522	1884	14.86%
						Integrated	[0,0,0,0]	480	940	184	1124	1604	
[4,4,6,2]	1.63	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	362	1440	82	1522	1884	12.69%
						Integrated	[2,0,1,0]	523	1040	82	1122	1645	
[5,3,6,2]	1.83	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	362	1440	82	1522	1884	12.69%
						Integrated	[2,0,1,0]	523	1040	82	1122	1645	
[4,4,7,1]	2.45	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	402	1440	84	1524	1926	13.50%
						Integrated	[1,0,1,0]	642	940	84	1024	1666	
[5,3,7,1]	2.58	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	442	1440	82	1522	1964	14.21%
						Integrated	[2,0,1,0]	563	1040	82	1122	1685	
[6,2,7,1]	2.94	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,2]	443	1340	82	1422	1865	7.83%
						Integrated	[0,0,1,0]	601	1040	78	1118	1719	
[4,4,8,0]	3.27	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	482	1440	84	1524	2006	14.96%
						Integrated	[1,0,1,0]	682	940	84	1024	1706	
[5,3,8,0]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	522	1440	82	1522	2044	15.61%
						Integrated	[2,0,1,0]	603	1040	82	1122	1725	
[5,3,0,8]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,1,0]	522	1140	80	1220	1742	0.00%
						Integrated	[1,0,1,0]	522	1140	80	1220	1742	
[5,0,3,8]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,0]	481	1140	78	1218	1699	2.53%
						Integrated	[0,0,0,0]	640	940	76	1016	1656	

### Variations in Resource Utilization Analysis Results (continued)

Resources						Type	Renewable Resource		Consumable Resource			Obj	Cost Saving
R1	SD	R2	SD	R3	SD		NL	Total	O*A	Inv*h	Total		
[5,0,8,3]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	482	1440	84	1524	2006	13.46%
						Integrated	[0,0,0,0]	720	940	76	1016	1736	
[5,8,3,0]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[0,0,0,1]	441	1440	74	1514	1955	10.74%
						Integrated	[1,0,1,0]	722	940	83	1023	1745	
[5,8,0,3]	3.37	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[0,0,1,1]	442	1440	73	1513	1955	12.43%
						Integrated	[0,0,0,0]	600	940	172	1112	1712	
[7,1,7,1]	3.46	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,2]	483	1340	82	1422	1905	9.76%
						Integrated	[0,0,1,0]	601	1040	78	1118	1719	
[6,2,8,0]	3.65	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	522	1340	79	1419	1941	9.33%
						Integrated	[0,0,0,0]	640	1040	80	1120	1760	
[7,1,8,0]	4.08	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[0,0,1,1]	562	1340	77	1417	1979	4.04%
						Integrated	[0,0,1,0]	781	1040	78	1118	1899	
[8,0,8,0]	4.62	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[1,0,0,1]	642	1340	79	1419	2061	8.83%
						Integrated	[0,0,1,0]	761	1040	78	1118	1879	
[16,0,0,0]	8.00	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[0,0,0,1]	761	1240	76	1316	2077	6.74%
						Integrated	[0,0,0,1]	921	940	76	1016	1937	
[0,16,0,0]	8.00	[1, 1, 1, 1]	0	[1, 1, 1, 1]	0	Sequential	[0,0,0,1]	761	1340	72	1412	2173	6.44%
						Integrated	[0,0,0,1]	921	1040	72	1112	2033	
[0,0,16,0]	8.00	[1, 1, 1, 1]	0.00	[1, 1, 1, 1]	0	Sequential	[0,0,0,1]	921	1340	86	1426	2347	13.68%
						Integrated	[1,0,1,0]	1002	940	84	1024	2026	
[0,0,16,0]	8.00	[1, 1, 1, 1]	0.00	[1, 1, 1, 1]	0	Sequential	[0,0,0,0]	880	1040	82	1122	2002	0.80%
						Integrated	[1,0,0,0]	961	940	85	1025	1986	
[4, 4, 4, 4]	0.00	[2 1, 1, 0]	0.82	[1, 1, 1, 1]	0	Sequential	[1,0,1,0]	322	1140	80	1220	1542	0.00%
						Integrated	[1,0,1,0]	322	1040	180	1220	1542	
[4, 4, 4, 4]	0.00	[2, 2, 0, 0]	1.15	[1, 1, 1, 1]	0	Sequential	[1,0,1,0]	322	1140	79	1219	1541	0.00%
						Integrated	[1,0,1,0]	322	1040	179	1219	1541	

### Variations in Resource Utilization Analysis Results (continued)

Resources						Type	Renewable Resource		Consumable Resource			Obj	Cost Saving
R1	SD	R2	SD	R3	SD		NL	Total	O*A	Inv*h	Total		
[4, 4, 4, 4]	0.00	[3, 1, 0, 0]	1.41	[1, 1, 1, 1]	0	Sequential	[1,0,1,0]	322	1140	76	1216	1538	0.00%
						Integrated	[1,0,1,0]	322	1140	76	1216	1538	
[4, 4, 4, 4]	0.00	[4, 0, 0, 0]	2.00	[1, 1, 1, 1]	0	Sequential	[1,0,1,0]	322	1140	73	1213	1535	0.00%
						Integrated	[1,0,1,0]	322	1140	73	1213	1535	
[4, 4, 4, 4]	0.00	[1, 1, 1, 1]	0.00	[2 1, 1,0]	0.82	Sequential	[1,0,1,0]	322	1140	85	1225	1547	0.00%
						Integrated	[0,0,1,1]	522	940	85	1025	1547	
[4, 4, 4, 4]	0.00	[1, 1, 1, 1]	0.00	[2, 2, 0, 0]	1.15	Sequential	[1,0,1,0]	322	1040	85	1125	1447	0.00%
						Integrated	[1,0,1,0]	322	1040	85	1125	1447	
[4, 4, 4, 4]	0.00	[1, 1, 1, 1]	0.00	[3, 1, 0, 0]	1.41	Sequential	[1,0,1,0]	322	1040	85	1125	1447	0.00%
						Integrated	[1,0,1,0]	322	1040	85	1125	1447	
[4, 4, 4, 4]	0.00	[1, 1, 1, 1]	0.00	[4, 0, 0, 0]	2.00	Sequential	[1,0,1,0]	322	1040	85	1125	1447	0.00%
						Integrated	[1,0,1,0]	322	1040	85	1125	1447	

## Appendix C

### Sequential Vs. Integrated Decisions Analysis Results

Analysis Type		Type	Renewable Resource					Consumable Resources				Obj	Cost Saving	
			NL	Split	CI*I	CD*D	CS*NL	Total	O	O*A	Inv*h			Total
Base	Constant	Sequential	[1 0 0 1]	2	120	0	2	122	[3 1]	340	183	523	645	0.00%
		Integrated	[1 0 0 1]	2	120	0	2	122	[3 1]	340	183	523	645	
	Variable	Sequential	[1 0 0 1]	2	260	100	2	362	[3 1]	340	178	518	880	0.11%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 1]	340	177	517	879	
Ordering Cost	R2=10	Sequential	[1 0 0 1]	2	260	100	2	362	[6 1]	160	118	278	640	1.09%
		Integrated	[1 0 1 0]	2	260	100	2	362	[6 1]	160	111	271	633	
	R2=30	Sequential	[1 0 0 1]	2	260	100	2	362	[4 1]	220	142	362	724	0.69%
		Integrated	[1 0 1 0]	2	260	100	2	362	[4 1]	220	137	357	719	
	R2=50	Sequential	[1 0 0 1]	2	260	100	2	362	[3 1]	250	178	428	790	0.13%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 1]	250	177	427	789	
	R2=100	Sequential	[1 0 0 1]	2	260	100	2	362	[2 1]	300	278	578	940	0.11%
		Integrated	[1 0 1 0]	2	260	100	2	362	[2 1]	300	277	577	939	
	R2=120	Sequential	[1 0 0 1]	2	260	100	2	362	[2 1]	340	278	618	980	0.10%
		Integrated	[1 0 1 0]	2	260	100	2	362	[2 1]	340	277	617	979	
	R3=10	Sequential	[1 0 0 1]	2	260	100	2	362	[3 3]	270	105	375	737	0.27%
		Integrated	[1 0 0 2]	3	260	100	3	363	[3 3]	270	102	372	735	
	R3=25	Sequential	[1 0 0 1]	2	260	100	2	362	[3 2]	290	123	413	775	0.77%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 2]	290	117	407	769	
	R3=35	Sequential	[1 0 0 1]	2	260	100	2	362	[3 2]	310	123	433	795	0.75%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 2]	310	117	427	789	
	R3=50	Sequential	[1 0 0 1]	2	260	100	2	362	[3 2]	340	123	463	825	0.73%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 2]	340	117	457	819	
	R3=65	Sequential	[1 0 0 1]	2	260	100	2	362	[3 1]	305	178	483	845	0.12%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 1]	305	177	482	844	

### Sequential Vs. Integrated Decisions Analysis Results (continued)

Analysis Type		Type	Renewable Resource					Consumable Resources				Obj	Cost Saving	
			NL	Split	CI*I	CD*D	CS*NL	Total	O	O*A	Inv*h			Total
Ordering Cost	R3=85	Sequential	[1 0 0 1]	2	260	100	2	362	[3 1]	325	178	503	865	0.12%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 1]	325	177	502	864	
	R3=120	Sequential	[1 0 0 1]	2	260	100	2	362	[3 1]	360	178	538	900	0.11%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 1]	360	177	537	899	
Holding Cost	R2=10	Sequential	[1 0 0 1]	2	260	100	2	362	[6 1]	580	316	896	1258	1.99%
		Integrated	[1 0 1 0]	2	260	100	2	362	[6 1]	580	291	871	1233	
	R2=30	Sequential	[1 0 0 1]	2	260	100	2	362	[9 1]	820	366	1186	1548	0.97%
		Integrated	[1 0 1 0]	2	260	100	2	362	[10 1]	900	271	1171	1533	
	R2=50	Sequential	[1 0 0 1]	2	260	100	2	362	[12 1]	1060	246	1306	1668	2.10%
		Integrated	[1 0 1 0]	2	260	100	2	362	[11 1]	980	291	1271	1633	
	R2=80	Sequential	[1 0 0 1]	2	260	100	2	362	[15 1]	1300	96	1396	1758	0.28%
		Integrated	[1 0 1 0]	2	260	100	2	362	[15 1]	1300	91	1391	1753	
	R2=100	Sequential	[1 0 0 1]	2	260	100	2	362	[15 1]	1300	96	1396	1758	0.28%
		Integrated	[1 0 1 0]	2	260	100	2	362	[15 1]	1300	91	1391	1753	
	R3=10	Sequential	[1 0 0 1]	2	260	100	2	362	[3 3]	540	312	852	1214	2.97%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 3]	540	276	816	1178	
	R3=25	Sequential	[1 0 0 1]	2	260	100	2	362	[3 4]	640	432	1072	1434	8.44%
		Integrated	[1 0 1 0]	2	260	100	2	362	[3 4]	640	311	951	1313	
	R3=35	Sequential	[1 0 0 1]	2	260	100	2	362	[3 6]	840	327	1167	1529	8.24%
		Integrated	[1 0 0 1]	2	260	100	2	362	[3 4]	640	401	1041	1403	
	R3=50	Sequential	[1 0 0 1]	2	260	100	2	362	[3 6]	840	432	1272	1634	6.79%
		Integrated	[1 0 0 0]	1	280	120	1	401	[3 7]	940	182	1122	1523	
	R3=65	Sequential	[1 0 0 1]	2	260	100	2	362	[3 7]	940	407	1347	1709	9.13%
		Integrated	[1 0 0 0]	1	280	120	1	401	[3 7]	940	212	1152	1553	
R3=85	Sequential	[1 0 0 1]	2	260	100	2	362	[3 7]	940	507	1447	1809	12.16%	
	Integrated	[0 0 0 0]	0	320	160	0	480	[3 7]	940	169	1109	1589		

### Sequential Vs. Integrated Decisions Analysis Results (continued)

Analysis Type		Type	Renewable Resource					Consumable Resources				Obj	Cost Saving		
			NL	Split	CI*I	CD*D	CS*NL	Total	O	O*A	Inv*h			Total	
Holding Cost	R3=100	Sequential	[1 0 0 1]	2	260	100	2	362	[3 12]	1440	82	1522	1884	14.86%	
		Integrated	[0 0 0 0]	0	320	160	0	480	[3 7]	940	184	1124	1604		
Releasing Cost	10	Sequential	[1 0 0 1]	2	260	50	2	312	[3 1]	340	178	518	830	0.12%	
		Integrated	[1 0 1 0]	2	260	50	2	312	[3 1]	340	177	517	829		
	30	Sequential	[1 0 0 1]	2	260	150	2	412	[3 1]	340	178	518	930	0.11%	
		Integrated	[1 0 1 0]	2	260	150	2	412	[3 1]	340	177	517	929		
	50	Sequential	[1 0 0 1]	2	260	250	2	512	[3 1]	340	178	518	1030	0.10%	
		Integrated	[1 0 1 0]	2	260	250	2	512	[3 1]	340	177	517	1029		
	80	Sequential	[1 0 0 1]	2	260	400	2	662	[3 1]	340	178	518	1180	0.08%	
		Integrated	[1 0 1 0]	2	260	400	2	662	[3 1]	340	177	517	1179		
	100	Sequential	[1 0 0 1]	2	260	500	2	762	[3 1]	340	178	518	1280	0.08%	
		Integrated	[1 0 1 0]	2	260	500	2	762	[3 1]	340	177	517	1279		
	Acquiring Cost	10	Sequential	[1 0 0 1]	2	130	100	2	232	[3 1]	340	178	518	750	0.13%
			Integrated	[1 0 1 0]	2	130	100	2	232	[3 1]	340	177	517	749	
30		Sequential	[1 0 0 1]	2	390	100	2	492	[3 1]	340	178	518	1010	0.10%	
		Integrated	[1 0 1 0]	2	390	100	2	492	[3 1]	340	177	517	1009		
50		Sequential	[1 0 0 1]	2	650	100	2	752	[3 1]	340	178	518	1270	0.08%	
		Integrated	[1 0 1 0]	2	650	100	2	752	[3 1]	340	177	517	1269		
80		Sequential	[1 0 0 1]	2	1040	100	2	1142	[3 1]	340	178	518	1660	0.06%	
		Integrated	[1 0 1 0]	2	1040	100	2	1142	[3 1]	340	177	517	1659		
100		Sequential	[1 0 0 1]	2	1300	100	2	1402	[3 1]	340	178	518	1920	0.05%	
		Integrated	[1 0 1 0]	2	1300	100	2	1402	[3 1]	340	177	517	1919		
Splitting Cost	10	Sequential	[1 0 0 1]	2	260	100	20	380	[3 1]	340	178	518	898	0.11%	
		Integrated	[1 0 1 0]	2	260	100	20	380	[3 1]	340	177	517	897		
	30	Sequential	[1 0 0 1]	2	260	100	60	420	[3 1]	340	178	518	938	0.11%	
		Integrated	[1 0 1 0]	2	260	100	60	420	[3 1]	340	177	517	937		

### Sequential Vs. Integrated Decisions Analysis Results (continued)

Analysis Type		Type	Renewable Resource					Consumable Resources				Obj	Cost Saving	
			NL	Split	CI*I	CD*D	CS*NL	Total	O	O*A	Inv*h			Total
<b>Splitting Cost</b>	35	Sequential	[1 0 0 1]	2	260	100	70	430	[3 1]	340	178	518	948	0.42%
		Integrated	[1 0 0 0]	1	280	120	35	435	[3 1]	340	169	509	944	
	40	Sequential	[1 0 0 1]	2	260	100	75	435	[3 1]	340	178	518	953	0.42%
		Integrated	[1 0 0 0]	1	280	120	40	440	[3 1]	340	169	509	949	
	45	Sequential	[0 0 0 0]	0	300	140	0	440	[3 1]	340	170	510	950	0.00%
		Integrated	[0 0 0 0]	0	300	140	0	440	[3 1]	340	170	510	950	
	50	Sequential	[0 0 0 0]	0	300	140	0	440	[3 1]	340	170	510	950	0.00%
		Integrated	[0 0 0 0]	0	300	140	0	440	[3 1]	340	170	510	950	



## **Vita**

Hessa Almatroushi was born in December 1990, in Dubai, United Arab Emirates. She was educated in local public schools and graduated from Amna Bint Wahab Secondary School in 2008. She directly then enrolled in the Electrical Engineering Undergraduate Program at the American University of Sharjah in Sharjah, United Arab Emirates, from which she graduated magna cum laude, in 2012. Following her graduation, Ms. Almatroushi began a Master's program in Engineering Systems Management at the American University of Sharjah.

Ms. Almatroushi joined Dubai Electricity and Water authority, Dubai, United Arab Emirates, after her graduation for 7 months. Following that, she joined the Emirates Institution for Advanced Science and Technology (EIAST), Dubai, United Arab Emirates, where she worked as an associate researcher in the Image Processing and Applications Department for a year. In that year, she was part of an important project that is considered a national milestone for the country, DubaiSat-2 an earth observation satellite project. Further, she was a presenting author at the SPIE Remote Sensing Conference (2014), Amsterdam, Netherlands, and at the Geospatial Scientific Summit (2013), Sharjah, United Arab Emirates. Currently she is working as an Engineer in the Application Development and Analysis Center at EIAST and as a Deputy Manager for the first CubeSat project in the United Arab Emirates, Nayif-1.