BUCKLING OF PRETWISTED STEEL COLUMNS

by

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## Approval Signatures

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Dedication

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Abstract

Buckling is a mode of failure that is mainly observed in compression members due to structural instability. Inducing a natural pretwist along the length of a column section makes the column have a different resistance at every point along its centroidal axis. A pretwisted column in 3D-space has its strong flexural plane weakened and its weak flexural plane strengthened, leading to a net favorable effect on the buckling strength of the pretwisted column. The proposed research focuses on studying the effect of pretwisting on the buckling capacity of steel columns. Experimental and numerical investigations of pretwisted steel columns with different slenderness ratios were carried out. A linear perturbation analysis was conducted for several universal column cross-sections for various lengths, a set of pretwisting angles ranging 0°-180° and different boundary conditions. A number of pretwisted columns were then tested to examine the elastic and inelastic behavior of one of the universal steel columns. The experimental results were utilized to verify and develop a set of non-linear finite element models including the material and geometric nonlinearities. The validated FE models were then extended to conduct a parametric study to include several more lengths and angles of twist. It was found that pretwisting is most effective with elastic buckling since it is mainly controlled by the moment of inertia of the column in opposition to inelastic buckling, which is additionally affected by material yielding. It was also noticed that fixed-ended conditions ensured better buckling capacity improvement as compared to pinned-ended columns in both elastic and inelastic buckling of pretwisted columns.

Search Terms: Pretwisting; elastic buckling; inelastic buckling; pinned-ended; fixed-ended; slenderness ratios; linear; nonlinear; Finite element analysis; material yielding; buckling improvement
Table of Contents

Abstract .......................................................................................................................... 6
List of Figures .................................................................................................................. 9
Chapter 1: Introduction .................................................................................................... 14
  1.1 Research Significance ......................................................................................... 14
  1.2 Research Objectives ........................................................................................... 14
  1.3 Introduction ......................................................................................................... 15
  1.4 Literature Review ............................................................................................... 18
  1.5 Thesis Structure and Methodology ...................................................................... 30
Chapter 2: Elastic Buckling Capacity of Pretwisted Steel Columns using Linear Perturbation Analysis ........................................................................................................ 32
  2.1 Linear Perturbation Analysis ............................................................................. 32
  2.2 FE Modeling Using Linear Perturbation ............................................................ 33
  2.3 Mesh Sensitivity Analysis .................................................................................... 35
  2.4 FE Model Verification ......................................................................................... 36
  2.5 Results and Discussions ..................................................................................... 38
    2.5.1 Improvement of buckling capacity ............................................................... 38
    2.5.2 Failure modes ............................................................................................... 38
    2.5.3 Discussion .................................................................................................... 40
    2.5.4 Linear perturbation analysis for $\phi$ up to $360^\circ$ .................................... 42
    2.5.5 Buckling improvement under pinned-pinned conditions ......................... 43
  2.6 Mathematical Model to Compute Critical Buckling Load ................................... 44
Chapter 3: Experimental Investigation of the Buckling Capacity of Pretwisted Columns ......................................................................................................................... 45
  3.1 Test Specimens .................................................................................................... 45
  3.2 Geometric Imperfections .................................................................................... 46
  3.3 Material Properties ............................................................................................ 46
  3.4 Test Setup ........................................................................................................... 48
  3.5 Results and Discussion: ..................................................................................... 49
    3.5.1 Axial compression ...................................................................................... 49
    3.5.2 Buckling modes ......................................................................................... 51
    3.5.3 Load versus strain plots .......................................................................... 52
    3.5.4 Load versus lateral deflection plots ......................................................... 55
Chapter 4: Nonlinear Finite Element Analysis of Buckling of Pretwisted Columns ...................................................................................................................... 57
  4.1 FE Model Description ........................................................................................ 57
  4.2 Material Properties ............................................................................................. 58
4.3 Buckling Analysis ........................................................................................................58
  4.3.1 Riks analysis ........................................................................................................58
  4.3.2 Displacement-based general static analysis .........................................................59
4.4 Mesh Sensitivity Analysis ........................................................................................59
4.5 FE Model Verification ...............................................................................................60
  4.5.1 Maximum buckling capacity ..............................................................................61
  4.5.2 Axial load versus displacement .........................................................................62
  4.5.3 Buckling mode shapes .......................................................................................62
  4.5.4 Axial load versus strain results ..........................................................................64
4.6 Expanded Parametric Study ......................................................................................67
  4.6.1 Fixed-ended pretwisted columns ......................................................................68
  4.6.2 Pinned-ended pretwisted columns .....................................................................70
4.7 Effect of Pretwisting on the Least Moment of Inertia of Column .........................72
Chapter 5: Conclusion ....................................................................................................75
  5.1 Summary and Conclusions ...................................................................................75
  5.2 Recommendations ..................................................................................................76
References .......................................................................................................................78
Vita ...................................................................................................................................86
List of Figures

Figure 1: Geometric description of the UC-sections used in the current study; (a) FE model, (b) fixed-fixed column, (c) cross-section, and (d) dimensions used in FE analysis ................................................................. 34
Figure 2: Samples of pretwisted geometries for UC100X100X17 columns at selected angles. ................................................................................................................. 35
Figure 3: Three different mesh configurations used in the mesh sensitivity analysis.... 36
Figure 4: Bar chart comparisons of the critical loads between FE analysis and Euler equation using three different mesh configurations ................................................... 36
Figure 5: FE model verifications for different column lengths: (a) non-boxed, (b) boxed sections ................................................................. 37
Figure 6: Flexural buckling modes for selected untwisted UC columns (ϕ = 0°), (a) UC100X100X17 with L = 5m and (b) UC150X100X21 at L = 4m. ......................... 37
Figure 7: Buckling improvement versus angle of twist for boxed (a & b) and non-boxed (c) sections. ............................................................................................... 39
Figure 8: Samples of buckling modes for the boxed sections (a) L = 4m, ϕ = 105°, (b) L = 6m, ϕ = 165°, and the non-boxed section (c) L = 5m, ϕ = 45° ................................. 40
Figure 9: Column charts of buckling improvement versus slenderness ratio up to ϕ = 150° ................................................................................................................. 41
Figure 10: Column charts of critical loads versus I/L ratio (the ratio of the weak moment of inertia to the specimen length) up to ϕ = 150° ........................................ 41
Figure 11: Increase in buckling capacity at pretwisting angles, ϕ up to 360° for boxed and non-boxed sections .................................................................................. 42
Figure 12: Buckling modes for selected sections at high pretwisting angles ............... 42
Figure 13: Increase in buckling capacity with pretwisting for pinned-pinned columns for (a) non-boxed and (b) boxed sections ................................................................. 43
Figure 14: Buckling modes of pin-ended columns (a) non-boxed, ϕ=90°, (b) boxed, ϕ=180° ................................................................................................................. 44
Figure 15: Comparison of FE-analysis against proposed Eq. 8 .................................... 44
Figure 16: Cross-section of the tested samples ................................................................ 45
Figure 17: Coupon Test Setup ..................................................................................... 47
Figure 18: Coupon Test Dimensions ............................................................................ 47
Figure 19: Tested Coupon Samples ............................................................................. 47
Figure 20: Stress-strain curves of coupon test specimens for flange and web ............ 48
Figure 21: Experimental Test Setup ........................................................................... 49
Figure 22: Special Test Setup for 2-meter steel columns ................................................ 49
Figure 23: Buckling load of experimentally tested prismatic and pretwisted columns ......................................................................................................................... 50
Figure 24: Load-deflection for tested columns (a) 1-meter tested columns, (b) 1.5-meter tested columns, (c) 2-meter tested columns .................................................. 51
Figure 25: Buckling mode shapes for 1-meter length columns, (a) CL1T0 and CL1T30, (b) CL1T45 ............................................................................................. 51
Figure 26: Buckling mode shapes of the 1.5-meter tested columns, (a) CL1.5T0, (b) CL1.5T20, (c) CL1.5T35, (d) CL1.5T45 ................................................................. 52
Figure 27: Buckling mode shapes for the tested 2-meter columns, (a) CL2T0, (b) CL2T30 and (c) CL2T60 ................................................................. 52
Figure 28: Load-strain graphs for 1-meter columns ........................................................ 53
Figure 29: Load vs. strain values for 1.5-meter columns ........................................... 54
Figure 30: Load vs. strain in 2-meter steel columns ..................................................... 55
Figure 31: Lateral deflection recorded by LVDT for tested columns (a) 1m-columns, (b) 1.5m-columns (c) 2m-columns ................................................................. 56
Figure 32: Samples of pretwisted geometries for the simulated experimentally tested specimens ........................................................................................................... 57
Figure 33: FE model used for the nonlinear finite element analysis ............................... 58
Figure 34: Three different meshes used in the mesh sensitivity analysis ....................... 60
Figure 35: Column chart comparisons of the critical loads between nonlinear FE analysis and experimental test results using three different mesh configurations. ..... 60
Figure 36: Column chart showing comparison between FE analysis, experimental results and AISC code values for prismatic columns ................................. 61
Figure 37: Column chart for comparison between different nonlinear FE analysis and experimental results ............................................................. 62
Figure 38: Axial load versus displacement of actual and simulated prismatic columns .. 63
Figure 39: Buckling mode shapes of CL1T0 (a) displacement control, (b) Riks analysis, (c) experimental test .......................................................... 63
Figure 40: Buckling mode shapes of CL1.5T0 (a) displacement control, (b) Riks analysis, (c) experimental test .......................................................... 63
Figure 41: Buckling mode shapes of CL2T0 (a) displacement control, (b) Riks analysis, (c) experimental test .......................................................... 64
Figure 42: Comparisons of buckling mode shapes for selected pretwisted steel columns ........................................................................................................ 64
Figure 43: Load-strain plots for simulated versus experimentally tested 1-meter columns .................................................................................................................. 65
Figure 44: Load-strain plots for simulated versus experimentally tested 1.5-meter columns ................................................................................................. 66
Figure 45: Load-strain graphs for simulated versus experimentally tested 2-meter columns ...................................................................................................... 67
Figure 46: Contours of von Mises stresses prior to buckling for selected fixed-ended columns with different lengths and pretwisting angles .......................... 69
Figure 47: Critical buckling load for fixed-ended simulated columns versus their slenderness ratios ................................................................. 70
Figure 48: Buckling capacity improvement versus angles of twist for fixed-ended modeled columns ........................................................................ 70
Figure 49: Von Mises contours of selected pinned-ended column simulations with different lengths and pretwisting angles prior to buckling .................. 71
Figure 50: Critical buckling load versus slenderness ratios for pinned-ended simulated columns .................................................................................. 72
Figure 51: Improvement in buckling capacity of simulated columns under pinned-ended conditions ................................................................. 72
Figure 52: Moment of inertia versus the applied angles of twist for fixed-ended columns ........................................................................................................... 73
Figure 53: Axial-load versus displacement graph for actual and simulated CL1T30 ..... 82
Figure 54: Axial-load versus displacement graph for actual and simulated CL1T45 ..... 82
Figure 55: Axial-load versus displacement graph for actual and simulated CL1.5T20 ... 83
Figure 56: Axial-load versus displacement graph for actual and simulated CL1.5T30 ... 83
Figure 57: Axial-load versus displacement graph for actual and simulated CL1.5T45 ... 84
Figure 58: Axial-load versus displacement graph for actual and simulated CL2T30 ..... 84
Figure 59: Axial-load versus displacement graph for actual and simulated CL2T60 ..... 85
List of Tables

Table 1: Dimensions of Steel columns tested ................................................................. 46
List of Abbreviations

A: Cross-sectional Area (mm$^2$)
E: Modulus of Elasticity (Gpa)
FE: Finite Element
F$_{cr}$: Compressive stress of the member (Mpa)
F$_e$: Stress based on Euler Buckling load (Mpa)
F$_y$: Yield stress of the material used (Mpa)
I: Moment of inertia (mm$^4$)
K: The effective length factor based on the boundary conditions
K$L/r$: Slenderness Ratio
L: Length of the member (mm)
LVDT: Linear Variable Differential Transformer
n: Buckling Mode number, used as 1 in this study
r: Radius of gyration of member
UC: Universal Column
UTM: Universal Testing Machine
Chapter 1: Introduction

1.1 Research Significance

Steel structures have proved to be more reliable than concrete on sustainability and durability levels provided that the appropriate protection measures against corrosion are taken. The use of structural steel eliminates the risk of creep and shrinkage, and minimizes construction waste as it is recyclable. However, buckling, whether elastic or inelastic, is the most prominent failure mode when steel compression members are used. Improving the buckling capacity of steel columns produces more reliable structural steel skeletons to be used in various construction projects such as high-rise buildings, bridges and stadiums. The technique introduced in this thesis, known as pretwisting, is assumed to increase the critical buckling load at which the member fails as it helps strengthen the weak axis and further weaken the strong axis. This thesis presents an experimental and numerical study to investigate the buckling capacity improvement of pretwisted steel members with different cross-sections, lengths and boundary conditions.

1.2 Research Objectives

The main objective of this research is to investigate the effect of pretwisting on the buckling capacity of steel columns (i.e. universal steel columns) under applied pure axial compression. The ultimate load-carrying capacity of the columns is investigated in three stages:

a. Elastic buckling of different universal steel sections (UC-sections) is studied via linear perturbation analysis through commercial ABAQUS software as a preliminary study.

b. Experimental investigation is conducted on a number of pretwisted columns to examine the elastic and inelastic buckling behavior for one of the UC sections.

c. Non-linear finite element analysis is conducted to simulate and verify the experimental test results of the investigated members and expand the developed FE model to accommodate different boundary conditions, lengths and a wider range of pretwisting angles.
1.3 Introduction

Columns are the backbone for any structure by supporting loads from all the other structural members. Thus, these vertical compression members are the most critical part of any structural design. Columns are usually subjected to pure axial compressive forces, or may involve other loadings such as a uniaxial or biaxial bending moment or even torque. The focus in this study is columns under a compressive axial force with no load eccentricity, as the primary aim herein is to investigate the buckling strength of columns under pure compression. Subjected to an increasing level of applied compressive load, a straight column may be shortened until a critical load is reached at which the column exhibits a deformed configuration (i.e. buckling takes place) [1].

Buckling of steel columns may be classified as elastic or inelastic. The Euler buckling equation defines the elastic buckling of slender columns quite well. For slender columns, the critical buckling load is usually of lower value than that of non-slender columns, which needs a greater applied load to buckle. If buckling is initiated in the elastic range, no increase in load is observed at the start of the buckling phase. On the other hand, if buckling of a column starts in the inelastic range, an increase in the load is observed with the initiation of the buckling stage. Basically, whether a column bends in the elastic or inelastic zone depends on its slenderness ratio [1]. Limits of slenderness ratio, as described by the American Institute of Steel Construction (AISC), are shown below.

\[ KL/r \leq 4.71 \sqrt{E/F_y} \]  \hspace{1cm} \text{for inelastic columns.}  \hspace{1cm} (1)

\[ KL/r > 4.71 \sqrt{E/F_y} \]  \hspace{1cm} \text{for elastic columns.}  \hspace{1cm} (2)

where \( K \) is the theoretical coefficient for end conditions, \( L \) is the column length, \( r \) is the radius of gyration, \( E \) is the modulus of elasticity and \( F_y \) is the yield stress of steel used.

According to provisions given by the AISC, the relationships describing the stress in a compression member under elastic and inelastic buckling are shown in equations (3) and (4):

\[ F_{cr} = 0.658 \frac{F_y}{F_e} F_y \]  \hspace{1cm} \text{for inelastic buckling}  \hspace{1cm} (3)

\[ F_{cr} = 0.877 F_e \]  \hspace{1cm} \text{for elastic buckling}  \hspace{1cm} (4)
In those equations $F_{cr}$ is the compressive stress of a compression member and $F_e$ is the stress based on the Euler buckling load equation (5).

$$P_o = \frac{n \pi^2 E I}{(KL)^2}$$

(5)

where $P_o$ denotes the critical buckling capacity corresponding to the first buckling mode, $I$ is the moment of inertia of the cross-section about the weak-axis, $K$ is the theoretical effective length factor.

Different modes of buckling may take place with columns under concentric axial compressive loads, classified into flexural buckling, torsional buckling and torsional-flexural buckling depending on the cross-sectional dimensions and shape of the member investigated. For an I-section, the shear center (i.e. point at which the transverse load needs to be applied in order to avoid twisting of the member) coincides with its centroidal axis, making it less prone to torsional buckling and mainly subjected to flexural buckling of its weak axis. On the other hand, torsional buckling may control the failure for very slender doubly-symmetric sections which are rarely addressed in the literature [2-3]. Moreover, equal-leg cruciform members are known to be mainly subjected to torsional buckling along with cold-formed and thin-walled open sections [4]. Nevertheless, pretwisted members are initially twisted about the centroidal axis; thus the buckling mode cannot be easily predicted as will be explained in the upcoming sections. However, flexural-torsional buckling is not usually witnessed for bi-symmetric sections. Single-symmetric cross-sections mainly fail under flexural buckling or flexural-torsional buckling. On the other hand, for non-symmetric sections, torsional-flexural buckling is the controlling buckling mode [5].

Local buckling is another mode of failure that depends more on the slenderness of the individual components of the steel member than on the slenderness ratio of the column as a whole. In other words, local buckling may be experienced with slender, intermediate or short columns depending on the width-to-thickness ratios of the flange and web of a certain steel member [5, 6]. This type of instability may take place before the tested specimen reaches its critical load and causes failure so that the sample is no longer structurally effective. Local buckling has to be considered for both stiffened and unstiffened elements. For the I- and H-sections, flanges are the unstiffened members and webs are the stiffened sections. Thus, a flange may reach the yield plateau and completely lose its stiffness leaving the elastic
core of the column to resist buckling with its remaining stiffness. However, the whole cross-section would have reduced stiffness, since one of its individual components has become structurally ineffective [7].

Pretwisting is mainly applying a rotational angle along the member's length, such that the principal axes of inertia rotate as a function of the axial coordinates (i.e. the centroidal axis) of the column. As simple as the definition may seem, interpreting the effect of pretwisting on the buckling capacity of a steel section is a challenging task. Hence, pretwisting would be considered as an introduced twist to the investigated specimen which would lead to bearing of a higher critical load prior to reaching its ultimate design strength. Implementation of pretwisting in a certain member leads to inducing a coupling effect on the weak and strong planes [8]. Moreover, the effect of pretwisting can be explained as witnessing a transition between the weak and strong axes of the member (i.e. the weak axis may be strengthened, while the strong axis may become weaker after being pretwisted). Another thoughtful interpretation would be that the natural pretwist applied induces a coupling effect on the two flexural planes of a naturally pretwisted compression member [9]. This induced coupling effect helps increase the first buckling load encountered by the member upon applying the axial compressive force, while reducing the buckling load of the second buckling mode. Since the second buckling mode is of relatively less importance than the first, the assumed coupling effect would evidently have an overall beneficial effect on the buckling capacity of the pretwisted member in the first buckling mode.

Given a pretwisted and a prismatic compression member may buckle under flexure, torsion or a combination of both (i.e. flexural-torsional buckling), the axis around which the member may fail (i.e. due to buckling) is no longer predictable. In other words, pretwisting results in changing the direction of least resistance (i.e. changing the definition of a pretwisted member's minor principal axis) at every point along the centroidal axis of a steel section [10]. Despite the fact that a compression member normally buckles around the direction of least resistance under flexural-buckling, this may no longer be valid for pretwisted members. Also, this configuration of the pretwisted member would exhibit higher stiffness and better static performance (i.e. buckling analysis) than the prismatic member. However, the
buckling mode shape of a pretwisted member is most similar to that experienced with the prismatic compression member [11].

In this study, the first buckling mode is the focus, and the second buckling mode will be ignored. The main contribution of pretwisting can thus be related to the second moment of area of the considered compression member. However, an extensive numerical study is conducted to achieve accurate correlations between the applied angle of twist and the different variables that exist in buckling analysis, as the primary stage in this thesis. The numerical investigation is conducted by simulating various cross-sections using the commercial finite element code ABAQUS [12]. For the initial stage of this thesis, buckling analysis of pretwisted slender steel sections are performed by using linear perturbation. Experimental investigation of the buckling capacity of pretwisted steel columns are also conducted at the second stage, considering different lengths and rotational angles. The final stage of this thesis includes the development of nonlinear finite element models, which will be verified using the experimental results and then extended to analyze the buckling improvement for a wider range of pretwisting angles, lengths and boundary conditions.

1.4 Literature Review

The topic of pretwisting was introduced in the literature a long time ago. Pretwisting was massively used with beams in helicopter rotor blades, turbine blades and gear teeth [13]. Most researchers seem to have greater interest in the dynamic analysis of the pretwisted member than its static performance. On the other hand, the nature and effect of pretwisting on the buckling capacity of axially loaded compression members was seldom covered in research over the years. Hence, the available research on how pretwisting strengthens the considered compression member to increase its buckling capacity is limited.

Lin et al. [13] replaced the Bernoulli-Euler beam system with a Timoshenko beam system to study the forced vibration of non-uniform pretwisted beams. Elastic boundary conditions were considered. The deformations of the considered beams were assumed to be small, and the stress components built within the beams were linearly related to the strain components, i.e., yielding does not take place. A static solution of the general system of equations with the help of a modified transfer matrix
was introduced. The shear deformation effect and rotary inertia were involved in the analysis along with the effect of the applied pretwist (unlike the Bernoulli-Euler system which ignores the effect of shear deformation). It was found that the effect of pretwisting was more obvious in the higher modes than in the lower modes, and that the influence of non uniform pretwisting is greater than that of uniform pretwisting.

Madhusudhana et al. [14] investigated the buckling capacity of uniformly and non-uniformly pretwisted beams (i.e. continuously, discontinuously and oppositely pretwisted beams) with fixed-end conditions. The derived system of equations was based on a Cartesian global coordinate system linked to a local coordinate system (i.e. defining the principal axis of a beam) with a rotational angle. The Galerkin's technique and the Hermite polynomials were then implemented to solve the derived system of equations. Consequently, Gaussian four point integration was used to compute the developed stiffness and mass matrices. These matrices were then assembled into a global matrix (i.e. describing the entire structure) that was solved with an Eigen-value solver. The study found that the optimum twist was 225°. Also, the unidirectional pretwist applied along the member's length was revealed to yield greater buckling capacities than when various pretwist combinations were implemented in opposite directions throughout the centroidal axis from one end of the member to the other. Furthermore, the analysis showed that with uniform pretwisting, the principal axis which governs buckling is the weaker axis (i.e. y axis). Moreover, the investigation exposed the oscillatory nature of the increase in the buckling load of a pretwisted member and linked it to the dependence of the buckling load on the buckling mode which is also influenced by the twist. The study also linked the increase in buckling load of a beam to the position of the pretwist from the centroid of the section to one end, concluding that the maximum buckling load is encountered when the twist is exactly at the center of the beam with a minimum buckling load at the beam end for both opposite and unidirectional twists. Finally, it was also concluded that when a unidirectional twist (symmetric about the center of a discontinuous beam) is applied in two portions, a greater buckling load is achieved.

Chen [15] developed two sets of equations to describe the lateral vibration of rotating pre-twisted Timoshenko beams under applied axial compression taking into account two different directions of axial force. The first set of equations described the action line of the applied compressive axial load to be normal to the shear force, while
the second set depicted the load direction to be tangential to the beam axis. The study conducted by Chen [15] also investigated the buckling stability of the pretwisted beams and considered the effect of various factors, namely, width-to-thickness ratio, angle of pretwist, and the spinning speed applied to the cantilever beams. Two Eigenvalue problems were of primary concern to solve the developed gyroscopic system of ordinary differential equations: the natural frequency and the buckling load of the beams. The results revealed that the buckling capacity of the beams increased with the decreased thickness-to-width ratio and the spinning speed. Furthermore, the direction of the axial load proved to have a significant role in the buckling loads of the higher mode for beams with width-to-thickness ratios equal to 1, while it had a negligible effect on the critical loads of beams with width-to-thickness ratios equal to 0.125. Moreover, the ultimate buckling capacity always increased with increasing angle of pretwist for the first buckling mode, yet the critical load decreases with increased pretwist for the higher modes. Additionally, the investigation also ascertained that the natural frequencies of the system always decreased upon application of compressive axial load regardless of the applied pretwist, spinning speed or width-to-thickness ratios. Likewise, an increase in the axial load caused greater deviation in the natural frequencies between the two sets. The first four natural frequency parameters for a cantilever untwisted Timoshenko beam achieved by Chen [15] exhibited adequate agreement with Farchaly and Shebl's findings [16]. The buckling load parameters of a clamped-free twisted Timoshenko beam attained by Chen [15] also proved to agree with the results of Sabuncu and Evran [17]. Likewise, the perceived dimensionless natural frequencies of a rotating fixed-free Timoshenko beam with a rectangular cross-section showed strong agreement with the Banerjee and Su models [18].

Sina et al. [19] investigated the dynamic behavior of rotating pretwisted thin-walled composite box beams (i.e. with cross-sections characterized by two-fold symmetry to eliminate coupling of axial extensions and torsion with bending). Axial-torsional vibrations were induced by a centrifugal force in order to ensure better performance schemes of the rotor blades designed for turbo-machinery. Anisotropic material was used, which proved to induce static torque in the presence of the applied angle of pretwist. Consequently, pretwisting had a significant effect on the torsional vibrations of composite blades. Nonlinear equations were derived to describe the
coupled axial-torsional motion of the blades utilizing the Hamilton's principle. In order to solve the equations for the equilibrium state, the nonlinear equations were linearlized. The extended Galerkin’s technique was then implemented to solve for the Eigenvalue problems and construct the mass and stiffness matrices. The separate and combined effect of each of the applied axial loads, linearly varying angles of pretwist, and stagger and fiber angles on torsional behavior was thoroughly investigated by Sina et al. [19]. The main assumption claimed that the axial stress induced in the filament produced a couple in the twisted blades about their neutral axis, namely the "Wagner effect". Consequently, the induced static torque caused a static equilibrium state in torsion, without which the torsional behavior of the blades is believed to exhibit both qualitative and quantitative errors. In opposition, for isotropic material, the static torque induced had negligible influence on the hardening of the natural frequencies. Furthermore, pretwist angles below 45° had a marginal effect on the natural frequencies which, therefore, decreased until θ=60°. Consequently, natural frequencies were found to increase as θ exceeded 60°. The nonlinear static torsion of the thin-walled beam under steady tension force and varying torsional moment (i.e. torque) presented in [19] concurred with the results of Rosen [20], proving the validity and accuracy of the results.

McGee et al. [21] probed the free torsional vibrations of a pretwisted isotropic thin-walled I-beam acting as a cantilever, for which a multi-filament model was developed to cover a wide range of length-to-leg ratios (L/b) and leg-to-thickness ratios (b/t). The beams considered were under elastic warping, inertial warping deformation and large angles of twist about its axis of shear centers. The separate and combined effects of the major parameters were thoroughly examined for the first three modes of torsional vibration. The non-linear boundary-value equations were first linearlized to derive the exact solution. Elastic warping was found to increase the non-dimensional torsional frequencies (i.e. ratio of circular frequency of free vibration to the St. Venant torsional frequency) reaching a maximum at the higher torsional modes of thin-walled I-beams with small L/b and relatively large b/t ratios. On the other hand, the inertial warping term was proved to have a reduced effect on the effective torsional resistance of thin-walled I-beams as the L/b ratio increased. Additionally, inertial warping proved to be independent of both the uniformly applied pretwist
angle and the $b/t$ ratio. Furthermore, the applied pretwist showed a significant positive effect on the basic torsional mode of the I-beams with low $L/b$ and high $b/t$ ratios.

Hodges et al. [22] investigated the non-linear cross-sectional modeling of a beam in an attempt to interpret the nonlinear trapeze effect (i.e. nonlinear coupling of extension and twist) through the "classical laminated shell theory". The implemented theory presented a simplified configuration characterized by an anisotropic/composite laminated strip-like beam with a relatively small angle of pretwist. Furthermore, the strain energy was derived in terms of its 1D quantities only in order to perform the geometrical reduction from 2D to 1D successfully. The developed model was extended to anti-symmetric beams made up of a combination of pretwisted anisotropic laminates, confirming the significance of the nonlinear extension-twist coupling. Applying the principle of virtual work proved significant for deriving the torsional buckling of a cantilever column with an untwisted cruciform cross-section made of prismatic "anti-symmetric laminates", under the sole application of an axial load at its tip. However, the results confirmed that a small angle of pretwist had no effect on the magnitude of the torsional buckling load of the column. The theory presented in [22] was found to reduce to the same analytical model developed by Armanios et al. [23] when used for anti-symmetric laminates unloaded with either bending or torsional loads.

Liao et al. [24] analyzed the structural characteristics of a rotating pretwisted orthotropic beam with an axisymmetric cross-section mainly used as a fluted cutting tool or end milling cutter. Moreover, a pretwisting angle was applied along its centroidal axis, and it possessed two different orthogonal principal axes. Finite element analysis, namely the Galerkin's technique, was implemented for the investigation. The study involved the derivation of the motion equations that described the lateral vibration and elastic stability of the considered beams under the effect of applied axial force, rotating speed, angle of pretwist, and length-to-thickness ratio. The main concern was the natural frequency, buckling load, critical speed and stable region of the beam. The orthotropic beam followed up on Euler and Timoshenko’s beam theories. The buckling load of a square cross-section proved to be independent of the applied angle of pretwist, as the $I_{xz}=0$, leaving $I_{yz}=I_{zx}$. Nevertheless, the buckling load was also found to be constantly increasing with any other cross-section until it reached a fixed value at a relatively large angle of pretwist,
where the beam looked like a round beam. Furthermore, the buckling load calculated using the Timoshenko beam theory seemed to converge to that estimated by using the Euler equation for relatively large values of length-to-thickness ratios \(L/t\). Conversely, the load perceived through the Timoshenko beam theory proved to be lower than that achieved using the Euler equation at relatively low \((L/t \leq 10)\) ratios for the short pretwisted beams, due to the considered transverse shear deformations. Moreover, the effect of the pretwist on the natural frequencies in the first mode proved to be lower than that in the higher vibration modes. Additionally, the results showed that increasing the angle of pretwist helped improve the elastic stability of the rotating pretwisted orthotropic beams. Conversely, an increase in axial load proved to have an adverse effect on the natural frequencies of the pretwisted orthotropic beams.

Liao and Huang [25] presented a finite element model to investigate the parametric instability of a rotating cantilever pretwisted beam having two orthogonal planes and unequal principal moments of inertia. The considered beam is usually used to describe fluted cutting tools with helix angles under the action of a time-dependent periodic axial compressive load applied at its tip. Euler-Bernoulli's beam theory was first incorporated along with Hamilton's principle to derive the equations of motion. Galerkin's technique then helped formalize the generated equations into a finite element model. The final set of simultaneous first-order differential equations was then solved by the multiple scales method, which decomposed the applied end axial load into a steady force and a small periodic perturbation superimposed on it. The effects of pretwisting angle \(\theta\), rotating speed and steady-state end axial force were simulated in the simple perturbation function \(F(t) = F \cos \theta t\), where \(F\) is always lower than the critical load \(P_{cr}\). The results then revealed that on increasing the angle of pretwist \(\theta\), larger unstable regions of main resonance were encountered near \(2\omega_2\) (region \(C\)) and the summed-resonance \(\omega_1 + \omega_2\) (i.e. region \(A\)) while smaller unstable regions of main resonance existed near region \(A\) (i.e. \(2\omega_1\)) where \(\omega_1\) refers to the first mode of frequency and \(\omega_2\) refers to the second mode of frequency. It was also found that increasing the angles of pretwist caused the main resonance of the unstable regions \(A\) and \(C\) to converge where the first two natural frequencies were closer to each other.
Lee [26] scrutinized the lateral/torsional vibrations of a spinning and axially moving pretwisted beam with an axisymmetric cross-section, to avoid coupling between torsional and lateral vibrations. The Euler method was applied along with Hamilton's principle to develop the motion equations used to describe pretwisted beams having time-dependant lengths, fixed to a rotating base. A coupling effect between the lateral deflections \( u_2 \) and \( u_3 \) was witnessed with the induced spinning action and achieved only when the elastic stiffness in \( j \) and \( k \) directions were unequal. Furthermore, an oscillatory nature of the tip lateral deflection was witnessed for a relatively low axial and spinning motion. Eventually, the fluctuating trend of the tip displacement in both orthogonal directions was reduced with increasing axial and spinning movements of the pretwisted beam.

Liu et al. [27] thoroughly investigated the coupled axial-torsional vibrations of pretwisted beams to model the aircraft and helicopter rotor blades. The equations of motion controlled by extension, torsion and cross-sectional warping were derived with the help of the Hamilton's principle and a scaling analysis. The broadly used assumption that the same warping function used for prismatic beams could similarly be incorporated for pretwisted beams was carefully inspected. More importantly, the warping function was significant for exhibiting the coupling effect between torsion and extension which cannot be ignored for pretwisted beams. This assumption appeared to be the reason for the inaccurate estimation of the axial resonance frequencies by previous researchers. The contradictions in Rosen's equations of motion [28] were also addressed and modified by Liu et al. [27] to include fourth-order terms with relatively small coefficients in the new developed equations of motion, allowing for non-uniform torsion and dynamic state analysis. This modification, in turn, becomes more significant with higher harmonics and larger angles of pretwist. The study also included a finite element analysis performed via the ANSYS software, the results of which were also compared to the analytical model developed by Liu et al. [27] and Rosen [28].

Giannakopoulos et al. [29] considered the use of a non-linearly varying angle of pretwist applied along the centroidal axis of a beam. The study aimed at revising the classical linear elasticity theory of torsion of beams and introducing a structural gradient theory for thin-walled beams. Significantly, thin-walled beams proved to be the actual microstructure of the DNA molecule, falsely assumed to have a circular
cross-section by previous researchers over the years [30]. The investigation used a non-local estimate of the average value of the applied pretwist, inducing a shear gradient consistent with the developed bimoment in the beam, following the proposed theory by Vardoulakis and Giannakopoulos [31]. The study revealed that, due to the initially applied angle of pretwist, the high axial tension force required to unwind/straighten the DNA molecule produced excessive shear stresses. Consequently, disorder of the bonding of the base pairs followed and helped detach the DNA strands. In other words, the high axial force provided a coupling effect between the gradient of the applied twist and the average axial-strain of the cross-section which eventually led to the destruction of the DNA structure. The results also demonstrated that pretwisting increased the torsional stiffness of thin-walled beams. Furthermore, shorter beams showed stiffer responses in torsion than longer beams. The proposed structural gradient theory by Giannakopoulos et al. [29] exhibited adequate agreement with Vlasov’s theory of thin-walled open elastic beams [32].

Frisch-Fay [33] analytically studied the stability of pretwisted columns under the application of a compressive axial load to develop charts that link the main variables included in the investigation. The boundary conditions considered for the permanently pretwisted bar were spherical hinges at the two ends. The main parameters included in the analysis were the ratio of buckling load of a pretwisted column with respect to a prismatic one (α), pretwist angle per unit length of the column (w) and the ratio of the two moments of inertia (I₂/I₁). The analytical model presented therein proved that the maximum increase achieved by a pretwisted column could go beyond two times that of a straight one, provided the k₂ < k₁, where k₁ = P/EI₁ and k₂ = P/EI₂. However, it was also shown that the ratio (α) could reach a maximum of 4 given k₁² ≠ w₁². Nevertheless, such high values of critical buckling loads do not provide stable equilibrium of the column and thus this much higher compressive strength is not all beneficial [33]. Another interesting finding was that at k₁=k₂, the applied pretwisting had no contribution on the critical buckling load of the column. On the other hand, discontinuities were observed in α but not thoroughly considered.

Celep [34] studied the stability of simply-supported pretwisted columns subjected to static and periodic axial load. Pretwisting was defined as the rotation of the principal axes of the column around its undeformed axis. The effect of the rigidity ratio (i.e. ratio of the two principal moments of inertia of the section) was highlighted
along with the effect of pretwisting on the static performance and the dynamic stability of the column. The analytical model was solved by using the Galerkin's method. The study revealed that as the rigidity ratio of a certain cross-section approaches unity, the effect of pretwisting almost vanishes. For the purpose of the study done by Celep [34], the first five modes of buckling were considered. The analysis also showed that the first buckling load is not much affected by the rigidity ratio, while the loads from the second and third buckling modes do vary slightly with a change in the rigidity ratio. Moreover, it was shown that as the rigidity ratio is increased, the buckling loads approach each other. It was also revealed that the load from the second buckling mode reaches a minimum before the first buckling load is reached. Furthermore, the study showed that medium pretwisting had the greatest effect on the lowest critical loads.

Celep [35] investigated the stability of a pretwisted Leipholz’ column. The Leipholz’ column is a cantilever elastic column with a linear viscous damping system under distributed and vertical follower loading conditions. Galerkin's technique was used for studying the flexural deformation configuration in both principal axes of the considered column, where only the first 3 mode shapes of the untwisted column were taken. It was shown that when the applied pretwist is considerably high, a significant increase in the divergence load is achieved. The study also revealed that the flutter load increases at a faster rate with higher pretwisting angles.

Tabarrok et al. [9] performed an analytical study on the buckling capacity of pretwisted columns implementing the principle of total potential energy to derive the desired equilibrium equations and the corresponding boundary conditions. The analysis involved both statically determinate and indeterminate cases. The equilibrium equations were adequate for directly obtaining the solution of the statically determinate columns (i.e. pinned-pinned and clamped-free columns). For statically indeterminate columns (i.e. clamped-clamped and clamped-pinned columns), force-displacement relations were needed along with the equilibrium equations. Specifically, this research involved slender columns for which the shear strain was ignored and Euler-type beam equations were used instead of the generalized Timoshenko beam theory. A significant increase in the buckling capacity of the column for the first mode and a faster decrease in the buckling strength of the second mode were observed for almost all boundary conditions. Furthermore, the first and the
second mode shape converged as the angle of twist increased. Moreover, the study showed that statically indeterminate cases exhibited more oscillatory nature than the statically determinate cases in the presented graphs of strength ratios of pretwisted and prismatic columns versus the applied pretwist.

Serra [10] studied the flexural buckling of pretwisted columns with rectangular cross-sections using the direct variational method with Fourier series. This analytical model aimed at quantifying the positive effect of the change of curvature (pretwist) on the tested columns which was claimed to increase the stiffness and behavior of a thin column under axial compression. The pretwist angles considered in the investigation ranged from 0°-360° with an increase in the number of terms needed in Fourier series for higher angles of rotation. The analysis showed that pretwisting is an effective technique for strengthening thin columns as far as its static performance to buckling is concerned.

Steinman et al. [11] presented an analytical solution in an attempt to link the effect of pretwisting on both the buckling load and the frequency on the axially-loaded pretwisted rods. Based on this analytical investigation, it was found that the buckling and frequency modes of the pretwisted bars were quite similar to those of prismatic bars independent of the applied natural pretwist, given that boundary conditions are kept constant. This finding applies to bars under compressive and moderate tensile loadings. Only at relatively high tensile loads does the mode shape of a pretwisted member differ from that of a prismatic bar.

Rosen et al. [36] applied a different technique for investigating the elastic stability of linearly pretwisted rods called "principle curvature transformation". Two cases of loading were considered: buckling under compressive axial load and buckling due to application of lateral loads at the tip of the rod, and each were dealt with separately. The perturbation analysis consisted of two states, calculation of the basic state with deformation, followed by calculation of the buckling load based on the basic state with the help of a load multiplier for each case. The Eigen value problem was used for the solution convenience. The analysis confirmed that pretwisting increases the axial buckling load of rods under pure compression but this increase gets flattened at higher angles of twist (i.e. pretwist > 270°) for both simply-supported and clamped-free conditions. It was also shown that for relatively high ratios of the major and minor principal bending stiffness, the buckling load decreased
for an increase in the applied pretwist. Moreover, it was revealed that the linear solution does not describe the lateral buckling of rods and the nonlinear approach is a must for more accurate results, specifically for lower ratios of bending stiffness and higher degrees of pretwist.

Yang and Yau [37] investigated the stability of pretwisted bars with various end torques. The approach used in this study was derivation of differential equations using the global coordinate system which was then reduced to a local coordinate system for a more convenient solution. The end torques applied were divided into two major types: the semi-tangential (i.e. made up of two couples of direct forces with equal magnitude and opposite direction) and the quasi-tangential (i.e. consisted of a single couple of forces). Having a bar with applied end torques restricts its failure mode to only torsional-flexural [37]. The results of the study showed that an increase in the applied pretwist until it approaches infinity forced the section to behave as a circular cross-section regardless of the type of torque initially applied. Also, it was noted that an initial pretwist had no effect on the buckling strength of a member with equal moments of inertia. However, natural pretwist would have a great positive effect on a bar with unequal moments of inertia (i.e. the bar gets stiffer).

Tabarrok and Xiong [38] developed a system of general equilibrium equations for curved and spatial rods. The model included the internal forces and moments, applied forces and moments, along with the translational displacements and rotational translations for a rod with finite length. The equilibrium equations were derived for both the deformed and undeformed configuration. For the undeformed configuration, the Lagrangian equilibrium equation was defined. Moreover, kinematic and constitutive relations linking the internal applied forces to the shear strains of a spatial rod were needed for defining the required equilibrium equation. Furthermore, the equilibrium equation for special cases was also defined. An equilibrium equation generalizing the Timoshenko-beam theory for straight columns and another equation for pretwisted Euler columns were also derived. Other special cases included buckling of helical springs; cylindrical arches were also part of the study.

Steinman et al. [39] studied the effect of naturally applied pretwist on the buckling capacity of slender columns both statically determinate and indeterminate cases. The equations used in the analytical model were derived as a subset of the general stability equations for a spatial rod. Basically, a simple iterative technique was
implemented to get the exact numerical solutions of the governing differential equations. Also, the analytical forms of the buckled mode shape equations, mode shape coefficients, and the calculated buckling loads were used in the analytical model. The consequent equation is of fourth-order and reduces to the Euler equation when the applied pretwist equals 0° (i.e. prismatic Euler column). The main parameters are the applied angle of pretwist and the ratio of second moments of area around the strong and weak axes of the slender columns considered. Four boundary conditions were considered in this study, involving hinged-hinged, clamped-hinged, clamped-clamped and clamped-free. The assumption behind this research was that a column in 3D-space buckles around the stronger flexural plane albeit its original plane of flexure being the weak plane and that pretwisting works on coupling these two flexural planes. It was found that the buckling mode of a pretwisted column resembles that of a prismatic column. The study revealed that for statically indeterminate columns, the optimum buckling capacity is reached for a range of pretwist angles between 90° and 270° followed by a decrease for the angles 270°-360°. The results also showed that the capacity of a pretwisted statically indeterminate column may be twice as much as that of the corresponding prismatic column.

Karami et al. [40] used finite element modeling for pretwisted rods, for which the governing equations were the equilibrium equations, the kinematic relations and the constitutive relations. The study involved the derivation of a complete stiffness matrix with a shape function of linearly elastic spatial elements applied to different types of boundary conditions and loadings. Various loading patterns (i.e. concentrated or uniformly distributed loading pattern) could be involved with the developed spatial elements. Moreover, the model could be expanded to simulate spatial elements with different cross-sectional dimensions and support conditions while keeping the degrees of freedom equal to 12 at both ends. The natural pretwist was applied at the z-axis which is considered to be the centerline of the member, while keeping the principal axes to be the xy in the local coordinate system. The kinematic relations implemented in the study linked the displacement and rotation angle vectors to the derived curvature matrix and the constant matrix [J] through Kirchhoff's assumption to get the change in curvature-twist vector {k} and the strain vector {ε}. Then, the area and moment of inertia of the section were linked to the shear and curvature-twist vectors through constitutive relations. Three types of elements were considered for the study:
prismatic, pretwisted with linear curvature change, and pretwisted with constant curvature change by varying the values of \( k_x, k_y, k_z \) and \( \varepsilon_x, \varepsilon_y, \varepsilon_z \). In conclusion, few elements were needed to obtain an adequate level of accuracy for pretwisted elements, in contrast to beam elements in which more elements lead to higher accuracy.

Recently, Barakat and Abed [41] conducted an experimental study to investigate the effect of pretwisting on the axial load capacity and stability of fixed-ended pretwisted steel bars with rectangular cross-sections. More than 200 specimens of different cross-sections, lengths, and widths were first twisted with several angles by applying pure torque using a torsion machine, then exposed to axial compression using an MTS machine. It was observed that the pretwisted bars claimed a non-planar deformed shape during buckling. Moreover, at buckling, the axial stiffness of the twisted bars decreased gradually until the critical load was reached. The experimental results revealed that the critical buckling load of a pretwisted bar was always of higher value than that of its corresponding prismatic bar. Furthermore, this experimental study showed that the effect of pretwisting was greater on sections with higher second moments of area for specific pretwist angles.

Abed et al. [42] then expanded the experimental study by using a nonlinear finite element analysis to include a wider range of pretwisting angles up to 270°. Both the experimental and numerical results concluded that pretwisting increases the buckling capacity of thin columns; the buckling load capacity becomes higher with higher ratios of principle moments of inertia for a specific set of pretwisting angles. It was also observed that the buckling load of a pretwisted bar is always higher than that of the corresponding prismatic bars with unequal principal moments of inertia. Also, the highest increase in buckling capacity of the bars was observed at a pretwisting angle close to 90°.

1.5 Thesis Structure and Methodology

The significance of this research and the objective of this study are thoroughly explained in Chapter 1. Moreover, Chapter 1 includes a comprehensive background and literature review that outlined the earliest and most recent research reported about the topic. Furthermore, a preliminary study utilizing linear finite element analysis is thoroughly explained in Chapter 2. Chapter 2 presents a numerical investigation of the effect of pretwisting on elastic buckling of members with different cross-sectional dimensions (i.e. boxed and non-boxed sections), lengths (i.e. From 4 meter up to 7
meter with an increment of 1 meter) and applied pretwist (i.e. From 0° up to 180° with an increment of 15°) through linear perturbation analysis on ABAQUS software which in turn implements the Euler elastic buckling equation. Furthermore, the detailed experimental test program and setup is presented in Chapter 3, along with the results achieved with the help of the UTM machine, strain gauges and LVDTs. Chapter 4 presents the development of a nonlinear finite element model to simulate the experimentally tested columns. Moreover, a comparison between the results from the nonlinear finite element model and experimental results against AISC provisions for inelastic buckling is also provided. Additionally, Chapter 4 discusses two different types of nonlinear buckling finite element analyses (i.e. RIKS and general static analysis) considered in this research and compares the two types in terms of maximum buckling load reached, axial load-strain graphs and axial load versus displacement plots. Moreover, an expanded parametric study established through extending the verified FE models is also presented in Chapter 4, where a wider range of lengths is used (i.e. from 3 meter up to 6 meter with an increment of 1 meter) and a greater range of pretwisting angles applied (i.e. From 0° up to 180° with an increment of 30°) on columns sharing the same cross-sectional dimensions and material properties. Finally, Chapter 5 concludes the research along with providing a summary and recommendations for future work.
Chapter 2: Elastic Buckling Capacity of Pretwisted Steel Columns using Linear Perturbation Analysis

In this chapter, elastic buckling of prismatic and pretwisted members under pinned-ended and fixed-ended boundary conditions is numerically investigated by using a linear perturbation analysis technique that is available in the finite element software ABAQUS. The material used is assumed to be elastic with $E=200$ Gpa and Poisson's ratio ($\nu$) = 0.3; the plastic portion of the stress-strain curve is not taken into account. The members under investigation in this chapter are classified into boxed and non-boxed sections based on the unique cross-sectional dimensions of each universal column (UC-section) used. Different pretwisting angles and lengths are considered with each cross-section as will be discussed in further detail in the later sections of this chapter. The governing equation used in this chapter is the Euler buckling equation, against which the developed Finite Element models are validated.

2.1 Linear Perturbation Analysis

The linear perturbation analysis step is created such that the response can only be linear, estimating elastic buckling by the use of Eigen value extraction. Eigen-value buckling analysis is utilized in the linear perturbation analysis step to analyze preloaded or unloaded stiff structures. This type of analysis step is, therefore, considered suitable for the focus of the present research dealing with elastic (Euler) buckling of stiff and unloaded compression members.

The Euler column responds very stiffly to the applied compressive load until the critical buckling load is reached. A sudden failure of the column, referred to as buckling, is then observed, and the column shows a much lower stiffness value. The buckling load is usually calculated on the base state of a column. Thus, an estimation using the general Eigen value extraction is useful since the perturbation load is elastic before the buckling occurs. The key point in an Eigen value problem is making the model stiffness matrix singular, such that the problem is described by the following relation (6):

$$K^{MN}u^M = 0$$

where $K^{MN}$ is the tangent stiffness matrix and $u^M$ is the displacement vector.
In this buckling analysis step, an incremental load pattern, of which magnitude is not of great importance, will be scaled by the load multipliers such that the Eigen value problem can be defined by a more general equation:

$$(K_0^{NM}+\lambda_iK_\Delta^{NM})u_i^M=0.$$  \hspace{1cm} (7)

where $K_\Delta^{NM}$ is related to the differential loading pattern while $K_0^{NM}$ corresponds to the initial loading condition. The superscripts $^M$ and $^N$ are the degrees of freedom for the whole system while the subscript $i$ denotes the $i^{th}$ buckling mode. Here, $K_0^{NM} = 0$, since the UC-sections used in this study are not preloaded. In equation (7), $\lambda_i$ are the Eigen values and the vectors $u_i^M$ are normalized so that the maximum displacement is equal to 1.0, which represents the different buckling modes, and not the actual deformation values at the critical buckling load. The subspace iteration Eigen solver is used in this analysis step, based on the number of Eigen values specified. However, the critical buckling load is taken as the load that corresponds to the first buckling mode for each twisting angle.

### 2.2 FE Modeling Using Linear Perturbation

Finite element (FE) modeling of pretwisted columns was developed using the commercial software ABAQUS 6.10 [12]. The FE simulations were performed using the (*BUCKLE) procedure that is available in the ABAQUS library [43], which adopts the Eigen value analysis procedure described in the previous section. The aim of the FE analysis was to investigate the improvement of the axial capacity in pretwisted universal steel columns. Only the first (critical) buckling mode predicted from the Eigen value analysis was considered. Three different universal column (UC) sections with different lengths were investigated: UC100×100×17, UC152×152×30, and UC150×100×21. Figure 1 shows a detailed description of the sections used in this FE analysis. As can be noticed, out of the three columns, only UC150×100×21 has a cross-section with unequal depth and width, i.e. not a boxed-section.

Each column was modeled using a three-dimensional shell part and was meshed with 4-node three-dimensional shell elements of S4R type. Two analytical rigid plates were created and attached to the original column. A reference point was introduced for each rigid plate to identify the boundary conditions on each end. A
fixed-fixed ends condition was introduced such that the rotation and translations were restricted at the supports. One support end, however, was allowed to translate in the direction at which the unit load was applied.

![Diagram](image)

**Figure 1:** Geometric description of the UC-sections used in the current study; (a) FE model, (b) fixed-fixed column, (c) cross-section, and (d) dimensions used in FE analysis

Different rotational angles and column lengths were considered in the present FE analysis. The minimum column length in this study was 4 meter and an increment of 1 meter was then taken until a maximum column height of 7 meter was reached. For each column length, a set of pretwisted angles between 0° and 180° with an increment of 15° were considered. Figure 2 shows examples of the overall geometry of a pretwisted column at selected rotations.

Eigen value analysis was then performed to obtain the buckling capacity and failure mode for each column. This was done by applying a unit load at one end of the rigid support. From this particular analysis, buckling modes and critical loads were obtained and the expected improvement in the column axial capacity was recorded.
2.3 Mesh Sensitivity Analysis

To decide on the appropriate mesh to give the most accurate results while minimizing the computational time, different mesh configurations were examined. Three mesh configurations were considered, as shown in Figure 3: Mesh 1, Mesh 2 and Mesh 3 with a global element size of 20 mm, 25 mm and 50 mm, respectively. The three meshes were used to simulate the buckling capacity of an untwisted and twisted UC152×152×30 column of 6 meter length. Mesh 1 and Mesh 2 were found to give similar accuracy when compared to the calculated values using the Euler equation as illustrated by the bar chart in Figure 4. The total difference between the results given by the linear perturbation analysis using Mesh 1 or Mesh 2 was less than 3%, whereas it exceeds 6% when using Mesh 3. Therefore, Mesh 2 was selected in the present FE analysis since it gives an acceptable combination of accuracy and reduced computational time.
Figure 3: Three different mesh configurations used in the mesh sensitivity analysis.

Figure 4: Bar chart comparisons of the critical loads between FE analysis and Euler equation using three different mesh configurations.

2.4 FE Model Verification

The elastic buckling load for straight columns (i.e., at an initial pretwisting angle of 0°) was obtained analytically using the well-known Euler equation (5) provided earlier and repeated here as:

\[ P_o = \frac{n\pi^2EI}{(KL)^2} \]  

where \( P_o \) denotes the critical buckling capacity corresponding to the first buckling mode, \( E \) is the modulus of elasticity (200GPa), \( I \) is the moment of inertia of the cross-section about the weak-axis, \( K \) is the theoretical effective length factor, and \( L \) is the column length. Results obtained from the FE buckling analysis for untwisted columns were compared with the analytical solution obtained via the Euler equation for each member. The main aim of this comparison was to validate the accuracy of the finite
element model. Figure 5 presents the comparison results for the case of the non-boxed section and the two boxed-sections. It can be noticed that good correlations have been achieved through FE modeling for the three sections and for several length increments.

![Figure 5](image1.png)

**Figure 5**: FE model verifications for different column lengths; (a) non-boxed, (b) boxed sections

In these comparisons, the length increments were selected such that columns were slender assuming elastic buckling. Figure 6 shows samples of the expected flexural buckling modes for the fixed-ended untwisted columns.

![Figure 6](image2.png)

**Figure 6**: Flexural buckling modes for selected untwisted UC columns ($\phi = 0^\circ$), (a) UC100X100X17 with $L = 5m$ and (b) UC150X100X21 at $L = 4m$. 

2.5 Results and Discussions

2.5.1 Improvement of buckling capacity

Utilizing the linear perturbation analysis through the FE simulations of pretwisted columns, critical buckling loads were recorded to study the improvement in buckling capacity due to pretwisting. Figure 7 shows the percentage increase in the buckling capacity for the three UC-sections with respect to the critical loads at $\phi = 0^\circ$ ($P_o$). Assessment of the FE results for pretwisting up to $\phi = 180^\circ$ indicates that the buckling capacity is always higher than $P_o$. The percentage increase in the critical loads is therefore always rising until the optimum pretwisting angle is reached, after which the improvement decreases relatively, but the critical buckling load remains of larger value than the reference $P_o$.

The increase in the buckling capacity was higher for the case of the non-boxed section (about 85%) as compared to the two boxed sections (between 55% and 65%). On the other hand, the optimum rotational angle, at which the buckling improvement is the maximum, took a different trend. For UC100X100X17, the optimum angle was around 120°, while for the other boxed-section UC152X152X30, it was 150°. For the non-boxed section, however, the maximum improvement was achieved at $\phi = 120^\circ$, similar to the smaller boxed-section. Further twisting seems to give less axial capacity for all sections as shown in Figure 7.

It can also be noticed from Figure 7 that the percentage and rate of increase in the buckling capacity for a column remains more or less the same with any given length. This can be observed for all UC sections. Thus, the original length of the member would not be correlated with the pretwisting angle, $\phi$ in justifying the buckling improvement. Consequently, it can be concluded that the variation in the section’s moment of inertia about the two axes with $\phi$ controls the increase in buckling capacity of the pretwisted member, for a given boundary condition (i.e., $K$ is assumed constant).

2.5.2 Failure modes

The proposed FE model was capable of accurately predicting the failure modes of the pretwisted members, mainly flexural buckling.

During buckling, the pretwisted steel sections assumed deformed configurations normal to the plane where the axial compressive load is applied. For
the purpose of the linear perturbation analysis, columns were assumed to remain in the elastic zone, failing due to buckling and not yielding. The compression members were assumed to be slender enough to claim global buckling as shown in the deformed shapes presented in Figure 8.

![Graphs showing buckling improvement versus pretwist angle for different sections.](image-url)

*Figure 7: Buckling improvement versus angle of twist for boxed (a & b) and non-boxed (c) sections.*
Figure 8: Samples of buckling modes for the boxed sections (a) L = 4 m, $\phi = 105^\circ$, (b) L = 6 m, $\phi = 165^\circ$, and the non-boxed section (c) L = 5 m, $\phi = 45^\circ$

2.5.3 Discussion

As per the definition of global buckling, a compression member is expected to go through extensive lateral translation in the weak-direction. As the pretwisting is applied, the weak and strong directions exchange roles along the length of the compression members. In other words, the weak direction is strengthened and the strong direction is weakened. This transition between the strong and weak axis leads to the overall increase in buckling capacity of the pretwisted member. As the angle of twist is increasing, one can no longer predict the direction about which failure would take place. However, about whichever direction buckling occurs, it will always be at a greater critical load than that for a non-pretwisted member failing by elastic buckling.

For all UC sections considered in this study, elastic buckling results predicted using linear perturbation analysis showed a sharp increase in buckling capacity up to 65-90% improvement as compared to non-twisted sections. The pretwisting angles, at which the critical buckling load is maximum, ranged between $120^\circ$ and $150^\circ$. This is clearly perceived in the column charts shown in Figure 9 and Figure 10. It is quite apparent that with higher slenderness ratios, the improvement in axial load capacity reaches its maximum at smaller pretwisting angles than for the case of lower slenderness ratios. Furthermore, the percentage of increase in the buckling capacity of members with higher slenderness ratios is much higher than that of sections with lower slenderness ratios. For example, while considering the elastic portion of the material stress-strain curve, maximum enhancement in the axial load capacity was reached at $\phi = 150^\circ$ for the lower slenderness ratio of $KL/r = 52$, while for the highest slenderness ratio ($KL/r = 148$), the maximum improvement was achieved at $\phi = 120^\circ$, as illustrated in Figure 9. Similarly, a sharp increase in the critical load up to $\phi =$
120°~150° is also noticed (see Figure 10) at larger ratios of $I/L$, where $I$ denotes the cross sectional moment of inertia about the weak axis and $L$ is the length of the specimen.

**Figure 9**: Column charts of buckling improvement versus slenderness ratio up to $\phi = 150°$

**Figure 10**: Column charts of critical loads versus $I/L$ ratio (the ratio of the weak moment of inertia to the specimen length) up to $\phi = 150°$
2.5.4 Linear perturbation analysis for $\phi$ up to 360°

For further investigation on the effect of pretwisting on the column’s axial capacity, the range of pretwisting angles chosen for analysis was extended up to $\phi = 360^\circ$. Accordingly, buckling analysis was conducted at pretwisting increments of 15° each on boxed and non-boxed sections for a fixed length of 6 meters as shown in Figure 11. As explained earlier, the buckling improvement for the non-boxed section keeps increasing until it reaches a maximum of 83% at $\phi = 120^\circ$, then slightly decreases between 120° and 165°. The buckling capacity starts increasing again up to 100% at $\phi = 240^\circ$, after which it decreases sharply as the pretwisting angles reach 360°. A similar trend was noticed for the case of the boxed section, but with less percentage of improvement at a higher pretwisting angle, $\phi =135^\circ$. However, the rotation angle $\phi =240^\circ$ remains the optimum for FE-analysis beyond $\phi =180^\circ$ for both sections (see Figure 11). Samples of buckling modes for the case of large pretwisting angles are also presented in Figure 12.

![Graph showing buckling capacity improvement for boxed and non-boxed sections](Figure 11)

**Figure 11: Increase in buckling capacity at pretwisting angles, $\phi$ up to 360° for boxed and non-boxed sections**

![Buckling modes for selected sections at high pretwisting angles](Figure 12)

**Figure 12: Buckling modes for selected sections at high pretwisting angles**
2.5.5 Buckling improvement under pinned-pinned conditions

Elastic buckling analysis on pretwisted columns with pinned-pinned ends conditions was also investigated to study the effect of the end conditions on the improvement in buckling capacity. The members picked for this investigation were UC100X100X17 and UC150X100X21. Applying pinned-pinned boundary conditions, however, gave relatively negligible improvement as compared to the fixed-fixed boundary condition. The improvement witnessed in the non-boxed section was slightly higher than that recorded for the boxed section, as can be seen from Figure 13. The increase in the buckling capacity, however, did not exceed 20% for either section. This low improvement may be attributed to the fact that under pinned-pinned conditions, the rotation is not restricted at the two ends. The member is free to rotate starting at the two supports, where the least effect of pretwisting is noted. On the other hand, under fixed-ended conditions, rotation of the member is restricted to only 0.5–0.65 of the whole length, mostly acting in the region where pretwisting is most effective. Samples of buckling modes for selected pin-ended sections are presented in Figure 14.

![Graph showing buckling capacity improvement with pretwisting for pinned-pinned columns](image1)

**Figure 13:** Increase in buckling capacity with pretwisting for pinned-pinned columns for (a) non-boxed and (b) boxed sections.

![Graph showing buckling modes of pin-ended columns](image2)

**Figure 14:** Buckling modes of pin-ended columns (a) non-boxed, $\phi=90^\circ$, (b) boxed, $\phi=180^\circ$. 43
2.6 Mathematical Model to Compute Critical Buckling Load

As mentioned earlier, Barakat and Abed [41] and Abed et al. [42] performed experimental and numerical investigations on buckling improvement for a set of steel bars with varying thicknesses, widths and lengths, and for pretwisting angles up to 270°. The maximum buckling improvements for the sections were achieved at optimum pretwisting angles between 75° and 90°. Consequently, Abed et al. [42] used multiple regression analysis to propose two definitions that relate the critical loads to pretwisting angles for the case of elastic and inelastic buckling. To serve the purpose of the current study, the equation for elastic buckling given in equation (8) was utilized for comparison with the linear perturbation analysis performed in this study.

$$P_{cr}^{pretwisted} = \left[1 + \frac{\phi}{360}\right]^4 P_o$$ (8)

The main aim behind equation (8) was to be able to predict the axial buckling capacity of a pretwisted (\(\phi \leq 90^\circ\)) compression member, regardless of its slenderness ratio, section properties or length. Figure 15 shows a comparison between the buckling improvements predicted by equation (8) and the present analysis for the three considered sections at a fixed length of 4 meter. By taking a closer look at the plotted graph, one can thus conclude that the proposed equation cannot be generalized for all compression members. The proposed equation (8) clearly overestimated the critical loads obtained by the present FE analysis for all sections considered.
Chapter 3: Experimental Investigation of the Buckling Capacity of Pretwisted Columns

An experimental investigation was conducted to study the effect of pretwisting on the buckling capacity of a pretwisted UC100x100x17 section, considering different lengths and angles of twist under pure axial compression. A brief description of the experimental program, the test setup, material properties as well as results will be presented next.

3.1 Test Specimens

The test program conducted in this research consisted of ten columns classified into three groups based on their lengths (L=1 meter, 1.5 meter and 2 meter). The specimens were labeled such that the length and the pretwisting angle could be easily recognized. The label C was given to all specimens; L1 for specimens with 1000-mm length, L1.5 for specimens with 1500-mm length, and L2 for specimens with 2000-mm length; T denotes the angle of twist for each sample (for example, CL1.5T20 refers to a column with length equal to 1.5 meter and pretwisted up to 20 degrees). Dimensions of the steel samples considered for this experimental program are illustrated in Table 1. Figure 16 shows the cross-sectional dimensions of the tested columns.

![Figure 16: Cross-section of the tested samples](image)
3.2 Geometric Imperfections

All samples were initially inspected to measure any geometric imperfections due to manufacturing, and a range of 1.0 to 3.0 mm was observed for all samples. Geometric imperfections are quite significant for developing the nonlinear FE simulations explained in detail in the upcoming chapter.

Table 1: Dimensions of Steel Columns tested

<table>
<thead>
<tr>
<th>SECTION</th>
<th>LENGTH (mm)</th>
<th>Flange width (mm)</th>
<th>Total depth (mm)</th>
<th>Flange Thickness (mm)</th>
<th>Web height (mm)</th>
<th>Web thickness (mm)</th>
<th>φ</th>
</tr>
</thead>
<tbody>
<tr>
<td>C L₁ T₀</td>
<td>1022.0</td>
<td>99.1</td>
<td>102.1</td>
<td>6.9</td>
<td>88.3</td>
<td>6.03</td>
<td>0.0</td>
</tr>
<tr>
<td>C L₁ T₃₀</td>
<td>990.0</td>
<td>98.7</td>
<td>102.0</td>
<td>7.7</td>
<td>86.6</td>
<td>6.03</td>
<td>32.4</td>
</tr>
<tr>
<td>C L₁ T₄₅</td>
<td>1019.0</td>
<td>99.6</td>
<td>99.1</td>
<td>7.3</td>
<td>84.5</td>
<td>6.03</td>
<td>44.0</td>
</tr>
<tr>
<td>C L₁₅ T₀</td>
<td>1526.0</td>
<td>98.9</td>
<td>100.5</td>
<td>7.1</td>
<td>86.3</td>
<td>6.03</td>
<td>0.0</td>
</tr>
<tr>
<td>C L₁₅ T₂₀</td>
<td>1523.0</td>
<td>98.4</td>
<td>99.9</td>
<td>7.2</td>
<td>85.5</td>
<td>6.03</td>
<td>22.8</td>
</tr>
<tr>
<td>C L₁₅ T₃₀</td>
<td>1523.0</td>
<td>99.5</td>
<td>99.5</td>
<td>7.2</td>
<td>85.1</td>
<td>6.03</td>
<td>35.7</td>
</tr>
<tr>
<td>C L₁₅ T₄₅</td>
<td>1528.0</td>
<td>98.5</td>
<td>101.2</td>
<td>7.7</td>
<td>85.8</td>
<td>6.03</td>
<td>45.0</td>
</tr>
<tr>
<td>C L₂ T₀</td>
<td>2022.0</td>
<td>100.0</td>
<td>99.7</td>
<td>7.1</td>
<td>85.5</td>
<td>6.03</td>
<td>0.0</td>
</tr>
<tr>
<td>C L₂ T₃₀</td>
<td>2031.0</td>
<td>99.9</td>
<td>100.4</td>
<td>7.2</td>
<td>86.0</td>
<td>6.03</td>
<td>33.7</td>
</tr>
<tr>
<td>C L₂ T₆₀</td>
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<td>101.7</td>
<td>7.9</td>
<td>85.9</td>
<td>6.03</td>
<td>58</td>
</tr>
</tbody>
</table>

3.3 Material Properties

Coupon tests were carried out on six samples using an INSTRON universal testing machine shown in Figure 17 in order to get the yield stress and to identify the complete elastic-plastic behavior of the material. The coupon specimens were taken from the longitudinal direction for both the web and flange of the column. The dimensions of the coupon test specimens are shown in Figure 18 below.
Figure 17: Coupon Test Setup

Figure 18: Coupon Test Dimensions

<table>
<thead>
<tr>
<th>W₁ (mm)</th>
<th>L₁ (mm)</th>
<th>W₂ (mm)</th>
<th>L₂ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>100</td>
<td>20</td>
<td>89</td>
</tr>
</tbody>
</table>

Figure 19 shows the tested coupon samples after rupture. All samples attained a certain amount of necking prior to rupture. Failure took place in the gage length of all samples.

Figure 19: Tested Coupon Samples

The stress-strain curves of the coupon test samples taken from the flange and the web of the UC100x100x17 section were plotted in separate graphs as shown in Figure 20.
3.4 Test Setup

The prismatic and pretwisted steel columns were tested under pure axial compression using a universal testing machine (UTM) with a capacity of up to 1200 kN. Prior to testing, the centroid of the column was placed exactly under the centroid of the circular shaft through which the load is transferred to the column as shown in Figure 21. To ensure uniform distribution of the axial compression load on the cross-sectional area of the steel column, base plates were welded at both ends. Strain gauges were mounted around mid-height of the samples. In general, four strain-gauges were mounted on the web and top flange of the samples, although some specimens had six strain gauges installed on the top and bottom flanges as well as on the web. A few samples had only two strain-gauges fixed in the web. LVDTs were also attached to both web sides at mid-height of the column to record the lateral deflection experienced by the samples during buckling. The tested columns are all presumed to be under pinned-pinned boundary conditions.

The test setup shown in Figure 21 was used for the test specimens of 1 meter and 1.5 meter in length. However, additional safety precautions had to be taken for the 2 meter steel columns to avoid slippage and tripping off due to instability of the column during buckling. Thus, a special steel frame was manufactured to interlock the 2 meter steel column in-place while applying the axial compression load as shown in Figure 22.
3.5 Results and Discussion:

3.5.1 Axial compression

The steel columns were tested under pure axial compression at a loading rate of 2 mm/min using a 1200 kN capacity UTM. The column chart shown in Figure 23 gives a summary of critical buckling loads achieved for each of the tested columns. No consistent trend could be drawn from the fluctuating nature of the ultimate buckling capacities attained by the pretwisted columns as seen in the column chart of Figure 23. The pretwisted columns CL1T30 and CL1.5T30 carried the highest critical loads, showing buckling capacity improvement of around 10.7% and 6.3% respectively in the two length groups assessed separately (i.e. the 1 meter and 1.5 meter group of columns). However, CL1T45 and CL1.5T45 showed relatively lower
buckling capacities. Nevertheless, CL1T45 had its buckling capacity drop lower than $P_o$ of the prismatic CL1T0 exhibiting buckling capacity diminution of around 4.1%. On the other hand, CL1.5T45 possessed a lower buckling capacity (i.e. as compared to the earlier CL1.5T30) but remained higher than the reference $P_o$ of the prismatic CL1.5T0, demonstrating a minor buckling improvement of around 2.7%. The two-meter group of columns showed a gradual increase in the buckling capacities of the columns with the increasing angles of twist up to 60°. In this 2 meter length group, CL2T60 showed the highest buckling improvement of around 10.1% as compared to the reference $P_o$ of the untwisted CL2T0.

![Figure 23: Buckling load of experimentally tested prismatic and pretwisted columns](image)

The UTM-machine has built-in gauges which provided the data required to plot the axial load versus axial displacement graphs shown in Figure 24. The effect of pretwisting on the buckling capacity and stiffness of the tested specimens was investigated for each length group. Further investigation of Figure 24 illustrated that the stiffness of the pretwisted columns tested under pure axial compression is almost independent of the applied angles of twist. In other words, the twisted and untwisted steel columns shared the same slope of axial buckling load versus displacement up until the maximum critical load is reached for the relatively short length groups considered. Moreover, the graphs in Figure 24 indicate that stiffness of the columns decreased relatively with higher length groups; the 2 meter columns showed slightly lower stiffness than the 1.5 meter group which in turn exhibited relatively lower stiffness than the 1 meter columns.
3.5.2 Buckling modes

The dominant buckling mode shape for all prismatic and pretwisted steel columns was flexural buckling around the weak axes. Additionally, material rupture/yielding was noticed with the 1 meter and 1.5 meter groups. Furthermore, large plastic deformations (i.e. post-buckling behavior) took place, displaying signs of excessive distortion mainly in the flanges of the prismatic CL1T0 and the pretwisted CL1T30 (see Figure 25(a)). The pretwisted CL1T45, however, did not experience such large plastic deformations, as shown in Figure 25(b).

![Figure 24: Load-deflection for tested columns](image)

(a) L=1-m

(b) L= 1.5-m

(c) L=2-m

Figure 24: Load-deflection for tested columns (a) 1-meter tested columns, (b) 1.5-meter tested columns, (c) 2-meter tested columns

![Figure 25: Buckling mode shapes for 1-meter length columns](image)

(a) CL1T0 and CL1T30, (b) CL1T45
For the 1.5 meter and 2 meter tested columns, no signs of flange distortions were displayed with any of the columns, as shown in Figure 26 and Figure 27.

Figure 26: Buckling mode shapes of the 1.5-meter tested columns, (a) CL1.5T0, (b) CL1.5T20, (c) CL1.5T35, (d) CL1.5T45.

Figure 27: Buckling mode shapes for the tested 2-meter columns, (a) CL2T0, (b) CL2T30 and (c) CL2T60

3.5.3 Load versus strain plots

With an average material yield strain of 0.00146 mm/mm as per Figure 20, it was found that the tested prismatic and pretwisted steel columns of the 1 meter and 1.5 meter length groups experienced material yielding prior to buckling as shown in Figure 28 and Figure 29, respectively. The 2 meter column group, however, showed relatively low strain values, implying that the critical load was reached before witnessing signs of material yielding as illustrated in Figure 30.
Figure 28: Load-strain graphs for 1-meter columns
Figure 29: Load vs. strain values for 1.5-meter columns
As mentioned earlier, two LVDTs were installed at both sides of the tested members and fixed at the mid-height of the column to record the lateral deflection until the critical load was reached. Figure 31 shows the lateral displacement recorded in the tested columns. Upon closer examination of the graphs in Figure 31, the lateral deflection at failure ranged from 0.13 mm to 5 mm for the members considered. One of the sources of errors encountered with the readings shown below is that, in some cases the LVDT moved constantly during buckling. Consequently, the plots considered in Figure 31 are not fully reliable.
Figure 31: Lateral deflection recorded by LVDT for tested columns (a) 1m-columns, (b) 1.5m-columns, (c) 2m-columns
Chapter 4: Nonlinear Finite Element Analysis of Buckling of Pretwisted Columns

Nonlinear finite element (FE) modeling was conducted using the commercial software ABAQUS to simulate the buckling capacity of pretwisted UC100x100x17 columns. The FE model was first verified against experimental results and compared with AISC code and then expanded to carry out a parametric study in an attempt to investigate the effect of different slenderness ratios and angles of twists on the buckling improvement of columns.

4.1 FE Model Description

A three-dimensional shell part was used to model the steel columns involved, and a pitch describing the angle of twist was applied for each model prior to its extrusion in the longitudinal direction as seen in Figure 32. For application of the boundary conditions and loading pattern, two analytical rigid plates were attached to every column as shown in Figure 33 to simulate the same end conditions as the experiments discussed earlier. For fixed-ended, rotation was restricted at both ends and translation was only allowed in the direction of axial loading (i.e. along the longitudinal direction of the column) in one end only, while for pinned-ended simulations, the ends were free to rotate, and translation in the z-direction was allowed at one end only for application of the load. 4-node three-dimensional shell elements of S4R type and a general purpose linear element with four degrees of freedom was chosen for the mesh applied with every simulated member.

Figure 32: Samples of pretwisted geometries for the simulated experimentally tested specimens
4.2 Material Properties

The engineering stress-strain curve shown in Figure 20 was utilized to define the mechanical properties for the steel material in ABAQUS. Linear elastic behavior was defined with a modulus of elasticity $E = 200$ Gpa and Poisson’s ratio $\nu = 0.3$. The plastic deformation was defined by considering the true stress-true strain values which were calculated from the engineering values using the following definitions:

\[
\sigma_{\text{true}} = \sigma_{\text{Eng}} (1 + \varepsilon_{\text{plEng}}) \tag{9}
\]

\[
\varepsilon_{\text{pltrue}} = \ln(1 + \varepsilon_{\text{plEng}}) \tag{10}
\]

\[
\varepsilon_{\text{pl}} = (\varepsilon_{\text{total}} - \frac{\sigma_{\text{true}}}{E}) \tag{11}
\]

4.3 Buckling Analysis

Nonlinear buckling analysis was conducted using two different procedures that are available in ABAQUS; displacement-based General Static analysis and Riks analysis. A brief description of each procedure is discussed next.

4.3.1 Riks analysis

Riks is a load-deflection analysis that simultaneously solves for both load and displacement while keeping the load magnitude as an additional unknown. The arc-length factor is a significant characteristic of this step, which can be user defined although it is usually set to 1. Furthermore, it is also a static-stress step which is based on 1% extrapolation of strain values. Riks analysis is usually a continuation of the Eigen-value buckling analysis, whose loads are typically referred to as dead loads, while the loads defined within the Riks step are the reference loads. Dead loads and reference loads are linked through an equation to give the total load as shown in equation (12):
\[ P_{total} = P_0 + \lambda (P_{ref} - P_0) \]  \hspace{1cm} (12)

where \( P_0 \) is the dead load, \( P_{ref} \) is the reference load and \( \lambda \) is the load proportionality factor that is determined as part of the solution.

Incrementation is used in Riks analysis by implementing Newton's method. The initial value of incrementation is calculated based on the following equation (13):
\[ \Delta \lambda_{in} = \frac{\Delta l_{in}}{l_{\text{period}}} \]  \hspace{1cm} (13)

where \( l_{\text{period}} \) is the value of the arc length factor input initially and is usually set to 1.0 as mentioned earlier. \( \Delta l_{in} \) is the initial value of the increment used, and \( \Delta \lambda_{in} \) is the initial load proportionality factor used in the first iteration of the Riks analysis step. The load proportionality factor is then automatically calculated for the rest of the iterations. To end a Riks analysis step, \( \lambda_{\text{end}} \) or maximum displacement desired could be specified [43].

4.3.2 Displacement-based general static analysis

In this type of analysis, the load is applied in the form of axial displacement at one of the support ends where translation in the axial direction is allowed while the other end is not movable. A value of imperfection between L/1000 to L/2000 was considered for all columns by inducing an equivalent lateral displacement at the column mid-height as a separate step before applying the axial displacement. The reaction at the other end was recorded to measure the critical load at which buckling starts.

4.4 Mesh Sensitivity Analysis

Various mesh configurations were examined in an attempt to settle on the appropriate mesh that grants the most accurate results and ensures utilization of the minimum computational time. The conducted mesh sensitivity analysis considered three different mesh configurations as shown in Figure 34: Mesh 1, Mesh 2 and Mesh 3 with a global element size of 50 mm, 25 mm and 10 mm, respectively. The three meshes were implemented to simulate the buckling capacity of the experimentally tested prismatic CL2T0 and the pretwisted CL1.5T20 columns. Mesh 1 was found to highly underestimate the capacity of the sample when compared to the experimental
test result achieved in the lab, as demonstrated by the column chart shown in Figure 35. Mesh 2 and Mesh 3 showed an overall variation less than 0.5% for both considered columns when compared to the experimental results, whereas, Mesh 1 exceeded 4% for CL1.5T20 and reached around 12% when used with CL2T0. Nevertheless, Mesh 2 was chosen in the present FE analysis and the expanded parametric analysis, since it illustrated an adequate combination of precision and reduced computational time.

![Meshes](image)

**Figure 34: Three different meshes used in the mesh sensitivity analysis**

![Column Chart](image)

**Figure 35: Column chart comparisons of the critical loads between nonlinear FE analysis and experimental test results using three different mesh configurations.**

### 4.5 FE Model Verification

The finite element model was then verified against the results obtained from the experimental tests presented in Chapter 3 and also compared with the AISC code equations. The four main criteria for verifying the FE model were the maximum buckling capacity carried by each sample, failure mode shapes, the axial load versus deflection values and the load-strain diagrams plotted at mid-height of the column.
4.5.1 Maximum buckling capacity

The FE models and experimental results were first compared against the AISC code equations for the prismatic columns as shown in Figure 36. However, great variation between the verified models and the AISC code was witnessed, and was most obvious with the 2 meter length columns. Furthermore, this variation could be explained in terms of the supporting conditions of the tested columns. In other words, the actual tested columns were not fully pinned-ended as perceived by the AISC provisions. Consequently, the experimentally tested columns exhibited lower effective lengths and higher buckling capacities as compared to the AISC code calculations. Moreover, the experimental test results were considered as the main reference for FE model verification since the code is believed to be more conservative than real-life situations. The results obtained from general static analysis exhibited a great level of agreement with the values achieved using Riks analysis in terms of the maximum buckling capacity carried by each simulated column as seen in Figure 36. Consequently, only Riks analysis was considered for the purpose of this thesis. The simulated columns shared almost the same buckling capacities of the actual tested columns as shown in Figure 37.

Figure 36: Column chart showing comparison between FE analysis, experimental results and AISC code values for prismatic columns
4.5.2 Axial load versus displacement

Further assessment of the axial load-displacement graphs shown in Figure 38 proves an adequate level of compatibility between the FE results and the experimental test values for all specimens considered. Finite element results showed stiffer response in most cases as compared to the experimental behavior, which is usually anticipated with FE analysis. It can also be observed that the results from Riks and general static analyses almost overlap for each of the prismatic columns shown in Figure 38. The axial load versus displacement graphs of the remaining pretwisted columns are presented in the Appendix.

4.5.3 Buckling mode shapes

General static and Riks analyses gave similar buckling mode shapes to those perceived from the experimental test for the prismatic steel columns CL1T0, CL1.5T0 and CL2T0 as shown in Figure 39, Figure 40 and Figure 41 respectively. As discussed previously the main mode of failure was flexural buckling. Local distortions in the flanges of shorter columns appeared with increasing plastic deformation, which was perceived as post-buckling behavior. Similar compatibility levels between the modeled and experimentally tested pretwisted columns tested experimentally as shown in Figure 42 for selected samples.
Figure 38: Axial load versus displacement of actual and simulated prismatic columns

Figure 39: Buckling mode shapes of CL1T0 (a) displacement control, (b) Riks analysis, (c) experimental test

Figure 40: Buckling mode shapes of CL1.5T0 (a) displacement control, (b) Riks analysis, (c) experimental test
4.5.4 Axial load versus strain results

The load-strain plots extracted from ABAQUS show relatively reasonable agreement with the values obtained via experimental testing. Evaluation of the load-strain curves plotted in Figure 43, Figure 44 and Figure 45 further illustrates that the simulated columns exhibited similar behavior to the actual tested column under pure axial compression; the simulated and experimentally tested 1 meter and 1.5 meter groups of columns showed similar amounts of material yielding prior to buckling, while the modeled and actual 2 meter columns witnessed failure due to buckling before material yielding took place.
Figure 43: Load-strain plots for simulated versus experimentally tested 1-meter columns
Figure 44: Load-strain plots for simulated versus experimentally tested 1.5-meter columns.
Figure 45: Load-strain graphs for simulated versus experimentally tested 2-meter columns

4.6 Expanded Parametric Study

A parametric study was conducted to extend the previously verified FE model in order to include a wider range of pretwisted angles and lengths. Fixed-ended boundary conditions were also included in the expanded investigation along with the pinned-ended support condition to help inspect the effect of various boundary conditions on the axial buckling capacity of pretwisted columns. The range of lengths considered to build up on the previously modeled columns was 3 meter to 6 meter, and the range of pretwisting angles used started from the default 0° to 180° with
increments of 30°. Only FE results conducted using the Riks analysis procedure will be presented in this chapter.

4.6.1 Fixed-ended pretwisted columns

For the fixed-ended boundary condition, yielding occurred prior to buckling at all twisting angles for the case of 3 meter length columns, and at a range of angles from 60° to 180° for the case of 4 meter length columns. However, simulations of 5 and 6 meter length columns showed that buckling took place before yielding at all twisting angles considered. Figure 46 shows the contours of von Mises stresses for selected fixed-ended FE models prior to reaching the critical load, keeping in mind that the average yield stress for the steel material is around 292 Mpa.

Figure 47 presents a summary of the maximum critical load achieved by the several pretwisted and prismatic simulated columns under fixed-ended conditions versus their respective slenderness ratios, which mainly vary by changing the length used since all FE models shared common cross-sectional properties (i.e. UC100x100x17). A consistent increasing trend of the critical load with increasing pretwisting angles is a characteristic of the fixed-ended boundary condition, but is most apparent with higher slenderness ratios that witnessed failure mainly due to elastic buckling as shown in Figure 47.

Figure 48 shows the buckling capacity improvement versus different lengths and angles of the UC100x100x17 under fixed-ended boundary conditions. Further consideration of the graph shown below illustrates that improvement in buckling capacity was higher at greater lengths. Furthermore, a maximum enhancement of around 55% was encountered, with the highest length, 6 meter columns, failing mainly due to elastic buckling. Shorter column lengths, however, showed relatively low buckling improvement —not exceeding 10.6% for 3 meter columns and 19.3% for the 4 meter columns. The intermediate column length, 5 meters, witnessed an overall improvement of around 34% at the optimum angle of pretwist. Nevertheless, the optimum angle was 150°. Regardless of the length of the column, the buckling capacity then decreases relatively at $\phi=180°$, keeping the critical buckling load greater than the reference $P_o$. 

68
Figure 46: Contours of von Mises stresses prior to buckling for selected fixed-ended columns with different lengths and pretwisting angles
4.6.2 Pinned-ended pretwisted columns

For pinned-ended simulations, material yielding initiated the mode of failure prior to buckling at all pretwisted angles for the case of 1 meter length and 1.5 meter length columns. For the higher lengths (i.e. 2 meter to 5 meter columns), however, the maximum critical load was reached before yielding. Figure 49 shows the contours of von Mises stresses for selected pinned-pinned columns with different lengths and pretwisting angles before buckling takes place.

The maximum critical load carried by each of the simulated pinned-ended columns was plotted against the slenderness ratio of each FE model considered as shown in Figure 50. It was deduced that at higher slenderness ratios where columns failed mainly due to elastic buckling, a more consistent trend was perceived in
opposition to the conflicting trend associated with lower slenderness ratios failing mainly due to inelastic buckling. An overall slight improvement was witnessed with pinned-ended boundary conditions, which was confirmed by the buckling improvement versus applied angles of the pretwist graph shown in Figure 51.

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Further assessment of Figures 50 and 51 proves that improvement in critical load carried by the modeled 1 meter and 1.5 meter columns did not exceed 10% and 8.5% respectively, at the optimum angle, $\phi=30^\circ$. Nevertheless, the same figure shows relatively low levels of improvement for the higher length/slenderness ratios as well; specifically, a maximum of 11% was witnessed with the 2 meter modeled columns, and 15% was the maximum enhancement for 3 meter, 4 meter and 5 meter simulations respectively.
4.7 Effect of Pretwisting on the Least Moment of Inertia of Column

Correlations between the applied angle of twist and the parameters that control the elastic and inelastic buckling capacities as per AISC code equations (3) & (4), respectively, can be further investigated based on the extended FE simulation in an attempt to incorporate the pretwisting effect in these equations. By rearranging the AISC code equations, the moment of inertia can be defined for inelastic and elastic buckling as follows:

\[
I = \frac{A + K + L}{\pi^2E} f_y \sqrt{f_y \left( \ln 0.65 \frac{f_y}{f_y} \right)}
\]  

(14)
Elastic buckling \[
I = \frac{A + K + L}{\pi^2 E} \sqrt{\frac{\gamma_{cr}}{f_y}}
\] (15)

Upon pretwisting, it becomes harder to locate the weak and strong axes of a column due to the imposed geometric orientation by the linearly varying (i.e. uniform) angle of rotation, applied along the centroidal axis of the member. Pretwisting is believed to strengthen the weak axis of the column and weaken the strong axis in the same sense.

Figure 52 shows that the moment of inertia followed the same trend of the buckling load, where it kept consistently increasing until it reached the optimum angle at 150\(^\circ\), after which it dropped at \(\phi = 180\)^\(^\circ\) but remained higher than the reference \(I_0\) at \(\phi=0\)^\(^\circ\).

![Figure 52: Moment of inertia versus the applied angles of twist for fixed-ended columns](image)

A slight variation of the I-values between the four different column lengths can be noticed from the above figure, particularly at angles of twists beyond 60\(^\circ\). It is quite clear that pretwisting helped enhance the moment of inertia in the weak direction without exceeding the moment of inertia in the strong axis. Moreover, the expected reduction in the effective length of the fixed-ended pretwisted column could have also signified the improvement in the buckling capacity of the member under axial compression. Under fixed-ended conditions, specifically, the combined effect of the reduced effective length and the restricted rotation at the two ends of the column helped maximize the critical load of the section. Moreover, the most obvious effect of
pretwisting is most apparent in the mid-portion of the column, where buckling under fixed-ended conditions takes place (i.e. approximately around 0.25-0.3L away from the end supports where no rotation is allowed) which further enhanced its buckling capacity. More experiments and FE simulation of pretwisted columns considering a wide range of cross-sections, lengths and angles of twist are deemed necessary before arriving at a robust relationship between the buckling parameters and pretwisting angles.
Chapter 5: Conclusion

5.1 Summary and Conclusions

In this research, the buckling capacities of prismatic and pretwisted steel columns under the application of a concentric axial compressive load were experimentally and numerically investigated. The expected effect of pretwisting was to improve the buckling capacity of the compression members. Consequently, permanent pretwisting is to be perceived as an effective technique to increase the strength of any steel compression member.

Initially, an elastic buckling analysis was conducted as a preliminary study to investigate the effect of pretwisting on the buckling response of selected universal columns (UC). In this regard, elastic buckling of pretwisted fixed-ended steel columns was studied using linear perturbation analysis through finite element modeling. Universal columns with three different cross-sections of various lengths, initially twisted at angles from 0°-180°, were analyzed in this preliminary study. Results obtained via a linear perturbation analysis showed that there was a significant improvement in the critical buckling capacity for different slenderness ratios. A sharp increase in buckling capacity up to 90% as compared to non-twisted sections was demonstrated in most sections. However, the effect of various column lengths on the buckling improvement for a given UC section was insignificant.

Elastic buckling capacity of pretwisted UC sections with pinned-pinned end conditions was also investigated. The improvement in the axial capacity was found to be very small as compared to its fixed-ended counterparts. Only a 20% maximum increase in the buckling capacity was achieved for the three UC sections used in this preliminary study.

Experimental tests were conducted next to investigate the buckling capacity and improvement of pretwisted columns using a 1200 kN capacity UTM. The experimental program consisted of ten UC100x100x17 sections (UC100x100x17 was one of the sections used in the preliminary study) with three lengths: 1 meter, 1.5 meter and 2 meter, and a range of pretwisting angles between 0° and 60° under pinned-ended boundary conditions. The experimental test results were then used to develop and verify a nonlinear FE model capable of capturing the inelastic buckling of pretwisted columns using the commercial software ABAQUS. The nonlinear
numerical analysis was then extended to conduct a parametric study to cover the effect of pretwisting on a wider range of slenderness ratios by changing the column length and boundary conditions while keeping the cross-sectional dimensions constant.

Upon close examination of the experimental and extended FE results, it can be deduced that pretwisting seemed to have the least effect on pinned-ended columns failing mainly due to inelastic buckling. However, a more constantly increasing trend of the columns’ buckling capacities was observed with the higher length columns under pinned-ended boundary conditions, whose mode of failure shifted from inelastic to elastic buckling. Nevertheless, the maximum improvement in buckling capacity did not exceed 15% for the highest slenderness ratio encountered with pinned-ended conditions. This is mainly due to the allowed rotation at the two ends of the columns, which in turn reduced the effect of pretwisting on the overall buckling capacity of the simulated members. On the other hand, fixed-ended boundary conditions helped improve the buckling capacity of short columns whose main mode of failure was inelastic buckling. Considerable improvement exceeding 10% was witnessed with the fixed-ended short pretwisted columns. Fixed-ended boundary conditions also restrict rotation at the supporting ends of the column, which further helped increase the elastic stability of the column and its resistance to flexural buckling. Nonetheless, fixed-ended steel columns failing by elastic buckling witnessed the greatest positive effect of pretwisting. Furthermore, elastic buckling of slender columns is mainly controlled by the moment of inertia in the weak axis which gets strengthened upon applying higher angles of pretwist until the optimum angle is reached. Last but not least, this outcome further emphasizes the results obtained from the preliminary study considering the elastic buckling of columns.

5.2 Recommendations

The following suggestions could be implemented in order to expand this research and get better predictions of the effect of pretwisting on the buckling capacity of structural steel columns.

• Investigate a wider range of slenderness ratios by considering more wide-flanged sections.
• Consider several types of steel materials with different yield stresses to expand the range of elastic and inelastic buckling of structural columns.

• Develop multiple regression analyses to incorporate the applied pretwist into the buckling equations in the AISC code.
References


Appendix

Plots of Axial-Load Versus Displacement for the experimentally tested and simulated pretwisted columns are shown in the Appendix.

Figure 53: Axial-load versus displacement graph for actual and simulated CL1T30

Figure 54: Axial-load versus displacement graph for actual and simulated CL1T45
**Figure 55:** Axial-load versus displacement graph for actual and simulated CL1.5T20

**Figure 56:** Axial-load versus displacement graph for actual and simulated CL1.5T30
Figure 57: Axial-load versus displacement graph for actual and simulated CL1.5T45

Figure 58: Axial-load versus displacement graph for actual and simulated CL2T30
Figure 59: Axial-load versus displacement graph for actual and simulated CL2T60
Vita

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