OPTIMIZATION OF EXCESS OF LOSS REINSURANCE STRUCTURE

by

Mai Muhtaseb

A Thesis Presented to the Faculty of the
American University of Sharjah
College of Engineering
in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science in
Engineering Systems Management

Sharjah, United Arab Emirates

January 2016

Approval Signatures

We, the undersigned, approve the Master's Thesis of Mai Muhtaseb.

Thesis Title: Optimization of Excess of Loss Reinsurance Structure

Signature	**Date of Signature** (dd/mm/yyyy)
___________________________	_______________

Dr. Malick Ndiaye
Associate Professor, Engineering Systems Management Graduate Program
Thesis Advisor

___________________________ _______________

Mr. Shivash Bhagaloo
Consulting Actuary, Lux Actuaries & Consultants
Thesis Co-Advisor

___________________________ _______________

Dr. Hazim El-Baz
Associate Professor, Engineering Systems Management Graduate Program
Thesis Committee Member

___________________________ _______________

Dr. Daniel Simonet
Associate Professor, Department of Management
Thesis Committee Member

___________________________ _______________

Dr. Moncer Hariga
Director, Engineering Systems Management Graduate Program

___________________________ _______________

Dr. Mohamed El Tarhuni
Associate Dean, College of Engineering

___________________________ _______________

Dr. Leland Blank
Dean, College of Engineering

___________________________ _______________

Dr. Khaled Assaleh
Interim Vice Provost for Research and Graduate Studies

Acknowledgments

In the name of Allah, the most gracious, the most merciful.

I am grateful for Allah for empowering me with the strength, patience, perseverance and determination to pursue and achieve this research.

I would like to thank Dr. Malick for his continuous support and patience throughout the journey. It was an exciting learning journey; and without his support and encouragement to keep going regardless of the challenges and frustrations at times, the end result wouldn't have been successfully achieved.

I would also like to acknowledge and express my gratitude to Mr. Shivash for his generous time and support in guiding me to understand the principles of the topic, and the means to drive it to a valuable solution to the industry. His guidance and support played a primary role in my ability to absorb the topic and complete the research effectively. Extended thanks to the committee members, Dr. Hazim El Baz and Dr. Daniel Simonet, and Dr. Moncer Hariga, the Engineering Systems Management Graduate Program Director.

Dedication

This thesis is dedicated to my backbone support, my family.

To my parents who always encouraged me to accept nothing other than being distinguished and the best. To my Mother who kept pushing me towards success regardless of any difficulties faced in the journey. I am who I am today because of you, Mom and Dad.

To all my dear friends and relatives who believed in me and my capabilities, shared all my emotional ups and downs, and supported me with their love and care.

Abstract

In the current practice in the region, before purchasing a reinsurance contract, small to medium insurance companies rarely conduct internal analysis of their data and experiences in order to evaluate and achieve optimal reinsurance arrangements and contracts. Most companies settle their reinsurance agreements through reinsurance intermediary, broker, who acts as the link of communication, negotiation and settlement between both the reinsurers and the ceding insurer. Alternatively, the reinsurance companies or intermediaries evaluate and analyze the insurer's historical losses and offer reinsurance agreement and proposal accordingly. Therefore, the proposed reinsurance structure is not necessarily the insurer's optimal arrangement. In this thesis, excess of loss reinsurance optimization models are developed in order to enable insurers to utilize user-friendly and efficient tools to evaluate the optimal reinsurance arrangement depending on financial requirements, and to gain better value of their reinsurance contracts. The models are developed to define the insurer's optimal reinsurance retention and ceding limits for two objectives; minimizing insurer's retention variance and maximizing insurer's return on capital. The model maximizing the return on capital resulted in more realistic optimization solutions of retention limits. A sensitivity analysis to evaluate the impact of the model's parameters on the return on capital was also conducted, and it was concluded that the impact of the insurer's retention limit on the return on capital was significantly small. Moreover, the defined capital and gross premium safety loading had a major impact on the behavior of the return on capital.

Search Terms: Insurance, Reinsurance, Optimization, Excess of Loss, Return on Capital

Table of Contents

Abstract 6

List of Figures 9

List of Tables 10

Glossary 11

Chapter 1: Introduction 12

1.1 Overview 12

1.1.1 Reinsurance principles. 13

1.1.2 Non-proportional reinsurance. 15

1.2 Problem Description 19

1.3 Research Objective 21

1.4 Research Significance 22

1.5 Research Methodology 22

1.6 Thesis Organization 23

Chapter 2: Literature Review 24

2.1 Insurer's Optimal Reinsurance Structure 24

2.2 Insurer and Reinsurer's Optimal Reinsurance Structure 27

2.3 Investments, Capital and Optimal Reinsurance Structure 28

2.4 Contribution to the Literature – Classification of Reinsurance Agreements 30

Chapter 3: Optimization Models 33

3.1 Optimization Models Assumptions 33

3.2 Optimization Models Parameters 34

3.3 Model 1 - Optimization of the Retention Limit (M), Minimizing Retained Claims Variance Given ROC Constraints – Unlimited Single Layer Treaty *(XL, M, 1, ∞)* 34

3.3.1 Optimization model overview. 34

3.3.2 Model formulation. 35

3.3.3 Model description. 36

3.4 Model 2 -Optimization of the Retention Limit (M), Minimizing Retained Claims Variance Given ROC Constraints – Limited Single Layer Treaty *(XL, M, 1, m)* 40

3.4.1 Optimization model overview. 40

3.4.2 Model formulation. 41

3.4.3 Model description. 42

3.5 Model 3 - Optimization of the Retention Limit (M), Maximizing Insurer's Return on Capital – Unlimited Single Layer Treaty *(XL, M,1, ∞)* 44

3.5.1 Optimization model overview. 44

3.5.2 Model formulation. 44

3.5.3 Model description. 45

3.6 Model 4 - Optimization of the Retention Limit (M), Maximizing Insurer's Return on Capital – Limited Single Layer Treaty *(XL, M, 1, m)* 46

3.6.1 Optimization model overview. 46

3.6.2 Model formulation. 46

3.6.3 Model description. 47

Chapter 4: Optimization Models Discussion of Results 50

4.1 Sensitivity Analysis 50

4.2 Optimization Models Results 52

Chapter 5: Conclusion and Recommendations 55

References 57

Vita 62

List of Figures

Figure 1: Insurance Chain – Premium, Risk and Claims Flow Chart [4] 14
Figure 2: Types of Reinsurance [5] 15
Figure 3: Limited Single Layer Reinsurance Treaty 18
Figure 4: Re-insurance Source Chain [11] 20
Figure 5: Unlimited Single Layer Reinsurance Treaty 35
Figure 6: Historical Claims Distribution Fitting 37
Figure 7: Simulated Claims' Data Sample 38
Figure 8: Optimization Model Parameters Definition 39
Figure 9: RiskOptimizer - Optimization Model Definition 39
Figure 10: Optimization Model Solution 40
Figure 11: Simulated Claims' Data and Limited Layer Claims' Split Sample 43

List of Tables

Table 1: Reinsurance Agreement Classification Notation....................................... 31
Table 2: Example of Reinsurance Agreement Notations....................................... 32
Table 3: Retained and Ceded Claims Distribution 42
Table 4: Model 2 Optimization Results 44
Table 5: Retained and Ceded Claims Distribution 47
Table 6: Model 4 Optimization Results 49
Table 7: Model Parameters' Dependencies 50
Table 8: Sensitivity Analysis - Impact of Gross Premium Risk Loading on ROC...... 51
Table 9: Sensitivity Analysis - Impact of U on ROC 51
Table 10: Sensitivity Analysis - Impact of M on ROC.. 52
Table 11: Optimization Models' Optimal Solutions 52

Glossary

Reinsurance Contract: A mutual agreement between insurance companies and third-party reinsurance companies.

Facultative Reinsurance: Insurers select individual and specific loss exposures to be covered by the reinsurer subject to reinsurer's cover approval or rejection.

Treaty Reinsurance: Reinsurance treaty covers all loss exposures covered by an insurer's portfolio or line of business and all individual losses that are covered under the reinsurance contract treaty without exceptions.

Proportional Reinsurance: Insurers share risks and premiums of issued policies with reinsurers.

Non-proportional Reinsurance: Insurers cede risks and liabilities of issued policies exceeding a determined limit of loss, excess amount, or also known as insurer's retention limit, in return of a reinsurance premium.

Quota Share Reinsurance: Type of proportional reinsurance where the insurer cedes an agreed percentage of losses to the reinsurer.

Surplus Reinsurance: Type of proportional reinsurance where the insurer cedes losses proportional to the reinsurer's share of total coverage limit

Stop Loss Reinsurance: Type of non-proportional reinsurance where the insurer cedes total/ aggregate claims in excess of predetermined amount.

Excess of Loss Reinsurance: Type of non-proportional reinsurance where the insurer cedes individual claims in excess of predetermined amount.

Return on Capital: Ratio that measures a company's profitability and return on the capital and investments contributed by shareholders.

Chapter 1: Introduction

This chapter introduces and provides an overview of general insurance and reinsurance principles, followed by the problem description. The research objective, its significance to the industry, and methodology are further discussed. The last section introduces the structure of the thesis.

1.1 Overview

In exchange of a premium payment from the insured, insurance companies provide their clients with financial compensation and protection against loss, damage, health incidents or death; thus transferring the risks from the insured to the insurer. Consequently, insurance companies are subject to significant risks of financial loss occurrences, depending on the severity and frequency associated with these risks and loss occurrences. Insurance policies, sometimes requested individually by the insured or mandated by laws, can be categorized into two main groups; general and life insurance. General insurance covers properties and liabilities, whereas life insurance insures people. Most common forms of life insurance are life, medical and pension insurance. However, most common types of general insurance include, but not limited to, motor, fire, engineering, workman compensation, marine and professional indemnity covers [1].

Different from most industries in the UAE such as construction, insurance industry has experienced massive growth after the economic crisis in 2009. The rapid and huge growth and advancement in medical insurance specifically contributed to the rapid growth of the insurance industry outlook. The report published in 2011, "UAE Insurance Market Forecast to 2012" reveals that insurance in the UAE is expected to experience rapid developments and growth, and is one of the fastest growing industries in the MENA region [2].

According to "UAE Insurance Report Q1 2013" [3], the growth in the healthcare industry and increase in demand of medical insurance are expected to continue for at least the next five years. One of the primary factors behind this strong development is the increasing population and number of expatriates in the country. Consequently, private healthcare sector market share and growth are on the rise; thus the country's healthcare spend. In addition, the countries' health regulatory authorities, such as Health Authority of Abu Dhabi, HAAD, have been announcing plans of new regulations, which influence the pricings and costs of healthcare services and delivery. Overrated healthcare services such as medical tests have been imposed a decrease of approximately 24%; whereas consultations services have been increased by a range of 15 – 25 %.

UAE Nationals are provided coverage of medical services by the government. Compulsory regulations of medical insurance have been recently imposed for expatriates in both Dubai and Abu Dhabi, and are expected to be expanded and implemented in remaining Emirates in the future. New regulations further mandate organizations to provide their employees with medical insurance cover. Consequently, increased costs of healthcare services have been shed away from providers and the public; therefore, reducing the effects of the costs' increase and inflation on the public and healthcare providers. These regulations also increased insurers' share of medical insurance services within the country; thus generating more premiums as well as increasing their risk exposure portfolio. In order to ensure that organizations conform to providing all their employees with medical coverage; regulatory authorities, HAAD, had imposed fines on organizations which fail to conform [3].

1.1.1 Reinsurance principles. In many situations, insurance companies are insured by reinsurance companies, through reinsurance contracts; a mutual agreement between insurance companies and third-party reinsurance companies. Through reinsurance contracts, insurers cede or share risks and liabilities of their issued policies with reinsurers in return of a reinsurance premium or ceding commission as shown in Figure 1. The insurer is solely obligated with fulfilling the responsibilities, services, and handling of claims and benefits to the insured in accordance with policy terms. Consequently, reinsurance does not alter or impact insurance terms of the policies between the insurer and insured, yet it is a form of financial protection to insurance companies against insurance risks and uncertainties of policy premiums insufficiency to cover incurred losses. Reinsurance contracts benefit insurance companies in various ways. Reinsurance can benefit them through stabilizing and reducing the variability of their loss experiences associated with risk exposures. Similarly, reinsurance enhances the insurer's financial stability and allows for an increase in its large line capacity, and maximum amount of limit of liability on a single loss exposure. Depending on the existence and nature of a reinsurance structure, the insurer's premiums, risks and losses are defined [1].

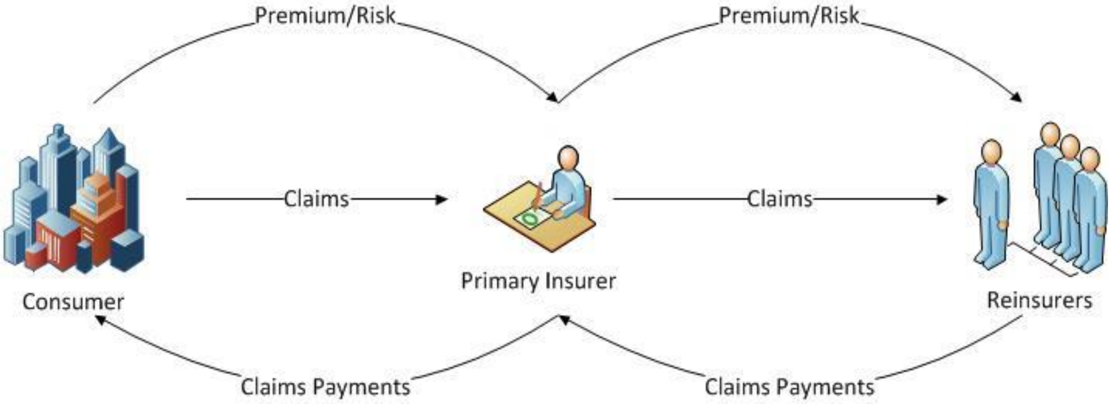

Figure 1: Insurance Chain – Premium, Risk and Claims Flow Chart [4]

Reinsurance contracts come in one of two forms: Facultative or Treaty Reinsurance. The difference between facultative and treaty reinsurance depends on loss exposures covered under the reinsurance contract. Reinsurance treaties cover all loss exposures covered by an insurer's portfolio or line of business and all individual losses are automatically covered and insured under reinsurance treaty, without any exceptions. On the other hand, insurers select individual and specific loss exposures to be covered under the facultative cover, and subject to reinsurer's cover approval or rejection. As shown in the Figure below, there are two main types of reinsurance; proportional or non-proportional; both of which fall under both facultative and treaty reinsurance. In proportional reinsurance, both parties, the insurer and the reinsurer, share proportions of the insurance premiums and losses either in the form of fixed percentages or fixed amounts. Most common forms of proportional reinsurance are quota share (QS) and surplus reinsurance (S). On the other hand, in non-proportional reinsurance, the reinsurer is liable of losses exceeding an insurer's determined limit of loss, excess amount or also known as insurer's retention limit. Within the contract, the reinsurer may also determine and specify the limits of maximum liability it is willing to be accounted for exceeding the excess of loss amount. Most common forms of non-proportional reinsurance are excess of loss (XL) and stop-loss (SL) reinsurance [1].

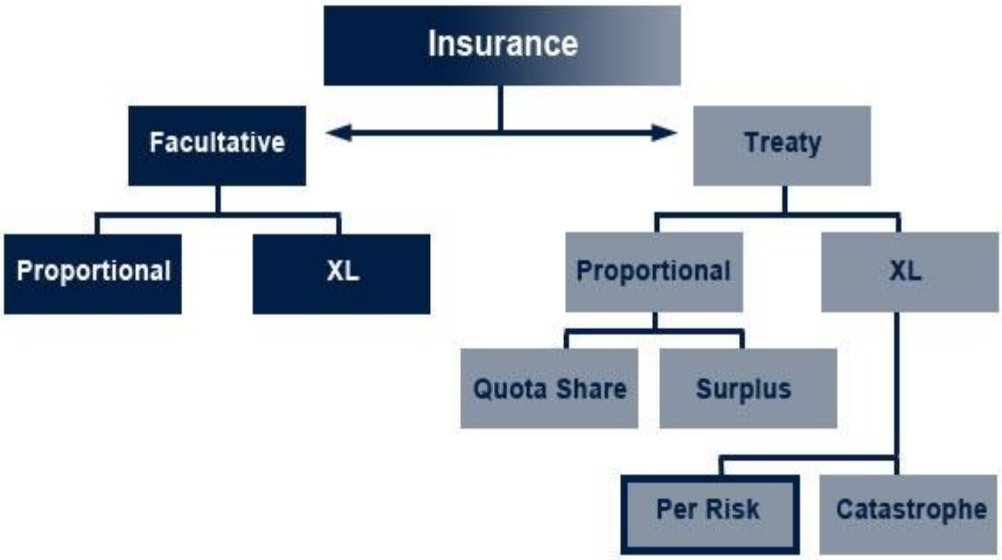

Figure 2: Types of Reinsurance [5]

1.1.2 Non-proportional reinsurance. In stop loss reinsurance, the loss ceded to the reinsurer is in excess of aggregate (total) claims incurred by the insurer. On the other hand, in excess of loss reinsurance, the reinsurer's share of losses is in excess of each individual loss; per risk (event) of each policy covered by the cedent, or per claim. The difference between both stop loss and excess of loss is further clarified below [1].

The notations used in the thesis are as follows;

N: Risk portfolio size – Number of Claims.

X_i: Non-negative Random Independent Individual Claim Incurred; $1 \leq i \leq N$.

X: Non-negative Random Total Loss Incurred – Aggregate Claims.

I: Amount of Loss retained by ceding insurer.

R: Amount of Loss ceded to reinsurer.

f(X): Probability density function of *X*.

F(X): Cumulative distribution function of *X*.

E(X): Expected value of losses incurred.

E(I): Expected value of losses retained.

E(R): Expected value of losses ceded.

M: Insurer's Retention limit – Deductible.

Such that:

$$X = \sum_{i=1}^{N} X_i$$

$$E(X_i) = \int_{0}^{\infty} x_i f(x_i) dx_i$$

$$E(X) = \sum_{i=1}^{\infty} E(X_i)$$

When an insurer purchases a reinsurance contract; *(X)* is a function of the loss retained by the insurer *(I)* and the loss ceded to the reinsurer *(R)*.

$$X = I + R$$

The definition of *(I)* and *(R)* depends on the type of non-proportional reinsurance contract in place. Losses in excess of the deductible amount *(M)* are ceded to the reinsurer in non-proportional reinsurance, stop loss and excess of loss contracts. The deductible/retention limit *(M)* is the insurer's maximum retention of claims before ceding to the reinsurer.

In ***stop loss reinsurance***, the loss retained and ceded respectively are:

$$I = \begin{cases} X, & X \leq M \\ M, & X > M \end{cases} = \min(X, M)$$

$$R = \begin{cases} 0, & X \leq M \\ X - M, & X > M \end{cases} = \max(0, X - M)$$

In ***excess of loss reinsurance***, the loss functions of the insurer and reinsurer, per claim or risk *(i)*, depending on whether evaluated per loss or per risk covered basis.

$$I(i) = \begin{cases} X_i, & X_i \leq M \\ M, & X_i > M \end{cases} = \min\ (X_i, M)$$

$$R(i) = \begin{cases} 0, & X_i \leq M \\ X_i - M, & X_i > M \end{cases} = \max\ (0, X_i - M)$$

Such that,

$$I = \Sigma\, I(i) \quad and \quad R = \Sigma R(i)$$

$$E(I) = \Sigma\, E(I(i)) \quad and \quad E(R) = \Sigma \mathrm{E}(R(i))$$

The expected value of total loss *(X)* is a function of the expected loss retained and the expected loss ceded

$$E(X) = E(I) + E\ (R)$$

The expected loss retained by the insurer and ceded to reinsurer respectively are expressed as

$$E(I) = E[\min(X, M)] = \int_0^M x\, f(x)dx + M\ [1 - F_x(M)]$$

$$E(R) = E[\max(X - M, 0)] = \int_M^\infty (x - M)\ f_x(x)dx = \int_0^\infty x\ f_x(x + M)dx$$

The purpose of acquiring a reinsurance contract is to reduce the impact and variability of claims incurred on an Insurer’s portfolio. Consequently, the sum of the variance of retained claims by the insurer and claims ceded to reinsurance shall be less than the variance of the total loss incurred; expressed as follows

$$\sigma_x^2 \geq \sigma_I^2 + \sigma_R^2$$

$$\sigma_I^2 = E(I^2) - E(I)^2$$

$$\sigma_I^2 = \int_0^M x^2\ f(x)dx + M^2[1 - F_x(M)] - (\int_0^M x\, f(x)dx + M\ [1 - F_x(M)])^2$$

$$\sigma_R^2 = E(R^2) - E(R)^2 = \int_M^\propto (x - M)^2 f_x(x)dx - (\int_M^\propto (x - M)\ f_x(x)dx)^2$$

Such that: σ_x^2 Variance of Claims Incurred

σ_I^2 Variance of Retained Claims

σ_R^2 Variance of Ceded Claims

Non-proportional reinsurance agreements may be either limited or unlimited. In unlimited excess of loss, the reinsurer covers all amounts of insurer's losses in excess of (M) without bounds, maximum amount limit. However, in limited excess of loss, by defining a maximum limit (L) the reinsurer limits its liability of losses per layer to the layer size (m). The size of each layer (m) defines the maximum amount of losses that a reinsurer is liable to the cedent in excess of the deductible (M) as per the Figure below. For losses exceeding the layer limit (X>L), the insurer retains the difference between the claim value and the layer size ceded to the reinsurer *(m)*.

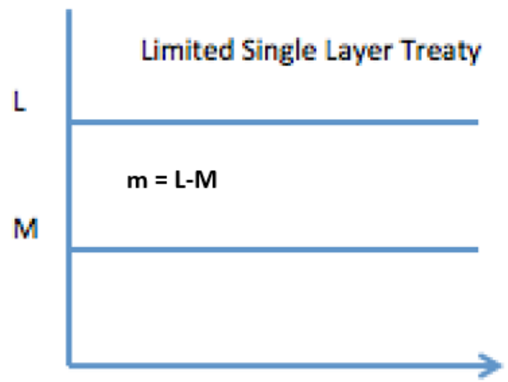

Figure 3: Limited Single Layer Reinsurance Treaty

Several insurance premium calculation principles have been proposed and implemented in the actuarial literature. Most commonly used general principles are: Expected value, variance, standard deviation, and exponential principles. The primary differences between each of the above common premium principles are related to how the risk is loaded. For instance, the expected value premium principle loads the risk – Expected loss (E(X))- proportionately by a number greater than or equal to one. The variance and standard deviation principles apply a risk proportional to the risk's variance and standard deviation respectively. Moreover, in the literature review context, some researchers attempted to use more convex premium principles such as the Wang, Dutch, semi-variance and semi deviation premium principles [6,7].

Assuming mean (expected value) premium calculation principle with a risk loading (θ) of 30% (20% and 10% expenses and profit respectively), the gross premium collected (P) is a function of the expected loss loaded by (θ) as follows

$$P = \sum_{i=1}^{\infty} P_i$$

$$P_i = (1 + \theta)\, E(X)$$

The net premium retained by the insurer (P_I) is expressed as the difference between the gross premiums insurer gains from its customers in exchange of insurance covers (P), and the reinsurance premium (P_R), the insurer incurs as cost of purchasing the reinsurance contract.

$$P = P_I + P_R$$

$$P_I = P - P_R$$

Insurance companies need to ensure business profitability, and satisfy solvency responsibility towards policyholders and shareholders. Capital supplied by shareholders is an essential requirement in the insurance industry. Hence, it should be maintained at a level that covers a company's risks and solvency [8]. In return of supplying the capital, shareholders and capital suppliers expect a targeted return on capital to be achieved by the end of a cycle and to deliver profitability of the business. Return on capital also evaluates the cost of capital to the company [9]. Assuming fixed insurer's capital (U), the insurer's return on capital (ROC) is expressed as

$$ROC = \frac{max\, \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

1.2 Problem Description

An insurance company mainly has two options for managing its risks and losses; full retention of premiums and losses without a reinsurance structure, or purchase a reinsurance contract, which limits its losses and risks by ceding portions of the risks to reinsurance companies.

Prior to the purchase of reinsurance contract, the current practice in the region, small to medium insurance companies rarely conduct internal analysis of their data and experiences in order to evaluate and achieve optimal reinsurance arrangements and contracts. Few insurance companies interact and deal directly with reinsurers. Most companies settle their reinsurance agreements through reinsurance intermediary, broker, which acts as the link of communication, negotiation and settlement between both the reinsurers and the ceding insurer. The broker provides different reinsurers with the ceding companies' historical loss and exposure data to be analyzed and evaluated in order to develop reinsurance terms accordingly. Through analyzing the insurer's data, reinsurance companies develop their proposed treaties, define layers and limit within each treaty, as well as the cost associated with the reinsurance agreement. The broker in turn communicates the proposed treaties and costs to the ceding company, and through negotiations, the reinsurance agreements are settled accordingly between both primary parties, ceding and reinsurance companies. In some cases, the intermediary conducts the analysis and accordingly negotiates the premiums and reinsurance agreements between the insurer and reinsurer [10].

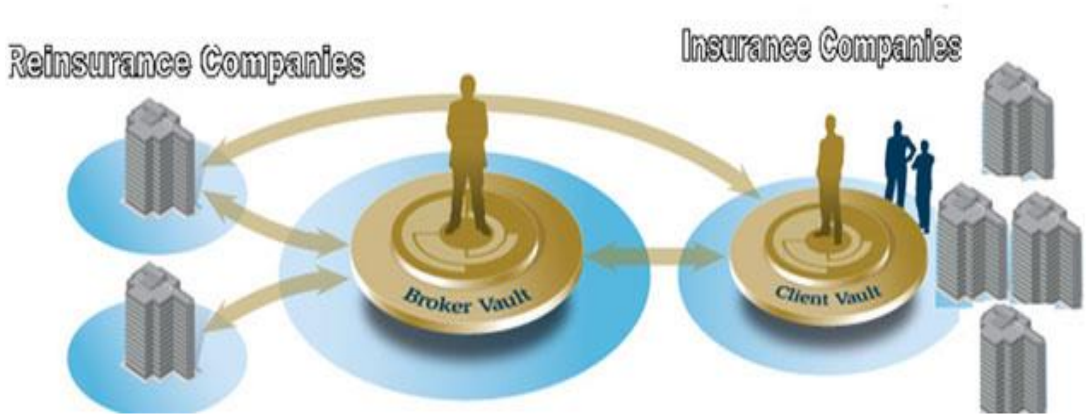

Figure 4: Re-insurance Source Chain [11]

Reinsurance companies offer their prices based on the analysis they have conducted on the insurer's experience and historical data. The reinsurance cost to the insurer includes an increase/loading to account for reinsurer's profitability or broker's commission. Prices offered from reinsurer's perspective and analysis, and agreed during the negotiation and agreement don't necessarily reflect the cedent's optimal reinsurance structure or cost [10].

Several studies related to insurance and reinsurance agreements have been conducted, especially on optimal reinsurance arrangements. Some studies explore the benefits of reinsurance agreements to insurers and approaches to defining the optimal reinsurance structure. However, the majority of the studies evaluate optimal reinsurance arrangements and contracts from a reinsurer's perspective or from a joint perspective.

Taking into account the different studies conducted related to optimal reinsurance contracts, the current common practice of reinsurance contracts' purchases in the regional insurance industry, a portfolio's past experiences and insurer's financial and risk requirements, financial models and optimization tools will be developed to allow insurance companies evaluate and estimate the optimal excess of loss reinsurance arrangement independently.

1.3 Research Objective

The objective of this research is to develop financial tools and models that allow insurance companies to evaluate the optimal excess of loss reinsurance structure that satisfies the risk appetite and financial requirements, by defining the optimal reinsurance contract limit, retention limit (M). Models developed evaluate the optimal arrangements of unlimited and limited single layer excess of loss reinsurance. Provided several assumptions and constraints, minimum insurance regulations, and financial ROC constraints/requirements, the optimal limit minimizes the insurer's variance of retained claims. The model that maximizes an insurer's return on capital is also evaluated, to get an understanding of the optimal approach to define the reinsurance agreement to be adopted. The focus of the study is on per claim excess of loss reinsurance structure; such that losses are ceded to the reinsurer per individual loss. A local insurance company's medical insurance historical claims, risk exposure and premiums will be used to build the model accordingly, assuming medical insurance is the company's primary line of business.

1.4 Research Significance

The research's primary contribution is to develop a simplified practical optimization model and financial tool which adds value to the insurance industry, specifically small to medium insurance companies. The objective is to design the model for insurance companies' use, in order to analyze and realize their optimal excess of loss reinsurance structure prior to settling their reinsurance agreements, while satisfying regulation's and shareholders' minimum financial requirements. Consequently, actual experience and exposure historical data of a local insurance company are utilized to develop and validate the model accordingly.

As noted from the literature review, most common research studies focused on stop-loss reinsurance optimality rather than excess of loss. Moreover, the literature review and studies related to minimizing the variance of insurer's retention implemented complicated mathematical and optimization approaches. Taking these observations into account, this approach will be tested and the optimization model will be formulated accordingly, in order to introduce to the industry a simplified approach of optimizing the excess of loss reinsurance structure. The models in the research are built on a user-friendly Risk simulation software, Palisade @Risk, which facilitates the application of the model in the industry.

1.5 Research Methodology

In order to address the research objective and develop the optimization model, the following steps will be followed.

Step 1: Researching literature review of reinsurance structure optimization

Step 2: Defining and formulating the optimization models as well as associated assumptions, parameters, and constraints

Step 3: Building and running the optimization models on @Risk

Step 4: Evaluating and comparing the optimization models' solutions

1.6 Thesis Organization

Chapter 1 introduces insurance and reinsurance principles, the problem description, research objective and methodology. Overview of the literature review related to reinsurance structure optimization is provided in Chapter 2. Chapter 3 presents and describes the model of optimization of insurer's retention limit (M), minimizing insurer's variance of retention given return on capital constraints for an unlimited single reinsurance layer. Followed by Chapter 4, which presents the optimization model for a limited single layer reinsurance structure, defining the optimal retention limit (M) and ceding limit (L), minimizing insurer's variance of retention given return on capital constraints. Chapter 5 presents the unlimited single layer optimization of insurer's retention limit, maximizing insurer's return on capital, and Chapter 6 addresses the insurer's retention and ceding limit optimization, maximizing return on capital for a limited single layer treaty. Chapter 7, which is the last one, evaluates the optimal solutions of the four-optimization models, summarizes the research and presents recommendations and future contribution.

Chapter 2: Literature Review

Optimal reinsurance has been frequently studied and addressed in the literature whether from the insurer's perspective or from the perspective and benefit of both the insurer and reinsurer. Researchers studied the optimum reinsurance strategies and types by varying different parameters such as; the method of premium calculation, the optimization objectives and the risk measures etc.

2.1 Insurer's Optimal Reinsurance Structure

Early studies primarily revolved around the optimization from an insurer's perspective. Some researchers adopted the expected premium principle to determine the optimal reinsurance. Borch [12] proved that stop loss reinsurance minimizes the variance of the retained losses. On the other hand, Arrow [13] proved that stop loss also maximizes an insurer's expected utility of terminal wealth. Furthermore, Cai et al. [14] have proved that, assuming increasing convex ceded loss function, and minimizing value at risk (VaR) and conditional tail expectation (CTE) risk measures of insurer's total cost, the optimum reinsurance strategy of ceded loss functions depends on the required level of confidence and safety loading of expected premium. The strategies that were taken into consideration in this study were stop loss, quota share and change loss. On the other hand, Balbas et al. [15] have attempted to find the optimum reinsurance problem and retention level that minimizes the risk of the total cost using general risk measures, including every deviation measure, expectation bounded risk measure, and most coherent and convex risk measures. They have concluded that, under the assumption of convex premium principles, quota share barely realizes optimization regardless of the risk function, whereas stop loss satisfies optimum conditions much more frequently. Extending their research results, they attempted to seek the "stable optimum retention level" which remains stable regardless of the risk measure used [16]. The conclusion of their research was that the optimum retention level of the stop-loss reinsurance is the stable optimum solution; thus it ensures a robust reinsurance plan regardless of the risk measure used for the optimization problem.

Extending Cai et al. [14] research, Chi and Tan [17] developed a simplified approach for finding the optimal reinsurance limits which minimize the VaR and conditional Value at Risk (CVaR) measures, by comparing stop loss, limited stop loss and truncated stop loss reinsurance. Stop loss reinsurance doesn't impose a maximum retention limit on the reinsurer, whereas the limited stop loss has a maximum limit of reinsurer liability and requires both the cedent and the insurer to be willing to pay more for larger losses. Meanwhile, truncated stop loss requires the cedent to transfer only moderate level losses to the reinsurer. They have studied the VaR model under two different constraints; *(1)* ceded and loss retained functions are increasing *(2)* loss retained function is increasing and left continuous.

Their study has proved that the VaR model is sensitive to the constraints set on ceded and retained loss functions, such that limited stop loss is optimum under the first constraint, whereas truncated stop loss is optimal under the second constraint. Nevertheless, the CVaR model proved to be robust such that regardless of the constraints, the stop loss reinsurance strategy remained optimum and consistent. Also, Tan and Weng [18] established an optimization problem which accounts for the insurer's optimization and tradeoff between risk and profitability. Subject to constrained insurer's profitability, the objective of the model was to derive the optimum reinsurance which minimizes the VaR of the insurer's net risk, using the expected value premium and assuming increasing and convex ceded loss functions. They have proved that the optimal reinsurance strategy would be quota share, pure stop loss, or a combination of stop loss and quota share, depending on the choice of level of confidence, safety loading of reinsurance premium, and expected insurer's profit.

Additionally, further studies have been conducted with respect to optimal reinsurance with premium calculation principles other than the expected value in the above mentioned studies. Kaluszka [19] derived the optimal reinsurance limits which minimize the cedent's variance of retained loss, using the mean- variance premium principles. In their research, Gajec and Zagrodny [20] proved that using symmetric and asymmetric general risk measures and the standard deviation premium principle, limited stop loss and change-loss reinsurance are the optimal strategies. Consequently, they derived the optimization problem of retention limits that minimize the insurer's risk measures. Furthermore, Kaluszka [21] proved that a combination of stop loss and quota share, or limited stop loss reinsurance are the most optimal strategies to minimize a cedent's convex measure of retained risk or maximize a utility function using different convex premium calculation methods. By setting a fixed reinsurance premium, the convex premium calculation methods he used in his research include exponential, p mean, semi deviation, semi-variance, Wang and Dutch. Extending their earlier research, Chi and Tan [22] have investigated optimization model, using the standard deviation and variance premium principles under VaR and CVaR. They proved that the layer reinsurance is the robust optimal solution for both VaR and CVaR, and defined the optimal layer's parameters.

On the other hand, in another study, assuming the reinsurance premium has distribution invariance, risk loading and stop loss ordering, Chi and Tan [23] excluded variance, standard deviation and Esscher principles from their study since they don't satisfy the stop loss ordering criterion. By using Wang and Dutch general premium principles and the constraint of increasing ceded and retained loss functions, they proved that layer reinsurance is always optimal for both VaR and CVaR models. Thus the optimization model defined both the retention level of the cedent and the maximum limit of reinsurance.

Jang [24] studied the factors that affect the catastrophe excess of loss reinsurance retentions and upper limits in the property liability line of business. Using two-stage least square regression, he identified and proved the hypotheses which support relationships among retentions, upper limits and co-insurance rates. The model's dependent variable was the reinsurance retention; whereas the independent variables were the upper limit co insurance rates, catastrophe exposures, catastrophe reinsurance price and other firm characteristics. Understanding the influence and relationship of these parameters on the reinsurance retentions help property-liability insurers in identifying and defining their reinsurance strategies. On the other hand, in further research, Kaluszka [25] proved that using mean-variance premium principles and minimizing the variance of the retained loss at a fixed expected cedent gain, quota share, excess of loss, or a combination of quota share and excess of loss are the optimal reinsurance strategies.

Alternatively, Centeno [26] defined the optimum excess of loss retention limits (m_1 & m_2) for two dependent risks using two objective functions; maximizing insurer's expected utility of wealth net of reinsurance with respect to an exponential utility function and maximizing the adjustment coefficient of retained business respectively. Both models were developed using the expected value premium principle along with the assumption of claims' bivariate passion distribution.

2.2 Insurer and Reinsurer's Optimal Reinsurance Structure

Other optimization problems were modeled taking into account the perspective and well-being of both the cedent and the reinsurer. Kaishev [27] addressed the optimization of excess of loss reinsurance under joint survival probability of parties, insurer and reinsurer, assuming Poisson claims distribution. Two optimization models with two risk measures were built, and each of which was implemented on two different constraints. The first optimization objective function was to maximize the joint survival probability, whereas the second model was minimizing the difference between the cedent's survival probability and the reinsurer's survival probability given cedent's survival probability). Similarly, the first constraint used in the model was fixing the premium proportion retained by the insurer and determining the optimum unlimited retention level accordingly. The second constraint was to fix the unlimited retention limit and determining the optimal reinsurance proportion accordingly. They have proved that second optimization model is better off to the insurer since the first model defines a higher optimum retention at a fixed premium proportion.

Kaishev and Dimitrova [28] further contributed to the above research by defining limited excess of loss retention level with optimal maximum limit of reinsurance liability and premium proportion respectively, which maximize the joint survival probability under the same constraints used in Kaishev's [27] research. They also added to their contribution by modeling dependent claims with copula functions, and observing the effect of varying the dependence parameters on the optimal solution. They have further extended their research and contribution by developing an efficiency frontier approach towards setting limiting and retention levels, which maximize the expected profits of both the reinsurer and cedent for a given level of joint survival probability [29]. The joint survival probability and the expected profit given joint survival were used as the risk measure and performance measure respectively. Assuming linear premium function as well as dependent and independent claim severities, the constraints used in the model included fixed premium distribution between the cedent and reinsurer and transfer the premium split into ratio of expected profits respectively. Optimal retention and limiting levels *(m)* and *(L);* which provide fair distribution of expected profits based on the premium allocation, were defined by the model accordingly.

Alternatively, Li [30] resorted to the expected value premium calculation method to develop optimal combinations of quota share and excess of loss reinsurance strategies under ruin-related optimization criteria. The developed models' optimization criteria were maximization of the joint survival probability of both parties and maximizing the lower bound of joint survival probability respectively. The optimal retentions of quota-share and excess-of-loss combined reinsurance under both optimization models have been defined, and the impact of economic and financial factors on the optimal retentions were explored; these factors are the influence of interest, dividends, commission, expense, and diffusion. Yusong and Jin [31] derived the optimum retention levels for both proportional and excess of loss reinsurance respectively which maximize the combination of rate of return of insurer and reinsurer correspondingly; such that they exceed amount of claim held by the reinsurer by a specific probability. These models were derived by assuming that investment funds follow log normal distribution, and using expectation premium principle for reinsurance premium.

2.3 Investments, Capital and Optimal Reinsurance Structure

On the other hand, many researches and studies have been conducted in the attempt to evaluate and assess the relationships between investments, capital and reinsurance. Some studies focused on optimizing the reinsurance structure taking into account investment related metric; whereas others studied optimal capital, risk and investment allocation considering reinsurance as a factor and optimization related parameter.

Many researchers examined the topic of optimal capital structure for insurance companies. Asmussen et al. [32] attempted to determine an insurer's optimal reinsurance structure that balances its expected profits and risks. Hence, they developed a model that determines the optimal excess of loss retention limit and dividend distribution policy, by maximizing the total expected discounted value of all paid out dividends. They have concluded that the excess of loss reinsurance is more optimal than proportional reinsurance, and it maximizes the insurer's adjustment coefficient in case of expected value premium calculation. In 2003, Froot [33] has developed a framework which analyzes the risk allocation, capital budgeting, and capital structure decisions facing both insurers and reinsurers. Using a three-factor model which maximizes the expected difference of dividend payments and the discounted costs of capital injection, the optimal amount of surplus capital held by a firm and the optimal allocation and the pricing of risky investments, underwriting, reinsurance and hedging opportunities are established. Moreover, a microeconomic financial model was developed by Laeven and Perotti [34], such that it designs the optimal solvency capital regulation as well as the optimal solvency capital that an insurance company should hold. The model analyzes and takes into account various aspects related to the economic trade-offs underlying the optimal design. Meng and Siu [35], investigated and developed a model for an insurer's optimal reinsurance, dividend and reinvestment strategies which maximize the difference between expected discounted dividends and expected discounted reinvestment until time of ruin. The model accounts for an excess of loss reinsurance and assumes that the insurer has both fixed and proportional costs. They developed a model and a solution for the optimal XL reinsurance as well an explicit expression of the optimal value's function, using an optimal impulse control approach and inventory control theory techniques.

Some researchers, such as Mitschele et al. [36] and Cortes et al. [4], approached the problem of insurer's reinsurance structure using multi-objective optimization methods. Insurance companies could either place a single, individual reinsurance type agreement, or a reinsurance program. Reinsurance program is composed of a number of reinsurance agreements. Most reinsurance optimization studies evaluated optimization problems for individual reinsurance agreements, and few researches evaluated opportunities of optimizing reinsurance programs. Hence, Mitschele et al. [36] developed a multi-objective optimization model of reinsurance contracts in a reinsurance program that minimizes the expense of the reinsurance contracts as well as the cedent's retained risks using the expected value premium principle. Developed for QS, SL and XL reinsurance types, Mitschele et al. used a modified Mean-Variance optimization criterion and multi-objective evolutionary algorithms, which optimize the allocation of reinsurance contracts taking into account the insurer’s tradeoff between risk and return. They concluded that applying the variance model, for a combination of QS, SL and XL contracts; SL is the optimal agreement. On a second note, using the VaR and CVaR risk measures, the combination of QS and XL are the optimal solution. At last but not least, on a reinsurance program of QS, as well as both unlimited XL and SL; the unlimited SL is the insurer’s optimal solution.

Similarly, Cortes et al. [4] also addressed an excess of loss reinsurance multi-objective optimization problem targeting both the risk value and the expected return. They developed a Pareto frontier that models an insurer’s optimal combinations of excess of loss reinsurance placements which minimize the risk value at a given expected return; optimizing the tradeoff between both. The modelling approach was based on discretized Population based incremental learning PBIL, an evolutionary heuristic search method assuming fixed number of treaty layers as well as simulated expected loss distribution. The discretized PBIL model developed solved reinsurance treaty optimization problems with higher time efficiency compared to exact enumeration methods. 7 to 15 layers were solvable in less than a day (minimum time frame of 1hr20 minutes) versus other approaches resolution of less than 7 layers in a day and more layers were solved within a week or resulted in unfeasible solutions. They have extended their research to explore the best metaheuristics approach, swarm and evolutionary logarithms, that determine the reinsurance layers and the share of these layers to be purchased by the cedent at the optimal time efficiency [37].

2.4 Contribution to the Literature – Classification of Reinsurance Agreements

This thesis attempts to contribute to the literature by introducing classification for reinsurance models. This notation classifies the reinsurance agreement structure based on the reinsurance contract type, number of layers, and the ceded layer size as explained and illustrated in an example in Tables 1 and 2. For each reinsurance agreement with defined values of agreement parameters, retention and ceding limit and layers’ size, the values of respective parameters can be replaced in the notation to provide a comprehensive overview of the reinsurance nature and parameters.

Table 1: Reinsurance Agreement Classification Notation

Reinsurance Type	Retention	No of Layers	Layers' Size	Notation
Proportional (QS, S)	*x* %	-	*(1-x)* %	(QS/S, *x* %, *[1-x]* %)
Non-proportional (XL, SL)	*M*	*(1,2,..j)*	$m_1 \ldots m_j$	(XL/SL, *M*, *j*, m_1,.., m_j)

j = number of reinsurance treaty layers

Reinsurance contracts notations' examples are presented in Table 2 and explained below:

1) Quota share reinsurance agreement where the insurer retains 40% of risks and premiums, and the remainder 60% is ceded to the reinsurer.
2) Excess of loss arrangement is contracted such that the insurer's deductible per loss is 50K. The reinsurance contract is an unlimited single layer treaty, hence the reinsurer's liability of claims' values in excess of 50K is unlimited.
3) Limited single layer stop loss contract is set with a deductible of 5M (Million) on the aggregate claims incurred in the portfolio to cap the insurer's retention. Once the aggregate claims on the portfolio exceed 5M, liability of the losses is ceded to the reinsurer with maximum limit of 49M.
4) Double layered excess of loss contract is defined by a deductible of 25K, and the size of each layer is 25K and 50K respectively as illustrated in the Table. In multiple layered reinsurance agreement, if the layer size is equal across all layers, then the notation would be represented as (XL, 25K, 2, 25K)

Table 2: Example of Reinsurance Agreement Notations

Reinsurance Type	Retention	No of Layers	Layers' Size	Notation
QS	*40* %	-	*60* %	(QS, 0.4, 0.6)
XL	50K	1	∞	(XL, 50K,1, ∞)
SL	5M	1	$m_1 = L - M$ =50– 5 = *49M*	(SL, 5M, 1, 45M)
XL	25K	2	$m_1 = L_1 - M$ $m_1 = 50 - 25 = 25K$ $m_2 = L_2 - L_1$ $= 100 - 50 = 50K$	(XL, 25K, 2, 25K, 50k)

Chapter 3: Optimization Models

This chapter introduces and discusses the optimization models developed separately. Two objectives are tested; minimizing insurer's variance of retained claims and maximizing insurer's return on capital respectively. For each objective function, unlimited and limited single layer reinsurance agreements are evaluated. Models' parameters and assumptions are applied consistently throughout all models, as well as majority of the approach and steps implemented. Primary differences lie around, the objective functions definition and some optimization constraints. This chapter explains the models and how they were built and run.

3.1 Optimization Models Assumptions

The models are built taking into account below assumptions:

1. Medical Insurance is the insurer's only line of business, with policy limit of AED 150,000.
2. Per claim excess of loss reinsurance structure for one-year period
3. Average insurer's exposure per year is 6,000 policy holders, and in average each insured claims 5 times a year. So number of claims (N) is 30,000.
4. Historical data used to determine and define the distribution function of insurer's claims, in order to simulate new claims, are two years old (2013). Inflation rate of 10% per year is applied on the simulated incurred claims, such that the model's simulated claims are normalized and consistent with current inflation and claims' rates [38].
5. Insurer's Capital (U) is greater than or equal to regulator's minimum capital requirements. UAE Insurance Authority minimum capital requirement is of AED 100M [39]. Values of the Insurer's Capital and Minimum Return on Capital are assumed throughout the model.
6. Premiums, gross premium and reinsurance premium, are calculated using expected value premium principle with risk loading (θ) of 30% (20% and 10% expenses and profit respectively).
7. For limited reinsurance structure, reinsurance maximum limits assumed and tested are 25,000, 35,000, and 50,000

3.2 Optimization Models Parameters

The models' parameters are:

σ_x^2: Variance of Incurred Claims.

σ_I^2: Variance of Retained Claims.

σ_R^2: Variance of Ceded Claims.

P: Gross Premium Collected from Policyholders.

P_R: Reinsurance Premium in Exchange of Reinsurance Agreement.

P_I: Insurer's Net Premium.

U: Insurer's Capital.

ROC: Insurer's Return on Capital.

3.3 Model 1 - Optimization of the Retention Limit (M), Minimizing Retained Claims Variance Given ROC Constraints – Unlimited Single Layer Treaty *(XL, M, 1, ∞)*

3.3.1 Optimization model overview. The model developed estimates an insurer's optimal retention limit (M) for unlimited single layer excess of loss reinsurance cover (XL, M, 1, ∞). Since the layer is unlimited (L= ∞); the size of the layer (m) tends towards ∞. The optimal decision variable is derived such that it minimizes the insurer's variance of retained claims satisfying regulator's minimum risk requirements as well as financial constraints. The model is developed using Palisade @Risk software and Risk Optimizer tool, which run the model and derive the optimal solution satisfying defined constraints accordingly. Insurer's historical exposure and loss experience data are used to evaluate the claims' distribution function to simulate expected claims to build and run the model.

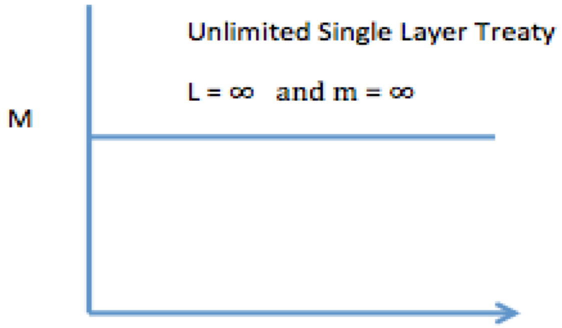

Figure 5: Unlimited Single Layer Reinsurance Treaty

3.3.2 Model formulation. The optimization model's objective function is to minimize the insurer's variance of retained claims by defining the optimal value of insurer's retention limit (M).

$$\boldsymbol{Min\ \sigma_I^2} = E(I^2) - E(I)^2 \tag{1}$$

$$\sigma_I^2 = \int_0^M x^2\, f(x)dx + M^2[1 - F_x(M)] - \left(\int_0^M x\, f(x)dx + M\,[1 - F_x(M)]\right)^2 \tag{2}$$

Subject to constraints of:

1) <u>Minimum Cedent Retention</u>: Insurance authorities impose on insurance companies 40% minimum retention on claims incurred [10]

$$\boldsymbol{M \geq M_{min} = 0.4\ E(X_i)} \tag{3}$$

2) <u>Insurer's Financial Requirement on Return on Capital:</u> to be greater than a minimum accepTable value (to be determined by the insurance company).

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

$$P_I = (P - P_R) = (1 + \theta_I)E(X) - (1 + \theta_R)E(R)$$

$$ROC = \frac{max\ \mathrm{E}\{0, U + P_I - I\}}{U} - 1$$

$$\boldsymbol{ROC \geq 2\%} \tag{4}$$

3) <u>Claims' Variance Constraints</u>

$$\sigma_I^2 + \sigma_R^2 \leq \sigma_X^2$$

$$\boldsymbol{\sigma_I^2 < \sigma_X^2 - \sigma_R^2} \tag{5}$$

3.3.3 Model description. The model was built using Excel and @Risk tools as follows:

I. The distribution function for the claims incurred f(X) is derived from the historical data of losses incurred using @Risk's Distribution Fitting tool as shown in Figure 6. The tool provides a list of the distribution functions that best fit the data set evaluated. Lognormal function was among the top best fit distributions, hence considered as the claims' incurred probability distribution function in the model. Claims incurred distribution function is further utilized to simulate individual claims' values as per the defined lognormal distribution.

II. Reference to Assumption 3 above, expected number of claims (N) are assumed and estimated as 30,000 claims, provided average exposure of 6,000 policyholders and expected average of 5 claims per insured. Expected values of individual claims $E(X_i)$ are simulated for the sample size (N) using the defined lognormal distribution f(X), and assumed as the insurers' expected portfolio of risks. As shown in Figure 7, simulated and generated values of expected incurred claims X_i are then fixed *(Column B)* and further used to define the model's parameters and develop the optimal reinsurance agreement for the respective claims' portfolio since the historical data used for the claims incurred distribution function is for 2013. The simulated claims are inflated by 10% for two years, to account for inflation and ensure consistency of the rates with the current year [38].

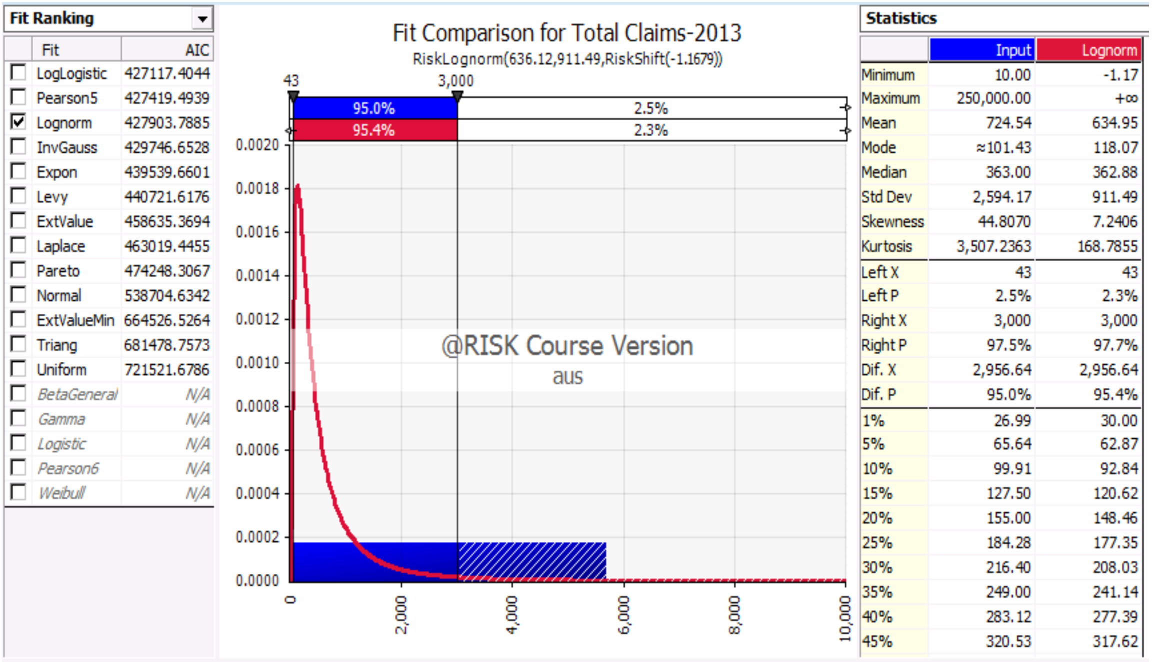

Figure 6: Historical Claims Distribution Fitting

III. Retained claims *(Column C)* are calculated from the expected claims incurred in (*Column B)*such that insurer retains minimum of the claim value (X_i) or the retention limit (M), which will be defined by the model during the optimization process. Value of (M) is randomly assumed for the sake of calculating and generating *(Column C)*

$$E(I(i)) = E(\min(X_i, M)).$$

IV. After calculating (*Column C),*ceded claims *(Column D)* are calculated such that

$$E\big(R(i)\big) = \max(0, X_i - M).$$

V. Now that expected individual incurred claims *(Column B),* retained claims *(Column C),* and ceded claims *(Column D)* are generated, the model is built accordingly. The total expected value of each type of claims E(*X)*, E(*I)* and E(***R)*** is the sum of the individual claims respectively. Similarly, the variance of the sample of individual type of claims' is calculated.

VI. In order to define the insurer's minimum retention limit (M_{min}), 40% of each E(X_i) is calculated as shown in *Column F* in Figure 7. The maximum value calculated is assumed as the value of (M_{min}).

	A	B	C	D	E	F
4			M	Variance		
5		1,329,890.8	20,227.0	46,576.4973		20,227.0
6	Claim Sr No.	Sample Simulated Claims Incurred E(Xi)	Retained Claims E(Ii)	Ceded Claims E(Ri)		40% (Xi)
7	1	1,070.8	1,070.8	-		428.3
8	2	157.8	157.8	-		63.1
9	3	428.5	428.5	-		171.4
10	4	1,004.8	1,004.8	-		401.9
11	5	300.6	300.6	-		120.2
12	6	3,259.3	3,259.3	-		1,303.7
13	7	320.0	320.0	-		128.0
14	8	103.8	103.8	-		41.5
15	9	327.9	327.9	-		131.1
16	10	570.7	570.7	-		228.3
17	11	1,466.4	1,466.4	-		586.5
18	12	92.9	92.9	-		37.1
19	13	1,294.8	1,294.8	-		517.9
20	14	1,047.7	1,047.7	-		419.1
21	15	102.6	102.6	-		41.0
22	16	344.6	344.6	-		137.8
23	17	1,324.5	1,324.5	-		529.8
24	18	182.7	182.7	-		73.1

Figure 7: Simulated Claims' Data Sample

VII. After calculating the expected incurred claims' values, the gross premium (P) is calculated using the expected value premium principle. Similarly, the reinsurance premium is calculated using expected ceded claims' values. The premium risk loading (θ) is assumed as 30%.

$$P = (1+ \theta)\, E(X)$$

$$P_R = (1+\theta)\, E(R)$$

VIII. The fixed parameters of insurer's capital (U) is assumed to be equal to regulator's minimum capital requirement (U_{min}) of 100M. Since all parameters of the ROC are computed, the ROC is calculated accordingly.

$$ROC = \frac{max\, \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

IX. The next step is defining the model parameters. Using Risk Optimizer tool, the optimization model's objective function, decision variable, and constraints are as shown in Figures 8 and 9. The number of trials and number of iterations per trial to derive the optimal solution are also defined, then the optimization model is ready to run.

	A	B	C	D	E	F	G	H	I
1	Assumptions & Fixed Input						Decision Variable		
2	Exposure	6,000					Excess Amount (M)	1,000	
3	Average Claim Per Person	5							
4	Number of Claims Incurred (N)	30,000							
5	Capital (U)	100,000,000					Output		
6	Premium Risk Loading (θ)	30%					Variance of Retained Claims (σI2)	114,090.2	
7							ROC	5%	
8									
9	Uncertain Input						Assumptions & Constraints		
10	Claims			Premiums			Variance Retained Non-negativity	>	0
11	Incurred Claims - E(X)	23,266,519.8		Gross Premiums (P)	30,246,475.8		Minimum Retention Limit	>	20,227
12	Variance Incurred Claims (σx2)	1,329,890.8		Reinsurance Premium (Pr)	9,920,402.3		Variance Constraint	<=	357,691.82
13							Minimum ROC	>=	2%
14	Retained Claims - E(I)	15,635,441							
15									
16	Ceded Claims - R	7,631,079							
17	Variance Ceded Claims (σR2)	972,199							

Figure 8: Optimization Model Parameters Definition

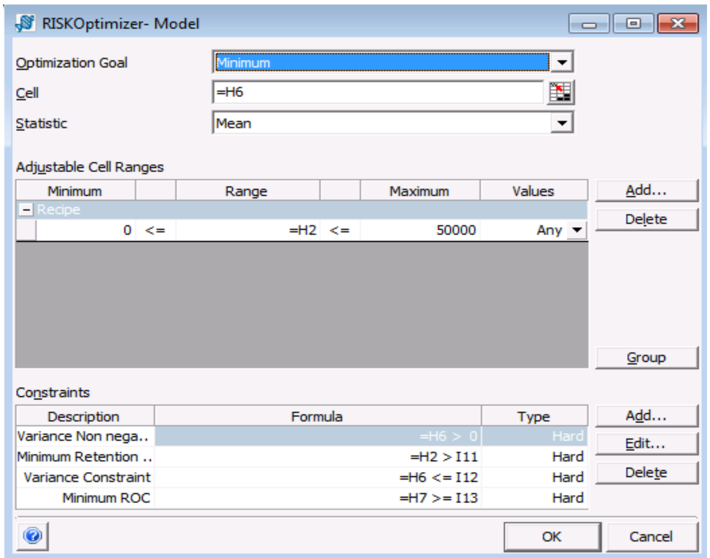

Figure 9: RiskOptimizer - Optimization Model Definition

X. Running the optimization model, the optimal retention limit (M) which minimizes the insurer's retention variance is defined on the model as displayed in Figure 10. Summary of the models' optimal solutions is illustrated and discussed in Chapter 4.

	A	B	C	D	E	F	G	H	I
1	Assumptions & Fixed Input						Decision Variable		
2	Exposure	6,000					Excess Amount (M)	20,227	
3	Average Claim Per Person	5							
4	Number of Claims Incurred (N)	30,000							
5	Capital (U)	100,000,000					Output		
6	Premium Risk Loading (θ)	30%					Variance of Retained Claims (σI2)	1,189,948.4	
7							ROC	7%	
8									
9	Uncertain Input						Assumptions & Constraints		
10	Claims			Premiums			Variance Retained Non-negativity	>	0
11	Incurred Claims - E(X)	23,266,519.8		Gross Premiums (P)	30,246,475.8		Minimum Retention Limit	>	20,227
12	Variance Incurred Claims (σx2)	1,329,890.8		Reinsurance Premium (Pr)	93,584.4		Variance Constraint	<=	1,283,314.29
13							Minimum ROC	>=	2%
14	Retained Claims - E(I)	23,194,532							
15									
16	Ceded Claims - R	71,988							
17	Variance Ceded Claims (σR2)	46,576							

Figure 10: Optimization Model Solution

3.4 Model 2 -Optimization of the Retention Limit (M), Minimizing Retained Claims Variance Given ROC Constraints – Limited Single Layer Treaty *(XL, M, 1, m)*

3.4.1 Optimization model overview. The optimization model is developed for a limited single layer excess of loss reinsurance cover such that the size of the layer m is given by *(L-M)*. The optimal decision variables (L and M) are derived such that the insurer's variance of retained claims is minimized satisfying regulator's minimum risk requirements as well as financial constraints. The model is built and simulated using Palisade @Risk software and Risk Optimizer tool, which run the model and derive the optimal solution satisfying defined constraints accordingly. Insurer's historical exposure and loss experience data are used to evaluate the claims' distribution function and build the model. The model is tested for two approaches:

1) Determine optimal retention limit (M) for different fixed values of ceding limit (L)
2) Determine optimal retention limit (M) and ceding limit (L) as decision variables

The solutions of both approaches are observed and evaluated to test and understand the model dynamics. Steps I and II from Model 3 explained above are duplicated for the limited layers' model, and the same set of simulated and inflated claims is used for the sake of comparing the optimal solutions of both models.

3.4.2 Model formulation. The optimization model's objective function is to minimize the insurer's variance of retained claims by defining the optimal values insurer's retention limit – deductible - (M) and the ceding limit (L)

$$\boldsymbol{Min\ \sigma_I^2} = E(I^2) - E(I)^2 \tag{6}$$

Subject to constraints of:

1) <u>Minimum Cedent Retention</u>: Insurance authorities impose on insurance companies 40% minimum retention on claims incurred [10]

$$\boldsymbol{M \geq M_{min} = 0.4\ E(X_i)} \tag{7}$$

2) <u>Insurer's Financial Requirement on Return on Capital</u>

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

$$P_I = (P - P_R) = (1 + \theta_I)E(X) - (1 + \theta_R)E(R)$$

$$ROC = \frac{max\ \mathrm{E}\{0, U + P_I - I\}}{U} - 1$$

$$\boldsymbol{ROC \geq 2\%} \tag{8}$$

3) <u>Claims' Variance Constraints</u>

$$\sigma_I^2 + \sigma_R^2 \leq \sigma_X^2$$

$$\boldsymbol{\sigma_I^2 < \sigma_X^2 - \sigma_R^2} \tag{9}$$

4) <u>Ceding Limit Constraint</u>

$$\boldsymbol{L > M} \tag{10}$$

3.4.3 Model description. The model was implemented using Excel and @Risk tools as follows:

I. The same set of claims simulated in steps I and II in Model 3 above is used to build and run the model for limited reinsurance layer.

II. In limited layer reinsurance structure, retained and ceded claims are calculated as per following scenarios, and the respective fields are generated:

Table 3: Retained and Ceded Claims Distribution

$E(X_i) \leq M$	$E(I_i)= \min (E(X_i), M)$	$E(R_i) = 0$
$M < E(X_i) < L$	$E(I_i) = M$	$E(R_i) = E(X_i) - M$
$E(X_i) > L$	$E(I_i) = M + (E(X_i) – (L-M))$	$E(R_i) = L – M$

III. As shown in Figure 11, expected individual incurred claims *(Column B),* total retained claims *(Column F),* and ceded claims *(Column D)* are generated, and the model is built accordingly. The total expected value of each type of claims $E(X)$, $E(I)$ and $E(R)$ is the sum of the individual claims respectively. Similarly, the variance of the sample of individual type of claims is calculated.

IV. In order to define the insurer's minimum retention limit (M_{min}), 40% of each X_i is calculated as shown in *Column I* in Figure 11. The maximum value calculated is assumed as the value of (M_{min}).

V. After calculating the expected incurred claims' values, the gross premium (P) is calculated using the expected value premium principle. Similarly, the reinsurance premium is calculated using expected ceded claims' values. The premium risk loading (θ) is assumed as 30%

$$P = (1+ \theta)\, E(X)$$

$$P_R = (1+\theta)\, E(R)$$

	A	B	C	D	E	F	G	H	I
4			M	L					Max
5			20,227.0	53,125.0					20,227
6	Claim Sr No.	Claims_Incurred (X)	Retained Claims (I)	Ceded Claims (R)	Net Claims Retained in Excess of M	Total Claims Retained			0.4E(Xi)
7	1	1,070.8	1,070.8	-	-	1,070.8			428.3
8	2	157.8	157.8	-	-	157.8			63.1
9	3	428.5	428.5	-	-	428.5			171.4
10	4	1,004.8	1,004.8	-	-	1,004.8			401.9
11	5	300.6	300.6	-	-	300.6			120.2
12	6	3,259.3	3,259.3	-	-	3,259.3			1,303.7
13	7	320.0	320.0	-	-	320.0			128.0
14	8	103.8	103.8	-	-	103.8			41.5
15	9	327.9	327.9	-	-	327.9			131.1
16	10	570.7	570.7	-	-	570.7			228.3
17	11	1,466.4	1,466.4	-	-	1,466.4			586.5
18	12	92.9	92.9	-	-	92.9			37.1
19	13	1,294.8	1,294.8	-	-	1,294.8			517.9
20	14	1,047.7	1,047.7	-	-	1,047.7			419.1
21	15	102.6	102.6	-	-	102.6			41.0
22	16	344.6	344.6	-	-	344.6			137.8

Figure 11: Simulated Claims' Data and Limited Layer Claims' Split Sample

VI. The fixed parameters of insurer's capital (U) is assumed equal to the regulator's minimum capital requirement (U_{min}) of 100M. Since all parameters of the ROC are computed, the ROC is calculated accordingly.

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

VII. The next step is defining the model parameters, the optimization model's objective function, decision variable, and constraints using the Risk Optimizer.

a. *Approach 1*: (M) and (L) are defined as the optimization model's decision variables. The model is defined to select optimal value of (L) from a range of 25,000-150,000, such that L >M.

b. *Approach 2*: (M) is defined as the optimization model's decision variable, and the model is tested for different assumed values of (L).

VIII. The solution for both approaches, as shown in Table 4 is evaluated and compared. From the results displayed. It can be concluded that the optimal limit (L) derived as the model's decision variable, resulted in the minimum retained variance compared to the assumed layer limits although different values of L had low to no impact on both ROC and M.

Table 4: Model 2 Optimization Results

Approach	L	M	σ_I^2	ROC
1	**53,125***	20,227	**1,189,948**	6.96%
2	25,000	20,227	1,275,916	6.97%
	35,000	20,227	1,222,794	6.96%
	50,000	20,227	1,190,695	6.96%

*Optimal decision variable

3.5 Model 3 - Optimization of the Retention Limit (M), Maximizing Insurer's Return on Capital – Unlimited Single Layer Treaty *(XL, M,1, ∞)*

3.5.1 Optimization model overview. The model developed estimates for an insurer's optimal retention limit (M) for an unlimited single layer excess of loss reinsurance cover (XL, M, 1, ∞). The optimal decision variable is derived such that it maximizes the insurer's return on capital satisfying minimum financial and risk requirements. Similar to other models, the model is developed using Palisade @Risk software and Risk Optimizer tool.

3.5.2 Model formulation. The optimization model's decision variable is the optimal retention limit (M) which is

$$\boldsymbol{Max\ ROC} = \frac{max\ \mathrm{E}\{0, U+P-I-P_R\}}{U} - 1 \quad (11)$$

Subject to constraints of:

1) Minimum Cedent Retention: Insurance authorities impose on insurance companies 40% minimum retention on claims incurred [10]

$$\boldsymbol{M \geq M_{min} = 0.4\ \mathrm{E}(X_i)} \quad (12)$$

2) Insurer's Financial Requirement on Return on Capital

$$\boldsymbol{ROC \geq 2\%} \quad (13)$$

3) <u>Claims' Variance Constraints</u>

$$\sigma_I^2 + \sigma_R^2 \leq \sigma_X^2$$

$$\boldsymbol{\sigma_I^2 < \sigma_X^2 - \sigma_R^2} \tag{14}$$

3.5.3 Model description. The model was implemented using Excel and @Risk tools as follows:

I. The same set of claims simulated in steps I and II in Model 3 is used to develop the model's parameters accordingly.

II. Retained claims are calculated from the expected claims incurred such that insurer retains minimum of the claim value (X_i) or the retention limit (M), which will be defined by the model during the optimization process. Value of (M) is randomly assumed for the sake of calculating and generating retained claims

$$E\big(I(i)\big) = E(\min(X_i, M))$$

III. After calculating retained claims, ceded claims are calculated such that

$$E(R(i)) = \max(0, X_i - M)$$

IV. Now that expected individual incurred claims, retained claims, and ceded claims are generated, the model is built accordingly. The total expected value of each type of claims E(*X*), E(*I*) and E(***R***) is the sum of the individual claims respectively.

V. In order to define the insurer's minimum retention limit (M_{min}), 40% of each $E(X_i)$ is calculated as shown in *Column F* in figure 7 above. The maximum value calculated is assumed as the value of M_{min}.

VI. After calculating the expected incurred claims' values, the gross premium (P) is calculated using the expected value premium principle. Similarly, the reinsurance premium is calculated using expected ceded claims' values. The premium risk loading (θ) is assumed as 30%.

$$P = (1+ \theta)\, E(X)$$

$$P_R = (1+\theta)\, E(R)$$

VII. The fixed parameter of insurer's capital (U) is assumed equal to regulator's minimum capital requirement (U_{min}) of 100M. Since all parameters of the ROC are computed, the ROC is calculated accordingly.

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

VIII. Using Risk Optimizer, the objective function, decision variable (M), and the constraints are defined, such that the model derives the optimal solution maximizing the insurer's ROC satisfying all constraints.

3.6 Model 4 - Optimization of the Retention Limit (M), Maximizing Insurer's Return on Capital – Limited Single Layer Treaty *(XL, M, 1, m)*

3.6.1 Optimization model overview. The model developed estimates an insurer's optimal retention limit (M) for limited single layer excess of loss reinsurance cover such that the size of the layer ***m= L-M***. The optimal decision variable is derived such that it maximizes the insurer's return on capital satisfying regulator's minimum risk requirements as well as financial constraints. The model is built and simulated using Palisade @Risk software and Risk Optimizer tool, which run the model and derive the optimal solution satisfying defined constraints accordingly. Insurer's historical exposure and loss experience data are used to evaluate the claims' distribution function and build the model. The model is tested for two approaches:

1) Determine optimal retention limit (M) and ceding limit (L) as decision variables
2) Determine optimal retention limit (M) for different fixed values of ceding limit (L)

The solutions of the same are observed and evaluated to test and understand the model dynamics. Steps I and II from Model 3 explained above are duplicated for the limited layers' model, and the same set of simulated and inflated claims is used, for the sake of comparing the optimal solutions of both models.

3.6.2 Model formulation. The optimization model's decision variables are the optimal retention limit (M) and ceding limit (L)

$$\boldsymbol{Max\ ROC} = \frac{max\ \mathrm{E}\{0, U+P-I-P_R\}}{U} - 1 \tag{15}$$

Subject to constraints of:

1) Minimum Cedent Retention: Insurance authorities impose on insurance companies 40% minimum retention on claims incurred [10]

$$M \geq M_{min} = 0.4\, E(X_i) \tag{16}$$

2) Insurer's Financial Requirement on Return on Capital

$$ROC \geq 2\% \tag{17}$$

3) Ceding Limit Constraint

$$L > M \tag{18}$$

4) Claims' Variance Constraints

$$\sigma_I^2 + \sigma_R^2 \leq \sigma_X^2$$

$$\boldsymbol{\sigma_I^2 < \sigma_X^2 - \sigma_R^2} \tag{19}$$

3.6.3 Model description. The model was implemented using Excel and @Risk tools as follows:

I. The same set of claims simulated in steps I and II in Model 3 is used to build the model and define its parameters accordingly.

II. In limited layer reinsurance structure, retained and ceded claims are calculated as per following scenarios in Table 5, and the respective fields are generated:

Table 5: Retained and Ceded Claims Distribution

$E(X_i) \leq M$	$E(I_i) = \min(E(X_i), M)$	$E(R_i) = 0$
$M < E(X_i) < L$	$E(I_i) = M$	$E(R_i) = E(X_i) - M$
$E(X_i) > L$	$E(I_i) = M + (E(X_i) – (L-M))$	$E(R_i) = L – M$

III. Now that expected individual incurred claims, retained claims, and ceded claims are generated, the model is built accordingly. The total expected value of each type of claims $E(X)$, $E(I)$ and $E(R)$ is the sum of the individual claims respectively.

IV. In order to define the insurer's minimum retention limit (M_{min}), 40% of each X_i is calculated as shown in *Column F* in figure 7 above. The maximum value calculated is assumed as the value of M_{min}.

V. After calculating the expected incurred claims' values, the gross premium (P) is calculated using the expected value premium principle. Similarly, the reinsurance premium is calculated using expected ceded claims' values. The premium risk loading (θ) is assumed as 30%.

$$P = (1+ \theta)\ E(X)$$

$$P_R = (1+\theta)\ E\ (R)$$

VI. The fixed parameter of insurer's capital (U) is assumed equal to regulator's minimum capital requirement (U_{min}) of 100M. Since all parameters of the ROC are computed, the ROC is calculated accordingly.

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

VII. The next step is defining the model parameters, the optimization model's objective function, decision variable, and constraints using the RiskOptimizer, such that the model derives the optimal solution maximizing the insurer's ROC satisfying all constraints.

a. *Approach 1*: (M) and (L) are defined as the optimization model's decision variables. The model is defined to select optimal value of (L) from a range of 25,000-150,000, such that L >M.

b. *Approach 2*: (M) is defined as the optimization model's decision variable, and the model is tested for different assumed values of (L)

VIII. The solution for both approaches is evaluated and compared in Table 6. It can be noted that the optimal solution from the optimization model provided a limit higher than the tested values of L. The ROC is also slightly increasing as the decision variables increase.

Table 6: Model 4 Optimization Results

Approach	L	M	σ_I^2	ROC
1	71,875*	46,137	1,315,837	6.98%
2	25,000	20,227	1,257,510	6.97%
	35,000	29,735	1,275,916	6.97%
	50,000	29,754	1,301,109	6.98%

*Optimal decision variable

Chapter 4: Optimization Models Discussion of Results

The chapter discusses, evaluates and compares the optimization models and their respective optimal solutions. It also provides an overview analysis of the impact of the model's independent parameters on the ROC.

4.1 Sensitivity Analysis

In order to get a better understanding of the solutions derived from the models, it is important to understand the dynamics and dependencies of the model parameters, and their impact on the optimization and optimal solutions.

After simulating the set of the expected claims incurred values $E(X_i)$, the parameter of E(X) was fixed throughout the models, and all model parameters depend on the severity and size of the portfolio of claims simulated. As for the remaining model parameters, Table 7 demonstrates the dependencies and relationships between these parameters. Values of θ and U, are also fixed and assumed depending on regulatory solvency requirements and insurer's financial and pricing requirements.

Table 7: Model Parameters' Dependencies

Independent Parameter	Dependent Parameter
M	σ_I^2, E(I), P(R), E(R)
θ	P, P(R), ROC
U	ROC

The impact of the independent parameters on the ROC can be better understood by testing the effect on the ROC of varying different values of independent parameters individually and keeping the other parameters fixed. Sample sensitivity analysis is illustrated below to demonstrate the same. The risk loading impact, in Table 8, is tested on the gross premium (P), since that contributes the major proportion of the premiums calculation.

It can be noted from the analysis results, in Tables 8 and 9, that the impact of the capital and gross premium risk loading on the ROC is significantly high. On the other hand, the impact of varying the values of M is significantly low on the ROC as shown in Table 10. Therefore, the defined parameters of the capital and premium risk loading play an important role in achieving and defining ROC targets as opposed to the value of M.

Table 8: Sensitivity Analysis - Impact of Gross Premium Risk Loading on ROC

% Change (θ)	θ	ROC	% Change (ROC)
-25%	23%	5.2%	-25%
-10%	27%	6.3%	-10%
0%	**30%**	**7.0%**	**0%**
10%	33%	7.7%	10%
25%	38%	8.7%	25%

Table 9: Sensitivity Analysis - Impact of U on ROC

% Change (U)	U	ROC	% Change (ROC)
0%	**100M**	**6.98%**	**0%**
10%	110M	6.34%	-9%
25%	125M	5.58%	-20%
50%	150M	4.65%	-33%
75%	175M	3.99%	-43%

Table 10: Sensitivity Analysis - Impact of M on ROC

% Change (M)	M	ROC	% Change (ROC)
-20%	16,182	6.95%	-0.14%
-10%	18,204	6.95%	-0.14%
0%	**20,227**	**6.96%**	**0%**
10%	22,250	6.96%	0%
20%	26,700	6.97%	0.14%
50%	40,049	6.98%	0.29%

4.2 Optimization Models Results

Four optimization models which address two objective functions; insurer's minimum retained claims' variance and maximum return on capital, have been built and run accordingly. The models' optimal solutions are summarized in Table 11 below. For each objective function, the optimal solution for both limited and unlimited layers is captured, and for the limited layer the values in the table indicate the values of the reinsurance agreement's deductible and ceding limit as the optimization problems' decision variables.

Table 11: Optimization Models' Optimal Solutions

Objective Function	Ceding Limit (L)	Deductible (M)	Retained Variance (σ_I^2)	ROC
Min σ_I^2	∞	20,227	1,189,948	6.96%
	53,125*	20,227	1,189,948	6.96%
Max ROC	∞	50,455	1,329,517	6.98%
	71,875	46,137	1,315,837	6.98%

On minimizing the variance of the insurer's retention, it can be concluded that both models resulted in a decision variable (M) that is equal to the minimum retention limit (deductible) that was defined in the model. The models' objective is to minimize the variance of retained claims given target return on capital. Since the impact of M on the ROC is significantly low, the minimum variance is achieved at the minimum value of M defined. Consequently, the model's optimal solution of the deductible didn't deviate from the minimum constraint. On the limited reinsurance structure model, limit (L) was defined in the optimization such that all losses in excess of M are covered under the limit L, hence insurer's retention is only limited to values capped by M, which is defined as the minimum value and delivers minimum retention. In conclusion, whether it's a limited or an unlimited reinsurance agreement, it is indifferent to the insurer as both solutions result in the same deductible, retention variance and ROC under the models' assumptions.

On the other hand, the behavior of the model of maximizing the ROC varied depending on the constraints defined on the variances.

1. $\sigma_I^2 \leq \sigma_X^2 - \sigma_R^2$ – The optimal solution presented by the model was in the form of full retention by the insurer and zero ceding to the reinsurer, since the constraint can be satisfied by $\sigma_I^2 = \sigma_X^2$ where $\sigma_R^2 = 0$
2. $\sigma_I^2 < \sigma_X^2 - \sigma_R^2$ – The optimal solution requires existence of reinsurance; hence the model provides a solution with significantly small number of claims ceded, such that majority of claims are covered under the insurer.

Since the objective of the research is to evaluate optimal options of reinsurance agreements, and compare effectiveness of both objective functions against each other, the second constraint was selected, regardless of the small ceding applied. It can be observed that by maximizing the ROC, the optimal value of M selected has deviated from the minimum value such that it satisfies the constraints accordingly. Consequently, an optimal deductible can be efficiently derived using this objective, taking into consideration that changes in the value of M will not significantly impact the ROC independently, and that Capital and premium risk loading are critical parameters that impact the behavior of the ROC. Comparing the results of both unlimited and limited reinsurance agreements, it can be noted that the limited layer reduces the insurer's deductible and variance of retained claims against the unlimited layer solution.

Evaluating both models, the model, which maximizes the return on capital, appears to deliver more efficient optimization results and solution for an insurer, against the latter model. The model takes into consideration the relationship and impact of M on the ROC parameters and derives the optimal value of a suggested reinsurance structure to meet insurers' and regulatory constraint and requirements. It also provides the insurer with an overview of, at given capital and premium risk loadings, what is the maximum return on capital achievable in order to meet shareholders' expectations. Consequently, given the example modeled and explained, the insurer's optimal excess of loss reinsurance structure would be a limited single layer treaty defined *(XL, 46.1K, 1, 26K)* which, given all parameters sustained, delivers maximum ROC of up to 6.98%. The value of delivered ROC highly depends on the defined values of the premium risk loading and capital.

Chapter 5: Conclusion and Recommendations

The subject of optimal reinsurance agreement has been an area of interest for many researches and studies. The literature has explored different reinsurance optimization approaches whether from the insurer, reinsurer or joint perspective.

In this study, the objective was to build and develop optimization model and tools that can be utilized by insurance companies in the industry to gain a better understanding and visibility of their optimal reinsurance structure prior to contract. Using both historical and expected loss experiences, these models allow insurers to understand their reinsurance agreements and facilitate decision making revolving around the reinsurance structure to be in place. Four models were developed and tested to meet pre-defined constraints with two main objective functions, minimizing insurer's retained variance and maximizing insurer's return on capital.

To minimize the variance of retained claims, a target return on capital and a minimum retention limit were defined as constraints, and the model attempted to define the optimal retention limit accordingly. Given the above, the optimization resulted in a retention limit equal to the defined minimum retention limit. The impact of the target return on capital constraint was insignificant on defining an optimal retention limit. This was interpretable through the sensitivity analysis evaluating the relationship and sensitivity between the return on capital and the retention limit. The analysis revealed that the impact of the retention limit on the target return on capital is significantly low; hence the constraints were satisfied and the minimal retention variance were satisfied on the defined minimum retention limit. In addition, the optimal solution was indifferent to whether the model was tested for a limited or unlimited reinsurance agreement. To satisfy the objective function, the solutions for both types of agreements were defined such that insurer retains the minimum retention limit and the net value of claims is ceded to the reinsurance.

The models that maximized the return on capital defined the insurer's retention limit more efficiently compared to the latter objective and deviated from the minimum retention constraint. However, given the models' assumptions and the claims' nature, the value of the maximum return on capital proved to be highly dependent on the defined parameters of the capital and gross premium safety loading rather than the retention limit. Therefore, for the model to run more efficiently, the relationship between the independent parameters (M), (U), and (θ) and the return on capital should be accounted for while developing the model.

Comparing both objective function, the models that maximize the return appear to address the optimization objective more efficiently, satisfying all constraints and defining more sensible optimal retention limits. However, the models have areas and opportunities for further improvements and to derive more efficient solutions.

To enhance the models' performance and value to the industry, the model can be tested on a larger portfolio exposed to higher severity of claims. Due to the nature of the medical insurance portfolio and data provided, low to medium severity and frequency, the distribution functions used to simulate the expected losses simulates significantly small sample of high severity claims. Portfolio with higher variation in claims' severity and risks may result in better optimization solutions for a reinsurance agreements and may generate a bigger demand/need for reinsurance structure in place. The impact of M and L on the ROC could also be tested in that case, if it could result in a more significant impact on the ROC, or if it will remain low compared to U and θ. Moreover, the probability distribution function used to simulate expected claims was derived from the distribution fitting of a one year claims' experience. Alternatively, the distribution function of several years of historical claims could be used to simulate expected claim. Moreover, reinsurance market related information can be accommodated to the model, such as reinsurance premium rates and different pricing approaches for different agreement types, limited and unlimited etc., to evaluate the cost of reinsurance more accurately. In this model, constant pricing approach was assumed such that reinsurer simply loads the expected ceded claims by 30%.

References

[1] B. Howe, R. Buchanan and D. Hart, *Actuarial practice of general insurance,* 7th edition, Australia:The Institute of Actuaries of Australia, 2007.

[2] Anonymous, "Health Insurance; Medical Insurance to Propel the UAE Insurance Market," NewsRx, Atlanta, USA, 2011.

[3] Anonymous, "United Arab Emirates Insurance Report - 2013 Q1," Business Monitor International, London, United Kingdom, 2013.

[4] O. Cort´es, A. Rau-Chaplin, D. Wilson, I. Cook and J. Gaiser-Porter, "Efficient optimization of reinsurance contracts using discretized PBIL," in *In Proceedings of the 2nd International Conference on Data Analytics*, Porto, Portugal, 2013.

[5] K. Riley, "Excess of Loss Reinsurance: Per-Risk Contracts," Willis Wire, 3 January 2014. [Online]. Available: http://blog.willis.com/2014/01/excess-of-loss-reinsurance-per-risk-contracts/. [Accessed 28 May 2014].

[6] G. Deelstra and G. Plantin, "Elements of Risk Theory," in *Risk Theory and Reinsurance*, Springer, pp. 2-4, 2014.

[7] V. R. Young, "Premium Principles," *Encyclopedia of Actuarial Science,* 15 September 2006.

[8] L. S. Ward and D. H. Lee, "Practical Application of the Risk-Adjusted Return on Capital Framework", CAS Forum Summer 2002, Dynamic Financial Analysis Discussion Papers, 2002. [Online]. Available at http://www.casact.org/pubs/forum/02sforum/02sftoc.htm.[Accessed: 20 March 2015]

[9] S. Feldblum and N. Thandi, "Financial Pricing Models for Property-Casualty Insurance Products: The Target Return on Capital", CAS Forum Winter 2003, Data Management, Quality and Technology Call Papers and Ratemaking Disucssion Papers, 2003. [Online]. Available at https://www.casact.org/pubs/forum/03wforum/03wf591.pdf. [Accessed: 15 May 2015]

[10] S. Bhagaloo, Interviewee, *Consulting Actuary, Lux Actuaries & Consultants.* [Interview]. 15 March 2014.

[11] Anonymous, "Trinity Insurance Brokers," Trinity Insurance Brokers, 2014. [Online]. Available: www.trinityinsurance.co.in/company-overview.php. [Accessed May 2014].

[12] K. Borch, "An attempt to determine the optimum amount of stop loss reinsurance," in *Transaction of the 16th International Congress of Actuaries*, 1960.

[13] K. J. Arrow, "Uncertainty and the Welfare Economics of Medical Care," *The American Economic Review,* vol. 53, pp. 941-973, 1963.

[14] J. Cai, K. S. Tan, C. Weng and Y. Zhang, "Optimal reinsurance under VaR and CTE risk measures," *Insurance: Mathematics and Economics,* vol. 43, pp. 185-196, 2008.

[15] A. Balbas, B. Balbaz and A. Heras, "Optimal reinsurance with general risk measures," *Insurance: Mathematics and Economics,* vol. 44, pp. 374-384, 2009.

[16] A. Balbas, B. Balbaz and A. Heras, "Stable solutions for optimal reinsurance problems involving risk measures," *European Journal of Operational Research,* vol. 214, pp. 796-804, 2011.

[17] Y. Chi and K. S. Tan, "Optimal Reinsurance under VaR and CVaR Risk Measures: A Simplified Approach," *Astin Bulletin,* vol. 41, pp. 487-508, 2011.

[18] K. S. Tan and C. Weng, "Enhancing Insurer Value Using Reinsurance and Value-at-Risk Criterion," *Geneva Risk and Insurance Review,* vol. 37, no. 1, pp. 109-140, 2012.

[19] M. Kaluszka, "Optimal reinsurance under mean-variance premium principles," *Insurance: Mathematics and Economics,* vol. 28, pp. 61-67, 2001.

[20] L. Gajek and D. Zagrodny, "Optimal reinsurance under general risk measures," *Insurance: Mathematics and Economics,* vol. 34, pp. 227-240, 2004.

[21] M. Kaluzka, "Optimal reinsurance under convex principles of premium calculation," *Insurance: Mathematics and Economics,* vol. 36, pp. 375-398, 2005.

[22] Y. Chi and K. S. Tan, "Optimal reinsurance under variance related premium principles," *Insurance: Mathematics and Economics,* vol. 51, pp. 310-321, 2012.

[23] Y. Chi and K. S. Tan, "Optimal reinsurance with general premium principles," *Insurance: Mathematics and Economics,* vol. 52, pp. 180-189, 2013.

[24] J. Jang, "Reinsurance retentions & limits for property-liability insurers - Doctoral Dissertation," Temple Univrsity , 2001.

[25] M. Kaluszka, "Mean-variance optimal resinurance arrangements," *Scandinavian Actuarial Journal,* vol. 1, pp. 28-41, 2004.

[26] M. d. L. Centeno, "Dependent risks and excess of loss reinsurance," *Insurance: Mathematics and Economics,* vol. 37, pp. 229-238, 2005.

[27] V. Kaishev, "Optimal retention levels given the joint survival of cedent and reinsurer," Sir John Cass Business School, London, 2003.

[28] V. Kaishev and D. S. Dimitrova, "Excess of loss reinsurance under Joint survival optimality," *Insurance: Mathematics and Economics,* vol. 39, pp. 376-389, 2006.

[29] V. Kaishev and D. S. Dimitrova, "Optimal joint survival reinsurance: efficient frontier approach," *Insurance: Mathematics and Economics,* vol. 47, pp. 27-35, 2010.

[30] Z. Li, "Optimal reinsurance retention under ruin related optimization criteria," M.S. thesis, Dept. Actuarial Science, University of Waterloo, Ontario, Canada, 2008.

[31] C. Yusong and X. Jin, "Proportional and excess-of-loss reinsurance under investment gains," *Applied Mathematics and Computation,* vol. 217, pp. 2546-2550, 2010.

[32] S. Asmussen, B. Højgaard and M. Taksar, "Optimal Risk Control and Dividend Distribution Policies," *Finance & Stochastics,* vol. 4, no. 3, p. 299, 2000.

[33] K. A. Froot, "Risk Management, Capital Budgeting, and Capital Structure Policy for Insurers and Reinsurers," *Journal of Risk and Insurance,* vol. 74, no. 2, pp. 273-299, 2007.

[34] R. J. Laeven and E. C. Perotti, "Optimal Capital Structure for Insurance Companies," 2010.

[35] H. Meng and T. K. Siu, "On optimal reinsurance, dividend and reinvestment strategies," *Economic Modelling,* vol. 28, no. (1-2), pp. 211-218, 2011.

[36] A. Mitschele, I. Oesterreicher, F. Schlottmann and D. Seese, "Heuristic optimization of reinsurance programs and implications for reinsurance buyers," in *Operations Research Proceedings*, 2006.

[37] O. A. C. Cortes, A. Rau-Chaplin, D. Wilson and J. Gaiser-Porter, "On PBIL, DE and PSO for Optimization of Reinsurance Contracts," in *Applications of Evolutionary Computation*, Granada, Spain, Springer Berlin Heidelberg, pp. 227-238, 2014.

[38] S. Bhagaloo, Interviewee, *Consulting Actuary, Lux Actuaries and Consultants.* [Interview]. November 2015.

[39] *Financial Regulations for Insurance Companies,* United Arab Emirates: Insurance Authority, 2014.

Vita

Mai Muhtaseb was born in 1990 in the United Arab Emirates. She obtained her Bachelor of Science in Civil Engineering at the American University of Sharjah, United Arab Emirates, and graduated in 2012. She has enrolled in the Master of Engineering Systems Management at the American University of Sharjah later after her graduation.

In 2012, Mai joined Noor Takaful for a year, a local insurance company in Dubai, United Arab Emirates as a Sales Coordinator and fulfilled roles as an Assistant Business Analyst as well. In that role, Mai has gained exposure and interest in analytics and financial risks. Later in the end of 2014, she joined Unilever Gulf FZE in Dubai, United Arab Emirates as a procurement intern. She is currently fulfilling the role of an Assistant Manager Global Procurement of Capital Expenditure focusing on Construction, Mechanical, Electrical and Installation services since May 2015. In her current role, she supports and drives value generation and savings as well as efficient management and execution of procurement activities and contracts related to Unilever's facilities and buildings' construction project on a global level.

Abstract

In the current practice in the region, before purchasing a reinsurance contract, small to medium insurance companies rarely conduct internal analysis of their data and experiences in order to evaluate and achieve optimal reinsurance arrangements and contracts. Most companies settle their reinsurance agreements through reinsurance intermediary, broker, who acts as the link of communication, negotiation and settlement between both the reinsurers and the ceding insurer. Alternatively, the reinsurance companies or intermediaries evaluate and analyze the insurer's historical losses and offer reinsurance agreement and proposal accordingly. Therefore, the proposed reinsurance structure is not necessarily the insurer's optimal arrangement. In this thesis, excess of loss reinsurance optimization models are developed in order to enable insurers to utilize user-friendly and efficient tools to evaluate the optimal reinsurance arrangement depending on financial requirements, and to gain better value of their reinsurance contracts. The models are developed to define the insurer's optimal reinsurance retention and ceding limits for two objectives; minimizing insurer's retention variance and maximizing insurer's return on capital. The model maximizing the return on capital resulted in more realistic optimization solutions of retention limits. A sensitivity analysis to evaluate the impact of the model's parameters on the return on capital was also conducted, and it was concluded that the impact of the insurer's retention limit on the return on capital was significantly small. Moreover, the defined capital and gross premium safety loading had a major impact on the behavior of the return on capital.

Search Terms: Insurance, Reinsurance, Optimization, Excess of Loss, Return on Capital

Table of Contents

Abstract 6

List of Figures 9

List of Tables 10

Glossary 11

Chapter 1: Introduction 12

1.1 Overview 12

1.1.1 Reinsurance principles. 13

1.1.2 Non-proportional reinsurance. 15

1.2 Problem Description 19

1.3 Research Objective 21

1.4 Research Significance 21

1.5 Research Methodology 22

1.6 Thesis Organization 22

Chapter 2: Literature Review 24

2.1 Insurer's Optimal Reinsurance Structure 24

2.2 Insurer and Reinsurer's Optimal Reinsurance Structure 27

2.3 Investments, Capital and Optimal Reinsurance Structure 28

2.4 Contribution to the Literature – Classification of Reinsurance Agreements 30

Chapter 3: Optimization Models 32

3.1 Optimization Models Assumptions 32

3.2 Optimization Models Parameters 33

3.3 Model 1 - Optimization of the Retention Limit (M), Minimizing Retained Claims Variance Given ROC Constraints – Unlimited Single Layer Treaty *(XL, M, 1, ∞)* 33

3.3.1 Optimization model overview. 33

3.3.2 Model formulation. 34

3.3.3 Model description. 35

3.4 Model 2 -Optimization of the Retention Limit (M), Minimizing Retained Claims Variance Given ROC Constraints – Limited Single Layer Treaty *(XL, M, 1, m)* 39

3.4.1 Optimization model overview. 39

3.4.2 Model formulation. 40

3.4.3 Model description. 40

3.5 Model 3 - Optimization of the Retention Limit (M), Maximizing Insurer's Return on Capital – Unlimited Single Layer Treaty *(XL, M,1, ∞)* 43

3.5.1 Optimization model overview. 43

3.5.2 Model formulation. 43

3.5.3 Model description. 44

3.6 Model 4 - Optimization of the Retention Limit (M), Maximizing Insurer's Return on Capital – Limited Single Layer Treaty *(XL, M, 1, m)* 45

3.6.1 Optimization model overview. 45

3.6.2 Model formulation. 45

3.6.3 Model description. 46

Chapter 4: Optimization Models Discussion of Results 48

4.1 Sensitivity Analysis 48

4.2 Optimization Models Results 50

Chapter 5: Conclusion and Recommendations 53

References 55

Vita 59

List of Figures

Figure 1: Insurance Chain – Premium, Risk and Claims Flow Chart [4] 14
Figure 2: Types of Reinsurance [5] 15
Figure 3: Limited Single Layer Reinsurance Treaty... 18
Figure 4: Re-insurance Source Chain [11]... 20
Figure 5: Unlimited Single Layer Reinsurance Treaty 34
Figure 6: Historical Claims Distribution Fitting.. 36
Figure 7: Simulated Claims' Data Sample... 37
Figure 8: Optimization Model Parameters Definition 38
Figure 9: RiskOptimizer - Optimization Model Definition 38
Figure 10: Optimization Model Solution.. 39
Figure 11: Simulated Claims' Data and Limited Layer Claims' Split Sample............ 42

List of Tables

Table 1: Reinsurance Agreement Classification Notation..30
Table 2: Example of Reinsurance Agreement Notations..31
Table 3: Retained and Ceded Claims Distribution 41
Table 4: Model 2 Optimization Results..43
Table 5: Retained and Ceded Claims Distribution 46
Table 6: Model 4 Optimization Results..47
Table 7: Model Parameters' Dependencies 48
Table 8: Sensitivity Analysis - Impact of Gross Premium Risk Loading on ROC......49
Table 9: Sensitivity Analysis - Impact of U on ROC 49
Table 10: Sensitivity Analysis - Impact of M on ROC...50
Table 11: Optimization Models' Optimal Solutions 50

Glossary

Reinsurance Contract: A mutual agreement between insurance companies and third-party reinsurance companies.

Facultative Reinsurance: Insurers select individual and specific loss exposures to be covered by the reinsurer subject to reinsurer's cover approval or rejection.

Treaty Reinsurance: Reinsurance treaty covers all loss exposures covered by an insurer's portfolio or line of business and all individual losses that are covered under the reinsurance contract treaty without exceptions.

Proportional Reinsurance: Insurers share risks and premiums of issued policies with reinsurers.

Non-proportional Reinsurance: Insurers cede risks and liabilities of issued policies exceeding a determined limit of loss, excess amount, or also known as insurer's retention limit, in return of a reinsurance premium.

Quota Share Reinsurance: Type of proportional reinsurance where the insurer cedes an agreed percentage of losses to the reinsurer.

Surplus Reinsurance: Type of proportional reinsurance where the insurer cedes losses proportional to the reinsurer's share of total coverage limit

Stop Loss Reinsurance: Type of non-proportional reinsurance where the insurer cedes total/ aggregate claims in excess of predetermined amount.

Excess of Loss Reinsurance: Type of non-proportional reinsurance where the insurer cedes individual claims in excess of predetermined amount.

Return on Capital: Ratio that measures a company's profitability and return on the capital and investments contributed by shareholders.

Chapter 1: Introduction

This chapter introduces and provides an overview of general insurance and reinsurance principles, followed by the problem description. The research objective, its significance to the industry, and methodology are further discussed. The last section introduces the structure of the thesis.

1.1 Overview

In exchange of a premium payment from the insured, insurance companies provide their clients with financial compensation and protection against loss, damage, health incidents or death; thus transferring the risks from the insured to the insurer. Consequently, insurance companies are subject to significant risks of financial loss occurrences, depending on the severity and frequency associated with these risks and loss occurrences. Insurance policies, sometimes requested individually by the insured or mandated by laws, can be categorized into two main groups; general and life insurance. General insurance covers properties and liabilities, whereas life insurance insures people. Most common forms of life insurance are life, medical and pension insurance. However, most common types of general insurance include, but not limited to, motor, fire, engineering, workman compensation, marine and professional indemnity covers [1].

Different from most industries in the UAE such as construction, insurance industry has experienced massive growth after the economic crisis in 2009. The rapid and huge growth and advancement in medical insurance specifically contributed to the rapid growth of the insurance industry outlook. The report published in 2011, “UAE Insurance Market Forecast to 2012” reveals that insurance in the UAE is expected to experience rapid developments and growth, and is one of the fastest growing industries in the MENA region [2].

According to “UAE Insurance Report Q1 2013” [3], the growth in the healthcare industry and increase in demand of medical insurance are expected to continue for at least the next five years. One of the primary factors behind this strong development is the increasing population and number of expatriates in the country. Consequently, private healthcare sector market share and growth are on the rise; thus the country’s healthcare spend. In addition, the countries’ health regulatory authorities, such as Health Authority of Abu Dhabi, HAAD, have been announcing plans of new regulations, which influence the pricings and costs of healthcare services and delivery. Overrated healthcare services such as medical tests have been imposed a decrease of approximately 24%; whereas consultations services have been increased by a range of 15 – 25 %.

UAE Nationals are provided coverage of medical services by the government. Compulsory regulations of medical insurance have been recently imposed for expatriates in both Dubai and Abu Dhabi, and are expected to be expanded and implemented in remaining Emirates in the future. New regulations further mandate organizations to provide their employees with medical insurance cover. Consequently, increased costs of healthcare services have been shed away from providers and the public; therefore, reducing the effects of the costs' increase and inflation on the public and healthcare providers. These regulations also increased insurers' share of medical insurance services within the country; thus generating more premiums as well as increasing their risk exposure portfolio. In order to ensure that organizations conform to providing all their employees with medical coverage; regulatory authorities, HAAD, had imposed fines on organizations which fail to conform [3].

1.1.1 Reinsurance principles. In many situations, insurance companies are insured by reinsurance companies, through reinsurance contracts; a mutual agreement between insurance companies and third-party reinsurance companies. Through reinsurance contracts, insurers cede or share risks and liabilities of their issued policies with reinsurers in return of a reinsurance premium or ceding commission as shown in Figure 1. The insurer is solely obligated with fulfilling the responsibilities, services, and handling of claims and benefits to the insured in accordance with policy terms. Consequently, reinsurance does not alter or impact insurance terms of the policies between the insurer and insured, yet it is a form of financial protection to insurance companies against insurance risks and uncertainties of policy premiums insufficiency to cover incurred losses. Reinsurance contracts benefit insurance companies in various ways. Reinsurance can benefit them through stabilizing and reducing the variability of their loss experiences associated with risk exposures. Similarly, reinsurance enhances the insurer's financial stability and allows for an increase in its large line capacity, and maximum amount of limit of liability on a single loss exposure. Depending on the existence and nature of a reinsurance structure, the insurer's premiums, risks and losses are defined [1].

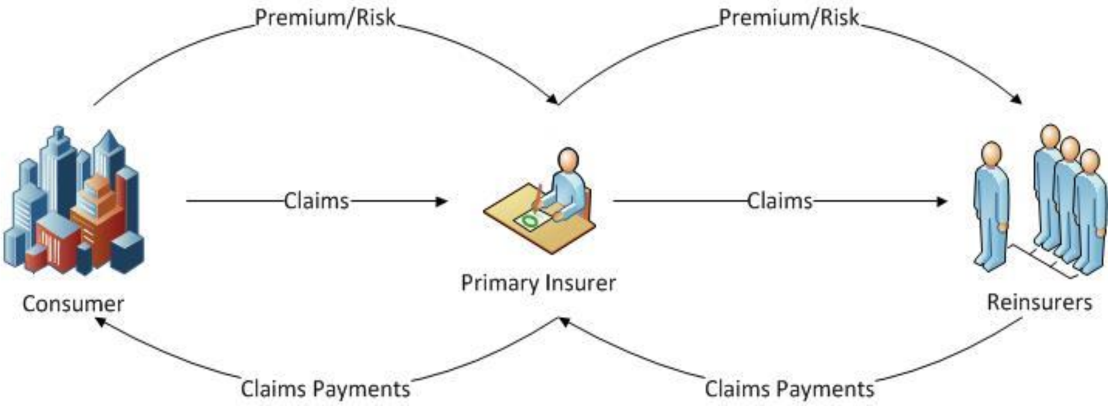

Figure 1: Insurance Chain – Premium, Risk and Claims Flow Chart [4]

Reinsurance contracts come in one of two forms: Facultative or Treaty Reinsurance. The difference between facultative and treaty reinsurance depends on loss exposures covered under the reinsurance contract. Reinsurance treaties cover all loss exposures covered by an insurer's portfolio or line of business and all individual losses are automatically covered and insured under reinsurance treaty, without any exceptions. On the other hand, insurers select individual and specific loss exposures to be covered under the facultative cover, and subject to reinsurer's cover approval or rejection. As shown in the Figure below, there are two main types of reinsurance; proportional or non-proportional; both of which fall under both facultative and treaty reinsurance. In proportional reinsurance, both parties, the insurer and the reinsurer, share proportions of the insurance premiums and losses either in the form of fixed percentages or fixed amounts. Most common forms of proportional reinsurance are quota share (QS) and surplus reinsurance (S). On the other hand, in non-proportional reinsurance, the reinsurer is liable of losses exceeding an insurer's determined limit of loss, excess amount or also known as insurer's retention limit. Within the contract, the reinsurer may also determine and specify the limits of maximum liability it is willing to be accounted for exceeding the excess of loss amount. Most common forms of non-proportional reinsurance are excess of loss (XL) and stop-loss (SL) reinsurance [1].

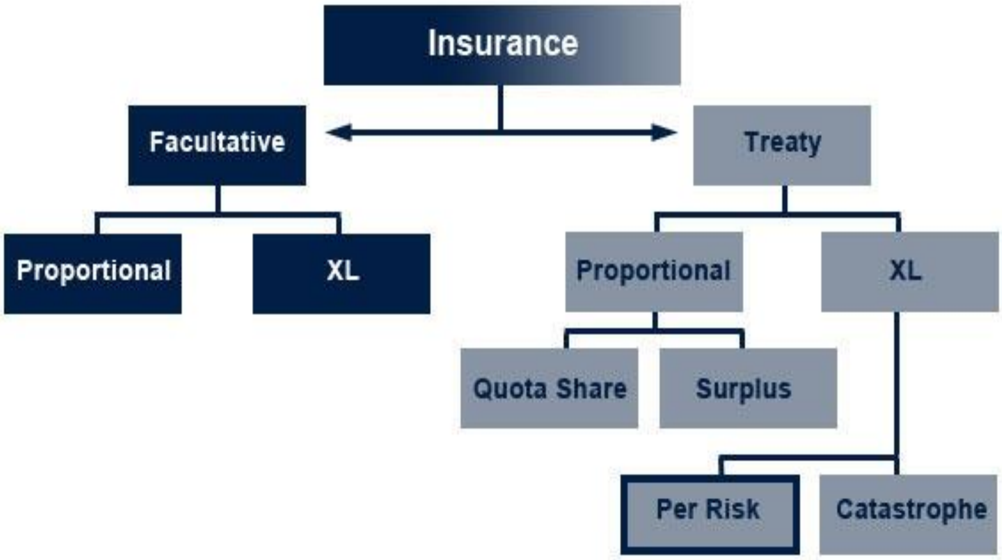

Figure 2: Types of Reinsurance [5]

1.1.2 Non-proportional reinsurance. In stop loss reinsurance, the loss ceded to the reinsurer is in excess of aggregate (total) claims incurred by the insurer. On the other hand, in excess of loss reinsurance, the reinsurer's share of losses is in excess of each individual loss; per risk (event) of each policy covered by the cedent, or per claim. The difference between both stop loss and excess of loss is further clarified below [1].

The notations used in the thesis are as follows;

N: Risk portfolio size – Number of Claims.

X_i: Non-negative Random Independent Individual Claim Incurred; $1 \leq i \leq N$.

X: Non-negative Random Total Loss Incurred – Aggregate Claims.

I: Amount of Loss retained by ceding insurer.

R: Amount of Loss ceded to reinsurer.

f(X): Probability density function of *X*.

F(X): Cumulative distribution function of *X*.

E(X): Expected value of losses incurred.

E(I): Expected value of losses retained.

E(R): Expected value of losses ceded.

M: Insurer's Retention limit – Deductible.

Such that:

$$X = \sum_{i=1}^{N} X_i$$

$$E(X_i) = \int_0^{\infty} x_i f(x_i) dx_i$$

$$E(X) = \sum_{i=1}^{\infty} E(X_i)$$

When an insurer purchases a reinsurance contract; *(X)* is a function of the loss retained by the insurer *(I)* and the loss ceded to the reinsurer *(R)*.

$$X = I + R$$

The definition of *(I)* and *(R)* depends on the type of non-proportional reinsurance contract in place. Losses in excess of the deductible amount *(M)* are ceded to the reinsurer in non-proportional reinsurance, stop loss and excess of loss contracts. The deductible/retention limit *(M)* is the insurer's maximum retention of claims before ceding to the reinsurer.

In ***stop loss reinsurance***, the loss retained and ceded respectively are:

$$I = \begin{cases} X, & X \leq M \\ M, & X > M \end{cases} = \min(X, M)$$

$$R = \begin{cases} 0, & X \leq M \\ X - M, & X > M \end{cases} = \max(0, X - M)$$

In ***excess of loss reinsurance***, the loss functions of the insurer and reinsurer, per claim or risk *(i)*, depending on whether evaluated per loss or per risk covered basis.

$$I(i) = \begin{cases} X_i, & X_i \leq M \\ M, & X_i > M \end{cases} = \min\ (X_i, M)$$

$$R(i) = \begin{cases} 0, & X_i \leq M \\ X_i - M, & X_i > M \end{cases} = \max\ (0, X_i - M)$$

Such that,

$$I = \Sigma I(i) \quad and \quad R = \Sigma R(i)$$

$$E(I) = \Sigma E(I(i)) \quad and \quad E(R) = \Sigma \text{E}(R(i))$$

The expected value of total loss *(X)* is a function of the expected loss retained and the expected loss ceded

$$E(X) = E(I) + E(R)$$

The expected loss retained by the insurer and ceded to reinsurer respectively are expressed as

$$E(I) = E[\min(X, M)] = \int_0^M x\, f(x)dx + M\,[1 - F_x(M)]$$

$$E(R) = E[\max(X - M, 0)] = \int_M^\infty (x - M)\, f_x(x)dx = \int_0^\infty x\, f_x(x + M)dx$$

The purpose of acquiring a reinsurance contract is to reduce the impact and variability of claims incurred on an Insurer's portfolio. Consequently, the sum of the variance of retained claims by the insurer and claims ceded to reinsurance shall be less than the variance of the total loss incurred; expressed as follows

$$\sigma_x^2 \geq \sigma_I^2 + \sigma_R^2$$

$$\sigma_I^2 = E(I^2) - E(I)^2$$

$$\sigma_I^2 = \int_0^M x^2\, f(x)dx + M^2[1 - F_x(M)] - (\int_0^M x\, f(x)dx + M\,[1 - F_x(M)])^2$$

$$\sigma_R^2 = E(R^2) - E(R)^2 = \int_M^\propto (x - M)^2 f_x(x)dx - (\int_M^\propto (x - M)\, f_x(x)dx)^2$$

Such that: σ_x^2 Variance of Claims Incurred

σ_I^2 Variance of Retained Claims

σ_R^2 Variance of Ceded Claims

Non-proportional reinsurance agreements may be either limited or unlimited. In unlimited excess of loss, the reinsurer covers all amounts of insurer's losses in excess of (M) without bounds, maximum amount limit. However, in limited excess of loss, by defining a maximum limit (L) the reinsurer limits its liability of losses per layer to the layer size (m). The size of each layer (m) defines the maximum amount of losses that a reinsurer is liable to the cedent in excess of the deductible (M) as per the Figure below. For losses exceeding the layer limit (X>L), the insurer retains the difference between the claim value and the layer size ceded to the reinsurer *(m)*.

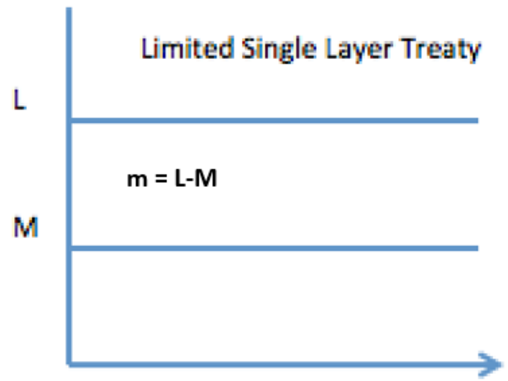

Figure 3: Limited Single Layer Reinsurance Treaty

Several insurance premium calculation principles have been proposed and implemented in the actuarial literature. Most commonly used general principles are: Expected value, variance, standard deviation, and exponential principles. The primary differences between each of the above common premium principles are related to how the risk is loaded. For instance, the expected value premium principle loads the risk – Expected loss (E(X))- proportionately by a number greater than or equal to one. The variance and standard deviation principles apply a risk proportional to the risk's variance and standard deviation respectively. Moreover, in the literature review context, some researchers attempted to use more convex premium principles such as the Wang, Dutch, semi-variance and semi deviation premium principles [6,7].

Assuming mean (expected value) premium calculation principle with a risk loading (θ) of 30% (20% and 10% expenses and profit respectively), the gross premium collected (P) is a function of the expected loss loaded by (θ) as follows

$$P = \sum_{i=1}^{\infty} P_i$$

$$P_i = (1 + \theta)\ E(X)$$

The net premium retained by the insurer (P_I) is expressed as the difference between the gross premiums insurer gains from its customers in exchange of insurance covers (P), and the reinsurance premium (P_R), the insurer incurs as cost of purchasing the reinsurance contract.

$$P = P_I + P_R$$

$$P_I = P - P_R$$

Insurance companies need to ensure business profitability, and satisfy solvency responsibility towards policyholders and shareholders. Capital supplied by shareholders is an essential requirement in the insurance industry. Hence, it should be maintained at a level that covers a company's risks and solvency [8]. In return of supplying the capital, shareholders and capital suppliers expect a targeted return on capital to be achieved by the end of a cycle and to deliver profitability of the business. Return on capital also evaluates the cost of capital to the company [9]. Assuming fixed insurer's capital (U), the insurer's return on capital (ROC) is expressed as

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

1.2 Problem Description

An insurance company mainly has two options for managing its risks and losses; full retention of premiums and losses without a reinsurance structure, or purchase a reinsurance contract, which limits its losses and risks by ceding portions of the risks to reinsurance companies.

Prior to the purchase of reinsurance contract, the current practice in the region, small to medium insurance companies rarely conduct internal analysis of their data and experiences in order to evaluate and achieve optimal reinsurance arrangements and contracts. Few insurance companies interact and deal directly with reinsurers. Most companies settle their reinsurance agreements through reinsurance intermediary, broker, which acts as the link of communication, negotiation and settlement between both the reinsurers and the ceding insurer. The broker provides different reinsurers with the ceding companies' historical loss and exposure data to be analyzed and evaluated in order to develop reinsurance terms accordingly. Through analyzing the insurer's data, reinsurance companies develop their proposed treaties, define layers and limit within each treaty, as well as the cost associated with the reinsurance agreement. The broker in turn communicates the proposed treaties and costs to the ceding company, and through negotiations, the reinsurance agreements are settled accordingly between both primary parties, ceding and reinsurance companies. In some cases, the intermediary conducts the analysis and accordingly negotiates the premiums and reinsurance agreements between the insurer and reinsurer [10].

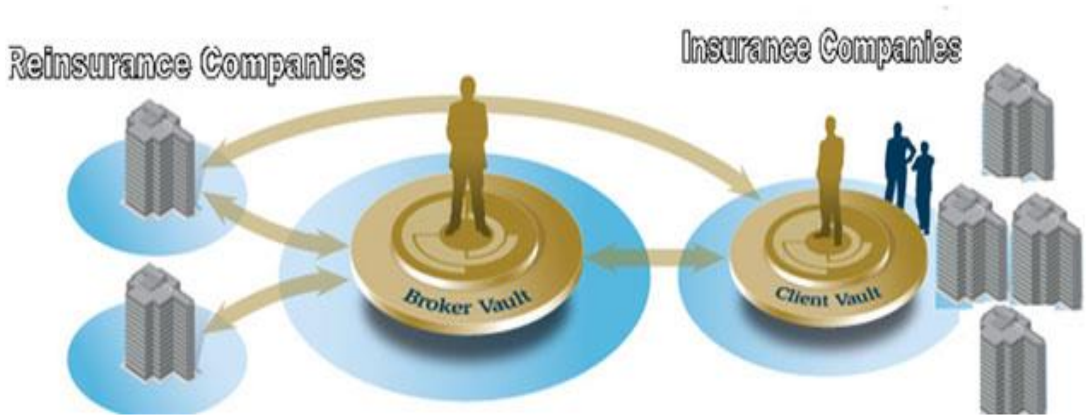

Figure 4: Re-insurance Source Chain [11]

Reinsurance companies offer their prices based on the analysis they have conducted on the insurer's experience and historical data. The reinsurance cost to the insurer includes an increase/loading to account for reinsurer's profitability or broker's commission. Prices offered from reinsurer's perspective and analysis, and agreed during the negotiation and agreement don't necessarily reflect the cedent's optimal reinsurance structure or cost [10].

Several studies related to insurance and reinsurance agreements have been conducted, especially on optimal reinsurance arrangements. Some studies explore the benefits of reinsurance agreements to insurers and approaches to defining the optimal reinsurance structure. However, the majority of the studies evaluate optimal reinsurance arrangements and contracts from a reinsurer's perspective or from a joint perspective.

Taking into account the different studies conducted related to optimal reinsurance contracts, the current common practice of reinsurance contracts' purchases in the regional insurance industry, a portfolio's past experiences and insurer's financial and risk requirements, financial models and optimization tools will be developed to allow insurance companies evaluate and estimate the optimal excess of loss reinsurance arrangement independently.

1.3 Research Objective

The objective of this research is to develop financial tools and models that allow insurance companies to evaluate the optimal excess of loss reinsurance structure that satisfies the risk appetite and financial requirements, by defining the optimal reinsurance contract limit, retention limit (M). Models developed evaluate the optimal arrangements of unlimited and limited single layer excess of loss reinsurance. Provided several assumptions and constraints, minimum insurance regulations, and financial ROC constraints/requirements, the optimal limit minimizes the insurer's variance of retained claims. The model that maximizes an insurer's return on capital is also evaluated, to get an understanding of the optimal approach to define the reinsurance agreement to be adopted. The focus of the study is on per claim excess of loss reinsurance structure; such that losses are ceded to the reinsurer per individual loss. A local insurance company's medical insurance historical claims, risk exposure and premiums will be used to build the model accordingly, assuming medical insurance is the company's primary line of business.

1.4 Research Significance

The research's primary contribution is to develop a simplified practical optimization model and financial tool which adds value to the insurance industry, specifically small to medium insurance companies. The objective is to design the model for insurance companies' use, in order to analyze and realize their optimal excess of loss reinsurance structure prior to settling their reinsurance agreements, while satisfying regulation's and shareholders' minimum financial requirements.

Consequently, actual experience and exposure historical data of a local insurance company are utilized to develop and validate the model accordingly.

As noted from the literature review, most common research studies focused on stop-loss reinsurance optimality rather than excess of loss. Moreover, the literature review and studies related to minimizing the variance of insurer's retention implemented complicated mathematical and optimization approaches. Taking these observations into account, this approach will be tested and the optimization model will be formulated accordingly, in order to introduce to the industry a simplified approach of optimizing the excess of loss reinsurance structure. The models in the research are built on a user-friendly Risk simulation software, Palisade @Risk, which facilitates the application of the model in the industry.

1.5 Research Methodology

In order to address the research objective and develop the optimization model, the following steps will be followed.

Step 1: Researching literature review of reinsurance structure optimization

Step 2: Defining and formulating the optimization models as well as associated assumptions, parameters, and constraints

Step 3: Building and running the optimization models on @Risk

Step 4: Evaluating and comparing the optimization models' solutions

1.6 Thesis Organization

Chapter 1 introduces insurance and reinsurance principles, the problem description, research objective and methodology. Overview of the literature review related to reinsurance structure optimization is provided in Chapter 2. Chapter 3 presents and describes the model of optimization of insurer's retention limit (M), minimizing insurer's variance of retention given return on capital constraints for an unlimited single reinsurance layer. Followed by Chapter 4, which presents the optimization model for a limited single layer reinsurance structure, defining the optimal retention limit (M) and ceding limit (L), minimizing insurer's variance of retention given return on capital constraints. Chapter 5 presents the unlimited single layer optimization of insurer's retention limit, maximizing insurer's return on capital, and Chapter 6 addresses the insurer's retention and ceding limit optimization, maximizing return on capital for a limited single layer treaty. Chapter 7, which is the last one, evaluates the optimal solutions of the four-optimization models, summarizes the research and presents recommendations and future contribution.

Chapter 2: Literature Review

Optimal reinsurance has been frequently studied and addressed in the literature whether from the insurer's perspective or from the perspective and benefit of both the insurer and reinsurer. Researchers studied the optimum reinsurance strategies and types by varying different parameters such as; the method of premium calculation, the optimization objectives and the risk measures etc.

2.1 Insurer's Optimal Reinsurance Structure

Early studies primarily revolved around the optimization from an insurer's perspective. Some researchers adopted the expected premium principle to determine the optimal reinsurance. Borch [12] proved that stop loss reinsurance minimizes the variance of the retained losses. On the other hand, Arrow [13] proved that stop loss also maximizes an insurer's expected utility of terminal wealth. Furthermore, Cai et al. [14] have proved that, assuming increasing convex ceded loss function, and minimizing value at risk (VaR) and conditional tail expectation (CTE) risk measures of insurer's total cost, the optimum reinsurance strategy of ceded loss functions depends on the required level of confidence and safety loading of expected premium. The strategies that were taken into consideration in this study were stop loss, quota share and change loss. On the other hand, Balbas et al. [15] have attempted to find the optimum reinsurance problem and retention level that minimizes the risk of the total cost using general risk measures, including every deviation measure, expectation bounded risk measure, and most coherent and convex risk measures. They have concluded that, under the assumption of convex premium principles, quota share barely realizes optimization regardless of the risk function, whereas stop loss satisfies optimum conditions much more frequently. Extending their research results, they attempted to seek the "stable optimum retention level" which remains stable regardless of the risk measure used [16]. The conclusion of their research was that the optimum retention level of the stop-loss reinsurance is the stable optimum solution; thus it ensures a robust reinsurance plan regardless of the risk measure used for the optimization problem.

Extending Cai et al. [14] research, Chi and Tan [17] developed a simplified approach for finding the optimal reinsurance limits which minimize the VaR and conditional Value at Risk (CVaR) measures, by comparing stop loss, limited stop loss and truncated stop loss reinsurance. Stop loss reinsurance doesn't impose a maximum retention limit on the reinsurer, whereas the limited stop loss has a maximum limit of reinsurer liability and requires both the cedent and the insurer to be willing to pay more for larger losses. Meanwhile, truncated stop loss requires the cedent to transfer only moderate level losses to the reinsurer. They have studied the VaR model under two different constraints; *(1)* ceded and loss retained functions are increasing *(2)* loss retained function is increasing and left continuous.

Their study has proved that the VaR model is sensitive to the constraints set on ceded and retained loss functions, such that limited stop loss is optimum under the first constraint, whereas truncated stop loss is optimal under the second constraint. Nevertheless, the CVaR model proved to be robust such that regardless of the constraints, the stop loss reinsurance strategy remained optimum and consistent. Also, Tan and Weng [18] established an optimization problem which accounts for the insurer's optimization and tradeoff between risk and profitability. Subject to constrained insurer's profitability, the objective of the model was to derive the optimum reinsurance which minimizes the VaR of the insurer's net risk, using the expected value premium and assuming increasing and convex ceded loss functions. They have proved that the optimal reinsurance strategy would be quota share, pure stop loss, or a combination of stop loss and quota share, depending on the choice of level of confidence, safety loading of reinsurance premium, and expected insurer's profit.

Additionally, further studies have been conducted with respect to optimal reinsurance with premium calculation principles other than the expected value in the above mentioned studies. Kaluszka [19] derived the optimal reinsurance limits which minimize the cedent's variance of retained loss, using the mean- variance premium principles. In their research, Gajec and Zagrodny [20] proved that using symmetric and asymmetric general risk measures and the standard deviation premium principle, limited stop loss and change-loss reinsurance are the optimal strategies. Consequently, they derived the optimization problem of retention limits that minimize the insurer's risk measures. Furthermore, Kaluszka [21] proved that a combination of stop loss and quota share, or limited stop loss reinsurance are the most optimal strategies to minimize a cedent's convex measure of retained risk or maximize a utility function using different convex premium calculation methods. By setting a fixed reinsurance premium, the convex premium calculation methods he used in his research include exponential, p mean, semi deviation, semi-variance, Wang and Dutch. Extending their earlier research, Chi and Tan [22] have investigated optimization model, using the standard deviation and variance premium principles under VaR and CVaR. They proved that the layer reinsurance is the robust optimal solution for both VaR and CVaR, and defined the optimal layer's parameters.

On the other hand, in another study, assuming the reinsurance premium has distribution invariance, risk loading and stop loss ordering, Chi and Tan [23] excluded variance, standard deviation and Esscher principles from their study since they don't satisfy the stop loss ordering criterion. By using Wang and Dutch general premium principles and the constraint of increasing ceded and retained loss functions, they proved that layer reinsurance is always optimal for both VaR and CVaR models. Thus the optimization model defined both the retention level of the cedent and the maximum limit of reinsurance.

Jang [24] studied the factors that affect the catastrophe excess of loss reinsurance retentions and upper limits in the property liability line of business. Using two-stage least square regression, he identified and proved the hypotheses which support relationships among retentions, upper limits and co-insurance rates. The model's dependent variable was the reinsurance retention; whereas the independent variables were the upper limit co insurance rates, catastrophe exposures, catastrophe reinsurance price and other firm characteristics. Understanding the influence and relationship of these parameters on the reinsurance retentions help property-liability insurers in identifying and defining their reinsurance strategies. On the other hand, in further research, Kaluszka [25] proved that using mean-variance premium principles and minimizing the variance of the retained loss at a fixed expected cedent gain, quota share, excess of loss, or a combination of quota share and excess of loss are the optimal reinsurance strategies.

Alternatively, Centeno [26] defined the optimum excess of loss retention limits (m_1 & m_2) for two dependent risks using two objective functions; maximizing insurer's expected utility of wealth net of reinsurance with respect to an exponential utility function and maximizing the adjustment coefficient of retained business respectively. Both models were developed using the expected value premium principle along with the assumption of claims' bivariate passion distribution.

2.2 Insurer and Reinsurer's Optimal Reinsurance Structure

Other optimization problems were modeled taking into account the perspective and well-being of both the cedent and the reinsurer. Kaishev [27] addressed the optimization of excess of loss reinsurance under joint survival probability of parties, insurer and reinsurer, assuming Poisson claims distribution. Two optimization models with two risk measures were built, and each of which was implemented on two different constraints. The first optimization objective function was to maximize the joint survival probability, whereas the second model was minimizing the difference between the cedent's survival probability and the reinsurer's survival probability given cedent's survival probability). Similarly, the first constraint used in the model was fixing the premium proportion retained by the insurer and determining the optimum unlimited retention level accordingly. The second constraint was to fix the unlimited retention limit and determining the optimal reinsurance proportion accordingly. They have proved that second optimization model is better off to the insurer since the first model defines a higher optimum retention at a fixed premium proportion.

Kaishev and Dimitrova [28] further contributed to the above research by defining limited excess of loss retention level with optimal maximum limit of reinsurance liability and premium proportion respectively, which maximize the joint survival probability under the same constraints used in Kaishev's [27] research. They also added to their contribution by modeling dependent claims with copula functions, and observing the effect of varying the dependence parameters on the optimal solution. They have further extended their research and contribution by developing an efficiency frontier approach towards setting limiting and retention levels, which maximize the expected profits of both the reinsurer and cedent for a given level of joint survival probability [29]. The joint survival probability and the expected profit given joint survival were used as the risk measure and performance measure respectively. Assuming linear premium function as well as dependent and independent claim severities, the constraints used in the model included fixed premium distribution between the cedent and reinsurer and transfer the premium split into ratio of expected profits respectively. Optimal retention and limiting levels (m) and (L); which provide fair distribution of expected profits based on the premium allocation, were defined by the model accordingly.

Alternatively, Li [30] resorted to the expected value premium calculation method to develop optimal combinations of quota share and excess of loss reinsurance strategies under ruin-related optimization criteria. The developed models' optimization criteria were maximization of the joint survival probability of both parties and maximizing the lower bound of joint survival probability respectively. The optimal retentions of quota-share and excess-of-loss combined reinsurance under both optimization models have been defined, and the impact of economic and financial factors on the optimal retentions were explored; these factors are the influence of interest, dividends, commission, expense, and diffusion. Yusong and Jin [31] derived the optimum retention levels for both proportional and excess of loss reinsurance respectively which maximize the combination of rate of return of insurer and reinsurer correspondingly; such that they exceed amount of claim held by the reinsurer by a specific probability. These models were derived by assuming that investment funds follow log normal distribution, and using expectation premium principle for reinsurance premium.

2.3 Investments, Capital and Optimal Reinsurance Structure

On the other hand, many researches and studies have been conducted in the attempt to evaluate and assess the relationships between investments, capital and reinsurance. Some studies focused on optimizing the reinsurance structure taking into account investment related metric; whereas others studied optimal capital, risk and investment allocation considering reinsurance as a factor and optimization related parameter.

Many researchers examined the topic of optimal capital structure for insurance companies. Asmussen et al. [32] attempted to determine an insurer's optimal reinsurance structure that balances its expected profits and risks. Hence, they developed a model that determines the optimal excess of loss retention limit and dividend distribution policy, by maximizing the total expected discounted value of all paid out dividends. They have concluded that the excess of loss reinsurance is more optimal than proportional reinsurance, and it maximizes the insurer's adjustment coefficient in case of expected value premium calculation. In 2003, Froot [33] has developed a framework which analyzes the risk allocation, capital budgeting, and capital structure decisions facing both insurers and reinsurers. Using a three-factor model which maximizes the expected difference of dividend payments and the discounted costs of capital injection, the optimal amount of surplus capital held by a firm and the optimal allocation and the pricing of risky investments, underwriting, reinsurance and hedging opportunities are established. Moreover, a microeconomic financial model was developed by Laeven and Perotti [34], such that it designs the optimal solvency capital regulation as well as the optimal solvency capital that an insurance company should hold. The model analyzes and takes into account various aspects related to the economic trade-offs underlying the optimal design. Meng and Siu [35], investigated and developed a model for an insurer's optimal reinsurance, dividend and reinvestment strategies which maximize the difference between expected discounted dividends and expected discounted reinvestment until time of ruin. The model accounts for an excess of loss reinsurance and assumes that the insurer has both fixed and proportional costs. They developed a model and a solution for the optimal XL reinsurance as well an explicit expression of the optimal value's function, using an optimal impulse control approach and inventory control theory techniques.

Some researchers, such as Mitschele et al. [36] and Cortes et al. [4], approached the problem of insurer's reinsurance structure using multi-objective optimization methods. Insurance companies could either place a single, individual reinsurance type agreement, or a reinsurance program. Reinsurance program is composed of a number of reinsurance agreements. Most reinsurance optimization studies evaluated optimization problems for individual reinsurance agreements, and few researches evaluated opportunities of optimizing reinsurance programs. Hence, Mitschele et al. [36] developed a multi-objective optimization model of reinsurance contracts in a reinsurance program that minimizes the expense of the reinsurance contracts as well as the cedent's retained risks using the expected value premium principle. Developed for QS, SL and XL reinsurance types, Mitschele et al. used a modified Mean-Variance optimization criterion and multi-objective evolutionary algorithms, which optimize the allocation of reinsurance contracts taking into account the insurer's tradeoff between risk and return. They concluded that applying the variance model, for a combination of QS, SL and XL contracts; SL is the optimal agreement. On a second note, using the VaR and CVaR risk measures, the combination of QS and XL are the optimal solution. At last but not least, on a reinsurance program of QS, as well as both unlimited XL and SL; the unlimited SL is the insurer's optimal solution.

Similarly, Cortes et al. [4] also addressed an excess of loss reinsurance multi-objective optimization problem targeting both the risk value and the expected return. They developed a Pareto frontier that models an insurer's optimal combinations of excess of loss reinsurance placements which minimize the risk value at a given expected return; optimizing the tradeoff between both. The modelling approach was based on discretized Population based incremental learning PBIL, an evolutionary heuristic search method assuming fixed number of treaty layers as well as simulated expected loss distribution. The discretized PBIL model developed solved reinsurance treaty optimization problems with higher time efficiency compared to exact enumeration methods. 7 to 15 layers were solvable in less than a day (minimum time frame of 1hr20 minutes) versus other approaches resolution of less than 7 layers in a day and more layers were solved within a week or resulted in unfeasible solutions. They have extended their research to explore the best metaheuristics approach, swarm and evolutionary logarithms, that determine the reinsurance layers and the share of these layers to be purchased by the cedent at the optimal time efficiency [37].

2.4 Contribution to the Literature – Classification of Reinsurance Agreements

This thesis attempts to contribute to the literature by introducing classification for reinsurance models. This notation classifies the reinsurance agreement structure based on the reinsurance contract type, number of layers, and the ceded layer size as explained and illustrated in an example in Tables 1 and 2. For each reinsurance agreement with defined values of agreement parameters, retention and ceding limit and layers' size, the values of respective parameters can be replaced in the notation to provide a comprehensive overview of the reinsurance nature and parameters.

Table 1: Reinsurance Agreement Classification Notation

Reinsurance Type	Retention	No of Layers	Layers' Size	Notation
Proportional (QS, S)	x %	-	$(1-x)$ %	(QS/S, x %, $[1-x]$ %)
Non-proportional (XL, SL)	M	$(1,2,..j)$	$m_1 \ldots m_j$	(XL/SL, M, j, m_1,.., m_j)

j = number of reinsurance treaty layers

Reinsurance contracts notations' examples are presented in Table 2 and explained below:

1) Quota share reinsurance agreement where the insurer retains 40% of risks and premiums, and the remainder 60% is ceded to the reinsurer.
2) Excess of loss arrangement is contracted such that the insurer's deductible per loss is 50K. The reinsurance contract is an unlimited single layer treaty, hence the reinsurer's liability of claims' values in excess of 50K is unlimited.
3) Limited single layer stop loss contract is set with a deductible of 5M (Million) on the aggregate claims incurred in the portfolio to cap the insurer's retention. Once the aggregate claims on the portfolio exceed 5M, liability of the losses is ceded to the reinsurer with maximum limit of 49M.
4) Double layered excess of loss contract is defined by a deductible of 25K, and the size of each layer is 25K and 50K respectively as illustrated in the Table. In multiple layered reinsurance agreement, if the layer size is equal across all layers, then the notation would be represented as (XL, 25K, 2, 25K)

Table 2: Example of Reinsurance Agreement Notations

Reinsurance Type	Retention	No of Layers	Layers' Size	Notation
QS	*40 %*	-	*60 %*	(QS, 0.4, 0.6)
XL	50K	1	∞	(XL, 50K,1, ∞)
SL	5M	1	$m_1 = L - M$ =50– 5 = *49M*	(SL, 5M, 1, 45M)
XL	25K	2	$m_1 = L_1 - M$ $m_1 = 50 - 25 = 25K$ $m_2 = L_2 - L_1$ $= 100 - 50 = 50K$	(XL, 25K, 2, 25K, 50k)

Chapter 3: Optimization Models

This chapter introduces and discusses the optimization models developed separately. Two objectives are tested; minimizing insurer's variance of retained claims and maximizing insurer's return on capital respectively. For each objective function, unlimited and limited single layer reinsurance agreements are evaluated. Models' parameters and assumptions are applied consistently throughout all models, as well as majority of the approach and steps implemented. Primary differences lie around, the objective functions definition and some optimization constraints. This chapter explains the models and how they were built and run.

3.1 Optimization Models Assumptions

The models are built taking into account below assumptions:

1. Medical Insurance is the insurer's only line of business, with policy limit of AED 150,000.
2. Per claim excess of loss reinsurance structure for one-year period
3. Average insurer's exposure per year is 6,000 policy holders, and in average each insured claims 5 times a year. So number of claims (N) is 30,000.
4. Historical data used to determine and define the distribution function of insurer's claims, in order to simulate new claims, are two years old (2013). Inflation rate of 10% per year is applied on the simulated incurred claims, such that the model's simulated claims are normalized and consistent with current inflation and claims' rates [38].
5. Insurer's Capital (U) is greater than or equal to regulator's minimum capital requirements. UAE Insurance Authority minimum capital requirement is of AED 100M [39]. Values of the Insurer's Capital and Minimum Return on Capital are assumed throughout the model.
6. Premiums, gross premium and reinsurance premium, are calculated using expected value premium principle with risk loading (θ) of 30% (20% and 10% expenses and profit respectively).
7. For limited reinsurance structure, reinsurance maximum limits assumed and tested are 25,000, 35,000, and 50,000

3.2 Optimization Models Parameters

The models' parameters are:

σ_x^2: Variance of Incurred Claims.

σ_I^2: Variance of Retained Claims.

σ_R^2: Variance of Ceded Claims.

P: Gross Premium Collected from Policyholders.

P_R: Reinsurance Premium in Exchange of Reinsurance Agreement.

P_I: Insurer's Net Premium.

U: Insurer's Capital.

ROC: Insurer's Return on Capital.

3.3 Model 1 - Optimization of the Retention Limit (M), Minimizing Retained Claims Variance Given ROC Constraints – Unlimited Single Layer Treaty *(XL, M, 1, ∞)*

3.3.1 Optimization model overview. The model developed estimates an insurer's optimal retention limit (M) for unlimited single layer excess of loss reinsurance cover (XL, M, 1, ∞). Since the layer is unlimited (L= ∞); the size of the layer (m) tends towards ∞. The optimal decision variable is derived such that it minimizes the insurer's variance of retained claims satisfying regulator's minimum risk requirements as well as financial constraints. The model is developed using Palisade @Risk software and Risk Optimizer tool, which run the model and derive the optimal solution satisfying defined constraints accordingly. Insurer's historical exposure and loss experience data are used to evaluate the claims' distribution function to simulate expected claims to build and run the model.

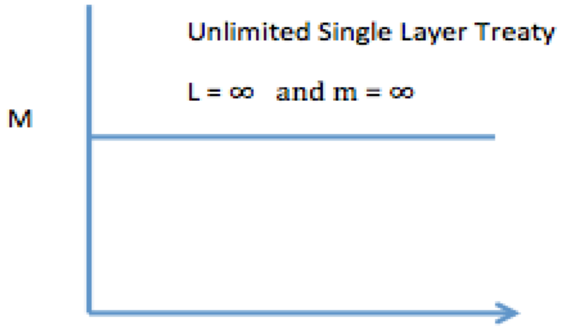

Figure 5: Unlimited Single Layer Reinsurance Treaty

3.3.2 Model formulation. The optimization model's objective function is to minimize the insurer's variance of retained claims by defining the optimal value of insurer's retention limit (M).

$$\boldsymbol{Min\ \sigma_I^2} = E(I^2) - E(I)^2 \tag{1}$$

$$\sigma_I^2 = \int_0^M x^2 f(x)dx + M^2[1 - F_x(M)] - (\int_0^M x\, f(x)dx + M\,[1 - F_x(M)])^2 \tag{2}$$

Subject to constraints of:

1) Minimum Cedent Retention: Insurance authorities impose on insurance companies 40% minimum retention on claims incurred [10]

$$\boldsymbol{M \geq M_{min} = 0.4\ E(X_i)} \tag{3}$$

2) Insurer's Financial Requirement on Return on Capital: to be greater than a minimum accepTable value (to be determined by the insurance company).

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

$$P_I = (P - P_R) = (1 + \theta_I)E(X) - (1 + \theta_R)E(R)$$

$$ROC = \frac{max\ \mathrm{E}\{0, U + P_I - I\}}{U} - 1$$

$$\boldsymbol{ROC \geq 2\%} \tag{4}$$

3) <u>Claims' Variance Constraints</u>

$$\sigma_I^2 + \sigma_R^2 \leq \sigma_X^2$$

$$\boldsymbol{\sigma_I^2 < \sigma_X^2 - \sigma_R^2} \quad (5)$$

3.3.3 Model description. The model was built using Excel and @Risk tools as follows:

I. The distribution function for the claims incurred f(X) is derived from the historical data of losses incurred using @Risk's Distribution Fitting tool as shown in Figure 6. The tool provides a list of the distribution functions that best fit the data set evaluated. Lognormal function was among the top best fit distributions, hence considered as the claims' incurred probability distribution function in the model. Claims incurred distribution function is further utilized to simulate individual claims' values as per the defined lognormal distribution.

II. Reference to Assumption 3 above, expected number of claims (N) are assumed and estimated as 30,000 claims, provided average exposure of 6,000 policyholders and expected average of 5 claims per insured. Expected values of individual claims $E(X_i)$ are simulated for the sample size (N) using the defined lognormal distribution f(X), and assumed as the insurers' expected portfolio of risks. As shown in Figure 7, simulated and generated values of expected incurred claims X_i are then fixed *(Column B)* and further used to define the model's parameters and develop the optimal reinsurance agreement for the respective claims' portfolio since the historical data used for the claims incurred distribution function is for 2013. The simulated claims are inflated by 10% for two years, to account for inflation and ensure consistency of the rates with the current year [38].

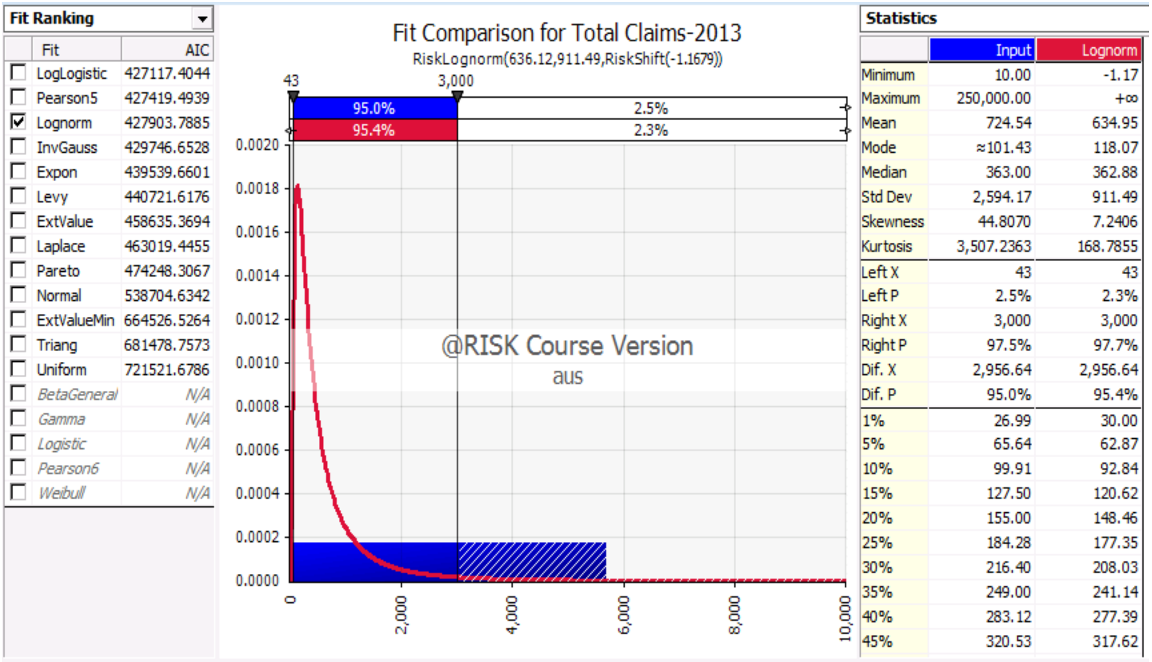

Figure 6: Historical Claims Distribution Fitting

III. Retained claims *(Column C)* are calculated from the expected claims incurred in (*Column B)*such that insurer retains minimum of the claim value (X_i) or the retention limit (M), which will be defined by the model during the optimization process. Value of (M) is randomly assumed for the sake of calculating and generating *(Column C)*

$$E(I(i)) = E(\min(X_i, M)).$$

IV. After calculating (*Column C)*,ceded claims *(Column D)* are calculated such that

$$E\big(R(i)\big) = \max(0, X_i - M).$$

V. Now that expected individual incurred claims *(Column B),* retained claims *(Column C),* and ceded claims *(Column D)* are generated, the model is built accordingly. The total expected value of each type of claims E(*X)*, E(*I)* and E(***R)*** is the sum of the individual claims respectively. Similarly, the variance of the sample of individual type of claims' is calculated.

VI. In order to define the insurer's minimum retention limit (M_{min}), 40% of each E(X_i) is calculated as shown in *Column F* in Figure 7. The maximum value calculated is assumed as the value of (M_{min}).

	A	B	C	D	E	F
4			M	Variance		
5		1,329,890.8	20,227.0	46,576.4973		20,227.0
6	Claim Sr No.	Sample Simulated Claims Incurred E(Xi)	Retained Claims E(Ii)	Ceded Claims E(Ri)		40% (Xi)
7	1	1,070.8	1,070.8	-		428.3
8	2	157.8	157.8	-		63.1
9	3	428.5	428.5	-		171.4
10	4	1,004.8	1,004.8	-		401.9
11	5	300.6	300.6	-		120.2
12	6	3,259.3	3,259.3	-		1,303.7
13	7	320.0	320.0	-		128.0
14	8	103.8	103.8	-		41.5
15	9	327.9	327.9	-		131.1
16	10	570.7	570.7	-		228.3
17	11	1,466.4	1,466.4	-		586.5
18	12	92.9	92.9	-		37.1
19	13	1,294.8	1,294.8	-		517.9
20	14	1,047.7	1,047.7	-		419.1
21	15	102.6	102.6	-		41.0
22	16	344.6	344.6	-		137.8
23	17	1,324.5	1,324.5	-		529.8
24	18	182.7	182.7	-		73.1

Figure 7: Simulated Claims' Data Sample

VII. After calculating the expected incurred claims' values, the gross premium (P) is calculated using the expected value premium principle. Similarly, the reinsurance premium is calculated using expected ceded claims' values. The premium risk loading (θ) is assumed as 30%.

$$P = (1+ \theta)\, E(X)$$

$$P_R = (1+\theta)\, E(R)$$

VIII. The fixed parameters of insurer's capital (U) is assumed to be equal to regulator's minimum capital requirement (U_{min}) of 100M. Since all parameters of the ROC are computed, the ROC is calculated accordingly.

$$ROC = \frac{max\, \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

IX. The next step is defining the model parameters. Using Risk Optimizer tool, the optimization model's objective function, decision variable, and constraints are as shown in Figures 8 and 9. The number of trials and number of iterations per trial to derive the optimal solution are also defined, then the optimization model is ready to run.

	A	B	C	D	E	F	G	H	I
1	Assumptions & Fixed Input						Decision Variable		
2	Exposure	6,000					Excess Amount (M)	1,000	
3	Average Claim Per Person	5							
4	Number of Claims Incurred (N)	30,000							
5	Capital (U)	100,000,000					Output		
6	Premium Risk Loading (θ)	30%					Variance of Retained Claims (σI2)	114,090.2	
7							ROC	5%	
8									
9	Uncertain Input						Assumptions & Constraints		
10	Claims			Premiums			Variance Retained Non-negativity	>	0
11	Incurred Claims - E(X)	23,266,519.8		Gross Premiums (P)	30,246,475.8		Minimum Retention Limit	>	20,227
12	Variance Incurred Claims (σx2)	1,329,890.8		Reinsurance Premium (Pr)	9,920,402.3		Variance Constraint	<=	357,691.82
13							Minimum ROC	>=	2%
14	Retained Claims - E(I)	15,635,441							
15									
16	Ceded Claims - R	7,631,079							
17	Variance Ceded Claims (σR2)	972,199							

Figure 8: Optimization Model Parameters Definition

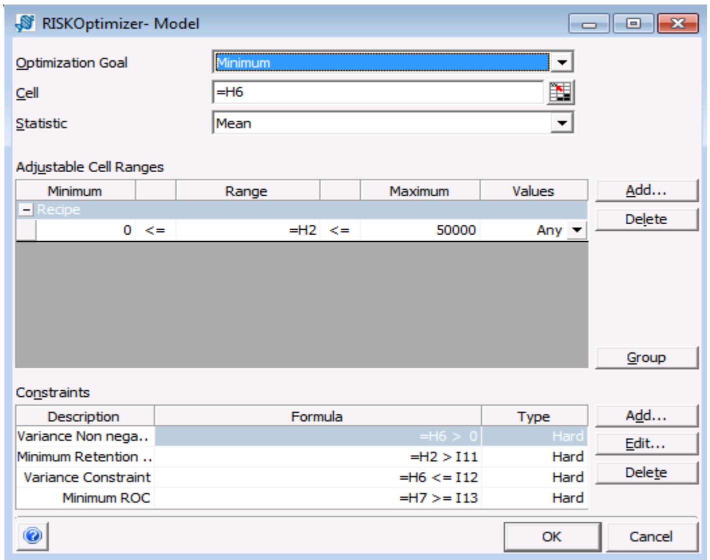

Figure 9: RiskOptimizer - Optimization Model Definition

X. Running the optimization model, the optimal retention limit (M) which minimizes the insurer's retention variance is defined on the model as displayed in Figure 10. Summary of the models' optimal solutions is illustrated and discussed in Chapter 4.

	A	B	C	D	E	F	G	H	I
1	Assumptions & Fixed Input						Decision Variable		
2	Exposure	6,000					Excess Amount (M)	20,227	
3	Average Claim Per Person	5							
4	Number of Claims Incurred (N)	30,000							
5	Capital (U)	100,000,000					Output		
6	Premium Risk Loading (θ)	30%					Variance of Retained Claims (σI2)	1,189,948.4	
7							ROC	7%	
8									
9	Uncertain Input						Assumptions & Constraints		
10	Claims			Premiums			Variance Retained Non-negativity	>	0
11	Incurred Claims - E(X)	23,266,519.8		Gross Premiums (P)	30,246,475.8		Minimum Retention Limit	>	20,227
12	Variance Incurred Claims (σx2)	1,329,890.8		Reinsurance Premium (Pr)	93,584.4		Variance Constraint	<=	1,283,314.29
13							Minimum ROC	>=	2%
14	Retained Claims - E(I)	23,194,532							
15									
16	Ceded Claims - R	71,988							
17	Variance Ceded Claims (σR2)	46,576							

Figure 10: Optimization Model Solution

3.4 Model 2 -Optimization of the Retention Limit (M), Minimizing Retained Claims Variance Given ROC Constraints – Limited Single Layer Treaty *(XL, M, 1, m)*

3.4.1 Optimization model overview. The optimization model is developed for a limited single layer excess of loss reinsurance cover such that the size of the layer m is given by *(L-M)*. The optimal decision variables (L and M) are derived such that the insurer's variance of retained claims is minimized satisfying regulator's minimum risk requirements as well as financial constraints. The model is built and simulated using Palisade @Risk software and Risk Optimizer tool, which run the model and derive the optimal solution satisfying defined constraints accordingly. Insurer's historical exposure and loss experience data are used to evaluate the claims' distribution function and build the model. The model is tested for two approaches:

1) Determine optimal retention limit (M) for different fixed values of ceding limit (L)
2) Determine optimal retention limit (M) and ceding limit (L) as decision variables

The solutions of both approaches are observed and evaluated to test and understand the model dynamics. Steps I and II from Model 3 explained above are duplicated for the limited layers' model, and the same set of simulated and inflated claims is used for the sake of comparing the optimal solutions of both models.

3.4.2 Model formulation. The optimization model's objective function is to minimize the insurer's variance of retained claims by defining the optimal values insurer's retention limit – deductible - (M) and the ceding limit (L)

$$\boldsymbol{Min\ \sigma_I^2} = E(I^2) - E(I)^2 \tag{6}$$

Subject to constraints of:

1) Minimum Cedent Retention: Insurance authorities impose on insurance companies 40% minimum retention on claims incurred [10]

$$\boldsymbol{M \geq M_{min} = 0.4\ E(X_i)} \tag{7}$$

2) Insurer's Financial Requirement on Return on Capital

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

$$P_I = (P - P_R) = (1 + \theta_I)E(X) - (1 + \theta_R)E(R)$$

$$ROC = \frac{max\ \mathrm{E}\{0, U + P_I - I\}}{U} - 1$$

$$\boldsymbol{ROC \geq 2\%} \tag{8}$$

3) Claims' Variance Constraints

$$\sigma_I^2 + \sigma_R^2 \leq \sigma_X^2$$

$$\boldsymbol{\sigma_I^2 < \sigma_X^2 - \sigma_R^2} \tag{9}$$

4) Ceding Limit Constraint

$$\boldsymbol{L > M} \tag{10}$$

3.4.3 Model description. The model was implemented using Excel and @Risk tools as follows:

I. The same set of claims simulated in steps I and II in Model 3 above is used to build and run the model for limited reinsurance layer.

II. In limited layer reinsurance structure, retained and ceded claims are calculated as per following scenarios, and the respective fields are generated:

Table 3: Retained and Ceded Claims Distribution

$E(X_i) \leq M$	$E(I_i) = \min(E(X_i), M)$	$E(R_i) = 0$
$M < E(X_i) < L$	$E(I_i) = M$	$E(R_i) = E(X_i) - M$
$E(X_i) > L$	$E(I_i) = M + (E(X_i) – (L-M))$	$E(R_i) = L – M$

III. As shown in Figure 11, expected individual incurred claims *(Column B)*, total retained claims *(Column F)*, and ceded claims *(Column D)* are generated, and the model is built accordingly. The total expected value of each type of claims E(*X)*, E(*I)* and E(***R)*** is the sum of the individual claims respectively. Similarly, the variance of the sample of individual type of claims is calculated.

IV. In order to define the insurer's minimum retention limit (M_{min}), 40% of each X_i is calculated as shown in *Column I* in Figure 11. The maximum value calculated is assumed as the value of (M_{min}).

V. After calculating the expected incurred claims' values, the gross premium (P) is calculated using the expected value premium principle. Similarly, the reinsurance premium is calculated using expected ceded claims' values. The premium risk loading (θ) is assumed as 30%

$$P = (1+ \theta)\, E(X)$$

$$P_R = (1+\theta)\, E\,(R)$$

	A	B	C	D	E	F	G	H	I
4			M	L					Max
5			20,227.0	53,125.0					20,227
6	Claim Sr No.	Claims_Incurred (X)	Retained Claims (I)	Ceded Claims (R)	Net Claims Retained in Excess of M	Total Claims Retained			0.4E(Xi)
7	1	1,070.8	1,070.8	-	-	1,070.8			428.3
8	2	157.8	157.8	-	-	157.8			63.1
9	3	428.5	428.5	-	-	428.5			171.4
10	4	1,004.8	1,004.8	-	-	1,004.8			401.9
11	5	300.6	300.6	-	-	300.6			120.2
12	6	3,259.3	3,259.3	-	-	3,259.3			1,303.7
13	7	320.0	320.0	-	-	320.0			128.0
14	8	103.8	103.8	-	-	103.8			41.5
15	9	327.9	327.9	-	-	327.9			131.1
16	10	570.7	570.7	-	-	570.7			228.3
17	11	1,466.4	1,466.4	-	-	1,466.4			586.5
18	12	92.9	92.9	-	-	92.9			37.1
19	13	1,294.8	1,294.8	-	-	1,294.8			517.9
20	14	1,047.7	1,047.7	-	-	1,047.7			419.1
21	15	102.6	102.6	-	-	102.6			41.0
22	16	344.6	344.6	-	-	344.6			137.8

Figure 11: Simulated Claims' Data and Limited Layer Claims' Split Sample

VI. The fixed parameters of insurer's capital (U) is assumed equal to the regulator's minimum capital requirement (U_{min}) of 100M. Since all parameters of the ROC are computed, the ROC is calculated accordingly.

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

VII. The next step is defining the model parameters, the optimization model's objective function, decision variable, and constraints using the Risk Optimizer.

a. *Approach 1*: (M) and (L) are defined as the optimization model's decision variables. The model is defined to select optimal value of (L) from a range of 25,000-150,000, such that L >M.

b. *Approach 2*: (M) is defined as the optimization model's decision variable, and the model is tested for different assumed values of (L).

VIII. The solution for both approaches, as shown in Table 4 is evaluated and compared. From the results displayed. It can be concluded that the optimal limit (L) derived as the model's decision variable, resulted in the minimum retained variance compared to the assumed layer limits although different values of L had low to no impact on both ROC and M.

Table 4: Model 2 Optimization Results

Approach	L	M	σ_I^2	ROC
1	**53,125***	20,227	**1,189,948**	6.96%
2	25,000	20,227	1,275,916	6.97%
	35,000	20,227	1,222,794	6.96%
	50,000	20,227	1,190,695	6.96%

*Optimal decision variable

3.5 Model 3 - Optimization of the Retention Limit (M), Maximizing Insurer's Return on Capital – Unlimited Single Layer Treaty *(XL, M,1, ∞)*

3.5.1 Optimization model overview. The model developed estimates for an insurer's optimal retention limit (M) for an unlimited single layer excess of loss reinsurance cover (XL, M, 1, ∞). The optimal decision variable is derived such that it maximizes the insurer's return on capital satisfying minimum financial and risk requirements. Similar to other models, the model is developed using Palisade @Risk software and Risk Optimizer tool.

3.5.2 Model formulation. The optimization model's decision variable is the optimal retention limit (M) which is

$$\boldsymbol{Max\ ROC} = \frac{max\,\mathrm{E}\{0, U+P-I-P_R\}}{U} - 1 \tag{11}$$

Subject to constraints of:

1) Minimum Cedent Retention: Insurance authorities impose on insurance companies 40% minimum retention on claims incurred [10]

$$\boldsymbol{M \geq M_{min} = 0.4\ \mathrm{E}(X_i)} \tag{12}$$

2) Insurer's Financial Requirement on Return on Capital

$$\boldsymbol{ROC \geq 2\%} \tag{13}$$

3) Claims' Variance Constraints

$\sigma_I^2 + \sigma_R^2 \leq \sigma_X^2$

$$\boldsymbol{\sigma_I^2 < \sigma_X^2 - \sigma_R^2} \quad (14)$$

3.5.3 Model description. The model was implemented using Excel and @Risk tools as follows:

I. The same set of claims simulated in steps I and II in Model 3 is used to develop the model's parameters accordingly.

II. Retained claims are calculated from the expected claims incurred such that insurer retains minimum of the claim value (X_i) or the retention limit (M), which will be defined by the model during the optimization process. Value of (M) is randomly assumed for the sake of calculating and generating retained claims

$$E\big(I(i)\big) = E(\min(X_i, M))$$

III. After calculating retained claims, ceded claims are calculated such that

$$E(R(i)) = \max(0, X_i - M)$$

IV. Now that expected individual incurred claims, retained claims, and ceded claims are generated, the model is built accordingly. The total expected value of each type of claims E(*X*), E(*I*) and E(***R***) is the sum of the individual claims respectively.

V. In order to define the insurer's minimum retention limit (M_{min}), 40% of each E(X_i) is calculated as shown in *Column F* in figure 7 above. The maximum value calculated is assumed as the value of M_{min}.

VI. After calculating the expected incurred claims' values, the gross premium (P) is calculated using the expected value premium principle. Similarly, the reinsurance premium is calculated using expected ceded claims' values. The premium risk loading (θ) is assumed as 30%.

$$P = (1+ \theta)\ E(X)$$

$$P_R = (1+\theta)\ E\ (R)$$

VII. The fixed parameter of insurer's capital (U) is assumed equal to regulator's minimum capital requirement (U_{min}) of 100M. Since all parameters of the ROC are computed, the ROC is calculated accordingly.

$$ROC = \frac{max\, \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

VIII. Using Risk Optimizer, the objective function, decision variable (M), and the constraints are defined, such that the model derives the optimal solution maximizing the insurer's ROC satisfying all constraints.

3.6 Model 4 - Optimization of the Retention Limit (M), Maximizing Insurer's Return on Capital – Limited Single Layer Treaty *(XL, M, 1, m)*

3.6.1 Optimization model overview. The model developed estimates an insurer's optimal retention limit (M) for limited single layer excess of loss reinsurance cover such that the size of the layer ***m= L-M***. The optimal decision variable is derived such that it maximizes the insurer's return on capital satisfying regulator's minimum risk requirements as well as financial constraints. The model is built and simulated using Palisade @Risk software and Risk Optimizer tool, which run the model and derive the optimal solution satisfying defined constraints accordingly. Insurer's historical exposure and loss experience data are used to evaluate the claims' distribution function and build the model. The model is tested for two approaches:

1) Determine optimal retention limit (M) and ceding limit (L) as decision variables
2) Determine optimal retention limit (M) for different fixed values of ceding limit (L)

The solutions of the same are observed and evaluated to test and understand the model dynamics. Steps I and II from Model 3 explained above are duplicated for the limited layers' model, and the same set of simulated and inflated claims is used, for the sake of comparing the optimal solutions of both models.

3.6.2 Model formulation. The optimization model's decision variables are the optimal retention limit (M) and ceding limit (L)

$$\boldsymbol{Max\ ROC} = \frac{max\, \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1 \tag{15}$$

Subject to constraints of:

1) <u>Minimum Cedent Retention</u>: Insurance authorities impose on insurance companies 40% minimum retention on claims incurred [10]

$$\boldsymbol{M \geq M_{min} = 0.4\ \mathrm{E}(X_i)} \tag{16}$$

2) <u>Insurer's Financial Requirement on Return on Capital</u>

$$\boldsymbol{ROC} \geq \mathbf{2\%} \tag{17}$$

3) <u>Ceding Limit Constraint</u>

$$\boldsymbol{L} > \boldsymbol{M} \tag{18}$$

4) <u>Claims' Variance Constraints</u>

$$\sigma_I^2 + \sigma_R^2 \leq \sigma_X^2$$

$$\boldsymbol{\sigma_I^2 < \sigma_X^2 - \sigma_R^2} \tag{19}$$

3.6.3 Model description. The model was implemented using Excel and @Risk tools as follows:

I. The same set of claims simulated in steps I and II in Model 3 is used to build the model and define its parameters accordingly.

II. In limited layer reinsurance structure, retained and ceded claims are calculated as per following scenarios in Table 5, and the respective fields are generated:

Table 5: Retained and Ceded Claims Distribution

$E(X_i) \leq M$	$E(I_i) = \min(E(X_i), M)$	$E(R_i) = 0$
$M < E(X_i) < L$	$E(I_i) = M$	$E(R_i) = E(X_i) - M$
$E(X_i) > L$	$E(I_i) = M + (E(X_i) – (L-M))$	$E(R_i) = L – M$

III. Now that expected individual incurred claims, retained claims, and ceded claims are generated, the model is built accordingly. The total expected value of each type of claims E(*X)*, E(*I)* and E(***R)*** is the sum of the individual claims respectively.

IV. In order to define the insurer's minimum retention limit (M_{min}), 40% of each X_i is calculated as shown in *Column F* in figure 7 above. The maximum value calculated is assumed as the value of M_{min}.

V. After calculating the expected incurred claims' values, the gross premium (P) is calculated using the expected value premium principle. Similarly, the reinsurance premium is calculated using expected ceded claims' values. The premium risk loading (θ) is assumed as 30%.

$$P = (1+\theta)\, E(X)$$

$$P_R = (1+\theta)\, E(R)$$

VI. The fixed parameter of insurer's capital (U) is assumed equal to regulator's minimum capital requirement (U_{min}) of 100M. Since all parameters of the ROC are computed, the ROC is calculated accordingly.

$$ROC = \frac{max\ \mathrm{E}\{0, U + P - I - P_R\}}{U} - 1$$

VII. The next step is defining the model parameters, the optimization model's objective function, decision variable, and constraints using the RiskOptimizer, such that the model derives the optimal solution maximizing the insurer's ROC satisfying all constraints.

a. *Approach 1*: (M) and (L) are defined as the optimization model's decision variables. The model is defined to select optimal value of (L) from a range of 25,000-150,000, such that L >M.

b. *Approach 2*: (M) is defined as the optimization model's decision variable, and the model is tested for different assumed values of (L)

VIII. The solution for both approaches is evaluated and compared in Table 6. It can be noted that the optimal solution from the optimization model provided a limit higher than the tested values of L. The ROC is also slightly increasing as the decision variables increase.

Table 6: Model 4 Optimization Results

Approach	L	M	σ_I^2	ROC
1	71,875*	46,137	1,315,837	6.98%
2	25,000	20,227	1,257,510	6.97%
	35,000	29,735	1,275,916	6.97%
	50,000	29,754	1,301,109	6.98%

*Optimal decision variable

Chapter 4: Optimization Models Discussion of Results

The chapter discusses, evaluates and compares the optimization models and their respective optimal solutions. It also provides an overview analysis of the impact of the model's independent parameters on the ROC.

4.1 Sensitivity Analysis

In order to get a better understanding of the solutions derived from the models, it is important to understand the dynamics and dependencies of the model parameters, and their impact on the optimization and optimal solutions.

After simulating the set of the expected claims incurred values $E(X_i)$, the parameter of E(X) was fixed throughout the models, and all model parameters depend on the severity and size of the portfolio of claims simulated. As for the remaining model parameters, Table 7 demonstrates the dependencies and relationships between these parameters. Values of θ and U, are also fixed and assumed depending on regulatory solvency requirements and insurer's financial and pricing requirements.

Table 7: Model Parameters' Dependencies

Independent Parameter	Dependent Parameter
M	σ_I^2, E(I), P(R), E(R)
θ	P, P(R), ROC
U	ROC

The impact of the independent parameters on the ROC can be better understood by testing the effect on the ROC of varying different values of independent parameters individually and keeping the other parameters fixed. Sample sensitivity analysis is illustrated below to demonstrate the same. The risk loading impact, in Table 8, is tested on the gross premium (P), since that contributes the major proportion of the premiums calculation.

It can be noted from the analysis results, in Tables 8 and 9, that the impact of the capital and gross premium risk loading on the ROC is significantly high. On the other hand, the impact of varying the values of M is significantly low on the ROC as shown in Table 10. Therefore, the defined parameters of the capital and premium risk loading play an important role in achieving and defining ROC targets as opposed to the value of M.

Table 8: Sensitivity Analysis - Impact of Gross Premium Risk Loading on ROC

% Change (θ)	θ	ROC	% Change (ROC)
-25%	23%	5.2%	-25%
-10%	27%	6.3%	-10%
0%	**30%**	**7.0%**	**0%**
10%	33%	7.7%	10%
25%	38%	8.7%	25%

Table 9: Sensitivity Analysis - Impact of U on ROC

% Change (U)	U	ROC	% Change (ROC)
0%	**100M**	**6.98%**	**0%**
10%	110M	6.34%	-9%
25%	125M	5.58%	-20%
50%	150M	4.65%	-33%
75%	175M	3.99%	-43%

Table 10: Sensitivity Analysis - Impact of M on ROC

% Change (M)	M	ROC	% Change (ROC)
-20%	16,182	6.95%	-0.14%
-10%	18,204	6.95%	-0.14%
0%	**20,227**	**6.96%**	**0%**
10%	22,250	6.96%	0%
20%	26,700	6.97%	0.14%
50%	40,049	6.98%	0.29%

4.2 Optimization Models Results

Four optimization models which address two objective functions; insurer's minimum retained claims' variance and maximum return on capital, have been built and run accordingly. The models' optimal solutions are summarized in Table 11 below. For each objective function, the optimal solution for both limited and unlimited layers is captured, and for the limited layer the values in the table indicate the values of the reinsurance agreement's deductible and ceding limit as the optimization problems' decision variables.

Table 11: Optimization Models' Optimal Solutions

Objective Function	Ceding Limit (L)	Deductible (M)	Retained Variance (σ_I^2)	ROC
Min σ_I^2	∞	20,227	1,189,948	6.96%
	53,125*	20,227	1,189,948	6.96%
Max ROC	∞	50,455	1,329,517	6.98%
	71,875	46,137	1,315,837	6.98%

On minimizing the variance of the insurer's retention, it can be concluded that both models resulted in a decision variable (M) that is equal to the minimum retention limit (deductible) that was defined in the model. The models' objective is to minimize the variance of retained claims given target return on capital. Since the impact of M on the ROC is significantly low, the minimum variance is achieved at the minimum value of M defined. Consequently, the model's optimal solution of the deductible didn't deviate from the minimum constraint. On the limited reinsurance structure model, limit (L) was defined in the optimization such that all losses in excess of M are covered under the limit L, hence insurer's retention is only limited to values capped by M, which is defined as the minimum value and delivers minimum retention. In conclusion, whether it's a limited or an unlimited reinsurance agreement, it is indifferent to the insurer as both solutions result in the same deductible, retention variance and ROC under the models' assumptions.

On the other hand, the behavior of the model of maximizing the ROC varied depending on the constraints defined on the variances.

1. $\sigma_I^2 \leq \sigma_X^2 - \sigma_R^2$ – The optimal solution presented by the model was in the form of full retention by the insurer and zero ceding to the reinsurer, since the constraint can be satisfied by $\sigma_I^2 = \sigma_X^2$ where $\sigma_R^2 = 0$
2. $\sigma_I^2 < \sigma_X^2 - \sigma_R^2$ – The optimal solution requires existence of reinsurance; hence the model provides a solution with significantly small number of claims ceded, such that majority of claims are covered under the insurer.

Since the objective of the research is to evaluate optimal options of reinsurance agreements, and compare effectiveness of both objective functions against each other, the second constraint was selected, regardless of the small ceding applied. It can be observed that by maximizing the ROC, the optimal value of M selected has deviated from the minimum value such that it satisfies the constraints accordingly. Consequently, an optimal deductible can be efficiently derived using this objective, taking into consideration that changes in the value of M will not significantly impact the ROC independently, and that Capital and premium risk loading are critical parameters that impact the behavior of the ROC. Comparing the results of both unlimited and limited reinsurance agreements, it can be noted that the limited layer reduces the insurer's deductible and variance of retained claims against the unlimited layer solution.

Evaluating both models, the model, which maximizes the return on capital, appears to deliver more efficient optimization results and solution for an insurer, against the latter model. The model takes into consideration the relationship and impact of M on the ROC parameters and derives the optimal value of a suggested reinsurance structure to meet insurers' and regulatory constraint and requirements. It also provides the insurer with an overview of, at given capital and premium risk loadings, what is the maximum return on capital achievable in order to meet shareholders' expectations. Consequently, given the example modeled and explained, the insurer's optimal excess of loss reinsurance structure would be a limited single layer treaty defined *(XL, 46.1K, 1, 26K)* which, given all parameters sustained, delivers maximum ROC of up to 6.98%. The value of delivered ROC highly depends on the defined values of the premium risk loading and capital.

Chapter 5: Conclusion and Recommendations

The subject of optimal reinsurance agreement has been an area of interest for many researches and studies. The literature has explored different reinsurance optimization approaches whether from the insurer, reinsurer or joint perspective.

In this study, the objective was to build and develop optimization model and tools that can be utilized by insurance companies in the industry to gain a better understanding and visibility of their optimal reinsurance structure prior to contract. Using both historical and expected loss experiences, these models allow insurers to understand their reinsurance agreements and facilitate decision making revolving around the reinsurance structure to be in place. Four models were developed and tested to meet pre-defined constraints with two main objective functions, minimizing insurer's retained variance and maximizing insurer's return on capital.

To minimize the variance of retained claims, a target return on capital and a minimum retention limit were defined as constraints, and the model attempted to define the optimal retention limit accordingly. Given the above, the optimization resulted in a retention limit equal to the defined minimum retention limit. The impact of the target return on capital constraint was insignificant on defining an optimal retention limit. This was interpretable through the sensitivity analysis evaluating the relationship and sensitivity between the return on capital and the retention limit. The analysis revealed that the impact of the retention limit on the target return on capital is significantly low; hence the constraints were satisfied and the minimal retention variance were satisfied on the defined minimum retention limit. In addition, the optimal solution was indifferent to whether the model was tested for a limited or unlimited reinsurance agreement. To satisfy the objective function, the solutions for both types of agreements were defined such that insurer retains the minimum retention limit and the net value of claims is ceded to the reinsurance.

The models that maximized the return on capital defined the insurer's retention limit more efficiently compared to the latter objective and deviated from the minimum retention constraint. However, given the models' assumptions and the claims' nature, the value of the maximum return on capital proved to be highly dependent on the defined parameters of the capital and gross premium safety loading rather than the retention limit. Therefore, for the model to run more efficiently, the relationship between the independent parameters (M), (U), and (θ) and the return on capital should be accounted for while developing the model.

Comparing both objective function, the models that maximize the return appear to address the optimization objective more efficiently, satisfying all constraints and defining more sensible optimal retention limits. However, the models have areas and opportunities for further improvements and to derive more efficient solutions.

To enhance the models' performance and value to the industry, the model can be tested on a larger portfolio exposed to higher severity of claims. Due to the nature of the medical insurance portfolio and data provided, low to medium severity and frequency, the distribution functions used to simulate the expected losses simulates significantly small sample of high severity claims. Portfolio with higher variation in claims' severity and risks may result in better optimization solutions for a reinsurance agreements and may generate a bigger demand/need for reinsurance structure in place. The impact of M and L on the ROC could also be tested in that case, if it could result in a more significant impact on the ROC, or if it will remain low compared to U and θ. Moreover, the probability distribution function used to simulate expected claims was derived from the distribution fitting of a one year claims' experience. Alternatively, the distribution function of several years of historical claims could be used to simulate expected claim. Moreover, reinsurance market related information can be accommodated to the model, such as reinsurance premium rates and different pricing approaches for different agreement types, limited and unlimited etc., to evaluate the cost of reinsurance more accurately. In this model, constant pricing approach was assumed such that reinsurer simply loads the expected ceded claims by 30%.

References

[1] B. Howe, R. Buchanan and D. Hart, *Actuarial practice of general insurance,* 7th edition, Australia:The Institute of Actuaries of Australia, 2007.

[2] Anonymous, "Health Insurance; Medical Insurance to Propel the UAE Insurance Market," NewsRx, Atlanta, USA, 2011.

[3] Anonymous, "United Arab Emirates Insurance Report - 2013 Q1," Business Monitor International, London, United Kingdom, 2013.

[4] O. Cort´es, A. Rau-Chaplin, D. Wilson, I. Cook and J. Gaiser-Porter, "Efficient optimization of reinsurance contracts using discretized PBIL," in *In Proceedings of the 2nd International Conference on Data Analytics*, Porto, Portugal, 2013.

[5] K. Riley, "Excess of Loss Reinsurance: Per-Risk Contracts," Willis Wire, 3 January 2014. [Online]. Available: http://blog.willis.com/2014/01/excess-of-loss-reinsurance-per-risk-contracts/. [Accessed 28 May 2014].

[6] G. Deelstra and G. Plantin, "Elements of Risk Theory," in *Risk Theory and Reinsurance*, Springer, pp. 2-4, 2014.

[7] V. R. Young, "Premium Principles," *Encyclopedia of Actuarial Science,* 15 September 2006.

[8] L. S. Ward and D. H. Lee, "Practical Application of the Risk-Adjusted Return on Capital Framework", CAS Forum Summer 2002, Dynamic Financial Analysis Discussion Papers, 2002. [Online]. Available at http://www.casact.org/pubs/forum/02sforum/02sftoc.htm.[Accessed: 20 March 2015]

[9] S. Feldblum and N. Thandi, "Financial Pricing Models for Property-Casualty Insurance Products: The Target Return on Capital", CAS Forum Winter 2003, Data Management, Quality and Technology Call Papers and Ratemaking Disucssion Papers, 2003. [Online]. Available at

https://www.casact.org/pubs/forum/03wforum/03wf591.pdf. [Accessed: 15 May 2015]

[10] S. Bhagaloo, Interviewee, *Consulting Actuary, Lux Actuaries & Consultants.* [Interview]. 15 March 2014.

[11] Anonymous, "Trinity Insurance Brokers," Trinity Insurance Brokers, 2014. [Online]. Available: www.trinityinsurance.co.in/company-overview.php. [Accessed May 2014].

[12] K. Borch, "An attempt to determine the optimum amount of stop loss reinsurance," in *Transaction of the 16th International Congress of Actuaries*, 1960.

[13] K. J. Arrow, "Uncertainty and the Welfare Economics of Medical Care," *The American Economic Review,* vol. 53, pp. 941-973, 1963.

[14] J. Cai, K. S. Tan, C. Weng and Y. Zhang, "Optimal reinsurance under VaR and CTE risk measures," *Insurance: Mathematics and Economics,* vol. 43, pp. 185-196, 2008.

[15] A. Balbas, B. Balbaz and A. Heras, "Optimal reinsurance with general risk measures," *Insurance: Mathematics and Economics,* vol. 44, pp. 374-384, 2009.

[16] A. Balbas, B. Balbaz and A. Heras, "Stable solutions for optimal reinsurance problems involving risk measures," *European Journal of Operational Research,* vol. 214, pp. 796-804, 2011.

[17] Y. Chi and K. S. Tan, "Optimal Reinsurance under VaR and CVaR Risk Measures: A Simplified Approach," *Astin Bulletin*, vol. 41, pp. 487-508, 2011.

[18] K. S. Tan and C. Weng, "Enhancing Insurer Value Using Reinsurance and Value-at-Risk Criterion," *Geneva Risk and Insurance Review,* vol. 37, no. 1, pp. 109-140, 2012.

[19] M. Kaluszka, "Optimal reinsurance under mean-variance premium principles," *Insurance: Mathematics and Economics,* vol. 28, pp. 61-67, 2001.

[20] L. Gajek and D. Zagrodny, "Optimal reinsurance under general risk measures," *Insurance: Mathematics and Economics,* vol. 34, pp. 227-240, 2004.

[21] M. Kaluzka, "Optimal reinsurance under convex principles of premium calculation," *Insurance: Mathematics and Economics,* vol. 36, pp. 375-398, 2005.

[22] Y. Chi and K. S. Tan, "Optimal reinsurance under variance related premium principles," *Insurance: Mathematics and Economics,* vol. 51, pp. 310-321, 2012.

[23] Y. Chi and K. S. Tan, "Optimal reinsurance with general premium principles," *Insurance: Mathematics and Economics,* vol. 52, pp. 180-189, 2013.

[24] J. Jang, "Reinsurance retentions & limits for property-liability insurers - Doctoral Dissertation," Temple Univrsity , 2001.

[25] M. Kaluszka, "Mean-variance optimal resinurance arrangements," *Scandinavian Actuarial Journal,* vol. 1, pp. 28-41, 2004.

[26] M. d. L. Centeno, "Dependent risks and excess of loss reinsurance," *Insurance: Mathematics and Economics,* vol. 37, pp. 229-238, 2005.

[27] V. Kaishev, "Optimal retention levels given the joint survival of cedent and reinsurer," Sir John Cass Business School, London, 2003.

[28] V. Kaishev and D. S. Dimitrova, "Excess of loss reinsurance under Joint survival optimality," *Insurance: Mathematics and Economics,* vol. 39, pp. 376-389, 2006.

[29] V. Kaishev and D. S. Dimitrova, "Optimal joint survival reinsurance: efficient frontier approach," *Insurance: Mathematics and Economics,* vol. 47, pp. 27-35, 2010.

[30] Z. Li, "Optimal reinsurance retention under ruin related optimization criteria," M.S. thesis, Dept. Actuarial Science, University of Waterloo, Ontario, Canada, 2008.

[31] C. Yusong and X. Jin, "Proportional and excess-of-loss reinsurance under investment gains," *Applied Mathematics and Computation,* vol. 217, pp. 2546-2550, 2010.

[32] S. Asmussen, B. Højgaard and M. Taksar, "Optimal Risk Control and Dividend Distribution Policies," *Finance & Stochastics,* vol. 4, no. 3, p. 299, 2000.

[33] K. A. Froot, "Risk Management, Capital Budgeting, and Capital Structure Policy for Insurers and Reinsurers," *Journal of Risk and Insurance,* vol. 74, no. 2, pp. 273-299, 2007.

[34] R. J. Laeven and E. C. Perotti, "Optimal Capital Structure for Insurance Companies," 2010.

[35] H. Meng and T. K. Siu, "On optimal reinsurance, dividend and reinvestment strategies," *Economic Modelling,* vol. 28, no. (1-2), pp. 211-218, 2011.

[36] A. Mitschele, I. Oesterreicher, F. Schlottmann and D. Seese, "Heuristic optimization of reinsurance programs and implications for reinsurance buyers," in *Operations Research Proceedings*, 2006.

[37] O. A. C. Cortes, A. Rau-Chaplin, D. Wilson and J. Gaiser-Porter, "On PBIL, DE and PSO for Optimization of Reinsurance Contracts," in *Applications of Evolutionary Computation*, Granada, Spain, Springer Berlin Heidelberg, pp. 227-238, 2014.

[38] S. Bhagaloo, Interviewee, *Consulting Actuary, Lux Actuaries and Consultants.* [Interview]. November 2015.

[39] *Financial Regulations for Insurance Companies,* United Arab Emirates: Insurance Authority, 2014.

Vita

Mai Muhtaseb was born in 1990 in the United Arab Emirates. She obtained her Bachelor of Science in Civil Engineering at the American University of Sharjah, United Arab Emirates, and graduated in 2012. She has enrolled in the Master of Engineering Systems Management at the American University of Sharjah later after her graduation.

In 2012, Mai joined Noor Takaful for a year, a local insurance company in Dubai, United Arab Emirates as a Sales Coordinator and fulfilled roles as an Assistant Business Analyst as well. In that role, Mai has gained exposure and interest in analytics and financial risks. Later in the end of 2014, she joined Unilever Gulf FZE in Dubai, United Arab Emirates as a procurement intern. She is currently fulfilling the role of an Assistant Manager Global Procurement of Capital Expenditure focusing on Construction, Mechanical, Electrical and Installation services since May 2015. In her current role, she supports and drives value generation and savings as well as efficient management and execution of procurement activities and contracts related to Unilever's facilities and buildings' construction project on a global level.