TOTAL DAMAGES QUANTIFICATION METHOD FOR FLOAT ALLOCATION

by

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Abstract

Construction projects involve several contracting parties such as clients, contractors, consultants and many others. These contracting parties tend to use the float of non-critical activities for their own benefits, such as the incorporation of change orders and for resource management purposes. Float ownership is one of the most debatable issues in the construction industry. In fact, using float by one party might yield in damages caused to the other party, such as increasing the risk towards finishing the project, disturbance of resource histogram and losses in the time value of money. This Thesis proposes a new method that can fairly allocate float ownership between all the project parties by considering all the damages that might occur when float is used. The total damages quantification method for float allocation aims to allocate float ownership for each party prior to signing the contract and specifies their liability towards using the float during the project execution. The float allocation process is performed using the total damages model that determines the optimum project schedule, with the lowest cost, considering the additional costs related to changes from that schedule due to the time value of money, resource leveling, and the increase of risk of delaying the project completion. Thus, contractors can set up their bid price based on the baseline schedule requirements and the liability of using float. Consumption of additional float can be quantified as damages using the total damages quantification model. Hence, float ownership for each party can be defined in the contract baseline schedule. Subsequently, unless all parties are committed to the baseline schedule, damages should be quantified as monetary values and the affected party should be compensated. As a result, by introducing this method, the contracting system clarifies all parties' rights and liabilities towards floats of non-critical activities, which in turn minimizes conflicts and claims that might end up by terminating projects and raising cases in courts of law. A case study is conducted in this research in order to show how each party of the construction project can suffer from damages due to the use of float, and how the Total Damages Method can be used in order to claim these damages.

Search Terms: Bids; Optimum Schedule; Float Consumption Risk; Resource Leveling; Time Value of Money; Float Allocation; Claims.

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Chapter 1: Introduction

1.1. Overview

The construction industry is considered as one of the most complicated industries. Its complexity is due to the fact that it involves a variety of processes that pass through different parties. Many cases of conflict have been recorded in this industry due to disagreements between the parties that are handling the project on their responsibility towards project's issues of delays. These cases usually end up in courts waiting for years due to their complexity and difficulty to determine which party is responsible for the mattered issues.

Float ownership is one of the most common issues that can cause conflicts and divergence between different parties. In fact, float can be beneficial to the owners as well as the contractors such that it can be used to accommodate for changes, contingency, resources allocation, procurement, and controlling cash flow. Float in many cases is valuable and can be equivalent to money [1].

Hence, the use of float's effect on time and cost shall be studied comprehensively in order to come up with constitutions that determine the rights of each party using it. Float use affects various parameters that contribute to the project costs and delays. Clients can specify the deadlines, milestones, payment methodology, while having flexibility of change orders. Meanwhile, contractors should propose their bidding prices based on these constraints.

Big efforts have been made in this field to come up with approaches that can solve these issues. Although many methods have been introduced for float pre-allocation, the applicability and efficiency of those methods are limited. As to be discussed later in this research, those methods are only based on the Critical Path Method (CPM) scheduling, which does not account for contingency, resourcing, and many other factors.

1.2. Statement of the Problem

The vagueness of each party's rights of using float for their own benefits might result in conflicts. In such cases, each party will end up claiming their damages due to the consumption of float by the other. However, it is difficult to judge on these cases with the absence of a clear premise of each party's liability towards using float.

Although many efforts have been made in this field, there is no comprehensive approach that explicitly considers the damages caused to each party due to float consumption.

1.3. Objectives

The objective of this research is to develop a methodology for float pre-allocation and reallocation prior to signing the contract by quantifying the cost damages to project parties introduced by changing the non-critical activities starting dates. The particular objectives are:

1. *To Quantify Damages Due to Cash Flow Plan Changes:* Changing the start of an activity results in changing the expenses and revenues of that activity. Based on the contracted payments methodology, the use of float will affect the cash flow and the rate of return of the capital assigned for the project. Different cases are to be investigated and the effect on the time value of money is to be introduced.
2. *To Quantify Damages Due to Resource Histogram Disturbance:* Float can be used for reducing the cost of project resources. In fact, resource leveling is mainly performed based on assigning the start date of non-critical activities. Resource leveling in most of the cases reduces the total cost spent on that resource. Therefore, resource allocation cost effect due to use of float shall be considered.
3. *To Quantify Damages Due to the Increase of Projects Risk*: According to CPM, the total float of an activity is the duration that the activity can be delayed without delaying the project’s total duration. However, in probabilistic analysis such as PERT, the delay of noncritical activities might result in increasing the project’s duration at the same certainty point; which in turn, will end up in having an extra cost introduced by either crashing activities or paying for liquidated damages.
4. *To Develop a Tri-Parameter Model:* As noticed, the use of float for a certain activity might increase the cost in one or two parameters while decreasing it in the others. However, there is an optimal value of the float that will introduce the lowest total cost at a certain point. Therefore, a model is to be developed in order to recognize the actual effect of changing any noncritical activity’s starting date due to any circumstances.

5. *To Propose a Methodology* that should be developed between owners and contractors in order to determine the validity of changing a non-critical activity’s starting or completion date.

1.4. Significance of the Research

Many researches have been reported in this field as discussed later in Chapter 2. However, the Total Damages Quantification Method (TDQM) for float allocation is the first method that develops float allocation process based on calculating the damages that will occur to each of the project’s parties when float is consumed to change the starting date of a non-critical activity. Obtaining the damages due to float consumption is performed in order to provide a fair judgment on how the harmed party should be compensated. Henceforth, this method will provide a clear acceptable solution to the conflicts that might occur due to the uncertainty of float ownership. Moreover, this method can be used to allocate float prior to signing the contract (float pre-allocation) and to re-allocate float during the project exaction (float re-allocation) in a way that is clear and agreed on by all project parties.

Furthermore, the quantification of the damages process developed in this research comprehensively considers the most important aspects of the project that affect the cost of changing non-critical activities. In fact, all other researches, developed earlier, considered only one or two parameters such as risk and resource leveling. The tri-parameter model, developed in this research, includes the costs encountered from changing the starting dates of non-critical activities due to the increase of risk in delaying the project handing over, changes in the time value of the project, and increase fluctuation of resources requirements. Then, the total damages are minimized using the model to obtain the optimum schedule that minimizes the cost of the project. This schedule is utilized as a reference to calculate all other damages that might occur when other schedule is followed. Moreover, the optimum schedule obtained is not only useful for the float allocation process, but it can be also useful for the construction process itself.

1.5. Research Methodology

The following steps are followed to achieve the research objectives:

Step 1: Review the literature. Literature was reviewed and utilized due to the fact that this research's topic is broad, and it includes a variety of fields such as scheduling and delay analysis. Some models already exist such as the model developed for float consumption impact on cost and schedule in construction projects [2]. Such models can be beneficial to this topic by modifying and utilizing them in the way that best serves the research.

Step 2: Develop models for each of the three parameters {risk, resources, and cash flow} that relate each of them to the consumption of float. In any project, delaying or advancing an activity, even if it is not critical, might impact the cost of the project due to the increase of probability of delaying the project, disturbing resources, and changing the cash flow of the plan for each of the client and the contractor. Hence, there should be a quantitative model, for each of the mentioned parameters, that examines the effect of deviating an activity's starting and ending date.

Step 3: Develop the tri-parameter model that will obtain the total effect of using float. Due to the fact that there are three parameters affected by float consumption, it can only be judged on each party's liability towards using float when the total effect on float consumption is determined. Therefore, a model that considers the three mentioned parameters' effect due to float consumption shall be developed. Once done, the optimum schedule can be determined in terms of cost that can be defined by using it and any deviation from that optimum schedule.

Step 4: Develop a systematic method for float pre-allocation and reallocation. Once the total effect of float consumption can be determined, project parties can agree on a baseline schedule that is to be followed in executing the project in the way that satisfies their needs. The baseline schedule might vary from the optimum schedule based on the client desire; however, contractors can propose their bidding price in accordance with the desired schedule. During the project execution, any party that deviates from the baseline agreed schedule shall compensate other parties in accordance to the float consumption effect model. The proposed method to be developed should comprise systematic steps to be followed on order to agree on projects schedule and float consumption liability.

Step 6: Presenting a case study. A case study that illustrates the applicability of the developed method is presented.

Chapter 2: Literature Review

2.1. Risk Cost Effect Due to Use of Float

The construction industry is risky. Construction projects are perceived to have more inherent risks due to the involvement of many contracting parties such as owners, designers, contractors, subcontractors, suppliers, etc. [3]. Even though delaying a non-critical activity within its float does not delay the project deterministically, one of the risk management techniques is to consider the probabilities of the consequences that might occur due to any occasion such as delaying these activities [4]. In fact, project activities might involve huge uncertainties in their costs and durations such highway projects which depend highly on the traffic conditions and feasibilities of making diversions [5].

Project risks and uncertainties cannot be considered in the CPM technique as this method assumes that all durations are deterministic. Thus, many researches have been conducted to develop new technique such as Program Evaluation and Review Technique (PERT) that deals with projects activities duration as probabilities [6]. Consequently, delaying any activity, even if it is not critical, increases the probability of delaying the project, which in turn introduces extra cost related to the project delay consequences.

Hence, the effect of float use on project's cost due to the increase of probability of delaying the project finish time has been discussed in various researches. Gong and Rowings [7] called attention to the fact that using the float of non-critical activities, especially those whose durations are larger in uncertainties, might result in schedule overruns. There is a limit on float use that should not be exceeded; otherwise, the project duration will be most probably increased. Accordingly, the amount of float that can be used is called "safe float". Calculation of safe float has been achieved by utilizing Monte-Carlo simulation that uses a probability stochastic analysis technique to find the point where the effect of using float can be negligible.

Zhong and Zhang [8] introduced a new definition of safe float using PERT analysis technique. Safe float for any path "*i*" is defined according to the authors as the total float for path "*i*" times the ratio of the variance of "*i*" to the variance of the critical path "*a*". This simple safe float calculation approach clarifies any misleading information that might be provided to managers in construction industry regarding the availability of using floats.

Afterwards, a new idea has come up to find a monetary value of the float loss due to uncertainty of non-critical activities involving the referred float. A two parameter probability stochastic analysis has been performed for time and cost to come up with a model that relates the delay of non-critical activities to cost change [2]. That analysis can be performed when having a database that estimates each activity's mean value and standard deviation.

Later on, the impact of float consumption is considered when time-cost trade-off (also known as activities durations crashing) is performed. When duration crashing is performed for any activity that merges in one or more paths, float is consumed in the other paths where the crashed activity does not exist. Therefore, the paths that are not crashed have more probability of delaying the project when probabilistic analysis is performed [9]. Moreover, risk of delaying the project has been also considered in the bidding process whenever time and cost are the criteria of awarding the project. In fact, when time is reduced in the bidding stage, the risk of float consumption increases, and that might cause delays to the project. The contractor then has to bear the liability of this delay by paying liquidated damages to the client and, this should be considered in the bidding price [10].

2.2. Resource Histogram Fluctuations Cost Effect Due to Use of Float

Many efforts have been done in the last three decades on the importance of float for resource allocation and leveling. In fact, resource leveling is usually performed to reduce the resource requirements histograms fluctuations by shifting non-critical activities starting dates within their floats [11]. Hence, float in traditional concept is a misleading figure when resource constraints exist [12]. Resource constraints yield in delaying activities in need of the limited resource that is being allocated for other activities. In other words, maintaining the resource constraints might yield in float consumption.

Gong [13] emphasized that floats are often used in project networks for resource allocations and as an alternative for reducing project costs without causing negative impact on project duration. However, when the time uncertainties of non-critical activities are great, the use of floats can lead to an increased risk of project schedule overruns, and accordingly an increase in project costs. [13] According to Gong [13], the optimum use of float would be at the point where the total cost of resources and over heads are minimized. In this article, the author illustrated how to optimize the use of float through a bridge construction project example using the tower crane as a dominant resource.

De La Garza and Kim [14] introduced a new deterministic approach for scheduling and calculation of float based on resource constraints. In fact, traditional schedule prepared using CPM are not necessarily realistic as many resources are limited in practice. In resource constrained project method (RCPM), the total float is not always free because of the possibility of having insufficient amount of resources; hence, real float should be calculated after taking into consideration the resources constraints while scheduling the activities as well as the technical precedence relationship [14].

Koulinas and Anagnostopoulos [15] introduced a new resourcing technique to allocate and level resources in resource constrained projects. This technique combines the objectives of RCPM and resource leveling models using Hyperheuristic algorithms to perform trade-off between resource leveling cost and project duration minimization considering resources constraints [15]. Hyperheuristic algorithm is a heuristic search method that seeks to automate, often by the incorporation of machine learning techniques, the process of selecting, combining, generating or adapting several simpler heuristics (or components of such heuristics) to efficiently solve computational search problems [16].

2.3. Cash Flow Effect Due to the Use of Float

Cash flow studies, during the last decades, were mostly isolated from float consumption impacts. However, these studies can be related to this thesis topic as they highlight the importance of maximizing the present worth of profit and handling the financial issues of the project. Tavakoli [17] developed a model to forecast cash flows for projects financed by banks (or any rate of return financing systems). The model can predict the net revenue and rate of return expected based on the initial project plan.

Moreover, a journal article discussion on schedule techniques based on the financial capabilities of the contractors has been published. This journal article emphasized on the importance of considering the amount of money that can be financed each period of the project when an execution schedule is to be made. One of the most common mistakes that are done by planners is setting up a plan schedule with financial requirements that exceeds their capabilities. Consequently, these schedules are not realistic and cannot be performed. Therefore, financial requirements of all project activities shall be highlighted and considered during the stage of planning [18].

Later, a model to evaluate the extent of risk occurrence and impact in case of occurrence on construction cost flow forecast had been developed by Odeyinka [19]. The model evaluates the forecasted risk based on surveying contracting firms. Various types of risk factors were investigated such as change orders from the clients, and each factor is evaluated based on its effectiveness and probability of occurrence.

Later on, cash flow models are developed using singularity function in order to support financial decision making in construction project that increase their net present worth of profit [20]. Singularity functions are defined as class of discontinuous functions that contain singularities, i.e. they are discontinuous at their singular points [21]. The analysis of cash flow is performed by connecting all cash inflow and out flow transactions to projects linear schedule through singularity functions. Cash flows that are processed through billings and payments are connected to the model which determines the net present worth of profit. Then, the project schedule is decided by maximizing the profit of cash flow [20].

Zayed and Liu [22] developed a model that forecasts risk factors on cash flow effects using probability stochastic method. Cash flow models are designed and implemented using the integration of the Monte Carlo simulation and AHP techniques to forecast cash flow and overdraft" [22].

2.4. Float Pre-Allocation Methods

Ownership of float has been a doubtful concern in construction projects in the past decades. All cases related to claims due to delays are mostly stuck because of the ambiguity of the referred matter. In fact, delays can be due to various reasons that are caused by the contractor, consultant of client. Approval of drawings, inadequate early planning and slowness of the owners' decision-making process are the top causes of delay [23]. Hence, ambiguity and conflicts occur when delays are introduces to non-critical activities that their float ownership is not clear.

Hence, many researches were made in the field of float allocation. The oldest and most common approach that is used to resolve conflicts in courts is that float is owned by the one who uses it first [24]. However, many efforts have been done by researchers in order to find solutions that clarify each of the contactor’s and the client’s right of utilizing float. Ponce de Leon [24] introduced the concept of no one has the right to use the float. This concept is called the bar approach; which was based on the assumption that float is represented as a bar on the bar chart. Hence, delaying any non-critical activity will result in extension of the project’s total duration, which in turn makes the party that delayed the non-critical activity held responsible for the delay.

Another concept has been introduced few years later suggesting that float should be distributed among parties based on their responsibilities towards project’s risks consequences [25]. For instance, if contractors have complete responsibility towards delays due to risks the project might encounter – mostly in lump sum contracts, they have 100% ownership of float. Similarly, in cases that client is involved partially or completely in taking the consequences of risks, float should be distributed accordingly as per each party share of risk taking.

Later on, De La Garza [26] introduced a method to trade float as a commodity. This method is simply applied by obtaining a monetary value of float for a particular case such that it can be traded among the project parties using that value. The monetary value would be considered as the equivalent losses that the contractor has incurred due to increase of risk. The trade value is calculated by dividing the difference of the late finish cost and early finish cost by the total float to obtain the float cost per day. Hence, float is assumed to belong to the contractor, and clients have to buy it whenever they need to use it.

Four years later, another approach called path distribution was introduced regarding float sharing. This method suggests that “the total float for each noncritical path can be distributed based on the duration of the activity. The method starts from the nearest critical path (second longest path after the critical path) and proceeds to the next-nearest critical path (third longest path after the critical path) until completion of all the noncritical paths [27]. In other words, any party has the right to delay a non-critical activity by a limited amount that will not affect other activities float. Any delay beyond that will be considered as delaying a critical activity.

Afterwards, De La Garza [1] suggested that amount and ownership of float is to be agreed on prior to signing the contract. In particular, they recommended contractual clauses that can be used in contracts to define each party's legibility of using float, taking into consideration the updates of the project's actual situation.

Al-Gahtani [28] introduced a systematic approach called - day-by-day - for updating project's plan on a periodic basis. After each update, projects duration, and float's amount and ownership would be updated. Any party will be held responsible for their delay right after the periodic analysis is conducted.

Two years later, the total risk approach to allocate float had been introduced. This approach is mainly based on utilizing all approaches and concepts mentioned earlier in this section. The approach starts from distributing float among parties before signing the contract according to the risk factor and based on the path distribution, and ends up with day-by-day and commodity approach to hold each party responsible towards projects delays [29].

Chapter 3: Total Damages Model

3.1. CPM-Delay Analysis Model

Studying delay of noncritical activities effect requires a model that performs the CPM calculation for a construction project. CPM has been chosen because it is the original method on which all other methods are based on. In this model, the delay aspect has been added to the CPM to calculate the starting and ending dates of non-critical activities after the delay.

3.1.1. CPM-delay analysis model algorithms. MS Excel spreadsheets have been used to develop the model which consists of the following:

3.1.1.1. *Data input.* The following data shall be provided in order to perform the analysis:

a. *Activity ID* – i.

b. *Activity Name/Description.*

c. *Activities duration* - D_i.

d. *Precedence Relation*– $P_{i,s}$, where s refers to the activities ID's from 1 to n that precede activity i.

e. *Delay* - DE_i**:** the number of days that an activity starts on is delayed from its early start. Delay shall not exceed the defined float.

3.1.1.2. *CPM analysis.* Based on the provided data, an MS Excel spreadsheet is used to perform a full CPM analysis and obtain the following:

a. *Early Start* - ES_i. The early start of the first activity will be set to zero ($ES_o = 0$), and following activities earliest start will be obtained as the maximum of the earliest finish of all its predecessors using Equation (1).

$$ES_i = MAX\left(EF_{P_{i,s=1,2,..n}}\right) \qquad Equation\ (1)$$

b. *Early Finish* - EF_i. The early finish is calculated using Equation (2).

$$EF_i = ES_i + D_i \qquad Equation\ (2)$$

c. *Late Finish* - LF_i. The late finish of the last activity is set equal to its early finish as $LF_n = EF_n$. Other activities' late finishes are set as the minimum of late start of following activities as per Equation (3)

$$LF_i = MIN(LS_{P_{i=1,2,..n,s}}) \qquad Equation\ (3)$$

d. *Late Start*– LS_i. The late start is calculated using Equation (4).

$$LS_i = LF_i - D_i \qquad \text{Equation } (4)$$

e. *Total Float* - F_i: The total float is calculated using Equation (5).

$$F_i = LS_i - ES_i \qquad \text{Equation } (5)$$

f. *Delayed Start* - DS_i The delayed start is calculated using Equation (6).

$$DS_i = ES_i + DE_i \qquad \text{Equation } (6)$$

g. *Delayed Finish* - DF_i The delayed finish is calculated using Equation (7).

$$DF_i = EF_i + DE_i \qquad \text{Equation } (7)$$

3.1.2. Illustrating example. A project consists of 8 activities: A, B, C, D, E, F, G, and H with total duration of 318 days and critical path {A, B, D, G, H} as demonstrated by the network diagram shown in Figure 1. Activities C and E were delayed by 93 and 3 days respectively.

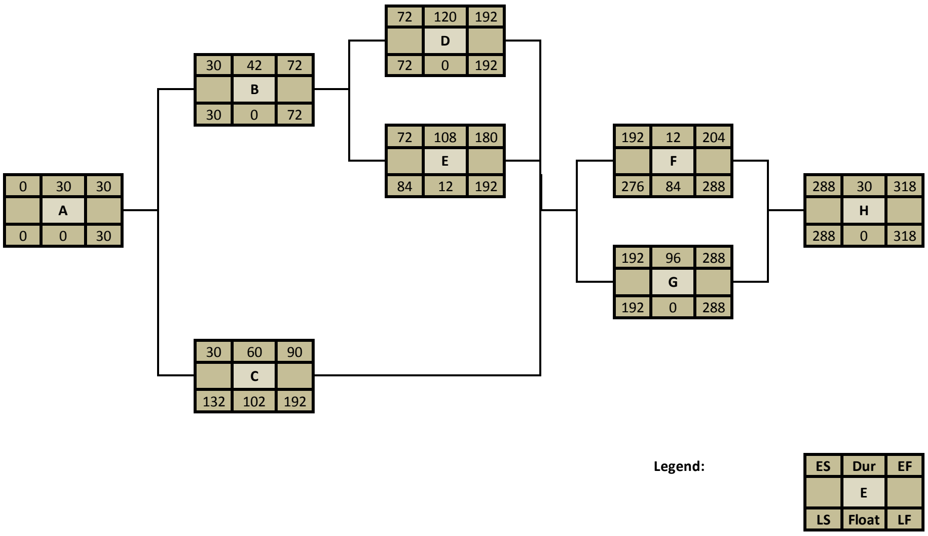


Figure 1: Network Diagram Example

CPM Delay Analysis model is used to perform the analysis as follows:

3.1.2.1. *Data input.* Table 1 represents the data input required to perform the analysis.

Table 1: Example CPM Analysis Data Input

Activities Data Input						
i	Activity	$P_{i,s}$				D_i
1	A	-	-	-	-	30
2	B	1	-	-	-	42
3	C	1	-	-	-	60
4	D	2	-	-	-	120
5	E	2	-	-	-	108
6	F	3	4	5	-	12
7	G	3	4	5	-	96
8	H	6	7	-	-	30

3.1.2.2. ***CPM analysis.*** Equations (1) through (7) are used to program a MS Excel spreadsheet and obtain the results shown in Table 2.

Table 2: Example CPM Model Results

i	Activity	$P_{i,s}$				D_i	ES_i	EF_i	LS_i	LF_i	F_i	DE_i	DS_i	DF_i
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	93	123	183
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	3	75	183
6	F	3	4	5	-	12	192	204	276	288	84	0	192	204
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

For example, activity "F" dates are obtained as follows:

$$i = 6$$

$$ES_6 = MAX(90,192,180) = 192 \qquad Equation\ (1)$$

$$EF_6 = 192 + 12 = 204 \qquad Equation\ (2)$$

$$LF_6 = MIN(288) = 288 \qquad Equation\ (3)$$

$$LS_6 = 288 - 12 = 276 \qquad Equation\ (4)$$

$$F_6 = 276 - 192 = 84 \qquad Equation\ (5)$$

$$DS_6 = 192 + 0 = 192 \qquad Equation\ (6)$$

$$DF_6 = 204 + 0 = 204 \qquad Equation\ (7)$$

3.2. Time Value of Money Delay Cost Model

The construction process depends highly on the cash flow diagram. Maintaining a high positive cash flow during the execution period of a project is beneficial due to the time value of money. In many cases, especially those that involve progress payment methods, delaying activities even if they are not critical yields in changes of the cash flow histogram. Hence, that will result in changes in the value of the profit for projects' parties due to time value of money.

3.2.1. Time value of money delay cost model algorithms. The cash flow model developed in this research demonstrates the effect of delaying activities on the Net Present Value of Profit from the contractors' perspective. This model consists of the following.

3.2.1.1. ***Data input.*** The following data shall be provided in order to perform the analysis:

a. *Contract Value* - CV_i: the amount of money to be paid by the client to the contractor upon the completion of activity i.

b. *Internal Payments* - IP_i: that include all expenses paid at the end of the month that the related work is accomplished by such as labor wages, fuel…etc.

c. *External payments* - EP_i: that include all expenses paid one month after the work accomplishment date such as material invoices, S/C payments…etc.

d. *Retention* - R_c: the percent of deduction to be made by the client on the contractors due payments.

e. *Retention Period* - t_c: the number of months after the completion date of the project that the client can hold the retention payment.

f. *Retention on suppliers/subcontractors* - R_s: the percent of deduction to be made by the contractor on the other suppliers/subcontractors due payments.

g. *Down payment* - DP_0: the percent of the total project value to be paid by the client to the contractor at the beginning of the project.

h. *Retention on suppliers/subcontractors Period* - t_s: the number of months after the completion date of the project that the contractor can hold the retention payment.

i. *Rate of Return/Interest* - ROR: the rate of return on the contractors investment – to be agreed on in the contract.

j. *Monthly Indirect cost* - IDC_t: the average monthly cost paid by the contractor (considering the time value of money) during the full duration of the project.

k. *Number of Working Days/Month* - n_w

3.2.1.2. *Cash break down schedules.* All revenues and expenses at the end of each month related to the execution of all activities are obtained as follows:

3.2.1.2.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, the cash breakdown for direct cost and earned value are obtained as follows:

a. *Monthly Direct Cost* - DC_t. It includes the monthly internal payments IP_{it} paid at the end of the work execution month t and the monthly external payments paid at the end of the following month $t+1$.

i. Activity Monthly Internal Payments IP_{it} are obtained by set of algorithms (1).

$If\ EF_i > t \text{ x } n_w > ES_i > (t-1)n_w,\ then\ IP_{it} = \frac{IP_i}{D_i}(t \text{ x } n_w - ES_i),\ else$

$If\ EF_i > t \text{ x } n_w > (t-1)n_w > ES_i,\ then\ IP_{it} = \frac{IP_i}{D_i} \text{ x } n_w,\ else$

$If\ t \text{ x } n_w > EF_i > ES_i > (t-1)n_w,\ then\ IP_{it} = IP_i,\ else$

$If\ t \text{ x } n_w > EF_i > (t-1)n_w > ES_i\ ,\ then\ IP_{it} = \frac{IP_i}{D_i} \text{ x } (\ EF_i - (t-1)n_w),\ else$

$IP_{it} = 0 \qquad\qquad Set\ of\ Algorithms\ (1)$

ii. Activity Monthly External Payments EP_{it} are obtained by set of algorithms (2).

$If\ EF_i > t \text{ x } n_w > ES_i > (t-1)n_w,\ then\ EP_{it} = \frac{EP_i}{D_i}(t \text{ x } n_w - ES_i)(1-R_s),\ else$

$If\ EF_i > t \text{ x } n_w > (t-1)n_w > ES_i,\ then\ EP_{it} = \frac{EP_i}{D_i} \text{ x } n_w(1-R_s),\ else$

$If\ t \text{ x } n_w > EF_i > ES_i > (t-1)n_w,\ then\ EP_{it} = EP_i(1-R_s),\ else$

$If\ t \text{ x } n_w > EF_i > (t-1)n_w > ES_i\ ,\ then\ EP_{it} = \frac{EP_i}{D_i} \text{ x } (\ EF_i - (t-1)n_w)(1-R_s),\ else$

$EP_{it} = 0 \qquad\qquad Set\ of\ Algorithms\ (2)$

The direct cost for any activity i expensed at month t is defined using Equation (8).

$$DC_{it} = IP_{it} + EP_{i(t-1)} \qquad Equation\ (8)$$

The Monthly Direct Cost expensed at month t is defined using Equation (9).

$$DC_t = \sum_{i=1}^{n} DC_{it} \qquad Equation\ (9)$$

where *n* is the total number of activities.

b. *Monthly Earned Value* - EV_t is amount of money earned as progress ratio of the activities' contract values. The Activity Monthly Earned Values EV_{it} are obtained by set of algorithms (3).

$If\ EF_i > t \text{ x } n_w > ES_i > (t-1)n_w,\ then\ EV_{it} = \frac{CV_i}{D_i}(t \text{ x } n_w - ES_i),\ else$

$If\ EF_i > t \text{ x } n_w > (t-1)n_w > ES_i,\ then\ EV_{it} = \frac{CV_i}{D_i} \text{ x } n_w,\ else$

$If\ t \text{ x } n_w > EF_i > ES_i > (t-1)n_w),\ then\ EV_{it} = CV_i,\ else$

$If\ t \text{ x } n_w > EF_i > (t-1)n_w > ES_i\ ,\ then\ EV_{it} = \frac{CV_i}{D_i}(\ EF_i - (t-1)n_w),\ else$

$EV_{it} = 0 \qquad\qquad Set\ of\ Algorithms\ (3)$

The *Monthly Earned Value* at month t is defined using Equation (10).

$$EV_t = \sum_{i=1}^{n} EV_{it} \qquad Equation\ (10)$$

3.2.1.2.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, the cash breakdown for direct cost and earned value are obtained as follows:

a. *Monthly Direct Cost* - DC_t . Includes the monthly internal payments IP_{it} paid at the end of the work execution month t and the monthly external payments paid at the end of the following month $t+1$.

i. Activity Monthly Internal Payments IP_{it} are obtained by set of algorithms (4).

$If\ DF_i > t \text{ x } n_w > DS_i > (t-1)n_w,\ then\ IP_{it} = \frac{IP_i}{D_i}(t \text{ x } n_w - DS_i),\ else$

$If\ DF_i > t \text{ x } n_w > (t-1)n_w > DS_i,\ then\ IP_{it} = \frac{IP_i}{D_i} \text{ x } n_w,\ else$

$If\ t \text{ x } n_w > DF_i > DS_i > (t-1)n_w,\ then\ IP_{it} = IP_i,\ else$

$If\ t \text{ x } n_w > DF_i > (t-1)n_w > DS_i\ ,\ then\ IP_{it} = \frac{IP_i}{D_i} \text{ x } (\ DF_i - (t-1)n_w),\ else$

$IP_{it} = 0 \qquad Set\ of\ Algorithms\ (4)$

ii. Activity Monthly External Payments EP_{it} are obtained by set of algorithms (5).

$If\ DF_i > t \text{ x } n_w > DS_i > (t-1)n_w,\ then\ EP_{it} = \frac{EP_i}{D_i}(t \text{ x } n_w - DS_i),\ else$

$If\ DF_i > t \text{ x } n_w > (t-1)n_w > DS_i,\ then\ EP_{it} = \frac{EP_i}{D_i} \text{ x } n_w,\ else$

$If\ t \text{ x } n_w > DF_i > DS_i > (t-1)n_w,\ then\ EP_{it} = EP_i,\ else$

$If\ t \text{ x } n_w > DF_i > (t-1)n_w > DS_i\ ,\ then\ EP_{it} = \frac{EP_i}{D_i} \text{ x } (\ DF_i - (t-1)n_w),\ else$

$EP_{it} = 0 \qquad Set\ of\ Algorithms\ (5)$

Values of DC_{it} and DC_t can be determined using Equations (8) and (9) respectively.

b. *Monthly Earned Value* - EV_t is amount of money earned as progress ratio of the activities contract values. The Activity Monthly Earned Values is EV_{it} are Obtained by set of algorithms (6).

$If\ DF_i > t \text{ x } n_w > DS_i > (t-1)n_w,\ then\ EV_{it} = \frac{CV_i}{D_i}(t \text{ x } n_w - DS_i),\ else$

$If\ DF_i > t \text{ x } n_w > DS_i > (t-1)n_w,\ then\ EV_{it} = \frac{CV_i}{D_i} \text{x } n_w,\ else$

$If\ t \text{ x } n_w > DF_i > DS_i > (t-1)n_w),\ then\ EV_{it} = CV_i,\ else$

$If\ t \text{ x } n_w > DF_i > (t-1)n_w > DS_i\ ,\ then\ EV_{it} = \frac{CV_i}{D_i}(\ DF_i - \ (t-1)n_w),\ else$

$EV_{it} = 0$ $\quad Set\ of\ Algorithms\ (6)$

Values of EV_t at each month can be determined using Equation (10).

3.2.1.3. ***Cash flow schedule.*** Calculation of present worth of profit is performed by obtaining all cash flows during each month for the two different cases.

3.2.1.3.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, the present worth of profit is obtained as follows:

a. *Monthly In-Direct Cost* IDC_t is obtained from 3.2.1.1.

b. *Monthly Direct Cost* DC_t is obtained from 3.2.1.2.1.1.

c. *Monthly Earned Value* EV_t is obtained from 3.2.1.2.1.2.

d. *Monthly Cash Out* is obtained using Equation (11).

$$CO_t = DC_t + IDC_t \qquad Equation\ (11)$$

e. *Monthly Retention and Down Payment Deduction* is obtained using Equation (12).

$$PD_t = (R_c + DP_0\ )EV_t \qquad Equation\ (12)$$

f. *Monthly Cash In* is obtained using Equation (13)

$$CI_t = \ EV_{t-1} - PD_{t-1} \qquad Equation\ (13)$$

g. *Cash In* and *Cash Out* at months $t = 0$, $t = t_c$ and , $t = t_s$ are obtained using Equations (14-16).

$$CI_0 = DP_0 \text{ x } \sum_{1}^{n} CV_i \qquad Equation\ (14)$$

$$CI_{t_c} = R_c \text{ x } \sum_{1}^{n} CV_i \qquad Equation\ (15)$$

$$CO_{t_c} = R_s \text{ x } \sum_{1}^{n} EP_i \qquad Equation\ (16)$$

h. *Monthly Profit* is obtained using Equation (17).

$$P_t = \ CI_t - CO_t \qquad Equation\ (17)$$

i. *The Monthly Present Worth of Profit* is obtained by Equation (18).

$$PW_t = \frac{P_t}{(ROR+1)^t} \qquad Equation\ (18)$$

j. *The Present Worth of Profit* is obtained using Equation (19).

$$PW_{Un-Delayed} = PW_o + PW_{t_c} + PW_{t_s} + \sum_{1}^{\mathrm{int}(EF_n/n_w)} PW_t \qquad Equation\ (19)$$

3.2.1.3.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, the present worth of profit is obtained as follows:

a. *Monthly In-Direct Cost* IDC_t is obtained from 3.2.1.1.

b. *Monthly Direct Cost* DC_t is obtained from 3.2.1.2.2.1.

c. *Monthly Earned Value* EV_t is obtained from 3.2.1.2.2.2.

Equations (11) to (18) are to be used in order to calculate the values in 3.2.1.3.1 for the delayed case

The Present worth of Profit is obtained using Equation (20).

$$PW_{Delayed} = PW_o + PW_{t_c} + PW_{t_s} + \sum_{1}^{\mathrm{int}(DF_n/n_w)} PW_t \qquad Equation\ (20)$$

3.2.1.4. *Time value of money delay cost.* The present worth of profit obtained in 3.2.1.3.1 shall differ from 3.2.1.3.2 due to the effect of time value of money. The difference is assumed to be the time value of money cost as described in Equation (21). In fact, this cost might be positive or negative and differs in magnitude depending on many factors as discussed in 3.2.3.

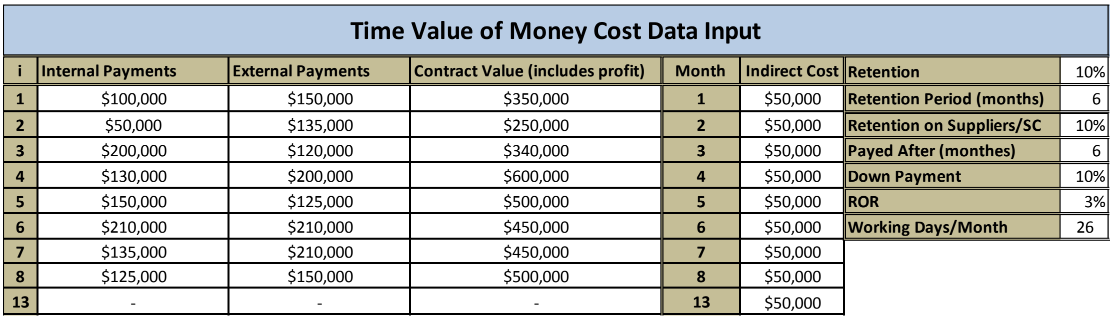
The Time Value of money Delay Cost is determined by Equation (21).

$$CTVM = PW_{Un-Delayed} - PW_{Delayed} \qquad Equation\ (21)$$

3.2.2. Illustrating example. Using the same example used in 3.1.2, the model solves the case as follows

3.2.2.1. *Data input.* Table 3 represents the data input required to perform the analysis.

Table 3: Example CTVM Data Input



Time Value of Money Cost Data Input

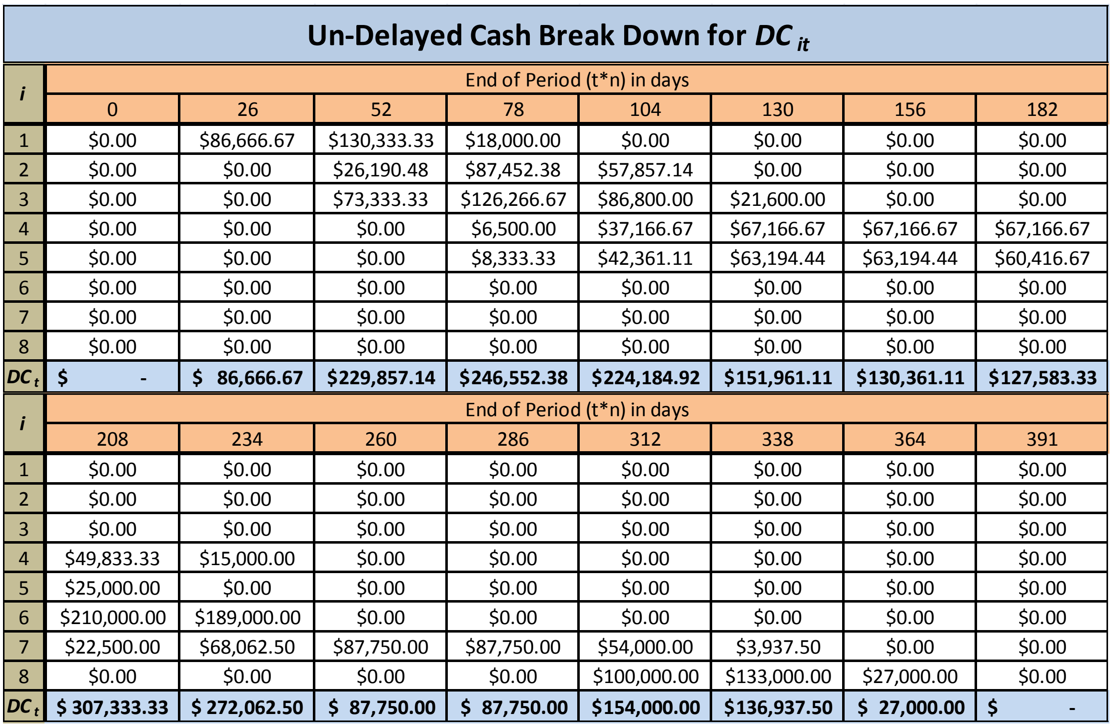
i	Internal Payments	External Payments	Contract Value (includes profit)	Month	Indirect Cost	Retention	10%
1	$100,000	$150,000	$350,000	1	$50,000	Retention Period (months)	6
2	$50,000	$135,000	$250,000	2	$50,000	Retention on Suppliers/SC	10%
3	$200,000	$120,000	$340,000	3	$50,000	Payed After (monthes)	6
4	$130,000	$200,000	$600,000	4	$50,000	Down Payment	10%
5	$150,000	$125,000	$500,000	5	$50,000	ROR	3%
6	$210,000	$210,000	$450,000	6	$50,000	Working Days/Month	26
7	$135,000	$210,000	$450,000	7	$50,000		
8	$125,000	$150,000	$500,000	8	$50,000		
13	-	-	-	13	$50,000		

3.2.2.2. ***Cash break down schedules:*** Using spreadsheets, Monthly Direct Costs and Earned Values are obtained for each activity at each month for both delayed and Un-delayed case as follows:

3.2.2.2.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, the cash breakdown for direct cost and earned value are obtained as follows:

a. *Monthly Direct Cost* - DC_t. Table 4 represents the calculation process using excel sheet to obtain the values of activities monthly direct cost DC_{it} and monthly direct cost DC_t.

Table 4: Example Monthly Direct Cost Calculations for Un-delayed Case



Un-Delayed Cash Break Down for DC_{it}

i	End of Period (t*n) in days							
	0	26	52	78	104	130	156	182
1	$0.00	$86,666.67	$130,333.33	$18,000.00	$0.00	$0.00	$0.00	$0.00
2	$0.00	$0.00	$26,190.48	$87,452.38	$57,857.14	$0.00	$0.00	$0.00
3	$0.00	$0.00	$73,333.33	$126,266.67	$86,800.00	$21,600.00	$0.00	$0.00
4	$0.00	$0.00	$0.00	$6,500.00	$37,166.67	$67,166.67	$67,166.67	$67,166.67
5	$0.00	$0.00	$0.00	$8,333.33	$42,361.11	$63,194.44	$63,194.44	$60,416.67
6	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
7	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
8	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
DC_t	**$ -**	**$ 86,666.67**	**$229,857.14**	**$246,552.38**	**$224,184.92**	**$151,961.11**	**$130,361.11**	**$127,583.33**

i	End of Period (t*n) in days							
	208	234	260	286	312	338	364	391
1	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
2	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
3	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
4	$49,833.33	$15,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
5	$25,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
6	$210,000.00	$189,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
7	$22,500.00	$68,062.50	$87,750.00	$87,750.00	$54,000.00	$3,937.50	$0.00	$0.00
8	$0.00	$0.00	$0.00	$0.00	$100,000.00	$133,000.00	$27,000.00	$0.00
DC_t	**$ 307,333.33**	**$ 272,062.50**	**$ 87,750.00**	**$ 87,750.00**	**$154,000.00**	**$136,937.50**	**$ 27,000.00**	**$ -**

For example, the direct cost of the executed work of activity “C” at the third month, to find DC_{33}, values of IP_{33} and EP_{32} should be obtained given that:

$$ES_3 = 30, EF_3 = 90, IP_3 = \$200{,}000, EP_3 = \$120{,}000, n_w = 26 \; and \;\; R_s = 10\%$$

Following set of algorithms (1) @ $(i = 3, t = 3)$

$$EF_3 = 90 > t \text{ x } n_w = 78 > (t-1)n_w = 52 > ES_i = 30, then$$

$$IP_{33} = \frac{\$200{,}000}{60} \text{ x } 26 = \$86{,}667$$

Following set of algorithms (2) @ $(i = 3, t = 2)$

$$EF_3 = 90 > t \text{ x } n_w = 52 > ES_i = 30 > (t-1) \text{ x } n_w = 26, then$$

$$EP_{33} = \frac{\$120{,}000}{60} (\ 2\text{x } 26 - 30\ )(1 - 0.10) = \$39{,}600$$

Using Equation (8):

$$DC_{33} = IP_{33} + EP_{32} = \$86{,}667 + \$39{,}600 = 126{,}266.67$$

The same procedure is programmed on a spreadsheet to obtain all the values of IP_{i3} and EP_{i2} as shown in Table 4.

DC_3 is obtained using Equation (9) as follows:

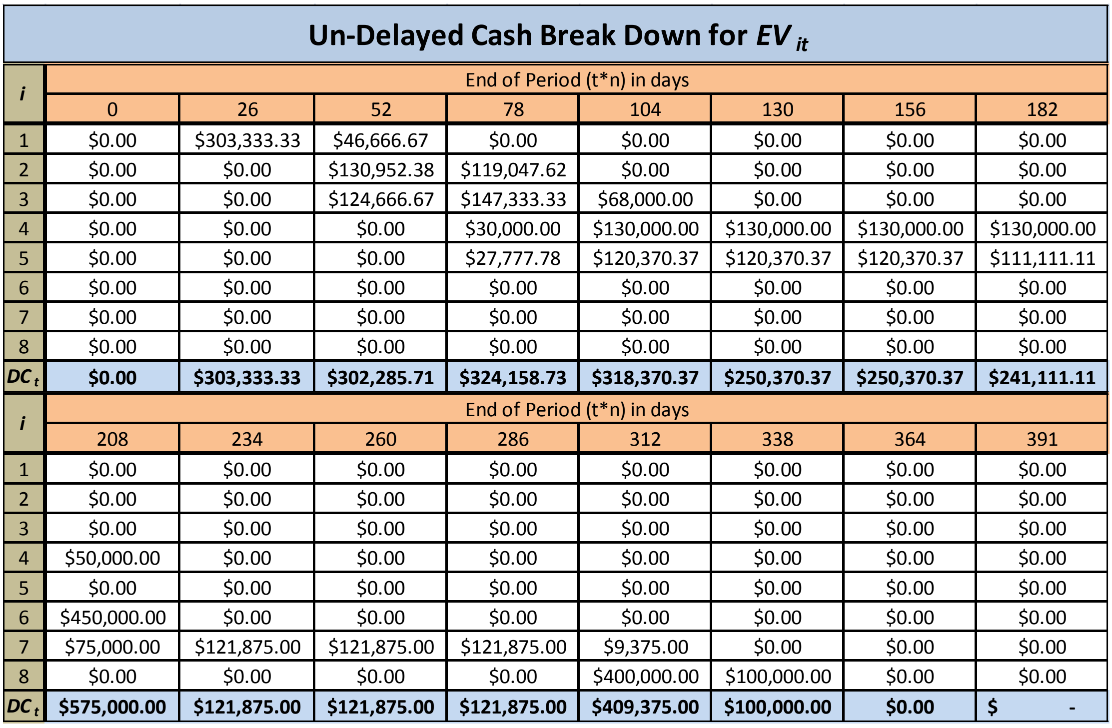
$$DC_3 = \sum_{i=1}^{8} DC_{i3}$$

$$= \$18{,}000 + \$87{,}452.38 + \$126{,}266.67 + \$6{,}500 + \$8{,}333.33 + 0 + 0 + 0$$

$$= \$246{,}552.38$$

b. *Monthly Earned Value* - EV_t. Table 5 represents the calculation process using excel sheet to obtain the values of activities monthly earned value EV_{it} and monthly earned value EV_t.

Table 5: Example Monthly Earned Value Calculations for Un-delayed Case



Un-Delayed Cash Break Down for *EV* $_{it}$

i	End of Period (t*n) in days							
	0	26	52	78	104	130	156	182
1	$0.00	$303,333.33	$46,666.67	$0.00	$0.00	$0.00	$0.00	$0.00
2	$0.00	$0.00	$130,952.38	$119,047.62	$0.00	$0.00	$0.00	$0.00
3	$0.00	$0.00	$124,666.67	$147,333.33	$68,000.00	$0.00	$0.00	$0.00
4	$0.00	$0.00	$0.00	$30,000.00	$130,000.00	$130,000.00	$130,000.00	$130,000.00
5	$0.00	$0.00	$0.00	$27,777.78	$120,370.37	$120,370.37	$120,370.37	$111,111.11
6	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
7	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
8	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
DC_t	**$0.00**	**$303,333.33**	**$302,285.71**	**$324,158.73**	**$318,370.37**	**$250,370.37**	**$250,370.37**	**$241,111.11**

i	End of Period (t*n) in days							
	208	234	260	286	312	338	364	391
1	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
2	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
3	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
4	$50,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
5	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
6	$450,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
7	$75,000.00	$121,875.00	$121,875.00	$121,875.00	$9,375.00	$0.00	$0.00	$0.00
8	$0.00	$0.00	$0.00	$0.00	$400,000.00	$100,000.00	$0.00	$0.00
DC_t	**$575,000.00**	**$121,875.00**	**$121,875.00**	**$121,875.00**	**$409,375.00**	**$100,000.00**	**$0.00**	**$ -**

For example, the earned value from the work execution of activity "C" at the third month. The Value of EV_{33} is obtained given that:

$$ES_3 = 30, EF_3 = 90, CV_3 = \$340{,}000.$$

Following set of algorithms (3) @ $(i = 3, t = 3)$

$$EF_3 = 90 > t \text{ x } n_w = 78 > (t-1)n_w = 52 > ES_i = 30, then$$

$$EV_{33} = \frac{\$340{,}000}{90 - 30} \text{ x } 26 = \$147{,}333.33$$

Same procedure is programmed on a spread sheet to obtain the all values of EV_{i3} as shown in Table 5.

EV_3 Is obtained using Equation (10) as follows:

$$EV_3 = \sum_{i=1}^{8} EV_{i3} = \; = 0 + \$119{,}047.62 + \$147{,}333.33 + \$30{,}000 + \$27{,}777.78 + 0 + 0 + 0$$

$$= \$324{,}158.73$$

3.2.2.2.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, the cash breakdown for direct cost and earned value is obtained as follows:

a. *Monthly Direct Cost - DC_t.* Table 6 represents the calculation process using excel sheet to obtain the values of activities monthly direct cost DC_{it} and monthly direct cost DC_t.

Table 6: Example Monthly Direct Cost Calculations for Delayed Case

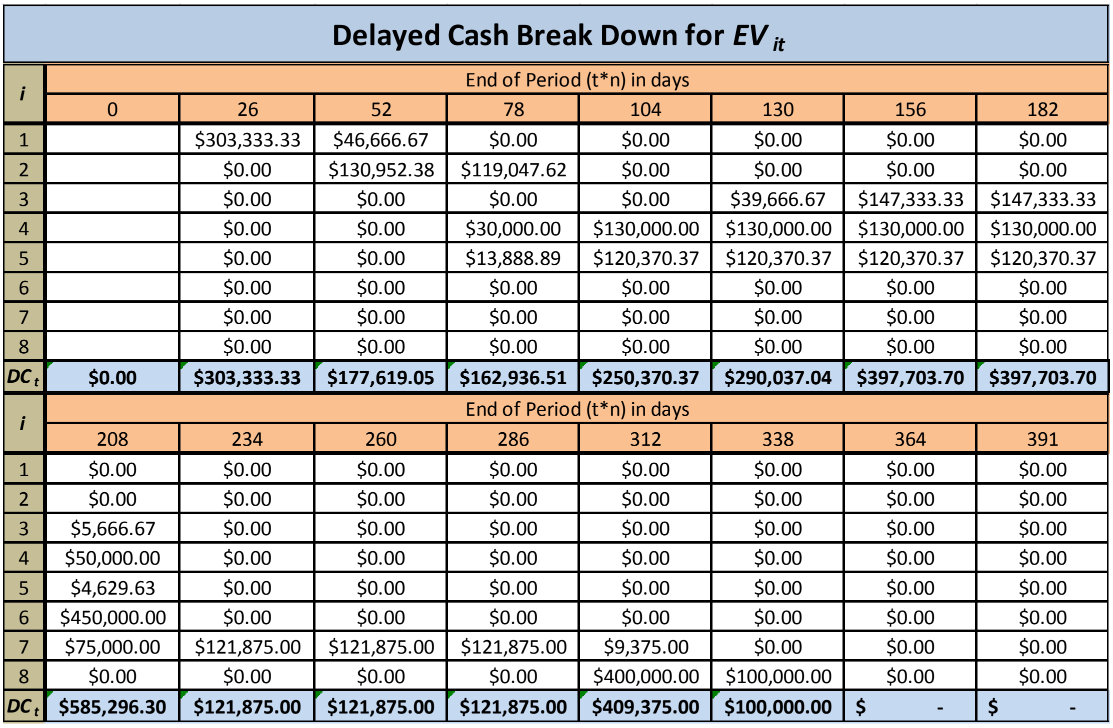
Delayed Cash Break Down for DC_{it}

i	End of Period (t*n) in days							
	0	26	52	78	104	130	156	182
1		$86,666.67	$130,333.33	$18,000.00	$0.00	$0.00	$0.00	$0.00
2		$0.00	$26,190.48	$87,452.38	$57,857.14	$0.00	$0.00	$0.00
3		$0.00	$0.00	$0.00	$0.00	$23,333.33	$99,266.67	$133,466.67
4		$0.00	$0.00	$6,500.00	$37,166.67	$67,166.67	$67,166.67	$67,166.67
5		$0.00	$0.00	$4,166.67	$39,236.11	$63,194.44	$63,194.44	$63,194.44
6		$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
7		$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
8		$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
DC_t	**$ -**	**$ 86,666.67**	**$156,523.81**	**$116,119.05**	**$134,259.92**	**$153,694.44**	**$229,627.78**	**$263,827.78**

i	End of Period (t*n) in days							
	208	234	260	286	312	338	364	391
1	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
2	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
3	$50,133.33	$1,800.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
4	$49,833.33	$15,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
5	$28,472.22	$1,041.67	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
6	$210,000.00	$189,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
7	$22,500.00	$68,062.50	$87,750.00	$87,750.00	$54,000.00	$3,937.50	$0.00	$0.00
8	$0.00	$0.00	$0.00	$0.00	$100,000.00	$133,000.00	$27,000.00	$0.00
DC_t	**$ 360,938.89**	**$ 274,904.17**	**$ 87,750.00**	**$ 87,750.00**	**$154,000.00**	**$136,937.50**	**$ 27,000.00**	**$ -**

As per Table 6, values of IP_{33} and EP_{32} are zero. This can be observed using a set of algorithms (4) and (5) where none of the four conditions applies. This is because activity “C” is delayed by a value that makes no work execution occurs at $t = 2$ and $t = 3$.

Table 7: Example Monthly Earned Value Calculations for Delayed Case



Delayed Cash Break Down for EV_{it}

i	End of Period (t*n) in days							
	0	26	52	78	104	130	156	182
1		$303,333.33	$46,666.67	$0.00	$0.00	$0.00	$0.00	$0.00
2		$0.00	$130,952.38	$119,047.62	$0.00	$0.00	$0.00	$0.00
3		$0.00	$0.00	$0.00	$0.00	$39,666.67	$147,333.33	$147,333.33
4		$0.00	$0.00	$30,000.00	$130,000.00	$130,000.00	$130,000.00	$130,000.00
5		$0.00	$0.00	$13,888.89	$120,370.37	$120,370.37	$120,370.37	$120,370.37
6		$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
7		$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
8		$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
DC_t	**$0.00**	**$303,333.33**	**$177,619.05**	**$162,936.51**	**$250,370.37**	**$290,037.04**	**$397,703.70**	**$397,703.70**

i	End of Period (t*n) in days							
	208	234	260	286	312	338	364	391
1	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
2	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
3	$5,666.67	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
4	$50,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
5	$4,629.63	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
6	$450,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
7	$75,000.00	$121,875.00	$121,875.00	$121,875.00	$9,375.00	$0.00	$0.00	$0.00
8	$0.00	$0.00	$0.00	$0.00	$400,000.00	$100,000.00	$0.00	$0.00
DC_t	**$585,296.30**	**$121,875.00**	**$121,875.00**	**$121,875.00**	**$409,375.00**	**$100,000.00**	**$ -**	**$ -**

As per Table 7, value of EV_{32} is zero. This can be observed by using a set of algorithms (6) where none of the four conditions applies. This is because activity “C” is delayed by a value that makes no work execution occurs at $t = 3$.

3.2.2.3. ***Cash flow schedule.*** Calculation of present worth of profit is performed by obtaining all cash flows during each month for the two different cases.

3.2.2.3.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, Table 8 represents the calculation process using excel sheet to obtain cash flows at each month t during the project life. For example, cash flows at the third month ($t = 3$) are obtained using Equations (11) through (13) discussed in 3.2.1.3 as follows:

a. $IDC_3 = \$50{,}000$ $\quad (obtained\ from\ 3.2.2.1)$

b. $DC_3 = \$246{,}552.38$ $\quad (obtained\ from\ 3.2.2.2.1.1)$

c. $EV_2 = \$302{,}285.71$ $\quad (obtained\ from\ 3.2.2.2.1.2)$

d. *Monthly Cash Out* is determined by

$$CO_3 = \$246,552.38 + \$50,000 = \$296,552.38 \qquad Equation\ (11)$$

e. *Monthly Retention and Down Payment Deduction* is determined by

$$PD_2 = (0.1 + 0.1\ )\ \text{x}\ \$302,285.71 = \$60,457.14 \qquad Equation\ (12)$$

f. *Monthly Cash In* is determined by

$$CI_3 = \ \$302,285.71\ - \$60,457.14 = \$241,828.57 \qquad Equation\ (13)$$

Table 8: Example Cash Flow Calculations Schedule for Un-delayed Case

Un-Delayed Activities Cash Flow								
Month *(t)*	**Indirect Cost (IDC_t)**	**Direct Cost (DC_t)**	**Cash Out (CO_t)**	**Earned Value (EV_t)**	**Retention + Down Payment Deduction (PD_t)**	**Cash In (CI_t)**	**Profit (P_t)**	**Present Worth of Profit (PW_t)**
0		$ -	$ -	$ -	$ -	$ 344,000.00	$ 344,000.00	$ 344,000.00
1	$ 50,000.00	$ 86,666.67	$ 136,666.67	$ 303,333.33	$ 60,666.67	$ -	$ (136,666.67)	$ (128,821.44)
2	$ 50,000.00	$ 229,857.14	$ 279,857.14	$ 302,285.71	$ 60,457.14	$ 242,666.67	$ (37,190.48)	$ (34,034.55)
3	$ 50,000.00	$ 246,552.38	$ 296,552.38	$ 324,158.73	$ 64,831.75	$ 241,828.57	$ (54,723.81)	$ (48,621.40)
4	$ 50,000.00	$ 224,184.92	$ 274,184.92	$ 318,370.37	$ 63,674.07	$ 259,326.98	$ (14,857.94)	$ (12,816.59)
5	$ 50,000.00	$ 151,961.11	$ 201,961.11	$ 250,370.37	$ 50,074.07	$ 254,696.30	$ 52,735.19	$ 44,164.89
6	$ 50,000.00	$ 130,361.11	$ 180,361.11	$ 250,370.37	$ 50,074.07	$ 200,296.30	$ 19,935.19	$ 16,209.13
7	$ 50,000.00	$ 127,583.33	$ 177,583.33	$ 241,111.11	$ 48,222.22	$ 200,296.30	$ 22,712.96	$ 17,929.82
8	$ 50,000.00	$ 307,333.33	$ 357,333.33	$ 575,000.00	$ 115,000.00	$ 192,888.89	$ (164,444.44)	$ (126,032.97)
9	$ 50,000.00	$ 272,062.50	$ 322,062.50	$ 121,875.00	$ 24,375.00	$ 460,000.00	$ 137,937.50	$ 102,638.45
10	$ 50,000.00	$ 87,750.00	$ 137,750.00	$ 121,875.00	$ 24,375.00	$ 97,500.00	$ (40,250.00)	$ (29,077.46)
11	$ 50,000.00	$ 87,750.00	$ 137,750.00	$ 121,875.00	$ 24,375.00	$ 97,500.00	$ (40,250.00)	$ (28,230.54)
12	$ 50,000.00	$ 154,000.00	$ 204,000.00	$ 409,375.00	$ 81,875.00	$ 97,500.00	$ (106,500.00)	$ (72,521.32)
13	$ 50,000.00	$ 136,937.50	$ 186,937.50	$ 100,000.00	$ 20,000.00	$ 327,500.00	$ 140,562.50	$ 92,928.37
14	$ -	$ 27,000.00	$ 27,000.00	$ -	$ -	$ 80,000.00	$ 53,000.00	$ 34,018.68
19		$ 130,000.00	$ 130,000.00	$ -	$ -	$ -	$ (130,000.00)	$ (71,977.85)
19						$ 344,000.00	$ 344,000.00	$ 190,464.46
							Total Present Worth of Profit (PW)	$ 290,219.69

The present worth of profit at the third month ($t = 3$) and total present worth of profit are obtained using Equations (14) through (19) discussed in cash flow developments theory (3.2.1.3) as follows:

g. *Cash In* and *Cash Out* at months $t = 0$, $t = 19$ and , $t = 19$ are determined by:

$$CI_0 = 0.1\ \text{x}\ \$3,440,000 = \$344,000 \qquad Equation\ (14)$$

$$CI_{19} = 0.1\ \text{x}\ \$3,440,000 = \$344,000 \qquad Equation\ (15)$$

$$CO_{19} = 0.1\ \text{x}\ \$1,300,000 = \$130,000 \qquad Equation\ (16)$$

h. *Monthly Profit @* $t = 3$ is determined by

$$P_3 = \ \$241,828.57 - \$296,552.38 = -\$48,621.40 \qquad Equation\ (17)$$

i. *The Monthly Present Worth of Profit @* $t = 3$ *and @* $ROR = 3\%$ is determined by:

$$PW_3 = \frac{-\$48,621.40}{(0.03 + 1)^3} \qquad Equation\ (18)$$

j. *The Present Worth of Profit* is determined by

$$PW_{Un-Delayed} = PW_o + PW_{19} + PW_{19} + \sum_{1}^{14} PW_t$$

$$= \$290{,}219.69 \qquad\qquad Equation\ (19)$$

3.2.2.3.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, Table 9 represents the calculation process using excel sheet to obtain cash flows at each month t during the project life.

Following same steps as in Un-delayed case (3.2.2.3.1), all cash flow values represented in Table 9 are obtained. Using Equation (20), the total present worth of profit for the delayed case is obtained as:

$$PW_{Delayed} = \$293{,}432.83$$

Table 9: Example Cash Flow Calculations Schedule for Delayed Case

Delayed Activities Cash Flow								
Month (*t*)	Indirect Cost (IDC_t)	Direct Cost (DC_t)	Cash Out (CO_t)	Earned Value (EV_t)	Retention + Down Payment Deduction (PD_t)	Cash In (CI_t)	Profit (P_t)	Present Worth of Profit (PW_t)
0		$ -	$ -	$ -	$ -	$ 344,000.00	$ 344,000.00	$ 344,000.00
1	$ 50,000.00	$ 86,666.67	$ 136,666.67	$ 303,333.33	$ 60,666.67	$ -	$ (136,666.67)	$ (128,821.44)
2	$ 50,000.00	$ 156,523.81	$ 206,523.81	$ 177,619.05	$ 35,523.81	$ 242,666.67	$ 36,142.86	$ 33,075.83
3	$ 50,000.00	$ 116,119.05	$ 166,119.05	$ 162,936.51	$ 32,587.30	$ 142,095.24	$ (24,023.81)	$ (21,344.84)
4	$ 50,000.00	$ 134,259.92	$ 184,259.92	$ 250,370.37	$ 50,074.07	$ 130,349.21	$ (53,910.71)	$ (46,503.86)
5	$ 50,000.00	$ 153,694.44	$ 203,694.44	$ 290,037.04	$ 58,007.41	$ 200,296.30	$ (3,398.15)	$ (2,845.90)
6	$ 50,000.00	$ 229,627.78	$ 279,627.78	$ 397,703.70	$ 79,540.74	$ 232,029.63	$ (47,598.15)	$ (38,701.65)
7	$ 50,000.00	$ 263,827.78	$ 313,827.78	$ 397,703.70	$ 79,540.74	$ 318,162.96	$ 4,335.19	$ 3,422.24
8	$ 50,000.00	$ 360,938.89	$ 410,938.89	$ 585,296.30	$ 117,059.26	$ 318,162.96	$ (92,775.93)	$ (71,105.02)
9	$ 50,000.00	$ 274,904.17	$ 324,904.17	$ 121,875.00	$ 24,375.00	$ 468,237.04	$ 143,332.87	$ 106,653.12
10	$ 50,000.00	$ 87,750.00	$ 137,750.00	$ 121,875.00	$ 24,375.00	$ 97,500.00	$ (40,250.00)	$ (29,077.46)
11	$ 50,000.00	$ 87,750.00	$ 137,750.00	$ 121,875.00	$ 24,375.00	$ 97,500.00	$ (40,250.00)	$ (28,230.54)
12	$ 50,000.00	$ 154,000.00	$ 204,000.00	$ 409,375.00	$ 81,875.00	$ 97,500.00	$ (106,500.00)	$ (72,521.32)
13	$ 50,000.00	$ 136,937.50	$ 186,937.50	$ 100,000.00	$ 20,000.00	$ 327,500.00	$ 140,562.50	$ 92,928.37
14	$ -	$ 27,000.00	$ 27,000.00	$ -	$ -	$ 80,000.00	$ 53,000.00	$ 34,018.68
19		$ 130,000.00	$ 130,000.00	$ -	$ -	$ -	$ (130,000.00)	$ (71,977.85)
19			$ -	$ -	$ -	$ 344,000.00	$ 344,000.00	$ 190,464.46
							Total Present Worth of Profit (PW)	$ 293,432.83

3.2.2.4. ***Time Value of money Delay Cost.*** The present worth of profit obtained in Un-delayed case (3.2.1.3.1) is less than the delayed case (3.2.1.3.2) due to the effect of time value of money. *The Time Value of money Delay Cost* is determined using Equation (21) as:

$$CTVM = \$290{,}219.69 - \$293{,}432.83 = -\$3{,}213.14$$

Table 10 represents the spreadsheet programmed to obtain the time value of money cost due to the delay in non-critical activities. In this case, note that negative value indicated that delaying the activities as per the example yields into cost saving rather than loss to the contractor.

Table 10: Example Time Value of Money Cost Calculation

Activities Data Input														
i	Activity	$P_{i,s}$				D_i	ES_i	EF_i	LS_i	LF_i	F_i	DE_i	DS_i	DF_i
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	93	123	183
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	3	75	183
6	F	3	4	5	-	12	192	204	276	288	84	0	192	204
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

Cash-Flow Results		
Present Worth Profit of Non-Delayed Schedule	$	290,219.69
Present Worth Profit of Delayed Schedule	$	293,432.83
Delay Cost	$	(3,213.14)

3.2.3. Discussion of results. In any construction project, the cash flow depends highly on the starting and ending dates of the construction activities because the last determines the amounts of expenses and payments encountered in each month during the project cycle. Taking into consideration the time value of money, delaying non-critical activities yields in change in the cash flow diagram, and as a result, changes the present worth of profit of the project.

Delaying non-critical activities might result in higher or lower present worth of profit compared to the present worth of the profit for the Un-delayed schedule. In fact, delaying a non-critical activity will affect the present worth of profit positively or negatively based on the following factors

a. The contract value of the activity - EV_i

b. The end of month payments of the activity - EP_i

c. The next month payments of the activity - NP_i

d. Retention hold by client - R_c

e. The down payment - DP_0

f. Retention hold by main contractor R_s

For any activity i, delaying a noncritical activity decreases the present worth of profit if and only if the periodical revenue of the execution of an activity is more than the periodical expenses of the same. When Equation (22) is satisfied, delaying the corresponding activity results in reducing the present worth of profit.

$$EV_i(1 - R_c - DP_0) > EP_i + NP_i.(1 - R_s) \qquad Equation\ (22)$$

In other words, delaying the activity will reduce the present worth of the profit for the project if and only if

Equation (22) is applied to the example in 3.2.2 as follows:

i. Activity “C” – $i = 3$

$$\$340{,}000(1 - 0.1 - 0.1) = \$272{,}000 < \$200{,}000 + \$120{,}000(1 - 0.1) = \$308{,}000$$

Hence, delaying activity C by 93 working days resulted in increasing the present worth of profit by $3,596.00 as shown in Table 11.

Table 11: CTMV for Delaying Activity "C"

Activities Data Input

i	Activity	*P_{i,s}*				*D_i*	*ES_i*	*EF_i*	*LS_i*	*LF_i*	*F_i*	*DE_i*	*DS_i*	*DF_i*
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	93	123	183
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	0	72	180
6	F	3	4	5	-	12	192	204	276	288	84	0	192	204
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

Cash-Flow Results

Present Worth Profit of Non-Delayed Schedule	$	290,219.69
Present Worth Profit of Delayed Schedule	$	293,815.70
Delay Cost	$	(3,596.00)

i. Activity “E” – $i = 5$

$$\$500{,}000(1 - 0.1 - 0.1) = \$400{,}000 > \$150{,}000 + \$125{,}000(1 - 0.1) = \$262{,}500$$

Hence, delaying activity E by 3 working days resulted in decreasing the present worth of profit $382.87 as shown in Table 12.

Table 12: CTMV for Delaying Activity "E"

Activities Data Input

i	Activity	*P_{i,s}*				*D_i*	*ES_i*	*EF_i*	*LS_i*	*LF_i*	*F_i*	*DE_i*	*DS_i*	*DF_i*
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	0	30	90
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	3	75	183
6	F	3	4	5	-	12	192	204	276	288	84	0	192	204
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

Cash-Flow Results

Present Worth Profit of Non-Delayed Schedule	$	290,219.69
Present Worth Profit of Delayed Schedule	$	289,836.83
Delay Cost	$	382.87

3.3. Resources – Delay Analysis Model

Resources such as labor, equipment, machineries, vehicles, etc, are part of the major items that should be considered when estimating the total cost of a project. The cost related to the resources required for any project can be minimized by maintaining the resources histograms as leveled as possible.

Leveling resources can be conducted by shifting the start date of non-critical activities using float; hence, any change in the start date of non-critical activities might result in change in the total cost of the resources as discussed in 3.3.1.

3.3.1. Resources – delay analysis model theory overview. This model calculates the cost related to having fluctuated resources histograms during the project lifetime. This additional cost is introduced due to the fact that for any resource required at the project lifetime, if the number/quantity required for this resource at any time decreases then increases later on, the contractor will have to choose either to demobilize/remobilize part of this resource or to set this part idle till it is required.

Both cases yield into extra cost added to the project total cost. This cost is calculated through the model as follows:

3.3.1.1. *Data input.* The following data shall be provided in order to perform the analysis.

a. *Resource Name* - j**:** Each resource has histogram that affects the cost of the project. This model is developed to perform analysis for maximum of 10 resources.

b. *Activities Daily Requirement* - Q_{ij}: each activity i in the project requires different types and quantity of resources j each day during its execution duration.

c. *Resource Cost/Day* - CR_j**:** the cost of one unit of resource/day in case of setting it idle.

d. *Mobilize/Demobilize Cost* - MC_j**:** Also called hiring and firing cost, is the cost encountered when a number/quantity of a certain resource is mobilized to the related project for some certain time and dismissed. This cost varies widely from a resource to another based on the nature of the resource itself. For example, hiring and firing a six-wheel truck costs much less than erecting and dismantling a tower crane in the same project.

3.3.1.2. *Resources break down schedules.* In which the model obtains the requirement of resources for each activity at each *Constant Period* - CP_t . The constant period is defined as end date of the time interval in which all the resources requirement for all the activities is constant. The break down is performed by the model as follows:

3.3.1.2.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, the constant periods and their corresponding activity daily resource requirements are obtained as follows:

a. *The constant period* - CP_t for any set of Activities $i = 1,2,3..n$ with Early Starts $ES_{i=1,2,3..n}$ and Early Finishes $EF_{i=1,2,3..n}$ each is determined using Equation (23).

$$CP_t = \boldsymbol{Small}\left[\left\{ES_{i=1,2,3..n} \mathrm{U} EF_{i=1,2,3..n}\right\}, \boldsymbol{Countif}(CP_t > CP_{t-1})\right] \qquad Equation\ (23)$$

Note that the value of CP_t refers to the end of the constant period t itself.

b. *The activity constant period resource requirement* - $Q_{t,ij}$ for any resource R_j is determined using Set of Algorithms (7).

$$If\ ES_i > CP_t,\ then\ Q_{t,ij} = 0,\ else$$

$$If\ CP_t > EF_i,\ then\ Q_{t,ij} = 0,\ else$$

$$Q_{t,ij} = Q_{ij} \qquad Set\ of\ Algorithms\ (7)$$

c. *The total resource daily requirement* for any resource *j* during the period CP_t is defined using Equation (24).

$$Q_{t,j} = \sum_{i=0}^{i=n} Q_{t,ij} \qquad Equation\ (24)$$

The resource histogram showing the fluctuation of the resource *j* can be plotted as a function of $Q_{t,j}$ vs. CP_t.

3.3.1.2.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, the constant periods and their corresponding activity daily resource requirements are obtained as follows:

a. *The Constant Period* - CP_t for any set of Activities $\mathrm{i} = 1,2,3..\mathrm{n}$ with Delayed Starts $DS_{i=1,2,3..n}$ and Delayed Finishes $DF_{i=1,2,3..n}$ each is using Equation (25).

$$CP_t = \boldsymbol{Small}\left[\left\{DS_{i=1,2,3..n} \mathrm{U} DF_{i=1,2,3..n}\right\}, \boldsymbol{Countif}(CP_t > CP_{t-1})\right] \qquad Equation\ (25)$$

b. *The activity constant period resource requirement* - $Q_{t,ij}$ for any resource *j* is determined using Set of Algorithms (8).

$$If\ DS_i > CP_t,\ then\ Q_{t,ij} = 0,\ else$$

$$If\ CP_t > DF_i,\ then\ Q_{t,ij} = 0,\ else$$

$$Q_{t,ij} = Q_{ij} \qquad Set\ of\ Algorithms\ (8)$$

c. *The total resource daily requirement* for any resource j during the period CP_t can be determined using Equation (24). The resource histogram showing the fluctuation of the resource j can be plotted as a function of $Q_{t,j}$ vs. CP_t.

3.3.1.3. *Resource histogram fluctuation cost calculation model.* Delays of non-critical activities might affect the project's resources total cost due to the change in the profile of the resource histogram. In fact, the more leveled resource histogram yields in less total resource cost and vice versa. This is due to the fact that fluctuations in the resource requirement mean that the contractor who is hiring these resources will have to either mobilize and demobilize a certain number/quantity of these resources or to keep them as idle for a certain duration until they are required again.

The cost of these fluctuations is determined in this section for the Un-delayed and delayed cases in order to compare them together and find the cost of the total resources fluctuation as discussed in calculation of delay effect (3.3.1.4).

3.3.1.3.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, the resource histogram fluctuation cost for any resource j is determined for all *Temporary Drop periods.*

A Constant Period $CP_{t=x,j}$ having resources requirement $Q_{t=x,j}$ is considered a *Temporary Drop Period* if and only if there exists a subsequent period $CP_{t>x,j}$, and a preceding period $CP_{t=(x-1),j}$ each having resource requirement more than the referred period. Temporary drop periods can be determined by set of algorithms (9).

```
$If\ Q_{t=x,j} < Q_{t>x,j} then,$
  $If Q_{t=x,j} < Q_{t=(x-1),j} then\ CP_{x,j} = CP_{t,j}\ else$
  $CP_{x,j} = 0$
$else, CP_{x,j} = 0$                                   $Set\ of\ Algorithms$ (9)
```

The cost of the resource fluctuation for all temporary drop periods is determined by finding the lowest cost whether to consider the resources as idle or to consider for a certain period or to demobilize and mobilize these resources again.

To analyze the cost encountered from the choice of keeping any as idle, the quantity and duration for keeping this resource should be determined. The quantity and duration for keeping any resource j at any temporary drop period $CP_{x,j}$ due to fluctuations in the resource histogram are defined as *Effective Fluctuated Quantity* -

$EFQ_{x,j}$ and *Effective Fluctuation Duration* - $EFD_{x,j}$. Values of $EFD_{x,j}$ and $EFQ_{x,j}$ are determined using Equations (26) and (27).

$$EFQ_{x,j} = min(Q_{t=(x-1),j}, Q_{t>x,j}) - Q_{t=x,j} \qquad Equation\ (26)$$

$$EFD_{x,j} = CP_{(t>x)-1,j} - CP_{t=x-1,j} \qquad Equation\ (27)$$

Values of $CP_{t=x-1,j}$ and $CP_{(t>x)-1,j}$ are used in Equation (27) to restore the value of the start dates of periods $CP_{t=x,j}$ and $CP_{(t>x),j}$ respectively.

Figure 2 illustrates different cases of the occurrence of temporary drop periods and their effective fluctuated quantities durations. Note that $CP_{t>x,j}$ is defined as the closest period that has more resource requirement than the temporary drop period. The cost of setting the resource idle $CRI_{x,j}$ is determined using Equation (28).

$$CRI_{x,j} = EFQ_{t=x,j} \text{ x } EFD_{t=x,j} \text{ x } CR_j \qquad Equation\ (28)$$

where CR_j is the cost of setting one number of the resource j idle for one day.

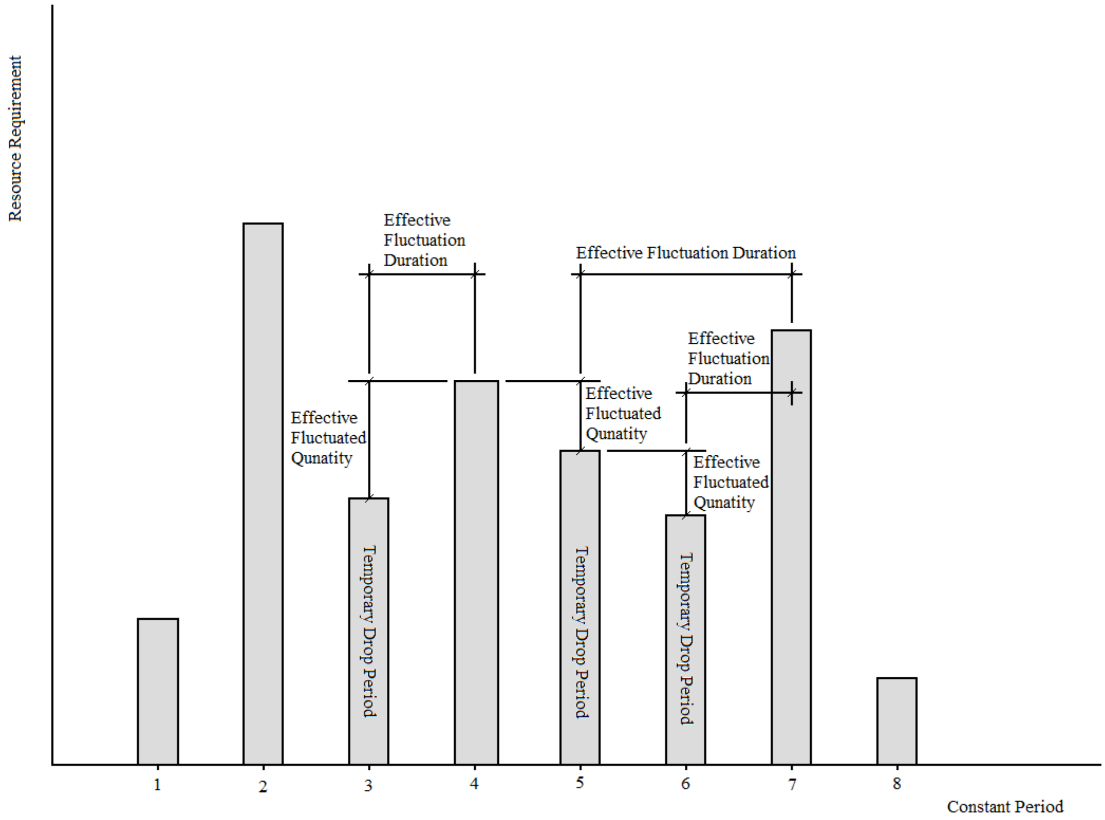


Figure 2: Resource Fluctuation Values Illustration

The Mobilize/Demobilize Cost of the Effective Fluctuated Quantity $CEM_{x,j}$ is determined using Equation (29).

$$CEM_{x,j} = EFQ_{t=x,j} \text{ x } MC_j \qquad Equation\ (29)$$

where MC_j is the cost of mobilizing and demobilizing one number of the resource j.

The Mobilize/Demobilize Cost of the Total Quantity $CTM_{x,j}$ is determined using Equation (30).

$$CTM_{t,j} = {}^1\!/_2 \left|Q_{t,j} - Q_{t,j}\right| MC_j \qquad Equation\ (30)$$

The Resource Fluctuation Cost $CRF_{t,j}$ is determined using Equation (31)

$$CRF_{t,j} = min\left(CTM_{t,j}, CRI_{x,j} + CTM_{x,j} - CEM_{x,j}\right) \qquad Equation\ (31)$$

For any resource histogram with T number of constant periods ($\text{t} = 1,2,3..T$), the Resource Histogram Fluctuation Cost CRF_j is determined using Equation (32).

$$CRF_j = \sum_{t=1}^{T} CRF_{t,j} \qquad Equation\ (32)$$

The Total Resource Histogram Fluctuation Cost CRF for a project having J resources is determined using Equation (33).

$$CRF_{Undelayed} = \sum_{j=1}^{J} CRF_j \qquad Equation\ (33)$$

3.3.1.3.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, the resource histogram fluctuation cost for any resource j is determined for all *Temporary Drop Periods.*

Values of $CP_{x,j}$ are determined by applying set of algorithms (9) to all values of $CP_{t,j}\ and\ Q_{t=x,j}$ obtained in 3.3.1.2.2.

Values of CRF_j for all values of j are obtained using Equations (26) through (32). Then, the value of $CRF_{Delayed}$ is determined using Equation (34).

$$CRF_{Delayed} = \sum_{j=1}^{J} CRF_j \qquad Equation\ (34)$$

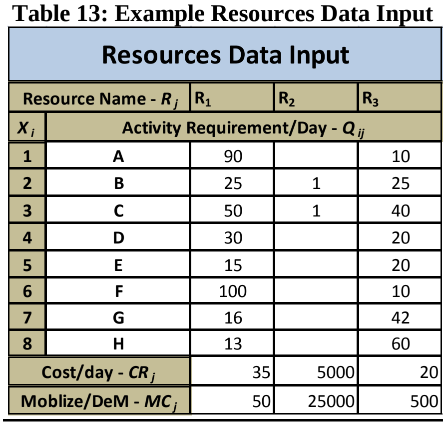
3.3.1.4. *Calculation of delay effect.* The total delay of non-critical activities cost due to the effect of the resource histogram CTRF is determined using Equation (35).

$$CTRF = CRF_{Undelayed} - CRF_{Delayed} \qquad Equation\ (35)$$

3.3.2. Illustrating example. Using the same example used in 3.1.2, the model solves the case as follows, where Activities C, E and F are delayed by 42, 10 and 60 days respectively.

3.3.2.1. *Data input.* Table 13 represents the data input for 3 resources assumed in this example.

Table 13: Example Resources Data Input



Resources Data Input

Resource Name - R_j		R_1	R_2	R_3
X_i	Activity Requirement/Day - Q_{ij}			
1	A	90		10
2	B	25	1	25
3	C	50	1	40
4	D	30		20
5	E	15		20
6	F	100		10
7	G	16		42
8	H	13		60
Cost/day - CR_j		35	5000	20
Moblize/DeM - MC_j		50	25000	500

3.3.2.2. *Resources break down schedules.* Constant Periods - CP_t and their corresponding resource requirement $Q_{t,ij}$ are calculated for both cases (Un-delayed and Delayed) by applying the model theory in 3.3.1 to a spreadsheet as follows:

3.3.2.2.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, the values of constant periods and their corresponding resource requirement $Q_{t,ij}$ are represented in Table 14.

For example, to find CP_3, Equation (23) is applied to values ES_i and EF_i obtained in 3.1.1.2 as follows:

$$CP_3 = \boldsymbol{Small}[\{0,30,30,72,72,192,192,180,30,72,90,192,180,204,288,318\}, \boldsymbol{Countif}(CP_t > 30)] = 72$$

Using set of algorithms 7, the third constant period requirement of resource “3” for activity “B” is calculated

$$If\ ES_2 = 30 \leq CP_3 = 72 \leq EF_2 = 72, then\ Q_{t,ij} = 25, else$$

Using Equation (24)

$$Q_{t,j} = 0 + 25 + 40 + 0 + 0 + 0 + 0 + 0 = 65$$

3.3.2.2.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, the values of constant periods and their corresponding resource requirement $Q_{t,ij}$ are represented in Table 15.

Table 14: Example of Constant Periods Calculations for Un-delayed Case

Resource R_1		***CP_t***								
X_i	***Activity***	***0***	***30***	***72***	***90***	***180***	***192***	***204***	***288***	***318***
1	**A**	0	90	0	0	0	0	0	0	0
2	**B**	0	0	25	0	0	0	0	0	0
3	**C**	0	0	50	50	0	0	0	0	0
4	**D**	0	0	0	30	30	30	0	0	0
5	**E**	0	0	0	15	15	0	0	0	0
6	**F**	0	0	0	0	0	0	100	0	0
7	**G**	0	0	0	0	0	0	16	16	0
8	**H**	0	0	0	0	0	0	0	0	13
$Q_{t,j}$		**0**	**90**	**75**	**95**	**45**	**30**	**116**	**16**	**13**

Resource R_2		***CP_t***								
X_i	***Activity***	***0***	***30***	***72***	***90***	***180***	***192***	***204***	***288***	***318***
1	**A**	0	0	0	0	0	0	0	0	0
2	**B**	0	0	1	0	0	0	0	0	0
3	**C**	0	0	1	1	0	0	0	0	0
4	**D**	0	0	0	0	0	0	0	0	0
5	**E**	0	0	0	0	0	0	0	0	0
6	**F**	0	0	0	0	0	0	0	0	0
7	**G**	0	0	0	0	0	0	0	0	0
8	**H**	0	0	0	0	0	0	0	0	0
$Q_{t,j}$		**0**	**0**	**2**	**1**	**0**	**0**	**0**	**0**	**0**

Resource R_3		***CP_t***								
X_i	***Activity***	***0***	***30***	***72***	***90***	***180***	***192***	***204***	***288***	***318***
1	**A**	0	10	0	0	0	0	0	0	0
2	**B**	0	0	25	0	0	0	0	0	0
3	**C**	0	0	40	40	0	0	0	0	0
4	**D**	0	0	0	20	20	20	0	0	0
5	**E**	0	0	0	20	20	0	0	0	0
6	**F**	0	0	0	0	0	0	10	0	0
7	**G**	0	0	0	0	0	0	42	42	0
8	**H**	0	0	0	0	0	0	0	0	60
$Q_{t,j}$		**0**	**10**	**65**	**80**	**40**	**20**	**52**	**42**	**60**

Note that values in Table 15 are obtained using algorithms and Equations in 3.3.1.2.2 which are different from those in Table 14. Hence, delaying non-critical activities might yield in changes in resource requirement histograms.

However, these changes might not be reflected on the actual resources histograms due to the fact that the contractor hiring these resources might rather choose to keep the resource and bear its cost by setting it idle. This topic is discussed more in 3.3.3.

3.3.2.3. ***Resource histogram fluctuation cost calculation model.*** Using the data obtained from resource breakdown schedules in 3.3.2.2, and the cost of the resources histogram fluctuation are calculated as follows:

3.3.2.3.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, Equations (26) through (33) are applied to the temporary drop periods defined by set of algorithms (9) using the data obtained in 3.3.2.2.1 in order to find the cost of fluctuation for each resource.

Table 15: Example of Constant Periods Calculations for Delayed Case

Resource R_1		***CP_t***										
X_i	***Activity***	***0***	***30***	***72***	***82***	***132***	***190***	***192***	***252***	***264***	***288***	***318***
1	**A**	0	90	0	0	0	0	0	0	0	0	0
2	**B**	0	0	25	0	0	0	0	0	0	0	0
3	**C**	0	0	0	50	50	0	0	0	0	0	0
4	**D**	0	0	0	30	30	30	30	0	0	0	0
5	**E**	0	0	0	0	15	15	0	0	0	0	0
6	**F**	0	0	0	0	0	0	0	0	100	0	0
7	**G**	0	0	0	0	0	0	0	16	16	16	0
8	**H**	0	0	0	0	0	0	0	0	0	0	13
$Q_{t,j}$		**0**	**90**	**25**	**80**	**95**	**45**	**30**	**16**	**116**	**16**	**13**
Resource R_2		***CP_t***										
X_i	***Activity***	***0***	***30***	***72***	***82***	***132***	***190***	***192***	***252***	***264***	***288***	***318***
1	**A**	0	0	0	0	0	0	0	0	0	0	0
2	**B**	0	0	1	0	0	0	0	0	0	0	0
3	**C**	0	0	0	1	1	0	0	0	0	0	0
4	**D**	0	0	0	0	0	0	0	0	0	0	0
5	**E**	0	0	0	0	0	0	0	0	0	0	0
6	**F**	0	0	0	0	0	0	0	0	0	0	0
7	**G**	0	0	0	0	0	0	0	0	0	0	0
8	**H**	0	0	0	0	0	0	0	0	0	0	0
$Q_{t,j}$		**0**	**0**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**
Resource R_3		***CP_t***										
X_i	***Activity***	***0***	***30***	***72***	***82***	***132***	***190***	***192***	***252***	***264***	***288***	***318***
1	**A**	0	10	0	0	0	0	0	0	0	0	0
2	**B**	0	0	25	0	0	0	0	0	0	0	0
3	**C**	0	0	0	40	40	0	0	0	0	0	0
4	**D**	0	0	0	20	20	20	20	0	0	0	0
5	**E**	0	0	0	0	20	20	0	0	0	0	0
6	**F**	0	0	0	0	0	0	0	0	10	0	0
7	**G**	0	0	0	0	0	0	0	42	42	42	0
8	**H**	0	0	0	0	0	0	0	0	0	0	60
$Q_{t,j}$		**0**	**10**	**25**	**60**	**80**	**40**	**20**	**42**	**52**	**42**	**60**

Table 16 represents the calculation of the resource fluctuations cost using the theory discussed in 3.3.1.3.1.

For example, the fluctuation cost of resource 3 at the fourth constant period (from day 72 to day 90) is obtained as follows:

Set of algorithms (9) is used to determine whether $CP_4 = 180$ is a temporary drop period or not as follows:

$Q_{4,3} = 40 < Q_{t>4,3}\ and\ Q_{4,3} = 40 < Q_{3,3} = 80,$

$CP_{4,3} = 180$ is a temporary drop period.

Note that $CP_{t>4,3} = CP_{6,3}$ because it is the closest constant period that has more resource requirement than $CP_{4,3}$.

Using Equations (26) through (31), the resource fluctuation cost is determined as follows:

Table 16: Example of Un-delayed Resource Histogram Fluctuation Cost Calculation

Resource: *1*

t		1	2	3	4	5	6	7	8	9
Start Day		*Start*	30	72	90	180	192	204	288	318
End Day - CP_t	0	30	72	90	180	192	204	288	318	*End*
Duration	0	30	42	18	90	12	12	84	30	-
$Q_{t,j}$	0	90	75	95	45	30	116	16	13	-
$EFQ_{t,j}$		0	15	0	50	15	0	0	0	0
$EFD_{t,j}$		0	42	0	102	12	0	0	0	0
$CRI_{t,j}$		$ -	$ 22,050.00	$ -	$ 178,500.00	$ 6,300.00	$ -	$ -	$ -	$ -
$CEM_{t,j}$		$0.00	$750.00	$0.00	$2,500.00	$750.00	$0.00	$0.00	$0.00	$0.00
$CTM_{t,j}$		$2,250.00	$375.00	$500.00	$1,250.00	$375.00	$2,150.00	$2,500.00	$75.00	$325.00
$CRF_{t,j}$		$2,250.00	$375.00	$500.00	$1,250.00	$375.00	$2,150.00	$2,500.00	$75.00	$325.00
CRF_j	$ 9,800.00									

Resource: *2*

t		1	2	3	4	5	6	7	8	9
Start Day		*Start*	30	72	90	180	192	204	288	318
End Day - CP_t	0	30	72	90	180	192	204	288	318	*End*
Duration	0	30	42	18	90	12	12	84	30	-
$Q_{t,j}$	0	0	2	1	0	0	0	0	0	-
$EFQ_{t,j}$		0	0	0	0	0	0	0	0	0
$EFD_{t,j}$		0	0	0	0	0	0	0	0	0
$CRI_{t,j}$		$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -
$CEM_{t,j}$		$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
$CTM_{t,j}$		$0.00	$25,000.00	$12,500.00	$12,500.00	$0.00	$0.00	$0.00	$0.00	$0.00
$CRF_{t,j}$		$0.00	$25,000.00	$12,500.00	$12,500.00	$0.00	$0.00	$0.00	$0.00	$0.00
CRF_j	$ 50,000.00									

Resource: *3*

t		1	2	3	4	5	6	7	8	9
Start Day		*Start*	30	72	90	180	192	204	288	318
End Day - CP_t	0	30	72	90	180	192	204	288	318	*End*
Duration	0	30	42	18	90	12	12	84	30	-
$Q_{t,j}$	0	10	65	80	40	20	52	42	60	-
$EFQ_{t,j}$		0	0	0	12	20	0	10	0	0
$EFD_{t,j}$		0	0	0	102	12	0	84	0	0
$CRI_{t,j}$		$ -	$ -	$ -	$ 24,480.00	$ 4,800.00	$ -	$ 16,800.00	$ -	$ -
$CEM_{t,j}$		$0.00	$0.00	$0.00	$6,000.00	$10,000.00	$0.00	$5,000.00	$0.00	$0.00
$CTM_{t,j}$		$2,500.00	$13,750.00	$3,750.00	$10,000.00	$5,000.00	$8,000.00	$2,500.00	$4,500.00	$15,000.00
$CRF_{t,j}$		$2,500.00	$13,750.00	$3,750.00	$10,000.00	-$200.00	$8,000.00	$2,500.00	$4,500.00	$15,000.00
CRF_j	$ 59,800.00									

$$EFQ_{4,3} = min(\,80, 52) - \ 40 = 12$$

$$EFD_{4,3} = \ 192 - \ 90 = 102$$

$$CRI_{4,3} = \ 102\text{x } 12 \text{ x } 20 = \$24{,}480.00$$

$$CEM_{4,3} = 12 \text{ x } 500 = \$6000.00$$

$$CTM_{4,3} = \ {}^{1}\!/_{2}\, |80 - 40| \text{ x } 500 = \$10{,}000.00$$

$$CRF_{4,3} = mi\,n(\$10{,}000.00, \$24{,}480.00 + \$10{,}000.00 - \$6000.00)$$

$$= \$10{,}000.00$$

Using Equation (32):

$$CRF_3 = \sum_{t=1}^{9} CRF_{t,3} = \$59{,}800.00$$

Using Equation (33):

$$CRF_{Undelayed} = \sum_{j=1}^{3} CRF_j = \$9{,}800.00 + \$50{,}000.00 + \$59{,}800.00$$

$$= \$119{,}600.00$$

3.3.2.3.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, Equations (26) through (34) are applied to the temporary drop periods defined by a set of algorithms (9) using the data obtained in 3.3.2.2.2 in order to find the cost of fluctuation for each resource.

Table 17:: Example of Delayed Resource Histogram Fluctuation Cost Calculation

Resource: *1*											
t	1	2	3	4	5	6	7	8	9	10	11
Start Day	***Start***	30	72	82	132	190	192	252	264	288	318
End Day - CP_t	30	72	82	132	190	192	252	264	288	318	***End***
Duration	30	42	10	50	58	2	60	12	24	30	-
$Q_{t,j}$	90	25	80	95	45	30	16	116	16	13	-
$EFQ_{t,j}$	0	55	0	0	50	15	14	0	0	0	0
$EFD_{t,j}$	0	42	0	0	120	62	60	0	0	0	0
$CRI_{t,j}$	$ -	$ 80,850.00	$ -	$ -	$ 210,000.00	$ 32,550.00	$ 29,400.00	$ -	$ -	$ -	$ -
$CEM_{t,j}$	$0.00	$2,750.00	$0.00	$0.00	$2,500.00	$750.00	$700.00	$0.00	$0.00	$0.00	$0.00
$CTM_{t,j}$	$2,250.00	$1,625.00	$1,375.00	$375.00	$1,250.00	$375.00	$350.00	$2,500.00	$2,500.00	$75.00	$325.00
$CRF_{t,j}$	$2,250.00	$1,625.00	$1,375.00	$375.00	$1,250.00	$375.00	$350.00	$2,500.00	$2,500.00	$75.00	$325.00
CRF_j	$ 13,000.00										

Resource: *2*											
t	1	2	3	4	5	6	7	8	9	10	11
Start Day	***Start***	30	72	82	132	190	192	252	264	288	318
End Day - CP_t	30	72	82	132	190	192	252	264	288	318	***End***
Duration	30	42	10	50	58	2	60	12	24	30	-
$Q_{t,j}$	0	1	1	1	0	0	0	0	0	0	-
$EFQ_{t,j}$	0	0	0	0	0	0	0	0	0	0	0
$EFD_{t,j}$	0	0	0	0	0	0	0	0	0	0	0
$CRI_{t,j}$	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -	$ -
$CEM_{t,j}$	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
$CTM_{t,j}$	$0.00	$12,500.00	$0.00	$0.00	$12,500.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
$CRF_{t,j}$	$0.00	$12,500.00	$0.00	$0.00	$12,500.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
CRF_j	$ 25,000.00										

Resource: *3*											
t	1	2	3	4	5	6	7	8	9	10	11
Start Day	***Start***	30	72	82	132	190	192	252	264	288	318
End Day - CP_t	30	72	82	132	190	192	252	264	288	318	***End***
Duration	30	42	10	50	58	2	60	12	24	30	-
$Q_{t,j}$	10	25	60	80	40	20	42	52	42	60	-
$EFQ_{t,j}$	0	0	0	0	2	20	0	0	10	0	0
$EFD_{t,j}$	0	0	0	0	60	2	0	0	24	0	0
$CRI_{t,j}$	$ -	$ -	$ -	$ -	$ 2,400.00	$ 800.00	$ -	$ -	$ 4,800.00	$ -	$ -
$CEM_{t,j}$	$0.00	$0.00	$0.00	$0.00	$1,000.00	$10,000.00	$0.00	$0.00	$5,000.00	$0.00	$0.00
$CTM_{t,j}$	$2,500.00	$3,750.00	$8,750.00	$5,000.00	$10,000.00	$5,000.00	$5,500.00	$2,500.00	$2,500.00	$4,500.00	$15,000.00
$CRF_{t,j}$	$2,500.00	$3,750.00	$8,750.00	$5,000.00	$10,000.00	-$4,200.00	$5,500.00	$2,500.00	$2,300.00	$4,500.00	$15,000.00
CRF_j	$ 55,600.00										

Table 17 represents the calculation of the resource fluctuations cost using the theory discussed in 3.3.1.3.2.

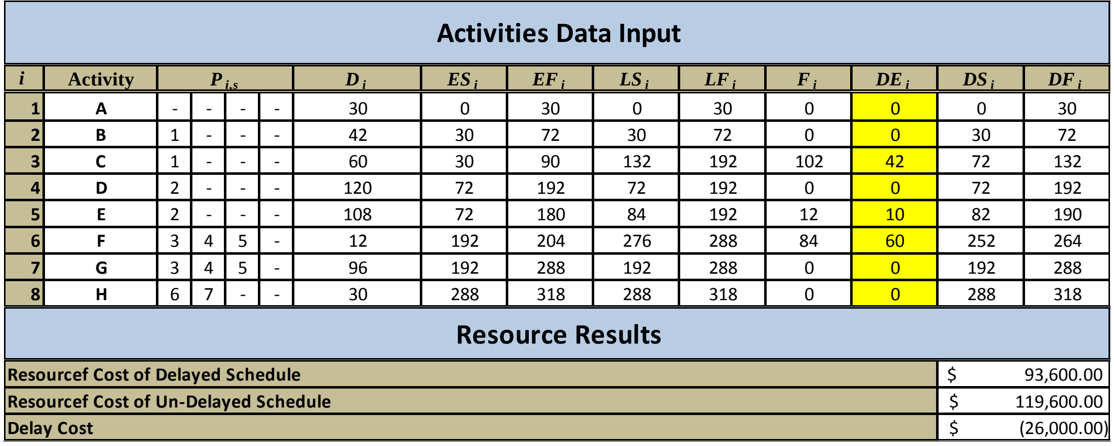
Using Equation (34):

$$CRF_{Delayed} = \sum_{j=1}^{3} CRF_j = \$13{,}000.00 + \$25{,}000.00 + \$55{,}600.00 = \$93{,}600.00$$

3.3.2.4. *Calculation of delay effect.* The total delay of non-critical activities cost due to the effect of the resource histogram $CTRF$ is determined using Equation (35) as shown in Table 18.

$$CTRF = \$93{,}600.00 - \$119{,}600.00 = -\$26{,}000.00$$

Table 18: Example Resource Histogram Fluctuation Delay Cost



Activities Data Input

i	Activity	$P_{i,s}$				D_i	ES_i	EF_i	LS_i	LF_i	F_i	DE_i	DS_i	DF_i
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	42	72	132
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	10	82	190
6	F	3	4	5	-	12	192	204	276	288	84	60	252	264
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

Resource Results

Resourcef Cost of Delayed Schedule	$	93,600.00
Resourcef Cost of Un-Delayed Schedule	$	119,600.00
Delay Cost	$	(26,000.00)

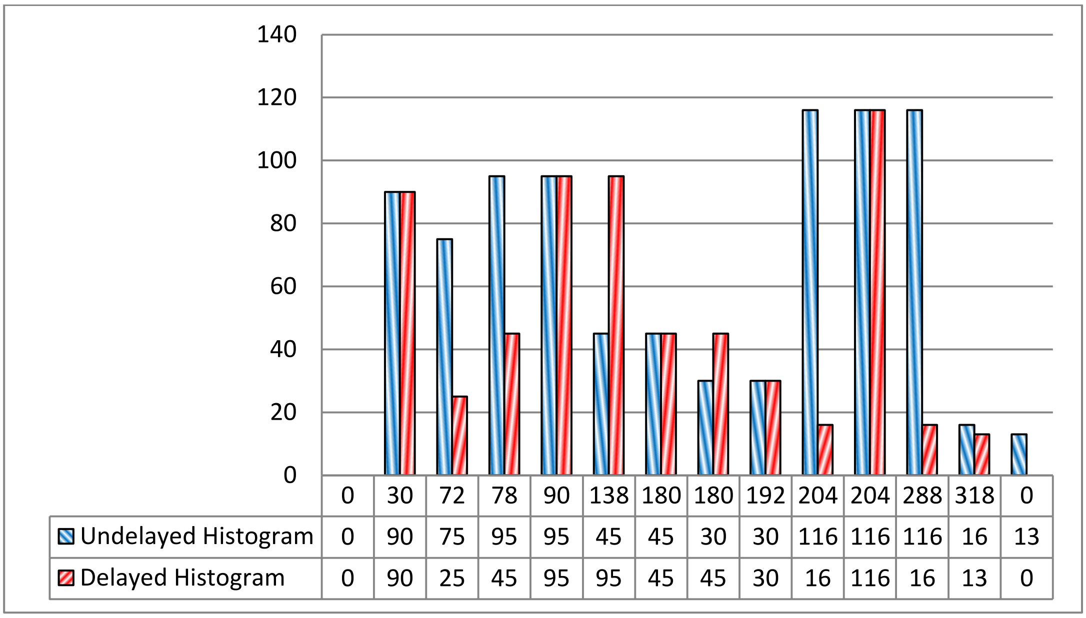
Note that as $CRF_{Delayed} < CRF_{Undelayed}$, this case of delay resulted generally in more leveled resources histograms. In fact, non-critical activities are usually delayed within their floats to save in the total cost of resources as known by resource leveling. However, this might not be always the case. In many cases, delaying non-critical activities might result in more fluctuation in the resource histograms. These cases are discussed more in 3.3.3.

3.3.3. Discussion of results. The resources histograms fluctuation cost has huge impact on the project cost. However, the sensitivity of the cost towards changes in non-critical activities dates is always different and hard to determine.

Due to the fact that float of non-critical activities is used to perform resource leveling, delaying non-critical activities might yield in cost saving and producing more leveled histograms. However, this is not always true. If the delay will yield in more fluctuations of resources requirement during the project life, the cost will be increasing.

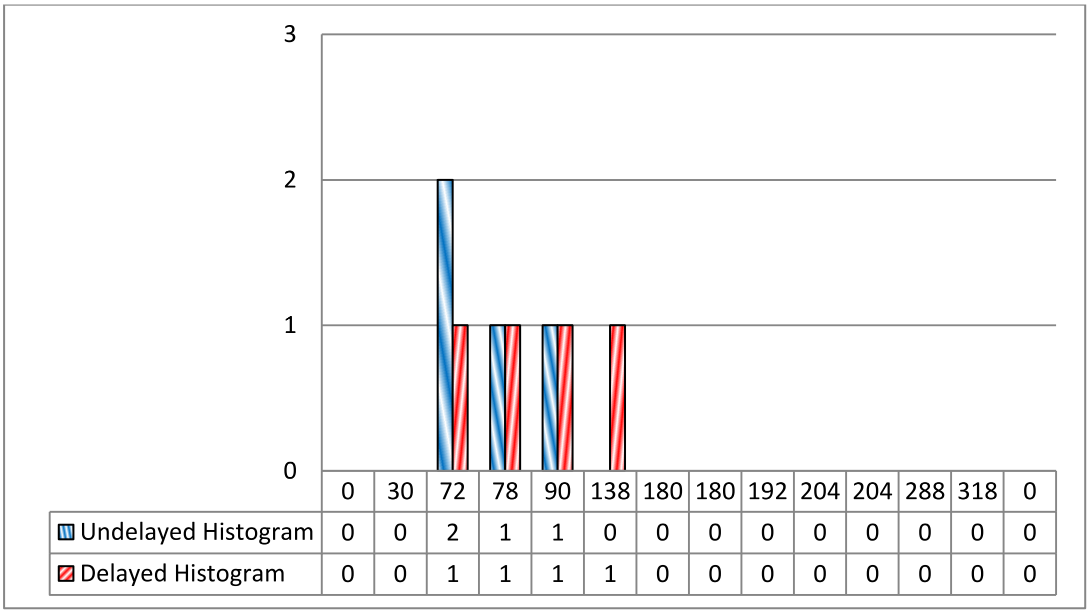
This can be determined by analyzing the delay effect on all the resources and estimating the cost encountered from the delay on each resource. Figures 3 and 4 compare the resource histograms before and after the delay for the resources 1 and 2 respectively.

Resource 1 histogram becomes more fluctuated after the delay, which in turn yields in more cost. See in Tables 16 and17 how Un-delayed $CRF_1 = \$9{,}800.00$ while delayed $CRF_1 = \$13{,}000.00$.



	0	30	72	78	90	138	180	180	192	204	204	288	318	0
▧ Undelayed Histogram	0	90	75	95	95	45	45	30	30	116	116	116	16	13
▨ Delayed Histogram	0	90	25	45	95	95	45	45	30	16	116	16	13	0

Figure 3: Example Resource 1 Histogram Comparison



	0	30	72	78	90	138	180	180	192	204	204	288	318	0
▥ Undelayed Histogram	0	0	2	1	1	0	0	0	0	0	0	0	0	0
▨ Delayed Histogram	0	0	1	1	1	1	0	0	0	0	0	0	0	0

Figure 4: Example Resource 2 Histogram Comparison

On the other hand, Resource 2 histogram becomes less fluctuated after the delay, which in turn yields in less cost. From Tables 16 and 17, Un-delayed $CRF_2 = \$50{,}000.00$ while delayed $CRF_2 = \$25{,}000.00$.

Moreover, the delay effect depends highly on the mobilize/demobilize cost. Having a resource with a high mobilize/demobilize cost compared to the daily cost of setting the resource idle makes the total fluctuation cost more sensitive to the change of non-critical activities starting date.

In fact, when encountering a change of starting date in a non-critical activity that requires a resource with a high mobilize/demobilize cost, the fluctuation cost will be changing rapidly at some critical points only. These critical points are the points which require the contractor to hire more resources in order two execute two activities that require the same resource.

To demonstrate, let's study the effect of change in activity "C" with values of delay ranging from 37 to 51 days and obtain the resource fluctuation cost CRF_2 after the delay. Figure 5 illustrates that the fluctuation cost remains constant with the change of delay in "C" before 42 days of delay; then, the cost drops at day 42 to the half suddenly, noting that 42 days of delay makes activity "C" starts at day 72, which means that the contractor does not require to hire more than one resource during the project life, and drop the number of mobilizing/demobilizing the resource 2.

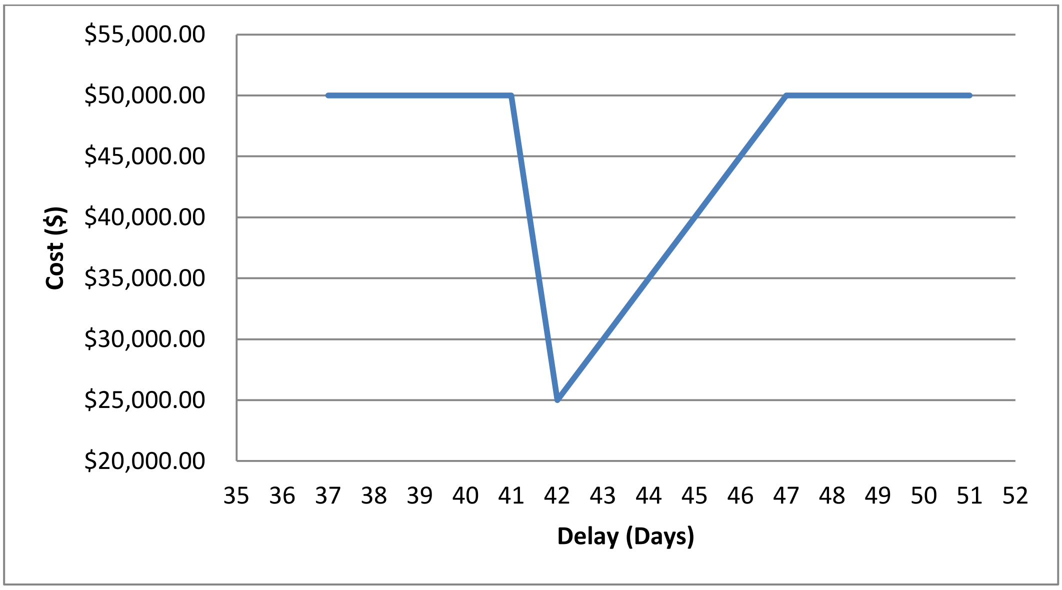


Figure 5: Example Activity "C" Cost vs. Delay

Furthermore, any increase in the number of days of activity “C” delay yields in constant increase in the histogram fluctuation cost. This is due to the fact the increasing the delay in “C” after the 42 days means resource 2 requirement is zero in the period between day 72 and day 72 + the effective fluctuated duration.

As a result, the contractor will have to choose to set the resource as idle for duration equal to the effective fluctuated duration $EFD_{t>72,3}$ as long as the resulted cost is less than demobilizing resource 2 and mobilizing it again. Henceforth, the fluctuation cost keeps increasing with the delay of “C” until the cost of setting the resource as idle is equal to the Mobilize/Demobilize cost which occurs at day 47.

3.4. Risk – Delay Analysis Model

CPM is a beneficiary method that can be used to determine the total project duration easily; however, the determined duration is obtained based on the precedence relations and duration of critical activities only. Critical path activities in CPM are assumed to have certain durations that can be added up using algebraic summation to obtain the total project duration.

What if noncritical activities are delayed to make one of their paths equal or close to the critical path in duration? And what if the project’s activities’ durations are uncertain and have probability of being more or less?

In fact, delaying a non-critical activity that has uncertain duration, even if the delay is less than its float, might result in delaying the project’s duration, especially when the uncertainty in the duration of this activity is high. Henceforth, the delay of non-critical activity might yield in extra costs due as liquidated damages, due to the delay of handing over the project or because of the crashing costs that are used to compensate for the delay.

The effect of delaying non-critical activities on the total project duration can be estimated using probabilistic methods of scheduling such as PERT and Monte Carlo Simulation. This section explains the model developed to determine the cost of the risk encountered when delaying non-critical activities using PERT method.

3.4.1. Risk – delay analysis model theory overview. PERT method has been chosen for this model due to the fact that it can be programmed deterministically in order to utilize the model in performing optimization as explained in 3.5.

The model has been developed to perform PERT analysis before and after the delay and to compare the use of MS Excel as follows:

3.4.1.1. ***Data input.*** The following data shall be provided in order to perform the analysis

a. *Activity Optimistic Duration* - t_{oi}: the minimum duration that activity i would require to be completed.

b. *Activity Pessimistic Duration* - t_{pi}: the maximum duration that activity i would require to be completed.

c. *Activity most likely Duration* - t_{mi}**:** the duration that activity i would most probably require to be completed.

d. *Path Combinations*: all the possible path combinations in the project for each path PC_r with R total number paths H total number of activities is described as $\{ PC_{r_1}, PC_{r_2}, PC_{r_3}, PC_{r_4} \ldots. PC_{r_H} \}$, where values of - PC_{r_h} represent the activity number i that starts after $h - 1$ activities in the same path.

e. *% Confidence* - $\% C$: the minimum probability required to describe how confident the project will complete by.

f. *Delay in Project Completion Date Cost* - C_{dp}: is defined amount of money to be paid as either daily liquidated damage due to the delay in handing over the project or the crashing cost per day which is ever less.

3.4.1.1.2. ***PERT values calculation model.*** To perform PERT analysis, the following values need to be determined.

3.4.1.1.3. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, PERT Values are obtained as follows*:*

a. *Activity Expected Duration* - t_{ei} is calculated using Equation (36).

$$t_{ei} = \frac{t_{oi} + 4 \text{ x } t_{mi} + t_{pi}}{6} \qquad Equation\ (36)$$

b. *Activity Standard Deviation* - σ_{ei} is calculated using Equation (37).

$$\sigma_{ei} = \frac{t_{pi} - t_{oi}}{6} \qquad Equation\ (37)$$

c. *Activity variance* - v_i is calculated using Equation (38).

$$v_{ei} = {\sigma_{ei}}^2 \qquad Equation\ (38)$$

d. *Path Expected Duration*- T_{er} is calculated using Equation (39).

$$T_{er} = \sum_{h=1}^{H} t_{er_h} \qquad Equation\ (39)$$

e. *Path Standard Deviation-* σ_{er} is calculated using Equation (40).

$$\sigma_{er} = \sqrt{\sum_{h=1}^{H} v_{er_h}} \qquad Equation\ (40)$$

3.4.1.1.4. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates. PERT values are obtained as follows:

Activity Expected Duration - t_{ei} *:* delays in each path are assumed as probability function with means equivalent to values of DE_i and standard deviations equivalent to the delayed path standard deviation σ_{er}. This assumption can be justified due to the fact that delays can also include uncertainties in construction projects.

Hence, delays are introduced to PERT analysis as increases in the delayed activities expected duration is as per Equation (41). On the other hand, the standard deviation of the activity itself and the delayed path will not be affected.

$$t_{ei} = \frac{t_{oi} + 4 \text{ x } t_{mi} + t_{pi}}{6} + DE_i \qquad Equation\ (41)$$

Values of σ_{ei},v_i, T_{er} , σ_{er} are obtained using Equations (37) through (40).

3.4.1.2. *Paths and project durations calculations.* All possible paths duration will be calculated based on their mean and standard deviations values at the % confidence specified in the data input. However, the projects duration is the duration that produces a probability of occurrence for all the paths which is equal to the % confidence. This duration is to be determined as follows:

3.4.1.2.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, the initial path duration PD_r at Confidence $\%C_0 = \%C$ is determined using Equation (42).

$$PD_{r,o} = Z(\%C_o\ ).\sigma_{er} + T_{er},\ \ k = 0 \qquad Equation\ (42)$$

where $Z(\%C_o)$ is the Z-score at $\%C_o$, k refers to the number of iteration.

Using Equation (43), the initial project expected duration is defined as the max path expected duration.

$$PD_0 = max\left[PD_{1,0}, PD_{2,0} PD_{3,0} \ldots\ldots PD_{R,0}\right] \qquad Equation\ (43)$$

The probability of occurrence for each path is determined by finding the corresponding probability of Z-score obtained from Equation (44).

$$Z(\%O_{r,0}) = \frac{PD_0 - T_{er}}{\sigma_{er}} \qquad Equation\ (44)$$

Assuming that all paths probability of occurrences are mutually independent; the probability of occurrence of all these events together is determined by product of all the probability of occurrences of all the paths as per Equation (45).

$$\%O_0 = \prod_{r=1}^{R} \%O_{r,0} \qquad Equation\ (45)$$

Unless all non-critical paths are certain, $\%\mathrm{O}_0$ will always be less than $\%\mathrm{C}_\mathrm{o}$. However, the aim is to find the project duration at probability of occurrence equal to the required confidence level. Therefore, iteration should be performed using set of algorithms (10)

```
$If\ k \leq 58, Set\ k = k + 1$
    $PD_{r,k} = Z(\%C_k\ ).\sigma_{er} + T_{er}$
    $PD_k = max[PD_{1,k}, PD_{2,k} PD_{3,k} \ldots\ldots PD_{R,k}]$
    $Z(\%O_{r,k}) = \frac{PD_k - T_{er}}{\sigma_{er}}$
    $\%O_k = \prod_{r=1}^{R} \%O_{r,k}$
        $If\ k < 12,$
            $If\ \%O_k < \%C_o, \%C_{k+1} = \%C_k + 10\%,$
            $Else\ \%C_{k+1} = \%C_k$
        $Else\ , If\ 12 \leq k < 24$
            $If\ \%O_k > \%C_o, \%C_{k+1} = \%C_k - 1\%,$
            $Else\ \%C_{k+1} = \%C_k$
        $Else\ , If\ 24 \leq k < 36$
            $If\ \%O_k < \%C_o, \%C_{k+1} = \%C_k + 0.1\%,$
            $Else\ \%C_{k+1} = \%C_k$
```

```
        $Else\ , If\ 36 \leq k < 48$
            $If\ \%O_k > \%C_o, \%C_{k+1} = \%C_k - 0.01\%,$
            $Else\ \%C_{k+1} = \%C_k$
        $Else, If\ 48 \leq k < 58$
            $If\ \%O_k < \%C_o, \%C_{k+1} = \%C_k + 0.001\%,$
            $Else\ \%C_{k+1} = \%C_k$
        $Else, End$
$Else, PD_{Undelayed} = PD_k$
$END$                                                   $Set\ of\ Algorithms\ (10)$
```

3.4.1.2.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, Equations (42) through (45) are applied to PERT values obtained in 3.4.1.2.2 to determine the initial project expected duration PD_0 and its corresponding set of probability of occurrence $\%O_0$.

Same iterations are required to find the project expected duration at the required confidence level using set of algorithms (10).

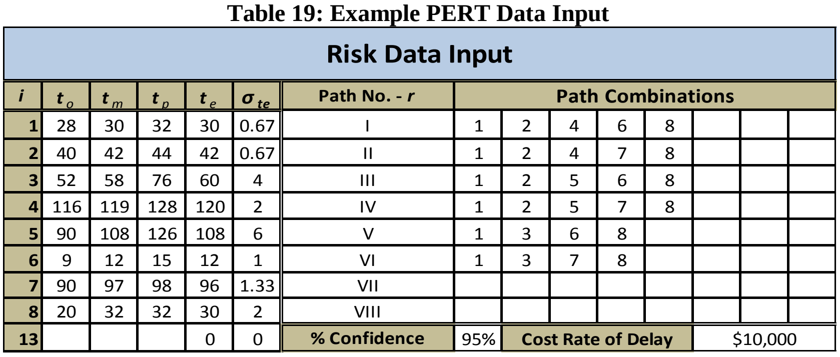
3.4.1.3. ***Risk of delay cost calculations.*** The Risk Cost due to delay of non critical activities is obtained using Equation (43).

$$CROD = C_{dp}.\left(PD_{Delayed} - PD_{Undelayed} \right) \qquad Equation\ (43)$$

3.4.2. Illustrating example. Using the same example used in 3.1.2, the model solves the case as follows, where activities C, E and F are delayed by 102, 12 and 84 days respectively.

3.4.2.1. ***Data input.*** Table 13 represents the data input for 3 resources assumed in this example.

Table 19: Example PERT Data Input



Risk Data Input

i	t_o	t_m	t_p	t_e	σ_{te}	Path No. - r	Path Combinations							
1	28	30	32	30	0.67	I	1	2	4	6	8			
2	40	42	44	42	0.67	II	1	2	4	7	8			
3	52	58	76	60	4	III	1	2	5	6	8			
4	116	119	128	120	2	IV	1	2	5	7	8			
5	90	108	126	108	6	V	1	3	6	8				
6	9	12	15	12	1	VI	1	3	7	8				
7	90	97	98	96	1.33	VII								
8	20	32	32	30	2	VIII								
13				0	0	% Confidence	95%	Cost Rate of Delay				$10,000		

3.4.2.2. *PERT values Calculation Model.* To perform PERT analysis, a spreadsheet is programmed using theory in 3.4.1.2 to obtain expected durations and standard deviations for both delayed and Un-delayed cases.

3.4.2.2.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, PERT values are represented in Table 20.

Table 20: Example Un-delayed PERT Values Calculations

PERT Values - Un-Delayed Activities												
Path - *r*	t_{er1}	t_{er2}	t_{er3}	t_{er4}	t_{er5}	v_{er1}	v_{er2}	v_{er3}	v_{er4}	v_{er5}	T_{er}	σ_{er}
1	30	42	120	12	30	0.44	0.44	4	1	4	234	3.14
2	30	42	120	96	30	0.44	0.44	4	1.78	4	318	3.27
3	30	42	108	12	30	0.44	0.44	36	1	4	222	6.47
4	30	42	108	96	30	0.44	0.44	36	1.78	4	306	6.53
5	30	60	12	30	0	0.44	16	1	4	0	132	4.63
6	30	60	96	30	0	0.44	16	1.778	4	0	216	4.71

Example: the expected duration and the variance for the third activity in the second path is calculated as follows:

$r = 2, h = 3$

From Table 19, $PC_{2_3} = 4$. Using values of $i = 4$

$$t_{e(i=4)} = \frac{116 + 4 \text{ x } 119 + 128}{6} = 120$$

$$\sigma_{e(i=4)} = \frac{128 - 116}{6} = 2$$

$$v_{e(i=4)} = (2)^2 = 4$$

Hence, $t_{er=2_{h=3}} = t_{e(i=4)} = 120$, $v_{er=2_{h=3}} = v_{e(i=4)} = 4$

Using all values of t_{e2_h} and v_{e2_h} in Table 20,

$$T_{e2} = \sum_{h=1}^{5} t_{e2_h} = 30 + 42 + 120 + 96 + 30 = 318$$

$$\sigma_{e2} = \sqrt{\sum_{h=1}^{5} v_{e2_h}} = \sqrt{0.44 + 0.44 + 4 + 1.78 + 4} = 3.27$$

3.4.2.2.2. *Delayed case.* Assuming that all non-critical activities start at their delayed start dates, PERT values are represented in Table 21.

Note that values of $t_e's$ for non critical activities have increased; each by the number of days delayed and standard deviations remain the same.

Table 21: Example Delayed PERT Values Calculations

PERT Values - Delayed Activities											
Path - r	t_{er1}	t_{er2}	t_{er3}	t_{er4}	t_{er5}	v_{er1}	v_{er2}	v_{er3}	v_{er4}	v_{er5}	T_{er}	σ_{er}
1	30	42	120	96	30	0.44	0.44	4	1	4	318	3.14
2	30	42	120	96	30	0.44	0.44	4	1.78	4	318	3.27
3	30	42	120	96	30	0.44	0.44	36	1	4	318	6.47
4	30	42	120	96	30	0.44	0.44	36	1.78	4	318	6.53
5	30	162	96	30	0	0.44	16	1	4	0	318	4.63
6	30	162	96	30	0	0.44	16	1.778	4	0	318	4.71

3.4.2.3. *Paths and project durations calculations.* Project expected duration at 95% confidence is obtained as follows:

3.4.2.3.1. *Un-delayed case.* Assuming that all critical and non-critical activities start at their early start date, all possible paths duration are calculated for initially ($k = 0$) at 95% confidence (the required confidence level).

Table 22: Example Un-delayed Paths and Project Duration Calculations

k=0		*k=24*		*k=36*		*k=48*		*k=58*		*END*
$PD_{r,o}$	$\%O_{r,o}$	$PD_{r,24}$	$\%O_{r,24}$	$PD_{r,36}$	$\%O_{r,36}$	$PD_{r,48}$	$\%O_{r,48}$	$PD_{r,58}$	$\%O_{r,58}$	PD_{r}
239.1725	1.000000	239.203243	1.000000	239.295310	1.000000	239.282986	1.000000	239.283633	1.000000	239.283633
323.3721	0.950000	323.403992	0.951000	323.499611	0.953900	323.486812	0.953520	323.487484	0.953540	323.487484
232.6458	1.000000	232.709021	1.000000	232.898508	1.000000	232.873144	1.000000	232.874475	1.000000	232.874475
316.7441	0.996088	316.807984	0.996144	316.999222	0.996309	316.973624	0.996287	316.974967	0.996288	316.974967
139.617	1.000000	139.662275	1.000000	139.797853	1.000000	139.779705	1.000000	139.780657	1.000000	139.780657
223.7539	1.000000	223.799991	1.000000	223.938005	1.000000	223.919531	1.000000	223.920500	1.000000	223.920500
$\%C_{o}$	$\%O_{o}$	$\%C_{24}$	$\%O_{24}$	$\%C_{36}$	$\%O_{36}$	$\%C_{48}$	$\%O_{48}$	$\%C_{58}$	$\%O_{58}$	$PD_{Undelayed}$
95%	0.946283	0.951000	0.947333	0.953900	0.950379	0.953520	0.949980	0.953540	0.950001	323.487484

For example, the second path initial expected duration is obtained as

$$PD_{2,0} = Z(95\%\,) \text{ x } 3.27 + 318 = 323.3721 days$$

Using the values in Table 22, the project initial expected duration is:

$$PD_0 = max[239.17,323.37,292.65,316.74,139.62,223.75] = 323.37$$

The probability of occurrence of the second path is same as the percent confidence because the second path is the one with the maximum duration. However, as per Table 22, the probability of occurrence of these durations assuming that required confidence level.

$$\%O_o = \prod_{r=1}^{6} \%O_{r,0} = 94.6283\%$$

58 iterations are made to yield a probability of occurrence that is equal to the required confidence level. Table 22 shows some of these iterations at different stages.

The project total duration at 95% confidence is 323.4875 days. Although the difference between the initial and final iteration is negligible in this case, it may be larger in so many other cases especially when non-critical activities are delayed as shown in 3.4.2.3.2.

3.4.2.3.2. *Delayed case.* . Assuming that all non-critical activities start at their delayed start dates, project duration at 95% confidence is calculated for the delayed case as shown in Table 23.

Table 23: Example Delayed Paths and Project Duration Calculations

k=0		k=24		k=36		k=48		k=58		END
$PD_{r,o}$	$\%O_{r,o}$	$PD_{r,24}$	$\%O_{r,24}$	$PD_{r,36}$	$\%O_{r,36}$	$PD_{r,48}$	$\%O_{r,48}$	$PD_{r,58}$	$\%O_{r,58}$	PD_{r}
323.1725	0.999683	323.961326	0.999959	324.269074	0.999983	324.264480	0.999982	324.265054	0.999983	324.265054
323.3721	0.999499	324.191323	0.999925	324.510944	0.999967	324.506174	0.999966	324.506770	0.999966	324.506770
328.6458	0.951548	330.269265	0.972140	330.902654	0.977889	330.893200	0.977811	330.894380	0.977821	330.894380
328.7441	0.950000	330.382647	0.971000	331.021889	0.976900	331.012348	0.976820	331.013539	0.976830	331.013539
325.617	0.989833	326.778626	0.996252	327.231814	0.997538	327.225050	0.997523	327.225894	0.997525	327.225894
325.7539	0.988672	326.936406	0.995690	327.397739	0.997131	327.390853	0.997113	327.391713	0.997115	327.391713
$\%C_{o}$	$\%O_{o}$	$\%C_{24}$	$\%O_{24}$	$\%C_{36}$	$\%O_{36}$	$\%C_{48}$	$\%O_{48}$	$\%C_{58}$	$\%O_{58}$	$PD_{Delayed}$
95%	0.883920	0.971000	0.936248	0.976900	0.950166	0.976820	0.949980	0.976830	0.950003	331.013539

3.4.2.4. ***Risk of delay cost calculations.*** The Risk Cost due to delay of non critical activities represented in Table 24 is determined by:

$$CROD = \$10{,}000 \text{ x } (\,331.0135 - 323.4875) = \$75{,}260.55$$

Table 24: Example Risk of Delay Cost

Activities Data Input															
i	Activity	$P_{i,s}$				D_i	ES_i	EF_i	LS_i	LF_i	F_i	DE_i	DS_i	DF_i
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	102	132	192
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	12	84	192
6	F	3	4	5	-	12	192	204	276	288	84	84	276	288
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

Risk of Delay Cost		
Un-Delayed Project Duration	$	323.49
Un-Delayed Project Duration	$	331.01
Delay Cost	$	75,260.55

Note that delayed and Un-delayed durations are not rounded to integers. Having portions of a day of delay might seem unrealistic; however, all the calculations made are based on probabilities and expectations. In other words, having a half day of delay can be translated as having 50% probability of one delay and liability to pay 50% of its liquidated damages.

3.4.3. Discussion of results. The risk delay analysis model has proven that delaying non-critical activities within their floats might result in delaying the total project duration. This delay can be translated into monetary value as liquidated damages paid due to the delay of handing over the project or as cost of crashing the following activities in order to achieve the targeted delay.

The relationship between the delay of non-critical activities and the total project duration varies and depends on the following factors.

a. ***The required confidence level* - $\%C$.** Using the same project in 3.4.2 with delay of 12 days in activity “E” only, Figure 6 represents the change in values of $CROD$ due to the increase in confidence requirement. As shown in Figure 6, the increase in confidence requirement increases the cost of delaying non-critical activities due to increase in risk. This is due to the fact that the increase in confidence level makes it harder to satisfy a probability of occurrence in all the paths when they are more close to the critical path.

Hence, delaying non-critical activities within their floats has more effect on the total project duration when the confidence requirement is higher.

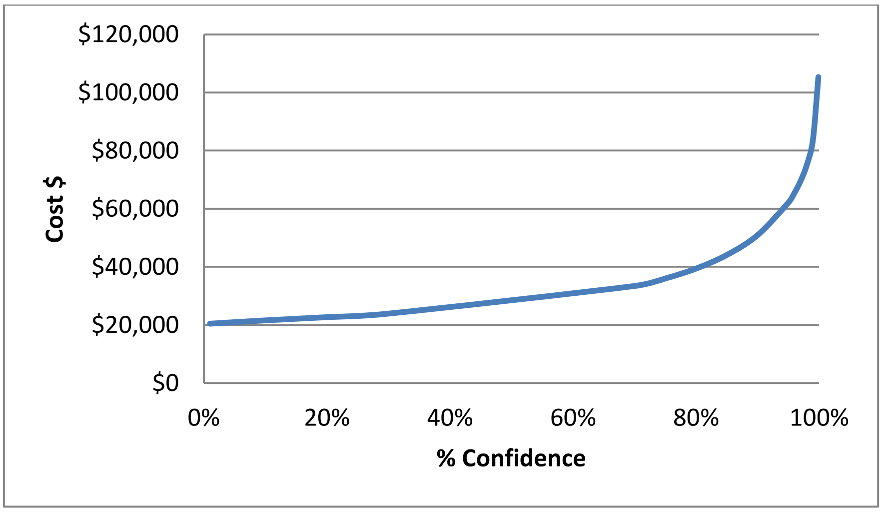


Figure 6: Risk of Delay Cost vs. % Confidence

b. ***Activities standard deviation.*** Activities “C”, “E” and “F” have standard deviations of 4, 6 and 1 respectively. Using the same example at 95% confidence, the relation between the risk cost and float consumption ratio (ratio of the delay to the float) is plotted in Figure 7. The increase in the activities standard deviation makes it more sensitive to the delay effect. As noticed, the cost starts to increase in the delay of activity “E” even at minor delay making it have 0 “safe float”; while on the other hand, activities “C” and “F” are still having more the 90% of their float safe.

c. ***Path standard deviation.*** In fact, the sensitivity to delaying non-critical activities does not depend on their standard deviations only, but it depends on the activities standard deviations that are on the same path of the delayed non-critical activity.

Path I consists of activities {A, B, D, F, G}, and these activities have standard deviations of {0.67, 0.67, 2, 1, 2}. The effect of changing the standard deviation path I on the risk cost is obtained at a delay value of 84 days in activity “F”. In order to get accurate results, the change of path I standard deviation is to be performed by changing the values of activity “D” standard deviation only. Activity “D” is presented only in path I and the critical path (path II), and any change in path II will be eliminated when the risk of delay cost is calculated (the duration of the original critical path will be the same in both the delayed and Un-delayed project duration).

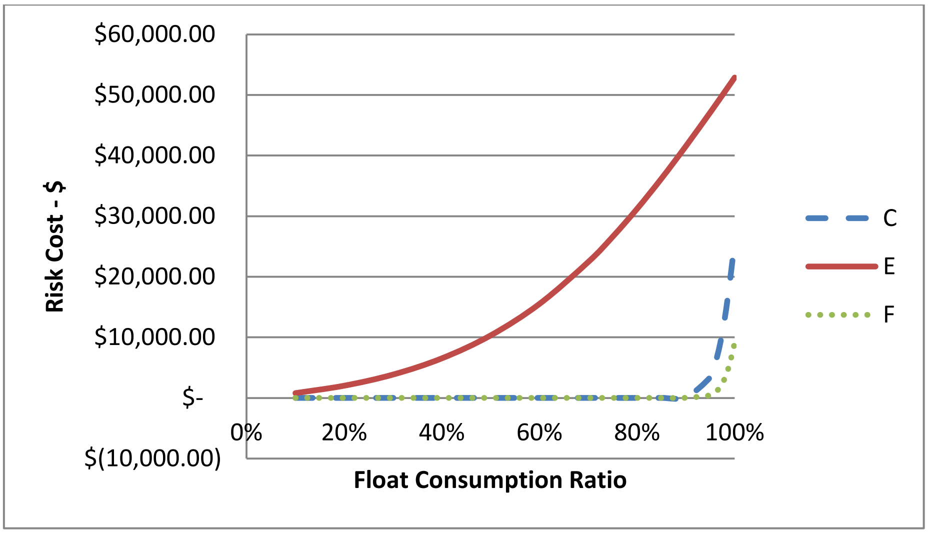


Figure 7: Risk of Delay Cost vs. Float Consumption of Non-Critical Activities

As shown in Figure 8, when delaying a non-critical activity, the risk of delay cost increases linearly with the increase of the standard deviation of the path that contains the delayed activity. This concludes that the risk cost of delaying a non-critical activity does not depend only on the activity's parameter itself, but it also depends on the other activities that share the same path of the delayed activity.

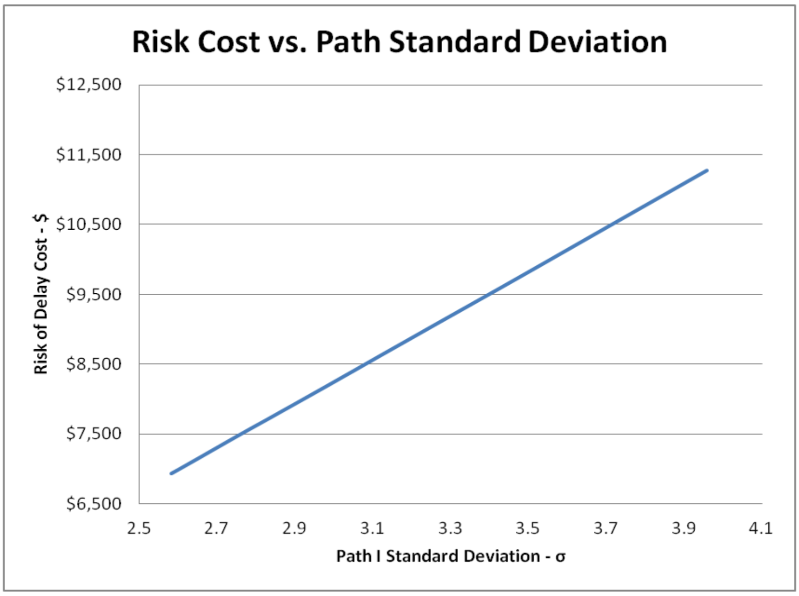


Figure 8: Risk of Delay Cost vs. Path Standard Deviation

3.5. Total Damages Quantification Model

As proven in Sections 3.2, 3.3, and 3.4, change in starting dates of non-critical activities might result in change in the cost of the project. This cost changes due to the changes in cash flow, resource histograms, and the total project duration.

3.5.1. Total damages quantification model theory overview. This section assembles all the theory developed in 3.1.1, 3.2.1, 3.3.1, and 3.4.1 in a single model to be utilized in float allocation prior to signing the contract and reallocation cost claims during the project execution as discussed in Chapter 4.

The float allocation and damages quantification process are developed by utilizing the following theory:

3.5.1.1. *Tri-Parameter cost calculation model.* Each of the three parameters might affect the cost of the project differently when one or more of the non-critical activities' starting dates are being changed. Henceforth, to determine the total cost effect of changes in non-critical activities starting dates, the effects of each of the three parameters should be added together when performing non-critical schedule variation analysis mentioned.

By combining all the models discussed in sections 3.1, 3.2, 3.3 and 3.4, the total cost due to changes in non-critical activities starting dates considering the three parameters is defined using Equation (46).

$$CTP = CTMV + CTRF + CROD \qquad Equation\ (46)$$

where the total cost (CTP) represents the change in the project total cost due to delaying non-critical activities from their early start date - ES_i by DE_i days.

If the minimum CTP occurs when all non-critical activities start at their earlier start ES_i, then any delay in these activities will result in increasing the total cost of the project and will be considered as damages that the contractor bears if not compensated. Note then CTP is always zero when all non-critical activities start at their early start ES_i.

3.5.1.2. ***Cost optimization model.*** The total cost – CTP is not always necessarily the minimum when all non-critical activities start at their early start $\mathrm{ES_i}$. In fact, float of non-critical activities can be used in many cases to reduce the projects total cost by performing resource analysis, or maintaining projects expenses to increase the present worth of profit.

Henceforth, the minimum CTP and its corresponding start dates of non-critical activities need to be determined in order to have a reference so that the calculation of damages due to any change can be calculated based on it.

For any project with n_{cr} number of non-critical activities defined as $i = (k_1, k_2, k_3, ..k_{\mathrm{n_{cr}}})$ having floats of $F_i = (F_{k_1}, F_{k_2}, F_{k_3}, ..F_{n_{cr}})$ and starting at $DE_i = (DE_{k_1}, DE_{k_2}, DE_{k_3}, ..DE_{n_{cr}})$ days after their early starts $ES_i = (ES_{k_1}, ES_{k_2}, ES_{k_3}, ..ES_{n_{cr}})$, the total cost CTP can be defined as a function of the number of days of delay as $CTP(DE_{k_1}, DE_{k_2}, DE_{k_3}, ..DE_{n_{cr}})$ where $CTP(0,0,0,..0) = 0$. Henceforth, there must be an absolute minimum value $CTP(DE_{k_1}, DE_{k_2}, DE_{k_3}, ..DE_{n_{cr}})$ where the corresponding values of delay $(DE_{k_1}, DE_{k_2}, DE_{k_3}, ..DE_{n_{cr}}) \in [(0,0,0,..0): (F_{k_1}, F_{k_2}, F_{k_3}, ..F_{n_{cr}})]$.

The objective function for performing this optimization process is defined as follows:

- Objective Function: minimize $CTP\ (DE_{k_1}, DE_{k_2}, DE_{k_3}, ..DE_{n_{cr}})$

- Variables: $\left(DE_{k_1}, DE_{k_2}, DE_{k_3}, .. DE_{n_{cr}}\right)$
- Subject to the following Constraints:
    1. $\left(DE_{k_1}, DE_{k_2}, DE_{k_3}, .. DE_{n_{cr}}\right) \geq (0,0,0,..0)$
    2. $\left(DE_{k_1}, DE_{k_2}, DE_{k_3}, .. DE_{n_{cr}}\right) \leq \left(F_{k_1}, F_{k_2}, F_{k_3}, .. F_{n_{cr}}\right)$
    3. $\left(DE_{k_1}, DE_{k_2}, DE_{k_3}, .. DE_{n_{cr}}\right)$ are all integers
    4. $(DF_1,\ DF_2, DF_3, .. DF_n) \leq T$

Where T refers to the project duration $T = EF_n$

This objective function is defined and solved by performing non-linear optimization using MS Excel Analytic Solver ® 2016 [30] as shown in section 4.5.2 and case study in Chapter 5. Note that the minimum CTP_{min} will be always less than or equal to zero.

3.5.1.3. *Total damages calculation.* Once the minimum total cost CTP_{min} is obtained, any case of changing non-critical activities starting date will yield in a higher total cost CTP_{new}. The total damages quantification TDQ can be defined using Equation (47).

$$TDQ = CTP_{new} - CTP_{min} \qquad Equation\ (47)$$

However, this might not be always the case. In many construction contracts, a baseline schedule may contain milestones and some activities execution dates are agreed on between the client and the contractor prior to signing the contract as discussed in Chapter 5.

Then, the total damages quantification calculation TDQ will be referred to a total cost CTP_o rather than CTP_{min} using Equation (48).

$$TDQ = CTP_{new} - CTP_o \qquad Equation\ (48)$$

Note that Equation (48) is a general form of Equation (47) as in the case of not having any milestones or schedule constraint, and then the baseline schedule will be defined as the optimum schedule itself using Equation (49)

$$CTP_o = CTP_{min} \qquad Equation\ (49)$$

Otherwise, CTP_o shall be obtained using MS Excel Analytic Solver ® 2016 to perform non-linear optimization for the same objective function added to it the additional *schedule Constraint* as follows:

a. Delay Constraint - DCN_k: such delays in providing designs of shop drawings approval by the client that delays the non-critical activity k. Delay Constraint are defined as:

$$\left(DS_{k_1}, DS_{k_2}, DS_{k_3}, .. DS_{n_{cr}}\right) \geq \left(DCN_{k_1}, DCN_{k_2}, DCN_{k_3}, .. DCN_{n_{cr}}\right)$$

b. Milestones Constraint - MSC_k: is defined as the deadline of completing the non-critical activity k. Delay constraints are defined as:

$$\left(DF_{k_1}, DF_{k_2}, DF_{k_3}, .. DF_{n_{cr}}\right) \geq \left(MSC_{k_1}, MSC_{k_2}, MSC_{k_3}, .. MSC_{n_{cr}}\right)$$

3.5.2. Illustrating example. Using the same example used in 3.1.2, the total damages model is used to perform cost optimization.

3.5.2.1. Optimal schedule. In this case, there are no schedule constraints defined to the objective function. The objective function is defined to MS Excel Analytic Solver ® 2016 as follows:

- Objective Function: minimize $CTP\ (DE_3, DE_5, DE_6)$
- Variables: (DE_3, DE_5, DE_6)
- Subject to the following Constraints:
  1. $(DE_3, DE_5, DE_6) \geq (0,0,0, ..0)$
  2. $(DE_3, DE_5, DE_6) \leq (102,12,84)$
  3. (DE_3, DE_5, DE_6) are all integers
  4. $(DF_1,\ DF_2, DF_3, DF_4, DF_5, DF_6, DF_7, DF_8) \leq 318$

The following results are obtained:

$DE_3 = 42,$

$DE_5 = 0$

$DE_6 = 79$

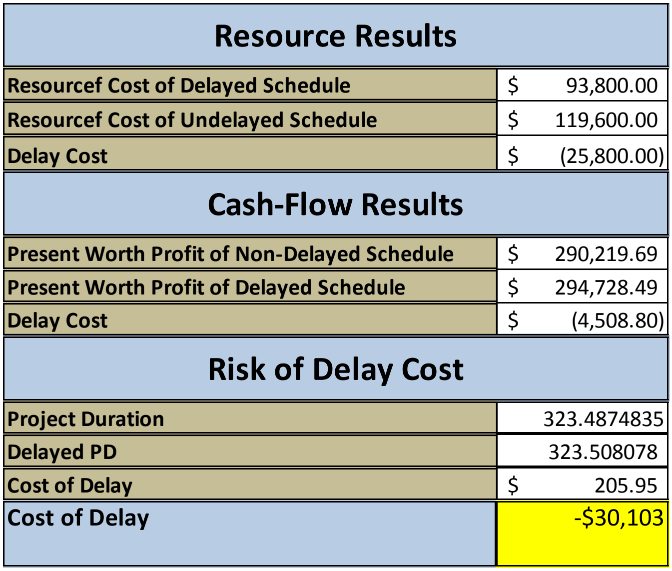
$CTP_{min} = CTP\ (42,0,79) = -\$30,103$

Tables 25 and 26 represent the data outcomes of the optimization process.

Table 25: Optimum Schedule Solution

Activities Data Input														
i	Activity	$P_{i,s}$				D_i	ES_i	EF_i	LS_i	LF_i	F_i	DE_i	DS_i	DF_i
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	42	72	132
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	0	72	180
6	F	3	4	5	-	12	192	204	276	288	84	79	271	283
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

Table 26: Optimum Schedule Cost



Resource Results	
Resourcef Cost of Delayed Schedule	$ 93,800.00
Resourcef Cost of Undelayed Schedule	$ 119,600.00
Delay Cost	$ (25,800.00)
Cash-Flow Results	
Present Worth Profit of Non-Delayed Schedule	$ 290,219.69
Present Worth Profit of Delayed Schedule	$ 294,728.49
Delay Cost	$ (4,508.80)
Risk of Delay Cost	
Project Duration	323.4874835
Delayed PD	323.508078
Cost of Delay	$ 205.95
Cost of Delay	-$30,103

3.5.2.2. ***Baseline schedule.*** In this case, all schedule constraints shall be defined to the objective function. The following schedule constraints are assumed in the baseline schedule.

$DCN_3 = 100,$

$DCN_5 = 82$

$MSC_6 = 214$

The objective function is defined to MS Excel Analytic Solver ® 2016 as follows:

- Minimize $CTP\ (DE_3, DE_5, DE_6)$
- Define Variables: (DE_3, DE_5, DE_6)
- Define the constraints:
  1. $(DE_3, DE_5, DE_6) \geq (0,0,0,..0)$
  2. $(DE_3, DE_5, DE_6) \leq (102,12,84)$
  3. (DE_3, DE_5, DE_6) are all integers
  4. $(DF_1,\ DF_2, DF_3, DF_4, DF_5, DF_6, DF_7, DF_8) \leq 318$
  5. $DS_3 \geq 100$
  6. $DS_5 \geq 82$
  7. $DE_6 \leq 214$

The following results are obtained:

$DE_3 = 96,$

$DE_5 = 10$

$DE_6 = 0$

$CTP_0 = CTP\ (96,10,0) = \$27,375$

Tables 27 and 28 represent the data outcomes of the optimization process.

Table 27: Baseline Schedule Solution

Activities Data Input														
i	Activity	$P_{i,s}$				D_i	ES_i	EF_i	LS_i	LF_i	F_i	DE_i	DS_i	DF_i
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	96	126	186
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	10	82	190
6	F	3	4	5	-	12	192	204	276	288	84	0	192	204
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

Table 28: Baseline Schedule Cost

Resource Results		
Resourcef Cost of Delayed Schedule	$	114,540.00
Resourcef Cost of Undelayed Schedule	$	119,600.00
Delay Cost	$	(5,060.00)
Cash-Flow Results		
Present Worth Profit of Non-Delayed Schedule	$	290,219.69
Present Worth Profit of Delayed Schedule	$	292,679.84
Delay Cost	$	(2,460.15)
Risk of Delay Cost		
Project Duration		323.4874835
Delayed PD		326.976983
Cost of Delay	$	34,894.99
Cost of Delay		$27,375

Note that $CTP_0 \geq CTP_{min}$ in all cases. The extra cost caused by the constraints of the schedule desired by the client shall be considered in the bidding price by the contractors who are interested in the project. This topic is discussed more in Chapters 4 and 5.

3.5.2.3. ***Activities delays during project execution stage.*** In this case, all delays and other events that might affect the baseline schedule shall be recorded and corresponding damages shall be quantified using the total damages model. The following events done by the client have been recorded during the project execution:

- Client requested to complete activity “C” before day 120
- Designs related to activity “E” has been delayed to day 84

Using total damages model, values of DE_k are defined as:

$$DE_3 = 90,$$

$$DE_5 = 12$$

$$DE_6 = 0$$

From Tables 29 and 30, new damages are determined.

$$CTP_{new} = \$45{,}924$$

Table 29: Actual Schedule Records

Activities Data Input														
i	Activity	$P_{i,s}$				D_i	ES_i	EF_i	LS_i	LF_i	F_i	DE_i	DS_i	DF_i
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	90	120	180
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	12	84	192
6	F	3	4	5	-	12	192	204	276	288	84	0	192	204
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

The total damages due to new schedule constraints are determined using Equation (48) as follows:

$$TDQ = \$45{,}924 - \$27{,}375 = \$18{,}549$$

Table 30: Actual Schedule Cost

Resource Results	
Resourcef Cost of Delayed Schedule	$ 114,630.00
Resourcef Cost of Undelayed Schedule	$ 119,600.00
Delay Cost	$ (4,970.00)
Cash-Flow Results	
Present Worth Profit of Non-Delayed Schedule	$ 290,219.69
Present Worth Profit of Delayed Schedule	$ 292,191.29
Delay Cost	$ (1,971.59)
Risk of Delay Cost	
Project Duration	323.4874835
Delayed PD	328.774018
Cost of Delay	$ 52,865.35
Cost of Delay	$45,924

3.5.3. Discussion of results. The total damages model integrates all the processes of obtaining damages related to changes of non-critical activities starting dates. The damages due to loss of time value of money, resource fluctuation cost, and increase in risk are built in one model to enable the process of developing optimum and baseline schedule and tracking all damages related to new deviation from the baseline agreement.

As obtained in 3.5.2, the cost of baseline schedule is always more than or equal to the optimum schedule. The difference between those two schedules in cost is due to the added schedule constraints to the baseline. In fact, the more constraints the client requests, the higher the difference in cost between baseline and optimum schedule.

However, not all constraints affect the schedule cost in the same amount. The cost of each constrain depends on the behavior of the three parameters of the affected activities. Hence, clients might reconsider adding costly constraints in order to receive lower bidding prices as discussed in 4.1.

Adding schedule constraints to the baseline is always cheaper than introducing them during the project execution period. The baseline cost is optimized through total damages model find the minimum cost schedule that satisfies the desired schedule constrain.

On the other hand, introducing these constraints later and during the project execution does not allow for optimization. This is due to the fact that during the project execution, many activities might have already been started or prepared for by the contractor at the time of constrain decision.

Following are two different scenarios for introducing the same schedule constraints at two different stages:

- ***Scenario I:*** using the same example in 3.5.2.2 with the same constraints, the baseline schedule cost is:

$$CTP_0 = CTP\ (96{,}10{,}0) = \$27{,}375$$

- ***Scenario II:*** Assuming that there are no schedule constraints as per contract agreement, the damages to the constraints introduced during the project execution are determined using the model by imposing the date of constraints on the optimum schedule as follows:

Using the total damages model, values of DE_k are defined as:

$DE_3 = 70,$

$DE_5 = 10$

$DE_6 = 10$

From Tables 31 and 32, new damages are determined.

$$CTP_{new} = \$32{,}736$$

Table 31: Scenario II Schedule

Activities Data Input														
i	Activity	$P_{i,s}$				D_i	ES_i	EF_i	LS_i	LF_i	F_i	DE_i	DS_i	DF_i
1	A	-	-	-	-	30	0	30	0	30	0	0	0	30
2	B	1	-	-	-	42	30	72	30	72	0	0	30	72
3	C	1	-	-	-	60	30	90	132	192	102	70	100	160
4	D	2	-	-	-	120	72	192	72	192	0	0	72	192
5	E	2	-	-	-	108	72	180	84	192	12	10	82	190
6	F	3	4	5	-	12	192	204	276	288	84	10	202	214
7	G	3	4	5	-	96	192	288	192	288	0	0	192	288
8	H	6	7	-	-	30	288	318	288	318	0	0	288	318

Table 32: Scenario II Cost

Resource Results	
Resourcef Cost of Delayed Schedule	$ 119,800.00
Resourcef Cost of Undelayed Schedule	$ 119,600.00
Delay Cost	$ 200.00
Cash-Flow Results	
Present Worth Profit of Non-Delayed Schedule	$ 290,219.69
Present Worth Profit of Delayed Schedule	$ 292,190.45
Delay Cost	$ (1,970.75)
Risk of Delay Cost	
Project Duration	323.4874835
Delayed PD	326.936869
Cost of Delay	$ 34,493.86
Cost of Delay	$32,723

Note that scenario II resulted in more cost although it has the same constraints of scenario I.

Chapter 4: Total Damages Quantification Method

Float ownership of non-critical activities has been discussed by many researchers and has been considered differently as discussed in Chapter 2. Some researchers concluded that float is free and can be owned by the one who uses it first, and other researchers refrained each party from using the float.

4.1. Float Allocation History Overview

Based on the models developed in Chapter 3, float can be utilized by the contractor to perform cost saving by performing cash flow enhancement, resource leveling and risk mitigation. Any constraint to using float introduced by the client can lead to extra cost related to the damages in these three parameters as the contractor will not be able to execute the project based on the optimum schedule. These conclusions support Al-Gahtani [29] concept discussed in the total risk approach to allocate float.

On the other hand, De La Garza, who highlighted the concepts of trading float as commodity [26] and float pre-allocation prior to signing the contract [1], did not introduce any quantitative method to buy or sell the float. The trade in value, mentioned in Section 2.4, did not cover all the damages related to change in non-critical activities starting date as explained in Chapter 3.

4.2. Total Damages Quantification Method

This chapter will combine De La Garza [26] and Al-Gahtani [29] concepts with the model developed in Chapter 3 (the total damages quatification model) to introduce a method that can be utilized in non-critical activities float allocation during the two different stages of the project.

Stage 1: Float pre-allocation. In the first stage, float pre-allocation process is to be conducted prior to signing the contract by an agreement such that each party of the project accepts. Figure 9 represents the flow chart followed to perform the pre-allocation of float as follows:

a. ***Client prepares request for proposal - RFP***. Unless the client is interested in a negotiated contact, the RFP is to be prepared including all the information required for contractors to prepare for the bidding. If any, all milestones or other schedule constraints that are required in the desired baseline schedule should be specified clearly in the Request for Proposal (RFP).

The client should also specify all liquidated damages related to delays in delivering milestones by the contractor. Schedule constraints might also include dates of some design documents, specification requirements, or approval of other submittals that might delay some of non-critical activities.

b. ***Contractors bidding proposals***. All contractors, who are interested to be awarded the project, shall submit their proposal including their bidding price, optimum schedule with its data input, and the baseline schedule. The submitted baseline schedule should be in accordance to what is specified in the RFP.

In case that there are no milestones or other schedule constraints, then the baseline will be, by default, the optimum schedule as it will yield in the most cost saving. In this case, contractors will be able to submit lower bidding prices equivalent to the total contract value $\sum CV_i$, which will also benefit the client.

Otherwise, contractors will increase their bidding price in case of having schedule constraints requested by the client. The bidding price for any schedule is determined by Equation (50).

$$Bidding\ Price = \sum CV_i + (CTP_o - CTP_{min}) \qquad Equation\ (50)$$

c. ***Proposal review and project award***. The client reviews all proposals by studying the submitted optimum schedules along with their data inputs. Then, the client evaluates which one best matches his benefits and decides whether to award the project to this contractor or to modify the schedule constraints in RFP and ask for new biddings again.

Noting that clients usually need some flexibilities in contracts, the choice shall not always be the lowest bidding price; however, the client might choose that proposal based on how costly will be introducing schedule constraints during the project execution stage which is discussed in 4.2.2.

d. ***Contractor and client liabilities***. Once the client awards the project to the contractor with the best proposal, float is pre-allocated as per the baseline schedule which makes the contractor liable to commit to all milestones and undertake all delays specified in the contract. The cost of these damages would have been already included in the contract.

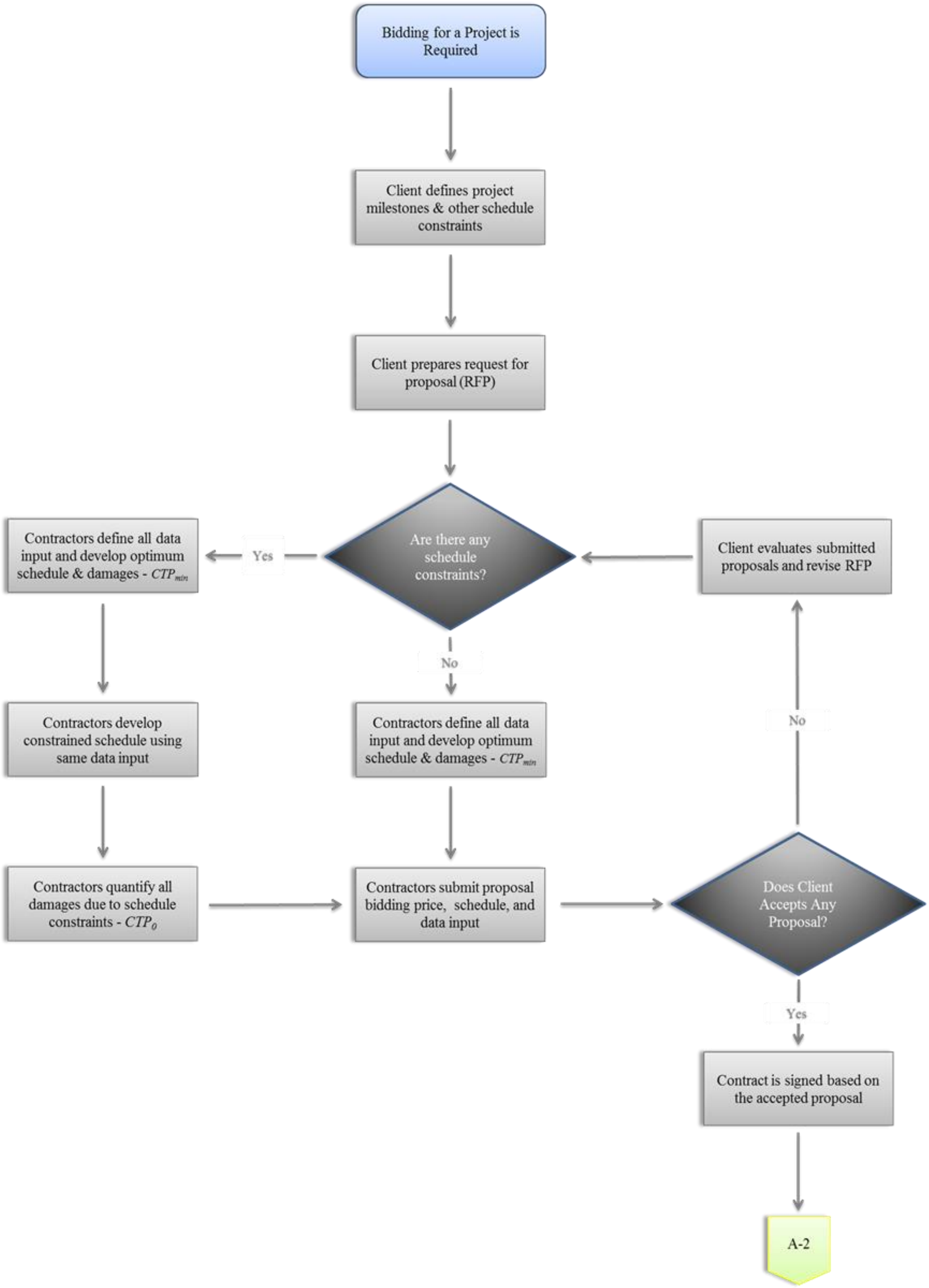


Figure 9: Float Pre-Allocation Flow Chart

On the other hand, client is responsible to obligate with all dates specified in baseline contract by providing all necessary designs and approval on time. Moreover, the contractor has the right to use all activities floats that are not constrained by the client in the contract and by delaying any non-critical activity for his interest or by accomplishing milestones before their specified dates.

Stage 2: Float re-allocation. In the second stage, re-allocation of float process during the execution of the project is conducted if required. In fact, construction projects involve too many ambiguities that cannot be determined prior to the commencement of the project. Henceforth, developing a process of re-allocating float can serve the interest of each party in a way that compensates the harmed party. Figure 10 represents the flow chart followed to perform the re-allocation of float as follows:

a. ***Contractor's liability towards original schedule* Constraints**. If there are no milestones of other schedule constraints agreed on in the contracts, which is called original constraints, the contractor has a full ownership of the project's floats as long as he delivers the project within the completion date.

However, if the contract of the project includes any milestones or other constraints, the contractor is fully liable to deliver the projects package specified in the contract as per the baseline schedule and to undertake all damages related to the deviations from the optimum schedule. In case that the contractor does not deliver the agreed milestones as per the contract, it is the client's right to claim against the liquidated damages or penalties as specified in the contract.

b. ***Contractor's liability towards new schedule constraints***. In the case that the client would like to introduce any new milestones or other schedule constraints, the contractor has the right to claim against the damages related to these new constraints. These damages shall be quantified using the total damages model and the same data input submitted in the proposal if and only if the contractor commits to these new constraints. Note that any changes to the Figures of data input such as increases in market prices shall be undertaken by the contractor.

c. ***Client's liability towards delay's to baseline***. If the client causes any delay to a non-critical activity rather than what was specified in the contract and baseline schedule such as delay in delivering a specific design, the contractor has the right to claim against all damages using total damages model and the same data input submitted in the proposal.

By using this process, float can be re-allocated in a way that reserves all parties' interests and compensates harmed parties.

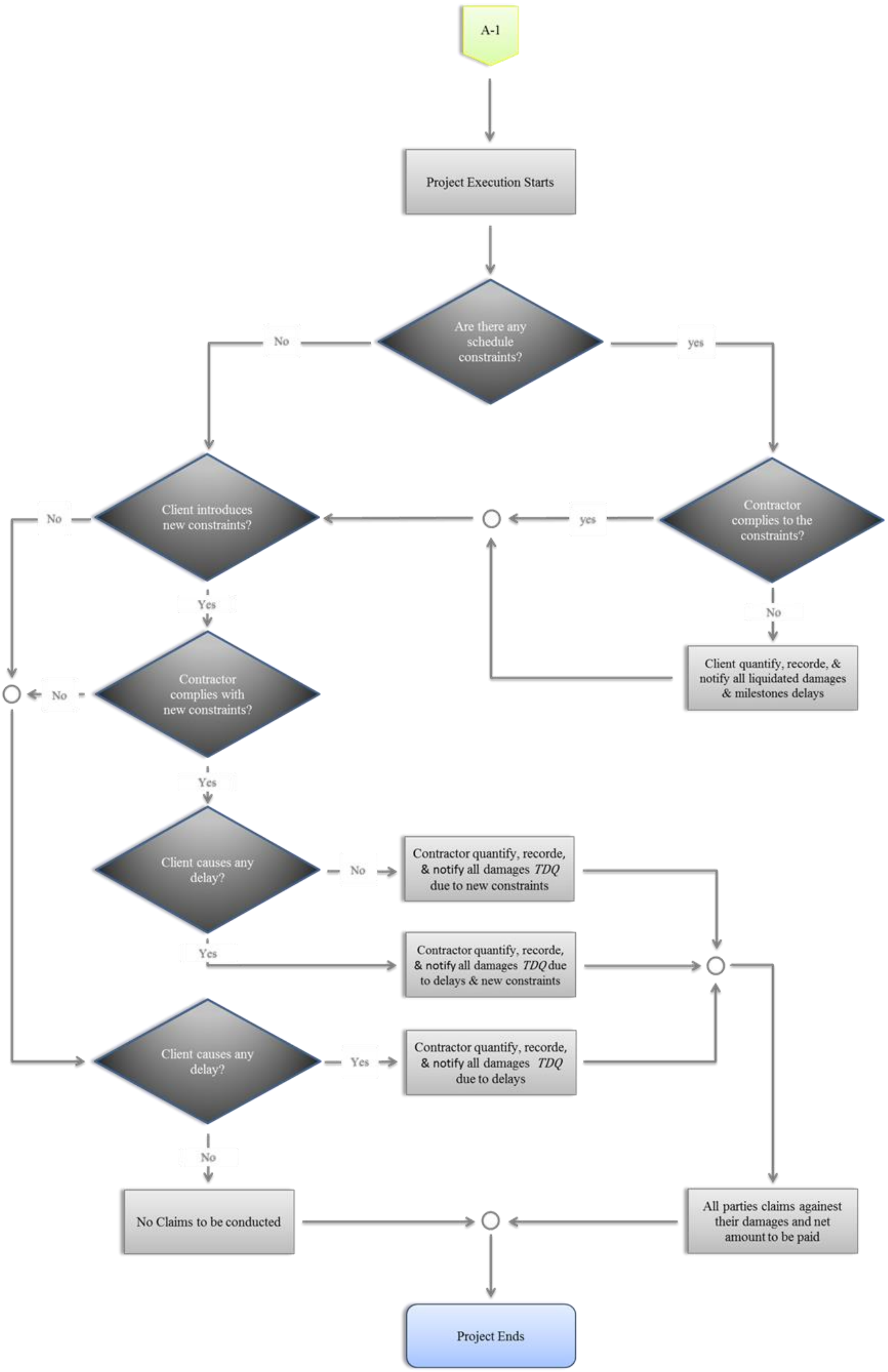


Figure 10: Float Re-Allocation Flow Chart

Chapter 5: Case Study

This chapter demonstrates the float allocation process using the Total Damages Quantification Method (TDQM) through a hypothetical case study that simulates real construction project. The case study represents a project in which the client has chosen to use the TDQM to proceed with project bidding and execution through the selected contractor.

5.1. Project Description

Company A real estate development is planning to develop a 2B+G+M+3P+60+2R residential tower with 2B+G+M+3P podium. The desired main contract scope is to procure and construct the following items within a period of 900 working days (1039 calendar day):

- Tower Substructure Works: include related earth work, waterproofing systems, and 3500mm thick reinforced concrete raft foundation at area of 2400m^2.
- Tower Superstructure: includes structural concrete works of slabs, beams, columns, and shear/core walls for 60 floors 17000m^2 each.
- Tower Residential Units Internal Initial Finishing/Civil Works: include walls block and plaster works, ceiling plaster works, internal water proofing systems, and concrete topping for floor leveling for 600 residential units.
- Tower Residential Units Internal Initial MEP Works: include all electrical connections and DBs, installation drainage systems and water supplies, and AC connections for 600 residential units.
- External Finishing Works: includes external waterproofing systems, installation of curtain walls, façade glazing, and GRC cladding.
- Tower Crown Works: includes related steel structure and Roof Coverings.
- Tower Residential Units Internal Final Finishing Works: includes painting, tiling, marble, false ceiling, doors, and joineries works for 600 residential units.
- Tower Residential Units Final MEP Works: includes sanitary works, vanities, bathtubs, showers, lights, AC, and elevators for 600 residential units.

- Podium Substructure Works: includes related earth work, waterproofing systems, 175 reinforced concrete pile cap 1600mm thick each, and 500 thick reinforced concrete slab on grade at area of 10,000m^2.
- Tower Superstructure: includes structural concrete works of slabs, beams, columns, and shear/core walls, ramps for 7 floors 10,000m^2 each.
- Podium Internal Initial Finishing/Civil Works: includes walls block and plaster works, ceiling plaster/surface treatment works, internal water proofing systems, and concrete topping for floor leveling all podium utilities.
- Podium Internal Initial MEP Works: includes all electrical connections and DBs, installation drainage systems and water supplies, and AC connections for all podium utilities mentioned below.
- Car Park Works: includes parking walls/columns/ceiling painting, traffic paving, curbstones, interlock, road marking, wheel stoppers, and lights for 950 designed car parks.
- Utilities and Services Rooms: includes walls/ceiling paint, flooring, steel doors, mechanical/electrical equipment (generators, pumps, transforms, ventilation fans, air handling units, garbage chutes) that will serve the tower units.
- Landscaping: includes curbstone, interlock paving, planters, irrigation systems, and lights for 2000m^2.
- Retails: include MEP supplies (AC/electricity/plumping) for 12 retails core finished with suitable access to hand them over for retailers.
- Hardscape works: includes 3 swimming pools, a tennis court, a basket ball field, planters, and irrigation systems located on podium roof.

The main contractor is liable to pay amount of $25,000/day as liquidated damages in the case of failing to deliver the project within the specified period that starts at the day of the project award.

5.2. Float Pre-Allocation Stage

Once the client decides to use the total damages quantification method to pre-allocate the referred project float, flow chart illustrated in Figure 9 shall be followed as follows:

The client defined the following schedule constraints:

- Delay Constraint: Tower internal finishing design to be provided at day 210
- Delay Constraint: Podium substructure design to be provided at day 120
- Delay Constraint: Podium internal finishing design to be provide at day 270
- Delay Constraint: Hardscape design to be provided at day 700
- Milestone: Parking design to be provided at day 680
- Milestone: Retails to be handed over before day 600 - $2000/day liquidated damages
- Services and Utilities rooms to be handed over at day 60 - $1000/day liquidated damages

The request for bidding through RFP that included all information mentioned above.

Since the client requires schedule constraints, each contractor who is interested in the project shall submit his data input for the total damages model, optimum schedule, and baseline schedule with the extra damages added to the bidding price.

Company “Y” general contracting, which is interested in the project, has submitted the following in its proposal:

- Data input as represented in Tables 33 through 36.
- Network Diagram
- Optimum Schedule and minimum damages cost represented in Figure 11 and Table 37.

$$CTP_{min} = -\$1{,}409{,}546.42$$

- Baseline Schedule and its damages cost represented in Figure 12 and Table 38.

$$CTP_0 = -\$459{,}670.82$$

Contractors submit their proposals including all data inputs along with the optimum schedule and baseline schedule. The bidding price is calculated and submitted by company “Y” using Equation (50) as follows:

$$Bidding\ Price = \$462{,}237{,}900 + (-\$459{,}671 - (-\$1{,}409{,}546))$$
$$= \$463{,}187{,}776$$

The client has reviewed the proposals and has chosen to modify schedule constraints as follows:

- Delay Constraint: Tower internal finishing design to be provided at day 180
- Delay Constraint: Podium substructure design to be provided at day 60
- Delay Constraint: Podium internal finishing design to be provided at day 210
- Delay Constraint: Hardscape design to be provided at day 700
- Delay Constraint: Podium internal finishing design to be provided at day 270
- Milestone: Parking design to be provided at day 480
- Delay Constraint: Retails to be handed over remain before day 630 - $2000/day liquidated damages
- Milestone to complete utilities and services rooms is canceled

Contractors re-submit their proposals with same data inputs along with the same optimum schedule and revised baseline schedule represented in Figures 11 and 13. The new bid price is calculated and submitted by company "Y" using Equation (50) as follows:

$$\begin{aligned} Bidding\ Price &= \$462{,}237{,}900 - (-\$1{,}349{,}892 + \$1{,}409{,}546) \\ &= \$462{,}297{,}554 \end{aligned}$$

The client has reviewed the proposals and has chosen to award the project for company "Y" general contracting as per the revised proposal and bid price.

The contractor owns the float of all non-critical activities that has no schedule constraints, while other constrained activities float ownership is as per the contract agreement. For activities that has delay constraints, the contractor has the right to delay it more than the agreed period while the client cannot; on the other hand, client can delay activities that have milestones as long as this delay does not cause damages to the contractor (before its optimum starting date) while the contractor cannot.

For example, for podium substructure works (activity 9) which has a delay constraint starting date at day 60 can be delayed by the contractor to its optimal date at day 91 to save cost.

On the other hand, for retail handing over (activity 16) which has a milestone end date at day 600, the client has the right to delay this until it reaches its optimal date (starting at day 795) while the contractor cannot.

Table 33: Case Study Activities Data Input

Activities Data Input

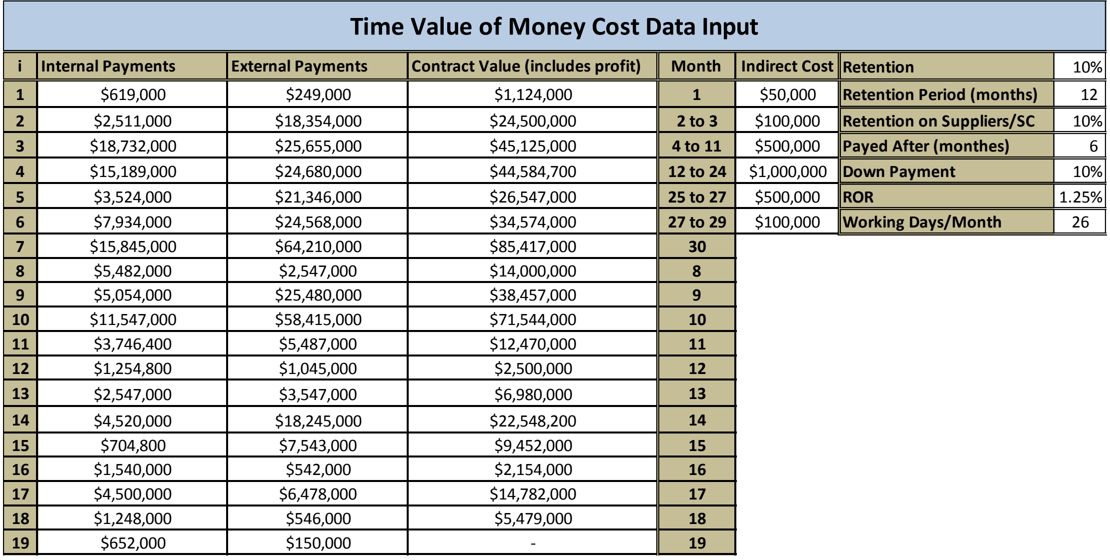
i	Activity	$P_{i,s}$				D_i
1	Moblization	-	-	-	-	30
2	Substructure Works - A	1	-	-	-	60
3	Superstructure Works - A	2	-	-	-	300
4	Units Internal Finishing/MEP Works - A	-	2	-	-	360
5	External Finishing Works - A	3	-	-	-	180
6	Crown Works - A	-	3	-	-	60
7	Final Finishing Works	4	5	-	-	240
8	Final Handing Over - A	6	7	-	-	60
9	Substructure Works - P	-	1	-	-	90
10	Superstructure Works - P	9	-	-	-	120
11	Internal Finishing/MEP Works - P	-	9	-	-	270
12	Lanscaping Works - P	-	-	9	-	90
13	Car Park Work - P	11	-	-	-	75
14	Utilities & Service Rooms Works - P	-	-	11	-	60
15	External Finishing Works- P	10	-	-	-	120
16	Retail Area Handing Over - P	12	11	-	-	30
17	Hardscape Works - P	-	10	-	-	120
18	Final Handing Over -P	13	14	15	17	30
19	Demobiliztion	8	16	18	-	30

Table 34: Case Study Risk Data Input

Risk Data Input

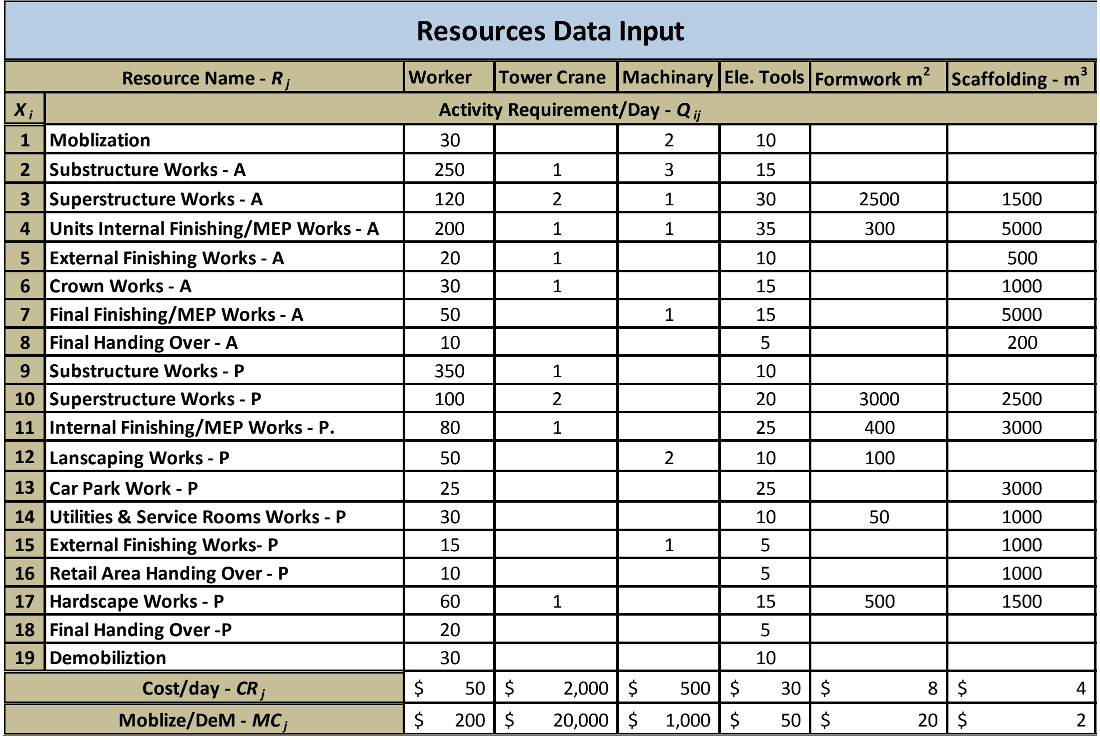
i	t_o	t_m	t_p	t_e	σ_{te}	Path No. - *r*	Path Combinations							
1	28	30	32	30.000	0.67	I	1	2	3	6	8	19		
2	45	55	95	60.000	8.33	II	1	2	3	5	7	8	19	
3	280	300	320	300.000	6.67	III	1	2	4	7	8	19		
4	350	355	390	360.000	6.67	IV	1	9	12	16	19			
5	170	180	190	180.000	3.33	V	1	9	11	16	19			
6	54	61	62	60.000	1.33	VI	1	9	11	13	18	19		
7	200	240	280	240.000	13.3	VII	1	9	11	14	18	19		
8	30	60	90	60.000	10	VIII	1	9	10	15	18	19		
9	65	80	155	90.000	15	IX	1	9	10	17	18	19		
10	110	120	130	120.000	3.33	X								
11	255	270	285	270.000	5	XI								
12	60	77	172	90.000	18.7	XII								
13	70	75	80	75.000	1.67	% Confidence	95%	Cost Rate of Delay				$25,000		
14	45	60	75	60.000	5									
15	105	120	135	120.000	5									
16	28	30	32	30.000	0.67									
17	105	120	135	120.000	5									
18	20	25	60	30.000	6.67									
19	29	30	31	30.000	0.33									

Table 35: Case Study TVM Data Input



Time Value of Money Cost Data Input							
i	Internal Payments	External Payments	Contract Value (includes profit)	Month	Indirect Cost	Retention	10%
1	$619,000	$249,000	$1,124,000	1	$50,000	Retention Period (months)	12
2	$2,511,000	$18,354,000	$24,500,000	2 to 3	$100,000	Retention on Suppliers/SC	10%
3	$18,732,000	$25,655,000	$45,125,000	4 to 11	$500,000	Payed After (monthes)	6
4	$15,189,000	$24,680,000	$44,584,700	12 to 24	$1,000,000	Down Payment	10%
5	$3,524,000	$21,346,000	$26,547,000	25 to 27	$500,000	ROR	1.25%
6	$7,934,000	$24,568,000	$34,574,000	27 to 29	$100,000	Working Days/Month	26
7	$15,845,000	$64,210,000	$85,417,000	30			
8	$5,482,000	$2,547,000	$14,000,000	8			
9	$5,054,000	$25,480,000	$38,457,000	9			
10	$11,547,000	$58,415,000	$71,544,000	10			
11	$3,746,400	$5,487,000	$12,470,000	11			
12	$1,254,800	$1,045,000	$2,500,000	12			
13	$2,547,000	$3,547,000	$6,980,000	13			
14	$4,520,000	$18,245,000	$22,548,200	14			
15	$704,800	$7,543,000	$9,452,000	15			
16	$1,540,000	$542,000	$2,154,000	16			
17	$4,500,000	$6,478,000	$14,782,000	17			
18	$1,248,000	$546,000	$5,479,000	18			
19	$652,000	$150,000	-	19			

Table 36: Case Study Resources Data Input



Resources Data Input							
	Resource Name - R_j	Worker	Tower Crane	Machinary	Ele. Tools	Formwork m^2	Scaffolding - m^3
X_i	Activity Requirement/Day - Q_{ij}						
1	Moblization	30		2	10		
2	Substructure Works - A	250	1	3	15		
3	Superstructure Works - A	120	2	1	30	2500	1500
4	Units Internal Finishing/MEP Works - A	200	1	1	35	300	5000
5	External Finishing Works - A	20	1		10		500
6	Crown Works - A	30	1		15		1000
7	Final Finishing/MEP Works - A	50		1	15		5000
8	Final Handing Over - A	10			5		200
9	Substructure Works - P	350	1		10		
10	Superstructure Works - P	100	2		20	3000	2500
11	Internal Finishing/MEP Works - P.	80	1		25	400	3000
12	Lanscaping Works - P	50		2	10	100	
13	Car Park Work - P	25			25		3000
14	Utilities & Service Rooms Works - P	30			10	50	1000
15	External Finishing Works- P	15		1	5		1000
16	Retail Area Handing Over - P	10			5		1000
17	Hardscape Works - P	60	1		15	500	1500
18	Final Handing Over -P	20			5		
19	Demobiliztion	30			10		
	Cost/day - CR_j	$ 50	$ 2,000	$ 500	$ 30	$ 8	$ 4
	Moblize/DeM - MC_j	$ 200	$ 20,000	$ 1,000	$ 50	$ 20	$ 2

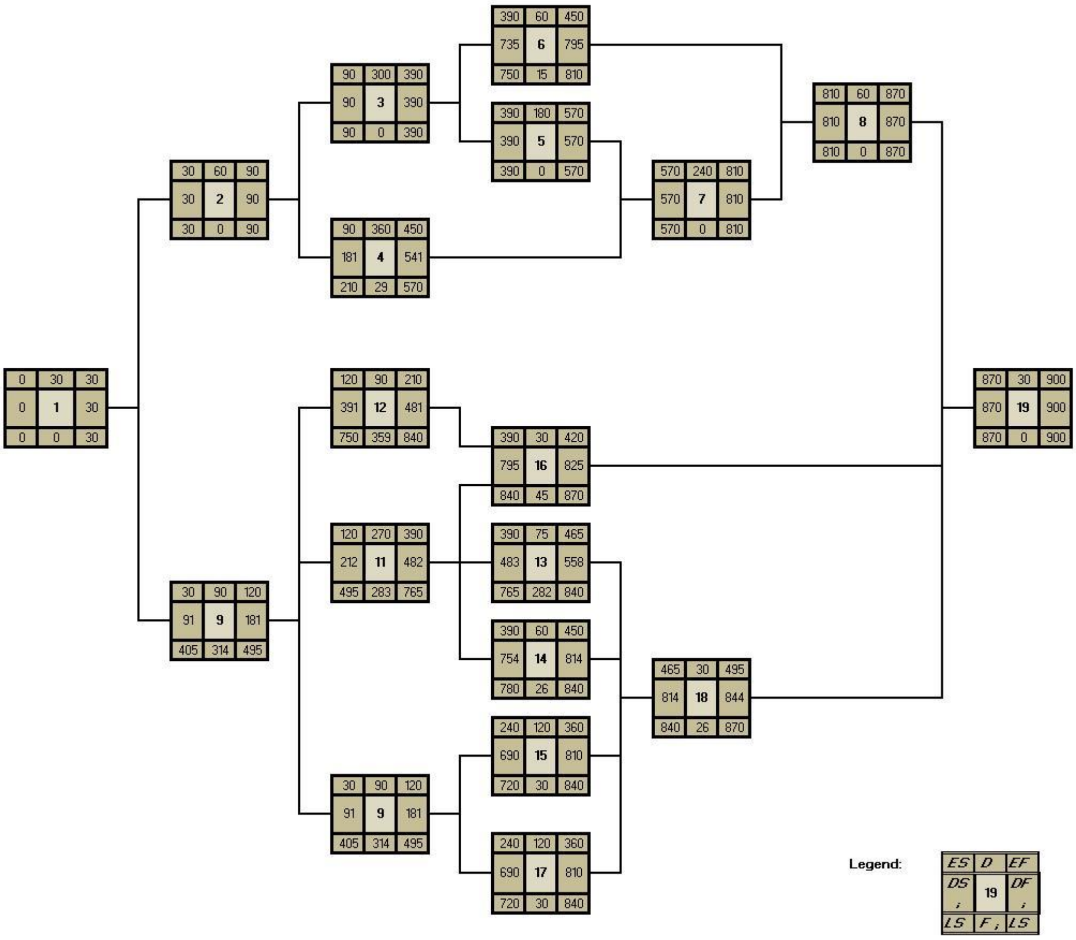


Figure 11: Case Study Optimum Schedule

Table 37: Case Study Optimum Cost

Resource Results	
Resourcef Cost of Delayed Schedule	$ 367,600.00
Resourcef Cost of Undelayed Schedule	$ 432,750.00
Delay Cost	$ (65,150.00)
Cash-Flow Results	
Present Worth Profit of Non-Delayed Schedule	$ 24,117,035.11
Present Worth Profit of Delayed Schedule	$ 25,473,043.93
Delay Cost	$ (1,356,008.82)
Risk of Delay Cost	
Project Duration	933.0358067
Delayed PD	933.500303
Cost of Delay	$ 11,612.40
Cost of Delay	$ (1,409,546.42)

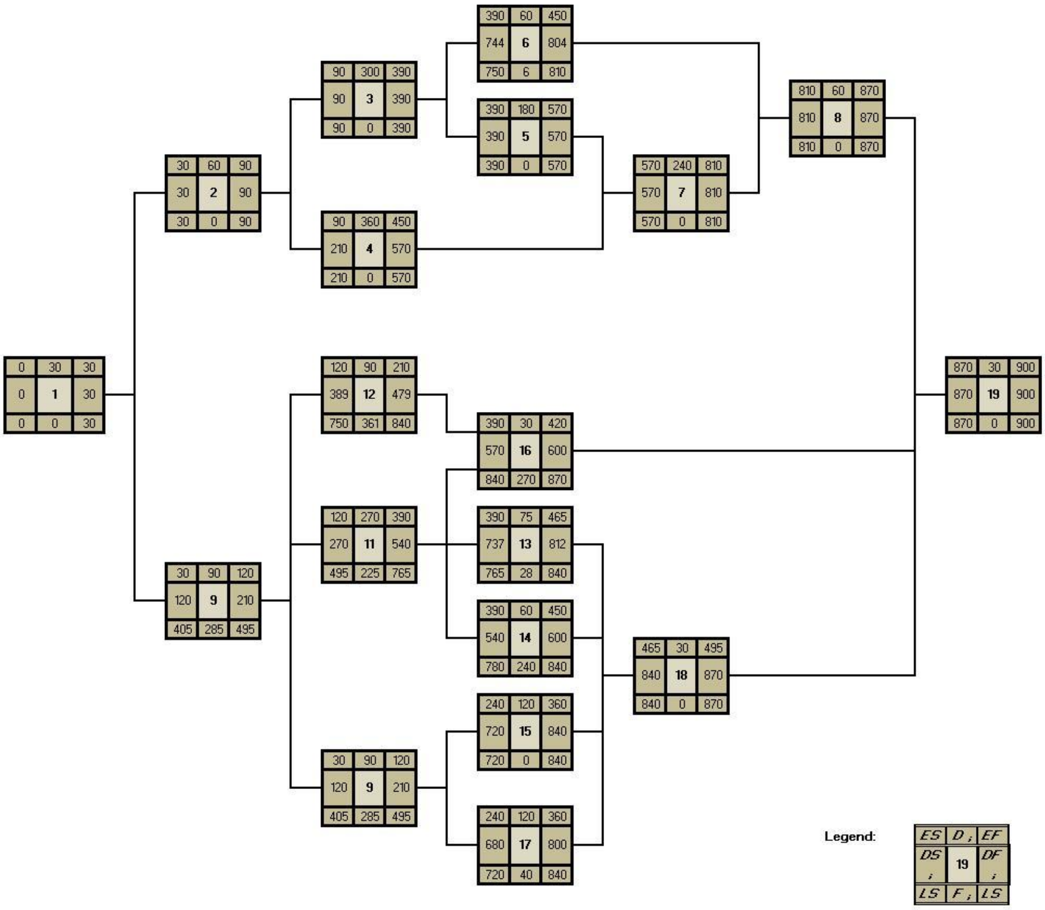
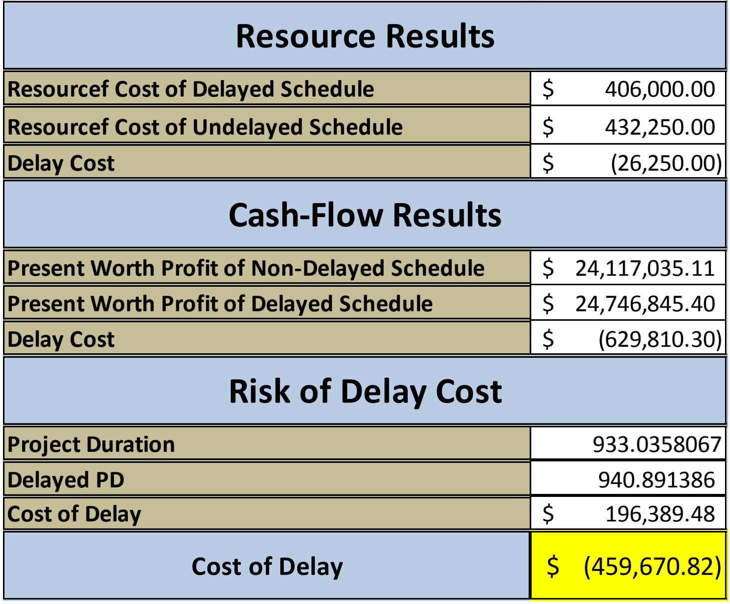


Figure 12: Case Study Baseline Schedule

Table 38: Case Study Baseline Schedule Cost



Resource Results	
Resourcef Cost of Delayed Schedule	$ 406,000.00
Resourcef Cost of Undelayed Schedule	$ 432,250.00
Delay Cost	$ (26,250.00)
Cash-Flow Results	
Present Worth Profit of Non-Delayed Schedule	$ 24,117,035.11
Present Worth Profit of Delayed Schedule	$ 24,746,845.40
Delay Cost	$ (629,810.30)
Risk of Delay Cost	
Project Duration	933.0358067
Delayed PD	940.891386
Cost of Delay	$ 196,389.48
Cost of Delay	$ (459,670.82)

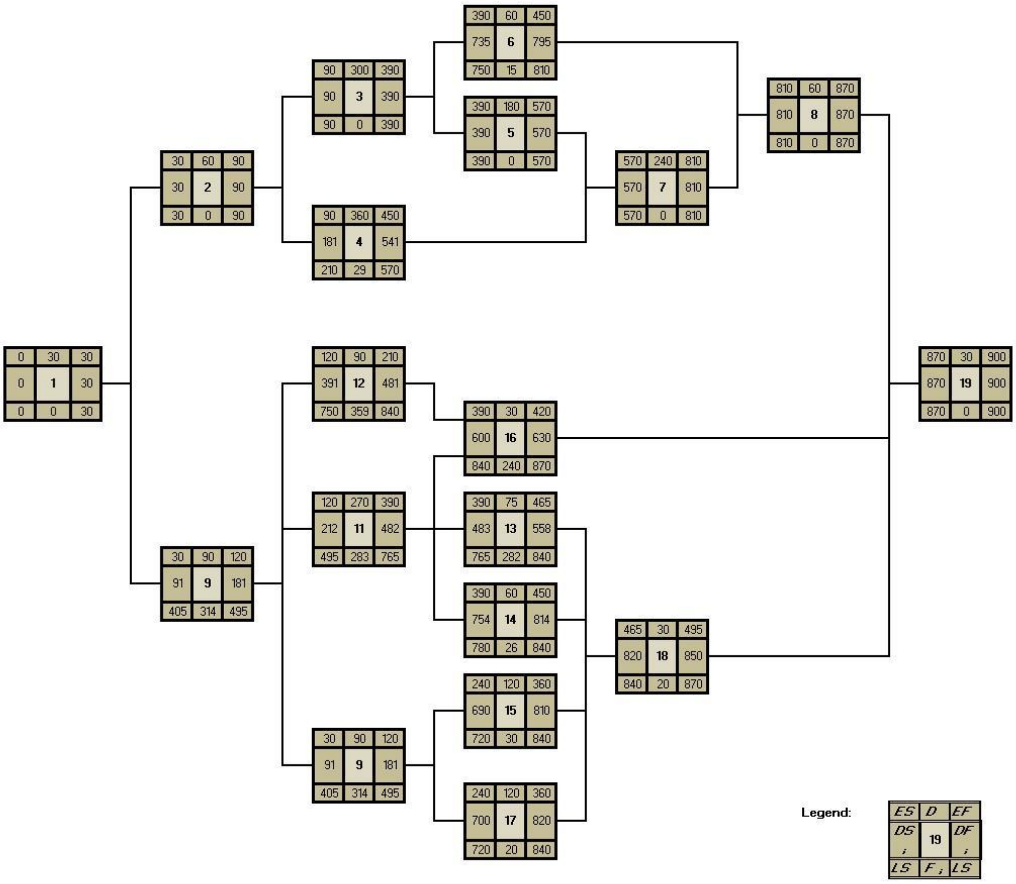


Figure 13: Case Study Revised Baseline Schedule

Table 39: Case Study Revised Baseline Schedule Cost

Resource Results	
Resourcef Cost of Delayed Schedule	$ 395,100.00
Resourcef Cost of Undelayed Schedule	$ 432,750.00
Delay Cost	$ (37,650.00)
Cash-Flow Results	
Present Worth Profit of Non-Delayed Schedule	$ 24,117,035.11
Present Worth Profit of Delayed Schedule	$ 25,443,123.87
Delay Cost	$ (1,326,088.76)
Risk of Delay Cost	
Project Duration	933.0358067
Delayed PD	933.589674
Cost of Delay	$ 13,846.68
Cost of Delay	$ (1,349,892.08)

5.3. Float Re-Allocation Stage

After project is awarded to company "Y", project execution starts at end of day 0. The contractor is supposed to start and end all activities as per the agreed revised baseline schedule while the client is supposed to provide all of the requirements in a way that complies with the same schedule. However, due to uncertainties in construction industry, the following scenario has occurred:

- Client has provided the units internal finishing design to day 200
- Client has provided the podium substructure design to day 80
- Client has provided the design of retails at day 620
- Client has provided the design of hardscape at day 720
- Client requested the contractor to complete utilities and service room before day 650
- Contractor handed over the retails on day 680.
- Contractor Completed utilities and service rooms at day 700

Following the flow chart illustrated in Figure 10, each part claim against their damages as follows:

a. Claims by the client. The contractor has delayed retail handing over to day 680; however, the client had already delayed its design to day 620. As per the baseline schedule, the duration of this activity is 30 working days, which means that the contractor could have completed it at day 650. Hence, the contractor is liable to 30 days of delay.

On the other hand, utilities and service rooms delay does not count in the client claim due to the fact that the requested delivery date is not an original milestone that is agreed on in the contract baseline schedule.

The client's total claim for the 30 days of delay in handing over the retails is obtained based on the contract liquidated damages agreement as follows:

Client Claim = 30 days x $2000/day = $60,000

b. Claims by the contractor: the contractor has the right to claim against all damages caused due to:

i. Client delays. Excepting the delay in retails design, the contractor has the right to claim against all damages that have been caused by the client through the total damages model using same data input agreed on in the contract.

Retails design delay is excepted due to the fact that its delivery date is a milestone defined and purchased by the client, and delaying it makes it closer to the optimal date.

Also, note that delaying the design of podium substructure will not affect the claim amount by the contractor as it does not cause any damages (its optimal date is 91).

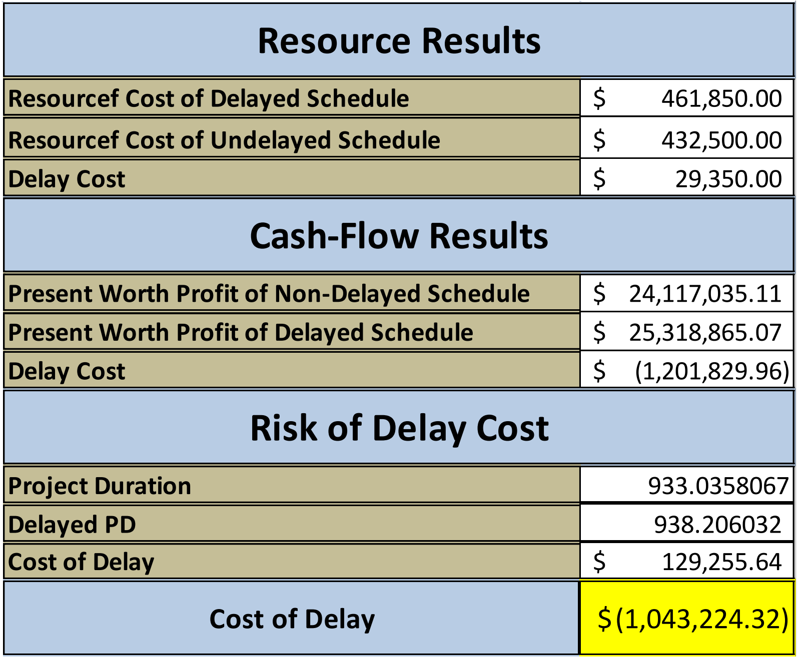
ii. New schedule constraints. The contractor has the right to claim against the damages due to schedule constraints that are not included in the original baseline schedule. The claim shall be obtained through the total damages model using same data input agreed on in the contract. Note that these claims can be considered only for what has been achieved by the contractor. In this case, the contractor can claim against what he achieved (completion at day 700) not what was requested (day 650).

Table 40 represents the cost of delay for the new schedule based on the actual data of project execution - CTP_{new}. The total damages to the contractor are quantified and claimed as per Equation (48) as follows:

$$TDQ = -\$1,043,224.32 - (-\$1,349,892) = \$306,667.68$$

The claims by each party are to be subtracted and the net amount should be paid to the party that endured more damages.

Table 40: Case Study Actual Schedule Cost



Resource Results	
Resourcef Cost of Delayed Schedule	$ 461,850.00
Resourcef Cost of Undelayed Schedule	$ 432,500.00
Delay Cost	$ 29,350.00
Cash-Flow Results	
Present Worth Profit of Non-Delayed Schedule	$ 24,117,035.11
Present Worth Profit of Delayed Schedule	$ 25,318,865.07
Delay Cost	$ (1,201,829.96)
Risk of Delay Cost	
Project Duration	933.0358067
Delayed PD	938.206032
Cost of Delay	$ 129,255.64
Cost of Delay	$ (1,043,224.32)

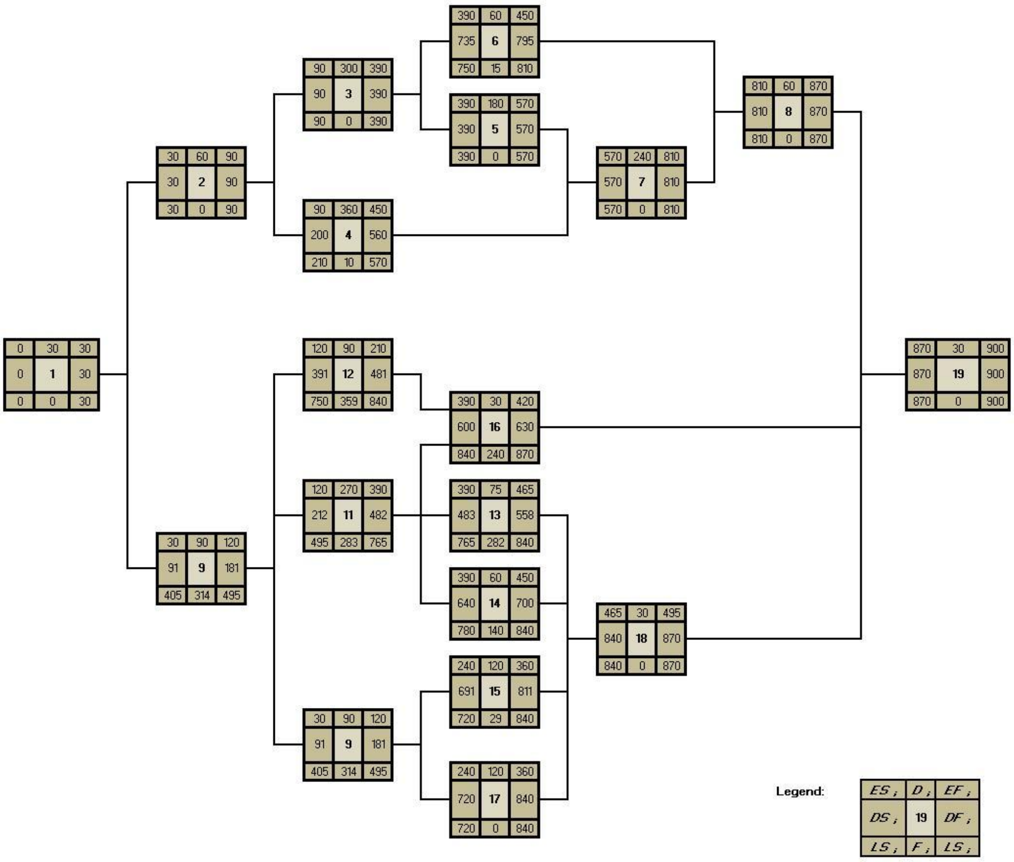


Figure 14: Case Study Actual Schedule

Chapter 6: Summary and Conclusions

Many float allocation methods have been developed in the last decades, which aim to provide an agenda that defines the right of using float for each of the construction parties. Nevertheless, the total damages quantification method is the first one that provides a float allocation methodology based on the quantitative analysis for the losses and benefits resulting from using the float.

6.1. Summary

The float pre-allocation process is performed by defining an optimum schedule that minimizes the total cost of the project by changing non-critical activities' starting dates. The cost of the contract baseline schedules, required by clients, can be determined and considered in the bidding price of the project by comparing the damages occurring due to it with the damages occurring when the optimum schedule is performed. Once the baseline schedule is agreed on, float ownership is determined and any damages due to changes that occur during the project execution to any party can be quantified and compensated.

The quantification of damages process is performed by obtaining the cash flow time value of money delay cost, resources fluctuation delay cost, and increase in project delay risk cost. Each of these three parameters is evaluated assuming that all non-critical activities start at their earliest and delayed starts in order to compare both scenarios with each other and find the marginal cost. The cost at the optimum (which is less than or equal to zero) is the lowest cost schedule that is used as a reference to determine the damages that occur due to schedule constraints prior to signing the contract and changes from baseline during project execution stage. The damages are calculated for each of the three parameters separately.

First, the delay cost of time value of money is obtained by breaking down all cash flows (expenses and revenues) of the project based on activities start and end dates. Each expense and revenue is obtained at each month based on the planned progress of the activity itself. Then, expenses and revenues at each month are obtained in order to obtain the profit. Using the rate of return that the contractor has, the net present value of the profit can be obtained. This process can be performed for any schedule in order to quantify the damages of losing the time value of money once any non-critical activity starting date changes.

Second, the delay cost of resources is calculated by breaking down all resources requirements for each activity during the project period, based on the planned schedule. Then, the total requirement of each resource at each day is obtained to determine each resource histogram. The cost of the resource histogram fluctuation is obtained as the minimum of mobilizing/demobilizing the resources and setting the resources idle at each fluctuation period. The cost of fluctuation for all the resources is summed up to the total resources fluctuations for the project schedule. This cost can be compared with the original (optimum or baseline) schedule in order to quantify the damages of the changes required in the non-critical activities.

Finally, the cost of the risk in delaying the handing over of the project is obtained using PERT analysis. By setting the required confidence level, the total project duration is obtained at the same confidence level through an iterative process. Then, the cost of risk to delay the project is obtained using the minimum of crashing cost of liquidated damages for the days that are delayed from the project handing over duration. The damages of changing the non-critical activity starting date can be quantified as the difference in risk cost between the baseline and new schedule.

6.2. Advantages of TDQM

This method aims to provide a solution to the float ownership conflict. TDQM is the first method that provides quantitative analysis to the losses and gains for each of the construction parties when float is utilized to change starting dates of non-critical activities. This has enabled TDQM to have the following advantages over other methods:

- Optimum schedule and baseline schedules are obtained by minimizing the cost of the project execution within the required constraints which enables bidders to provide lower bidding prices to clients which is in the interest of both.
- Float ownership is clearly agreed on prior to signing the contract, and both the client and the contractor are considering the consequences of this agreement in the bidding price.
- Float ownership can be re-allocated during the project execution, and harmed parties are compensated based on the damages occurred to them.
- The criticality of non-critical activities can be judged by the cost damages caused from changing their starting dates. Hence, all parties should deal with

non-critical activities with a high float cost as critical in order to avoid any compensation payments to the other parties.

6.3. Limitation of TDQM

TDQM works perfectly in all ordinary projects that consist of a definite number of activities with clear precedence relationships, cost, resource requirements, etc. However, for unique projects (that have no similar projects constructed before) and for projects that have uncertain events, it is difficult to determine the data inputs required for the total damages model. In fact, minor changes in some data inputs can severely affect the results of the total damages model.

Furthermore, this method can work only in lump sum contracts in which all designs and quantities are determined prior to signing the contract. This method cannot work in unit price or cost plus projects as data inputs and baseline schedules cannot be defined prior to signing the contract.

6.4. Recommendations

This thesis proposes a method that utilizes a model to perform float allocation. However, the application of this method in real life projects requires special contract forms to implement this method. Contract clauses and documents should include all the data related to the models and methodologies used for the float allocation and baseline agreements. Hence, the contracts forms should be adapted to contain this information and ensure that they have to be implemented by law.

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Vita

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