CLOSED LOOP SUPPLY CHAIN UNDER CONSIGNMENT STOCK PARTNERSHIP

by

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A Thesis Presented to the Faculty of the
American University of Sharjah
College of Engineering
in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science in
Engineering Systems Management

Sharjah, United Arab Emirates

May 2015

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Acknowledgements

I would like to thank Dr. Moncer Hariga for his support, intellectual advice and guidance during this research and throughout my graduate studies. I have learned a lot under his teaching both academically and professionally. I would also like to thank Dr. Rami for his support and encouragement during my thesis.

Besides my advisors, I would like also to thank the rest of my thesis committee: Dr. Mehmet Gumus, and Dr. Zied Bahroun, for serving on my committee and spending their precious time in reviewing my work and providing inspiring and valuable suggestions. Moreover, I would like to thank the ESM faculty for teaching me during my graduate studies.

I would like to thank my family for their never ending support and love as well as my classmates at AUS and colleagues at Philips

Dedication

This thesis is dedicated to my father, Mr. Mohammad Ali Khan, mother, Mrs. Kishwar Ara, sister, Ms. Saba Khan, and brother, Mr. Shuab Khan. Thank you for your continuous support, encouragement, and unconditional love.

Abstract

The emerging field of reverse logistics or closed loop supply chain management seeks to efficiently handle all the activities pertaining to the retrieval of products from end customers along with the proper reuse or disposal of those products. This thesis addresses a closed loop supply chain comprised of a single vendor and a single buyer operating under a consignment stock (CS) strategy. Following this sort of partnership, the buyer agrees to store the products at its premises where, in return, the payment for those products is made only after being sold to the end customer. We develop a mixed integer non-linear program that seeks to minimize the closed loop chain-wide total cost by jointly optimizing the length of the production cycle, the number and the sequence of the newly manufactured and the remanufactured batches, as well as the inventory levels of the finished and recovered products at the beginning of the cycle. Extensive numerical experiments are also conducted in order to assess the impact of key problem parameters on the behavior of the model. The results indicate that, for low values of the setup cost associated with newly manufactured batches, the vendor might be better off adopting intermittent schedules in which he would alternate between the production of new and remanufactured batches more than once. The economic merits of the generalized model are also assessed through calculating the cost percentage increase when adopting special production sequences such as consecutive remanufacturing batches followed by consecutive newly manufacturing batches, and other sequences with single manufactured or remanufactured batch.

Search Terms: Reverse logistics, integrated vendor-buyer, consignment stock, closed loop supply chain, heuristic solution.

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Chapter 1: Introduction

1.1 Supply Chain Overview

World class leading companies often gain competitive advantage over their peers based on the strength of their supply chains. Supply chain management is emerging as an extremely important topic in recent years, to the extent that some call it a 'backbone' or 'blood and muscle' of a company. Statistics show that companies managing their supply chains efficiently have 5% higher profit margin, 15% less inventory, up to 17% stronger perfect order ratings, and 35% shorter cash-to-cash cycle times than their peers [1]. The next sections will briefly discuss what underlies the dynamic and complex, yet very important concepts behind the supply chain.

1.1.1 Supply chain management.

The Association for Operations Management [2] defines supply chain management as:

> "design, planning, execution, control, and monitoring of supply chain activities with the objective of creating net value, building a competitive infrastructure, leveraging worldwide logistics, synchronizing supply with demand and measuring performance globally."

In other words, "supply chain" refers to all sets of activities that are required to convert a product from raw materials into a finished product, while maintaining efficiency. Our daily lives are surrounded by the results of supply chain activities. For example, a product at a retail store or apparel at a fashion store usually goes through a large chain of activities before it is bought by a customer. Any of these two products may go through the following stages as shown in Fig. 1.

Figure 1: Stages of Supply Chain

For example, once there is a need for shirts at an apparel store, suppliers will provide fabric to manufacturers. Once the shirts are produced, they are transported through a distributor to a retailer and finally to the customer. Each stage can bear multiple nodes; for instance, multiple manufacturers or distributors may be required.

Any particular chain may undergo only two, three, four, or all stages. This seemingly simple supply chain can become more complex when procurement of materials and retail shops are located in different countries. This is commonly called a "global supply chain".

Although supply chain has evolved from logistics management, the new integrated approach in supply chain management is a revolution to industries. It has tremendously improved responsiveness and productivity, reduced operational costs, and enhanced end-to-end visibility [4].

Logistics management means moving the right products, to the right place at the right time, with the right cost. The idea of 'lean' or efficient product flow was introduced through modern logistics, but supply chain management (SCM) means much more than logistics. Now, companies view suppliers and customers as an extension to their firm.

Traces of supply chains were evident as early as ancient times with the barter system, and later with trading within and among countries. The 1950's and 1960's became popular for farming and distribution of goods. It was known as the push era because more importance was given to the production of goods. During this period, value functions were disparate. In some cases, there were adversarial relationships between departments which led to conflicts of interest [5].

Starting from the 1970's, as management theories were developing, customer focus was increasingly receiving attention, leading a shift from the push era to the pull era. To fulfill customer needs, companies realized that distribution functions needed much improvement. As a result, companies gained interest in logistics, which was widely applied in militaries. It is interesting to note that the term "logistics" was coined in the military in response to maintaining on-time delivery of army supplies and weapons [4, 5].

The term "supply chain management" was coined in 1982 by Keith Oliver, an industrial consultant at Booz Allen Hamilton [6]. Having excellent quality standards and responsive systems became a key solution for companies to remain competitive in the growing consumer demand market. Firms began to see benefits of integrating firms externally to increase productivity and reduce operational costs. Advancement in technology further enabled integration. By the 2000s, people began to use information technology (IT) and information systems to increase efficiency of their logistics systems, and collaborate externally as well [4].

1.1.2 Supply chain applications.

One of the most successful applications of the supply chain can be seen in Wal-Mart. The success of Wal-Mart is mainly due to its emphasis on cost reduction and customer satisfaction; both were achieved through excellent strategic management techniques. Examples of supply chain practices through logistical and cross-functional drivers are described below:

- Facilities*:* Wal-Mart was the first to centralize its (19) distribution centers. Goods were ordered, assembled, and later dispatched to individual stores. This way the company was able to order in large quantities from suppliers at lower prices, and pass the savings on to the customers.
- Pricing: They minimized their purchasing costs by dealing with the manufacturers directly, and they only finalized deals with suppliers who offered the cheapest price. They also maintained a close relationship with the vendors, to understand their cost structure and to evaluate their manufacturing practices. By doing so, they ensured reduction in purchase costs.
- Information: They introduced a virtual world of e-commerce through the website WalMart.com to enhance merchandisers' online presence. Additionally, the company formed the largest satellite communication system, to provide point-of-sales data to its suppliers.
- Transportation: They maintained a fast and efficient transportation system by hiring highly experienced truck drivers and dealing with more than 3000 company trucks. Moreover, cross-docking was incorporated to increase efficiency of its distribution center. In cross-docking operations, finished goods were sorted according to customer orders and then directly delivered to them. As a result, this reduced the material handling cost for products, and practically eliminated the distribution center.
- Inventory Management**:** Wal-Mart used very effective IT systems (e.g. point-of-sale (POS), electronic data interchange (EDI), radio frequency identification devices (RFID), communication satellites, and hand-held computers) to monitor inventories through the supply chain and maintain accurate replenishment. For example, a hand-held computer that is linked to terminals at stores was used to track

inventory levels at distribution centers, stores and deliveries. They used this information to reduce overall inventory levels, but they ensured that appropriate levels of inventories were available for products with higher demands. In addition, they allowed stores to manage their stocks to reduce the package sizes of products.

In addition, accurate replenishment of stock was possible through the use of point-of-sale (POS) which could track stock levels at the shelves of stores. They further used IT and optimization to minimize floor plant layout to assist in quickly locating and replenishing goods. Wal-Mart was also behind several supply chain initiatives, such as vendor managed inventory and consignment stock which will be discussed later.

Dell computers, one of the largest PC manufacturers, is well known for its successful supply chain management, specifically in customer satisfaction, cash flow and inventory management techniques [3]. In fact, Dell has an amazingly fast response time because they ship their products directly from plants to customers. They maintain a close relationship with customers to get real time data and feedback. Moreover, Dell has only 5 weeks of inventory while its peers carry several weeks of inventory. They keep few centralized manufacturing plants. They also customize orders, which allows them to be flexible to customer demand. The applications show that many supply chain initiatives are used in industries to further increase profitability. Supply chain initiatives are discussed in the next section.

1.2 Supply Chain Initiatives

The pressure of competition and cost are driving companies to change their tactical and strategic approaches in order to remain profitable. Consumers are demanding cheaper products with better quality. As a result, organizations are turning toward initiatives that will lower their supply chain costs in an effort to offer cheaper prices to customers while remaining profitable.

Many companies are adopting technology such as information systems, hardware technology, and RFID to run their supply chain efficiently. The aim is to use technology to reduce operational costs and improve visibility. Many organizations are also reducing costs through their supply chain, forming collaborative partnerships, and forging supplier partnerships. In some cases, companies are taking this to an external level. They are forming partnerships through horizontal integration; that is partnering with similar companies, or vertical integrations (i.e. partnering with their suppliers and customers). Organizations that partner externally realize that customer satisfaction is the primary driver for success. Through collaboration and data sharing, they can reduce stock variability across the supply chain instead of focusing on individual supply chains to increase the customer service level. Overall, supply chain can be optimized. Individually, firms may reduce operational costs with improved visibility as they can better match supply with demand. Some examples of supply chain initiatives are vendor managed inventory (VMI) and consignment stock (CS). These two initiatives will be briefly described in the next sections, and the CS initiative will be the focus of this thesis.

1.2.1 Vendor managed inventory.

Vendor managed inventory (VMI) was introduced in the late 1980's to improve collaborative planning and integration between supply chain members. VMI agreement or continuous replenishment strategy is a source of partnership between a buyer and vendor to improve overall supply chain efficiency. Felix [7] stated that VMI can reduce total supply chain costs. In traditional systems, each member is concerned with individual profit. However, as they keep in mind the end target which is customer satisfaction, vendors and retailers undergo partnerships such as a VMI partnership to improve visibility and reduce demand uncertainty.

In a vendor managed inventory system, the vendor monitors the buyers' inventory levels and makes periodic resupply decisions regarding order quantities, shipping, and timing. In a VMI partnership, customers are required to share information with vendors so that vendor will closely monitor the inventory level of the customer. Usually, the vendor will know in advance the customer stock levels and demand forecasts through electronic data interchange (EDI) or internet [8].

In general we can expect to see cost savings for vendors under VMI, since they know in advance the stock forecasts. With less demand uncertainty, they can better plan their operational activities such as manufacturing and transportation to reduce lead time and inventory costs. The vendor will not have to worry about sudden surges in demand, so they can minimize safety stock. They can now better plan their production schedule and reduce manufacturing costs. In addition, they have more flexibility in choosing the transportation routes and they can prioritize shipments more effectively. Vendors can also wait to consolidate shipments. In both cases, transportation cost will decrease. Moreover, the transportation process used with VMI improves customer service because of better communication between buyers and distribution centers, so fewer rejections occur during shipments. From the customer's point of view, they will no longer have to worry about making the orders and paying major portions of the ordering costs.

There are several successful examples of VMI applications industry wide. Wal-Mart uses the VMI approach through EDI. Under this system, suppliers are able to download purchase orders and closely monitor sales information of Wal-Mart products. Suppliers now have a more accurate estimate of product demand, and can then ship the required products to Wal-Mart's distribution center [9]. Shell Chemical also implements VMI in an attempt to become the sole supplier for its customers. It provides the bill only once at the end of the month and supplies its products without any disruption [10].

Similarly, we can expect to see applications of VMI in a wide range of industries from large retailers, airline companies, as well as the manufacturing sector. VMI can be used in large retailers similar to Wal-Mart and Seven Eleven where stock-out levels and shorter lead times are critical. In addition, companies where short lead times are critical for success such as newspaper companies may also implement VMI. Healthcare centers and pharmacies also fall into this category since they cannot afford to delay selling medicines. In all these cases, VMI will be beneficial since the vendor will be closely monitoring the inventory levels and aligning their production to meet customer demands in advance. This will lead to less stock -outs and shorter lead times [7].

1.2.2 Consignment stock.

The consignment stock (CS) system is a similar type of supply chain partnership as the VMI. However, in CS, although the products are stocked at the buyer's storage facility, the products belong to the vendor until they are used by the buyer. Thus, the vendor is now responsible for opportunity cost instead of the customer. Buyers will be in charge of the storage costs, but they still have to decide the time and size of the orders. The ownership of products or goods will be transferred from vendor to buyer when the products and goods leave the customer's warehouse or are sold to the end customer. Thus, a buyer will have their stocks available at their warehouse well in advance, and will receive an invoice only upon use, which implies that the inventory capital cost is now incurred by the vendor. Usually, vendors and customers agree on the duration of contract and state of left over stocks. Sometimes, buyers also impose a maximum stock level upon their vendors. Table 1 shows the responsibility of each party for the different aspects of the inventory decision process.

The benefits of consignment inventory are usually seen in the buyers as they don't bear the cost of products until production or selling. This is especially appealing for the buyer if it is a new product or a product with less market demand. Buyers will not bear the opportunity cost, and they will have the luxury of having stocks available close to manufacturing. However, it is shown that there is opportunity for vendors to have benefits as well, as they can reduce their inventory costs through storing the product at the buyer's premises.

Table 1: Vendor and Buyer Responsibilities in the Inventory Decision Process [11]

Aspect	VMI	CS
Order quantity	Vendor	Buyer
Order initiation cost	Vendor	Buyer
Ordering receiving cost	Buyer	Buyer
Inventory capital cost	Buyer	Vendor
Storage cost	Buyer	Buyer

There are many successful examples of CS applications. For example, Tepe Home, a large retailer and manufacturer of retail stores in Turkey, has CS partnership with its vendors. With less powerful vendors, they pay for the supplies only after sale. Siemens Automation and Drives uses CS agreement for items that have high purchasing costs, which has helped them to reduce their inventory costs [12].

So far, we have described different initiatives that are being used by companies. Recently, green supply chain is being adopted by SC companies as an environmentally friendly and economically beneficial initiative. This will be described next.

1.3 Green Supply Chain

Environmental concern is becoming a serious threat as greenhouse gas emissions are increasing, and critical resources such as food, water, and minerals are decreasing. At the same time there is an increase in industry sectors due to economic growth. In fact, a third of the total energy consumption is used by industries in developed countries. The value of returned products in the U.S. is estimated to be $100 billion per annum [13]. Thus, there is great opportunity for organizations to reduce greenhouse gas emissions and be more socially and environmentally responsible [14]. Many governments are also imposing regulations on companies in order to improve waste disposal methods and reduce environmental impact, and consumers are increasingly demanding green products. As a result, several companies are now shifting their supply chain practices to address environment concerns, i.e. they are shifting toward green supply chain.

The impact of green supply chain is also leading to innovative solutions and increasing competitiveness of companies. Green supply chains or sustainable supply chains are defined by Oral [15] as "concepts that take a more holistic systems perspective on the total environment impacts of the supply chain on resources and ecological foot prints." In other words, a sustainable supply chain encompasses environmental impacts of supply chain processes. "Greening" a supply chain can include purchasing 'green' products, and evaluating and selecting suppliers that are environmentally friendly. To implement it, companies are taking many steps such as: environmental sourcing, life cycle analysis through energy and material reduction, and designing efficient processes [14]. Green product design is also being implemented in the early design stage of products to avoid disposal, repair, and reuse costs [16]. In addition, sophisticated supply chains also deal with product recovery, also known as reverse supply chains. The reverse supply chain is defined by Guide and Van [17] as "a series of activities required to retrieve used products from customers and either dispose of them or reuse them, and it requires managing those activities effectively and efficiently." Some applications of green supply chain will be presented in the next section.

1.3.1 Green supply chain applications.

Green supply chain is rampant in theory as well as in practice. Numerous examples exist in diverse industries today as presented in [19]. Schering AG, a pharmaceutical company, reuses its solvent and recycles its by-products during production. It has saved 11.5 million EUR per annum in production costs, and has achieved additional cost savings from waste disposal. Although their motivation was economic benefits, environment impact was also reduced. Another example is the reuse of containers in the Blue Container Line Company [19]. This feeder company realized the advantage of reusing containers since maintaining and storing them is cheaper than investing in new containers and disposal costs. However, stringent planning and use of IT was needed to optimize this reverse supply chain, especially because of differences in costs of storage in different sea ports.

Many applications can be seen in electronic companies as well. The NEC Corporation, Japan, an electronic manufacturer of desktops, notebooks, and servers increased its product return value from 10-20 % to 70–80 %. Their NSR (notebook server recover), provides a structured process for data gathering and analysis of consumer returns, production returns and end-of life components [19]. Many other electronic companies such as HP and Cannon are taking advantage of the reverse supply chain. Xerox Corporation saves 40% - 65% in manufacturing costs through a prepaid mailbox service which allows customers to return used cartridges. Kodak recycles 76% of disposed cameras that are received from large retailers [20].

In addition, batteries can also be recycled since they contain up to 65% lead at the end of the life cycle and plastic which is easily recyclable. It is stated that "about 35 kg of lead and 7.5 kg of plastic can be recovered from 100 kg of used battery [21]."

So far, we have seen complex supply chains including green supply chains. Many different cost parameters are measured to achieve optimality. As a result, many quantitative models are widely used, as shown below.

1.4 Supply Chain Mathematical Representations

Optimization models are commonly used in supply chain management. Efficiently managing a supply chain requires working with large data and a wide range of cost parameters. Examples include inventory, transportation, quality, raw materials, production, labor, and operational costs. Optimization models are beneficial as they can use large data in a convenient and realistic way to minimize cost or maximize profit, while balancing tradeoffs. Some common types of optimization models are linear programming, mixed integer programming, stochastic modeling and simulation. Examples of each are described below:

- Linear programming (LP): This is one of the most common optimization models. One of its many applications is resource utilization. Many firms deal with scare resources such as machines, labor, materials, and capacity. LP models optimize resources in the most economical way. They are also used in personnel scheduling problems (i.e. assigning labor to shifts), or in planning and distribution problems such as optimizing inventories to be used during production and optimizing shipping quantities.

- Mixed integer programming (MIP): This model is an extension of linear programming where some of the variables can only take integer values. Some examples include capacity planning, deciding transportation modes, and facility location planning. Minner and Lindner [22] developed a mixed integer programming model to optimize production and transportation quantities in a remanufacturing system.
- Stochastic programming*:* Stochastic models are used when some of the problem parameters cannot be estimated with certainty such as demand and lead time. For example, Santoso *et al*. [23] developed a stochastic model to optimize the supply chain network design by minimizing expected production and distribution costs under uncertain demand. Similarly, Mo *et al*. [24] designed a supply chain and reduced demand uncertainty through a two-stage stochastic model.

1.5 Problem Statement

There has been much research conducted on the operational benefits of implementing SC partnerships to determine under which conditions such initiatives are beneficial for the vendor, buyer or both. In addition, many researchers have optimized the forward supply chain. Some researchers have also conducted studies for both forward and reverse logistics in single and multi-echelon supply chain systems. However, to the best of our knowledge, only Jaber *et al.* [25] have conducted a study on a closed loop supply chain system integrated with CS agreement. Having forward and reverse (closed loop) supply chain simultaneously and partnership between supply chain parties can lead to improved supply chain performance. Thus, in this thesis, a mathematical model will be developed for a closed loop supply chain under CS agreement. Figure 2 below shows the multi-echelon reverse supply chain framework.

More specifically, we present a mathematical model that optimizes the total integrated supply chain cost by determining the optimum lot sizes, cycle time, number of batches to be produced from new and returned materials, as well as the sequence of the batches. We first review the related models that have been developed for closed loop supply chain and CS partnerships in Section 2. We then develop the optimization model for the study on hand (i.e. integrated closed loop supply chain model under CS partnership). Here, we present the problem description along with the stipulated assumptions, the derivations of the optimization model as well as the solution algorithm. In Section 4, we present the sensitivity analysis to study the effect of key parameters on the total supply chain cost. Lastly, we propose future research directions in Section 5 and summarize key results in the conclusion.

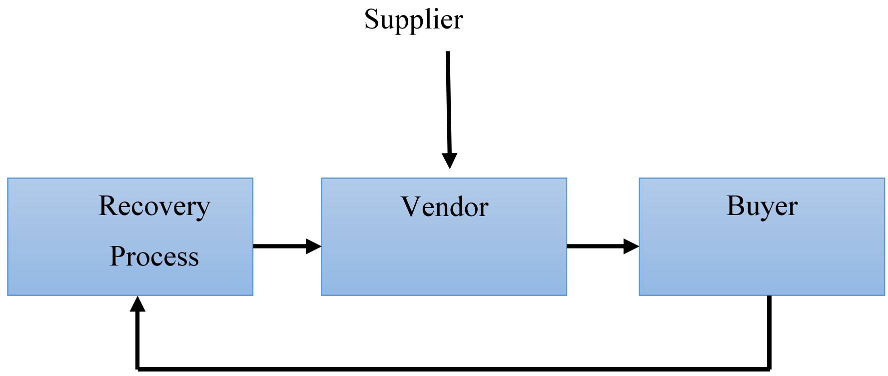

Figure 2: Closed Loop Supply Chain Framework

1.6 Contributions and Significance

The significance of this thesis is to provide mathematical models for remanufacturing systems integrated with CS initiatives. Therefore, the central contribution of the proposed research is the development of a mathematical model to minimize the chain-wide total cost by jointly optimizing the length of the production cycle, the number and the sequence of the newly manufactured and remanufactured batches, as well as the inventory levels of the finished and recovered products at the beginning of the cycle. The thesis also shows how certain parameters effect the overall supply chain cost as well as cycle time and lot sizes of the new and returned products.

1.7 Research Methodology

The following steps will be undertaken to achieve the research objectives:

Step 1: Review the literature related to green supply chain as well as CS partnerships.

Step 2: Develop and solve a mathematical model for CS agreement when implemented in a closed loop supply chain with one vendor and one buyer.

Step 3: Conduct computational experiments to determine the most critical parameters that affect the performance of the reverse supply chain operating under CS partnership

Chapter 2: Literature Review

In this chapter, the relevant literature related to the problem addressed in this research is reviewed. The chapter is divided into two parts, literature review of the joint economic lot size (JELS) problem and CS partnership, followed by review of green supply chain models. The chapter begins with a brief introduction to JELS models showing the benefits of considering the integrated vendor-buyer system. Operational benefits of CS are then presented respectively, for both single and multiple buyers. The chapter then summarizes relevant research papers which developed optimization lot sizing problems for forward and reverse supply chain models in a single echelon system followed by models in a multi-echelon system. Emphasis is given to the production sequence of new and returned items since the sequencing of batches is one of the key contributions in this research work.

2.1 Joint Economic Lot-Sizing

There has been extensive research work conducted to show benefits of integrated vendor-buyer models. We begin the literature review with the joint economic lot sizing (JELS) model, which is the foundation of integrated supply chain models. Banarjee [26] stated that the joint vendor-buyer policy adopted through a spirit of cooperation can be of economic benefit to both parties. In JELS models, the vendor manufactures multiple product batches which are shipped to single/multiple buyers. The objective is to determine the optimum number of shipments and batch sizes to reduce overall costs of the integrated supply chain.

One of the first JELS models was developed by Goyal [27], who assumed an infinite production rate, deterministic and constant demand, for a two-stage supply chain system. He showed that production of unequal batch sizes reduces total cycle time and inventory costs, thus minimizing total costs. Monahan [28] and Banarjee [26] later developed similar models assuming a finite production rate. Monahan [28] determined an optimal quantity discount pricing schedule to be used as incentive to encourage buyers to increase his/her order size. As a result of such cooperation, the vendor saves in processing orders, transportation costs as well as reduced set up costs, since there are less frequent but larger orders. At the same time, the buyer receives a discounted price for increased order sizes. Thus, both the vendor and the buyer receive a mutual benefit through this agreement.

Goyal [29] and Lu [30] proposed models where lot sizes are integer multiples of the order sizes. In all these models, it was assumed that shipments can only take place after batches are completed with equal shipment sizes. Lu [30] relaxed this assumption and found optimal and heuristic solutions when batches can be shipped before being completed. He considered single vendor and buyer as well as single vendor and multiple buyer scenarios. However, his solutions are likely to be used when the vendor has more power over the buyer. Later, Goyal [31] built on Lu's idea and showed that shipments with an increasing fixed factor within a production batch leads to a lower cost solution.

Unlike previous researchers, Hill [32] optimized the number of deliveries by considering important parameters like costs of delivery and production set-up costs, in addition to the inventory holding costs of vendor and buyer, for a single vendor and buyer scenario. He showed that the results are sensitive to the holding cost parameters of the buyer and vendor. If the vendor and buyer have similar holding costs, then Goyal's increasing successive shipment size model is better than Lu's equal shipment size model. Other JELS models have also been suggested. It was Hill [32] who developed the optimal solution to the vendor-buyer problem under general problem settings. Several contributions to the JSLS problem have also been made since then.

Ben-Daya *et al.* [11] developed a more general formulation of the JELS model. They evaluated existing models (i.e., lot-to-lot shipment, (non) delayed equal-sized shipments, the geometric shipment policy, and the geometric-then-equal-shipment policies), and showed that the latter outperformed the remaining shipment policies. Similarly, Glock [33] also presented a thorough review of the JELS models. He proposed extensions to the problems in several directions including multi-stage models considering multiple suppliers. He also emphasized the selection of suppliers, dynamic models, and conducted case studies to view the impact of integrated vendor-buyer models on the costs at companies.

Although there are many advantages of the JELS models, some researchers have stated that it may be difficult to carry out in practice as they require good coordination between the vendor and buyer.

2.2 Consignment Stock Partnership

Due to its many potential benefits, several researchers addressed the CS agreement in the context of a two-echelon supply chain. In particular, they extended

JELS models where a CS agreement takes place, in a single-vendor single-buyer situation, as well as in a single-vendor multi-buyer setting.

According to Valentini and Zavanella [34], CS is increasingly used in small and large industries in Italy. They analyzed the importance and shortcomings of a manufacturing company that undergoes CS partnership. They provided several reasons why industries benefit from CS partnerships. Buyers pay for items only upon use and have fewer stock-out issues. Vendors also have an advantage because they save on inventory carrying costs and have smoother production. However, it is necessary to share information between the vendor and buyers to maintain CS partnerships successfully. Their numerical results showed that CS is better than classical independent inventory models. However, their analysis only takes into account buyers' opportunity and storage cost but not the ordering cost.

Braglia and Zavanella [35] stated that integrated vendor-buyer models without CS partnership are optimum when demand and lead times are constant and deterministic, since inventory is held at the vendor's storehouse for a longer time, until it is required by buyer. This is better because inventories are more valuable downstream, leading to higher inventory holding costs at the buyer's warehouse. However, they showed that CS will be advantageous in environments where demand and lead times are varying. In this scenario, the vendor will ship as soon as they manufacture a lot size to the buyer. Inventory is stored at the buyer's warehouse, so that the buyer can have the stock available close to the production line. At the same time, the vendor guarantees a minimum stock level (s) and a maximum stock level (S) for the buyer. Their model will be the basis for the model developed in this thesis, and will be discussed again in more detail in the following chapter.

Gumus *et al.* [12] determined the operational benefits of CS under deterministic demand for a single item and integrated vendor-buyer system. They included buyers' ordering and carrying costs, and the vendor's setup, releasing, and holding costs. They showed that buyers always benefit under CS because they have definite advantage of paying for goods only after use. In addition, vendors can also achieve savings through a CS agreement, even though they bear the opportunity cost of the items. This is possible when the vendor becomes efficient and reduces the opportunity cost of its capital. In the case when only the buyer and overall system have savings, the buyer can give incentives to the vendor to undergo a CS partnership through a wholesale price increment.

Persona *et al*. [36] developed models for a single vendor who has a CS agreement with multiple buyers. They analyzed different transportation policies. Their results showed that overall cost savings depend on demand, cost parameters, and production schedules. They also considered the case when items are deteriorating. In this case, overall cost savings are lower with higher production variability.

Piplani and Visawanathan [37] referred to the CS policy as the supplier owned inventory (SOI) and they were the first to address the problem in the context of a single-vendor multi-buyer supply chain, where the vendor owns the inventory for only one particular buyer, making it similar to the single buyer case. They showed that the total supply chain cost is always lower under CS policy, and cost savings increase when the ratio of that specific buyer's demand to the vendor's total demand increases. The supplier also benefits under this partnership but it is sensitive to the holding and ordering costs. The CS partnership for a single-vendor multi-buyer supply chain was also addressed by Zavanella and Zanoni [38] where they handled a special case in which a buyer receives one shipment per cycle or consecutive shipments in case of more than one shipment per cycle. In addition, the sequence of deliveries to the buyers was assumed to be known in advance. As noted by the same authors, Zavanella and Zanoni [39], in a follow up paper, the sequencing of the shipments from the vendor to the various buyers is a complicated problem.

More recently, Ben-Daya et al. [40] addressed another special case where the buyers receive equal shipments and only one shipment in each cycle. Hariga *et al*. [41] tackled the single-vendor multi-buyer problem where they determined the optimum number of shipments, shipment lot sizes, and delivery sequence for a single vendor who undergoes a consignment agreement with multiple buyers. They assumed demand is deterministic, and manufacturing takes place for a single item at a finite production rate. They first solved the optimum scheduling and lot sizing problem using nonlinear mixed integer programming (MINP). They then proposed a heuristic solution that allows for the attainment of near optimal solutions. Through numerical experiments, they showed that the proposed heuristic solution results in substantial savings that increase with an increasing number of buyers. A recent review and classification of the literature pertaining to the CS partnerships is given by Sarker [42], where he reviewed different

CS models with a single vendor and single buyer as well as with a single vendor and multiple buyers.

2.3 Green Supply Chain

As environmental concerns are forcing companies to look for green solutions, another subject that is becoming a prominent topic of research is the green supply chain. On the other hand, some companies are transforming to become greener as they find it economically beneficial. Therefore, more research is now being conducted through analyzing green or closed loop supply chain systems with both single- and multi-echelon systems.

2.3.1 Closed loop supply chain in a single-echelon system.

In a forward supply chain, the economic order quantity (EOQ) model balances the setup and replenishment costs with the holding cost. Optimizing the forward and reverse supply chain simultaneously is more complex because there is additional tradeoff made for selecting between the supply modes: procured items and economically valuable returned items. In addition, a decision has to be made whether to dispose the returned items or store them in a serviceable or recoverable inventory, where recoverable items are returned items that are stored before being processed. Once they are remanufactured, these returned items are stored in a serviceable inventory with the new items.

Schrady [43] was the first to develop an EOQ model for a repair inventory system. He proposed a solution where one manufacturing batch is succeeded by R remanufacturing batches, denoted as the (R, 1) policy. However, he assumed that none of the returned materials are disposed. Later, Tuenter [44] relaxed this assumption and showed that the optimal policy is to either produce one lot of new or returned items or only one of both new and returned items are produced ($M = 1$, $R = 1$, or both M, $R = 1$). He further showed when $R = 1$, the optimal policy is to either recover all returned items or to dispose of them.

Similarly, Schulz [45] discussed the (R, 1) policy of Schrady and the (1, M) policy of Tuenter [44]. He developed a better approach for determining the total cost, and determined the optimal R for the (R, 1) policy and the optimal M for the (1, M) policy. He then developed a third policy by considering different remanufacturing batch sizes. In his paper, Schulz also evaluated the three policies by developing an optimization model. He generated a benchmark solution which required the input of the number of remanufacturing and manufacturing batches into a mixed-integer non-linear problem.

Minner and Lindner [22] determined the optimum batch sizes and number of recovered and newly procured items simultaneously, as well as the sequence of batch sizes. They assumed constant and deterministic demand and return rate, no backorders, infinite manufacturing and remanufacturing rate, and return rate to be less than demand. They first considered the case where a single lot size of a new item is manufactured first followed by identical batches of remanufactured lots (1, *R*). They then studied the reverse scenario where a single lot size of returned products is remanufactured first, followed by manufacturing of procured items with identical batch sizes (1, *M*). They found that returned products are remanufactured before new products if the reuse rate approaches 1. On the other hand, for small recovery returns, new products are manufactured before returned products. The authors then relaxed the assumption of having identical batches, and showed that the final remanufacturing batch size should be smaller than previous batch sizes. They also developed a model by considering finite remanufacturing rate and determined new optimum lot sizes of remanufactured and procured items.

Koh *et al*. [46] determined the optimum inventory level of recoverable items and procured items simultaneously, based on joint EOQ and economic production quantity (EPQ) models. They assumed a fixed return rate of items, and that parameters such as demand, recovery rate, repair capacity, and lead times were deterministic. In addition, repairing is assumed to be more cost effective than procurement. Unlike other researchers, their model takes into account the time it takes to transform products from recoverable to serviceable, and a finite repair capacity. However, their model has the following limitation. They assumed that only one order for procured item can be made with multiple setups for recovery, or only one setup for recovery can occur with multiple orders for procured items. In reality, multiple orders and setups for newly manufactured and remanufactured items can occur simultaneously, in which case the sequencing of the production batches becomes a major concern.

Choi *et al.* [47] developed a model to determine the total cost, inventory levels, number of procurement orders, number of setups, and optimum order quantities for new and returned items. They assumed a deterministic recovery rate and demand, and a fixed production and recovery rate. In addition, they assumed equal lot sizes of new and recovered items, and that demand rate is greater than recovery rate but less than production rate. Unlike other authors, they also developed a sequencing algorithm to determine the optimum sequence of orders. In their experiments, multiple lot sizes of new and returned items (R >1 and M > 1) gave a better solution in only 0.2% of the solved cases.

Other researchers, Feng and Vishwanathan [48], attempted to determine a better solution than Tuenter's [44] policies. They studied the general policy where new products are manufactured followed by remanufacturing of returned products, denoted by (P, R) policy, under deterministic and constant demand. Their deterministic model was unique as they developed two different heuristics based on interleaving the setup sizes in order to reduce the batch size of returned products. The first heuristic consisted of equal lot sizes of new and returned products, while their second heuristic model considered varying lot sizes of new and retuned products. However, the second heuristic gave a better solution only marginally.

2.3.2 Closed loop supply chain in a multi-echelon system.

For multi-echelon systems involving two or more stages of the supply chain, optimizing procurement policies are more complex due to the existing tradeoffs between set up and inventory costs for the supply chain members as well as the need to meet customer demand. Although much research has been done on multi-echelon systems in a forward supply chain, very little research is has been done with reverse logistics.

Mitra [49] developed a model for a multi-echelon system with forward and reverse supply chain, where orders are placed from distributors to the depot. He determined the optimum ordering quantity and number of cycles at the distributor level, by considering deterministic demand, setup and inventory costs at each level, and shortage costs for the serviceable inventory levels.

Chung *et al*. [50] studied a multi-echelon system where products are recovered in a reconditioning facility and then returned to retailer. They analyzed the effect of an integrated system consisting of a single supplier, a manufacturer, a retailer, and a third party provider. They showed that integrated policy leads to greater combined profits for the supplier, manufacturer, retailer, and distribution than a decentralized system.

Saadany and Jaber [51] developed optimization models for the production of new and returned items while considering waste disposal cost. Unlike other researchers, they assumed that the return rate of the item depends on its price and quality level. Under this system, they developed two optimization models. The first one considered one single manufacturing batch followed by one remanufacturing batch, while their second model relaxed this assumption and assumed multiple production and remanufacturing batches. Their findings showed that the model is not optimal when the numbers of manufacturing and remanufacturing batch sizes are even numbers. They also showed that a mixed strategy (i.e., a lot sizing system that includes producing new and return items) leads to a better solution than a policy with either production of new products only or production of return items only. A more recent and a comprehensive review of 382 research papers on reverse logistics and the closed supply chain is provided by Govindan *et al*. [52].

Recently, Jaber *et al.* [25] developed a closed-loop supply chain model under a CS agreement. They considered a CS integrated system where the vendor produces new items as well as returned items after undergoing the collection, sorting, inspection, and waste disposal process, and can ship either equal or different lot sizes to the buyer. They showed that it is more beneficial to satisfy demand with returned items when the remanufacturing cost is lower than the cost of manufacturing new items. Their results also showed that the total cost is affected by the collection and return rate. Although they developed mathematical models for the cases of equal and non-equal batch sizes, the sequencing of the batches is assumed to be predetermined where m remanufacturing batches are produced first followed by n manufactured batches. The work presented in the thesis also addresses the reverse supply chain system under consignment stock partnership. However, we are the first to jointly optimize the production sequence of multiple manufacturing and remanufacturing batches, the numbers and sizes of both batches, as well as the delivery scheduling decisions to the downstream buyer. Moreover, our model also considers the initial inventory levels of the finished product at the buyer stage and those of the recovered product at the vendor stage as decision variables in the model. In the following chapter, the detailed derivation of the optimization model that jointly optimizes the above-mentioned elements is developed along with an efficient solution algorithm.

Chapter 3: Closed Loop Supply Chain Models under Consignment Stock Partnership

In this chapter, we first review the forward single vendor, single buyer integrated model under a consignment stock (CS) partnership [35] as it is related to this thesis. We then formulate the mathematical model for closed loop supply chain under a consignment stock partnership. We also propose a solution procedure to solve the formulated problem.

3.1 A Supply Chain Model with Consignment Partnership without Returns

Over the past two decades, vendor-buyer integrated partnerships have become prominent as both vendors and buyers benefit from better coordination and sharing of information, thus saving overall supply chain costs. This is the case with consignment stock (CS), where many companies have implemented and benefited through this vendor-buyer integrated system. In a CS partnership, the vendor keeps his stocks under the consignee's premises and will bear the inventory's capital cost. The buyer will pay for the stock only once he uses it. This is especially attractive to the buyers because they will not bear the opportunity cost, and will have the luxury of having stocks available close to manufacturing. However, it can also benefit vendors who would otherwise not be able to sell their products as easily, as they can guarantee a minimum stock level. Moreover, vendors will be able to better plan their production (as they have better visibility of customers' demand through electronic data interchange, [EDI]), and save inventory costs.

Several models have been developed by researchers to show the benefits of the CS strategy. Braglia and Zavanella [35] developed a mathematical model for a single vendor and single buyer under CS strategy and showed the conditions under which it will be beneficial. Similarly, our research analyzes a single stage supply chain under CS policy, but, to the best of our knowledge, we are the second to analyze the model with returned products. Therefore, the problem we are addressing can be considered as an extension to Braglia and Zavanella's model. In this section, we will review their model for the sake of completeness since their work is relevant to our research.

The following are the parameters and equations as presented in their paper:

Notations

$A_{v:}$ vendor's batch set-up cost

$A_{b:}$ buyer's ordering cost

h_v:	vendor's holding cost per item and per time period

h_b:	buyer's holding cost per item and per time period

P:	vendor's production rate

n:	number of shipments per production batch

q:	quantity delivered per shipment, where production batch size $Q = n \cdot q$

C:	average total costs of the system per unit time

Under consignment stock strategy, the vendor will ship each batch as soon as it is manufactured to minimize inventory costs. Therefore, the inventories at the vendor and buyer facilities will vary over time according to the patterns as shown in Figure 3.

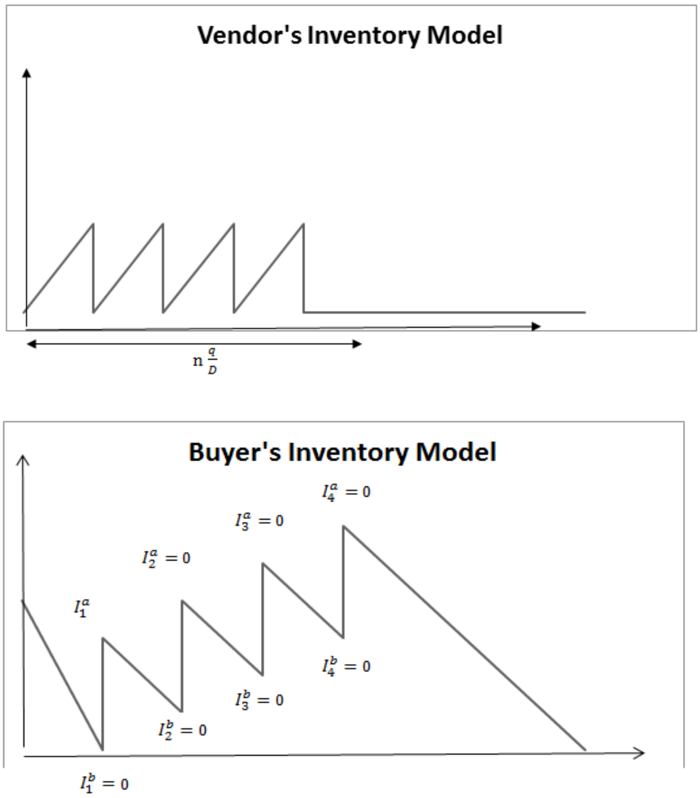

Figure 3: Inventory Levels of Vendor and Buyer

In the following, we will derive the mathematical expressions of the integrated vendor-buyer totals under the consignment stock partnership.

Given that the quantity produced per cycle should be equal to the quantity demanded, the cycle time is $T = nq/D$.

The vendor's set-up cost per unit of time is:

$$C_s^v = A_v/T = \frac{A_v D}{n q} \tag{1}$$

The vendor's holding cost per cycle is the sum of the areas of the *n* triangles in Figure 3. Therefore, the vendor's holding cost per unit time is:

$$C_h^v = h_v n \frac{q^2}{2PT} = h_v \cdot \left(\frac{q\,D}{2P}\right) \qquad (2)$$

The buyer's ordering cost per unit of time is:

$$C_o^b = nA_b \frac{D}{nq} = A_b \frac{D}{q} \qquad (3)$$

We next derive the mathematical expression for the buyer's holding cost per unit of time. The derivation of this cost is not presented in Braglia and Zavanella's paper or in any other research papers. Before that, we need to introduce the following notations.

$$\rho = 1 - \frac{D}{P}$$

I_j^b: Buyer's inventory level just before receipt of their j^{th} shipment (see Figure 4)

I_j^a: Buyer's inventory level just after the j^{th}shipment

Just before the receipt of the j^{th} shipment, the inventory level at the buyer facility is equal to the difference between the quantity received and the quantity sold over the (*j*-1) production runs. Therefore, given that the demand over a production interval is Dq/P , we have

$$I_j^b = (j-1)q - (j-1)D\frac{q}{P} = (j-1)q\left(1 - \frac{D}{P}\right) = (j-1)q\rho$$

Next, given that

$$I_j^a = I_j^b + q,$$

we get

$$I_j^a = jq - (j-1)D\frac{q}{p} = jq(1 - \tfrac{D}{P}) + q\,\frac{D}{P} = q\,(j\rho + \tfrac{D}{P})$$

Moreover,

$$I_j^a + I_{j+1}^b = jq\rho + q\,\frac{D}{P} + jq\rho = q\,(2j\rho + \tfrac{D}{P})$$

The total buyer's inventory per cycle is the area of (*n*-1) trapezoids and one triangle as shown in Figure 3.

$$\text{Total area} = \frac{q}{2p}\sum_{j=1}^{n-1}(I_j^a + I_{j+1}^b) + \frac{1}{2}\, I_n^a\, \frac{I_n^a}{d},$$

After substituting the above derived equations for the inventory levels and some algebraic simplifications, we get

$$\text{Total area} = \frac{1}{2}\, I_n^a\, \frac{q}{D} = \frac{1}{2}\, I_n^a\, T$$

As a result, the buyer's holding cost per unit of time is

$$C_b^h = \frac{h_b}{2}\, I_n^a = \frac{h_b}{2}\left[n\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]q \tag{4}$$

The overall system cost, which is the sum of vendor's setup, holding cost, and buyer's ordering and holding costs, is given by:

$$C = (A_v + \mathrm{n}\, A_b)\cdot\frac{\mathrm{D}}{n\cdot q} + \frac{h_v}{2}\frac{\mathrm{qD}}{P} + \frac{h_b}{2}\left[n\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]q$$

The total cost per unit of time can be rewritten as function of the cycle time as follows

$$C(T) = \frac{A_v + \mathrm{n}\, A_b}{T} + \frac{\mathrm{DT}}{2}[\ (h_b + h_b)\frac{D}{nP} + \ h_b\left(1-\frac{D}{P}\right) \tag{5}$$

Therefore, the optimum cycle time *T* is obtained by setting the first derivative of *C*(*T*) with respect to *T* equal to zero and solving the resulting equation for *T*.

$$T^* = \sqrt{2\frac{(A_v + n\, A_b)}{D[\ (h_v + \ h_b)\frac{D}{nP} + h_b\left(1-\frac{D}{P}\right)]}} \tag{6}$$

Substituting T^* into (5) the total optimum cost C (T^*) is then:

$$C(T^*) = \sqrt{2\mathrm{D}\ (A_v + \mathrm{n}\, A_b)\left[(h_v + \ h_b)\frac{D}{n\, P} + h_b\right) 1 - \frac{\mathrm{D}}{\mathrm{P}})]} \tag{7}$$

The optimum shipment frequency is the one minimizing the mathematical expression under the root of Eq. (7). It is the lower $\underline{n}$, or upper $\overline{n}$, rounded integer of

$$n^* = \sqrt{\frac{(h_v + \ h_b)\ DA_v}{A_b\ P\ h_b\ (1-\frac{D}{P})}} \tag{8}$$

with the smallest total cost per unit of time.

To illustrate Braglia and Zavanella's model, consider the following example with $D = 2000$, $P = 4000$, $A_v = 200, A_b = 100, h_v = 3$ and $h_b = 4$. Using equation (8), we have $n^* = 1.87$. Then substituting $n = 1$ and $n = 2$ into (7), we get C (n = 1) = 2569 and C (n = 2) = 2449. Therefore, the optimal number of shipments is 2.

3.2 Closed Loop Supply Chain Model under CS Partnership

Consider a centralized supply chain composed of a single manufacturer and a single buyer operating under a consignment stock partnership. We assume that both supply chain parties have generated cost savings under this partnership. Due to the growing environmental concerns, as confirmed by recent rigid environmental legislations, supply chain partners are investigating the economic impact of recovering used products through remanufacturing on the supply chain.

In this section, we extend Braglia and Zavanella's model by assuming that a fraction of the recovered products can be remanufactured through disassembly, cleaning, testing, and machining activities. Therefore, the end consumer demand is satisfied by manufacturing newly purchased materials and remanufacturing returned products. We assume that the quality of remanufactured products is as good as that of newly manufactured products. The other assumptions of the mathematical model developed in this section are as follows:

1- The two-stage supply chain manufactures, remanufactures, and delivers a single finished product to customers.
2- The vendor orders new materials from an external source with an infinite capacity.
3- The demand rate, D, for the finished product is known and constant, and must be met without backordering or lost sales.
4- Used products are returned for remanufacturing at a constant and known rate, r.
5- The demand rate is greater than return rate ($D > r$). Therefore, r units of the demand are met through the remanufacturing of recovered items and (D - r) units are manufactured using newly purchased material.
6- The manufacturing and remanufacturing rates are finite and known, P and R, respectively.
7- All returned products can be remanufactured (i.e. the disposal fraction is zero).
8- The remanufacturing time includes disassembly time of the returned product and the processing time.

9- The manufacturing rate is greater than the remanufacturing rate ($P > R$).

10- Both manufacturing and remanufacturing rates are greater than the demand rate ($P \geq D$ and $R \geq D$).

11- Setup times for the manufacturing and remanufacturing processes are negligible.

12- All cost parameters are known and constant.

13- The same production cycle T is repeated over a long planning horizon.

In order to simplify the notations, we consider the items manufactured using new and returned finished items as different products, as shown in Figure 4, and we number them as products 1 and 2, respectively.

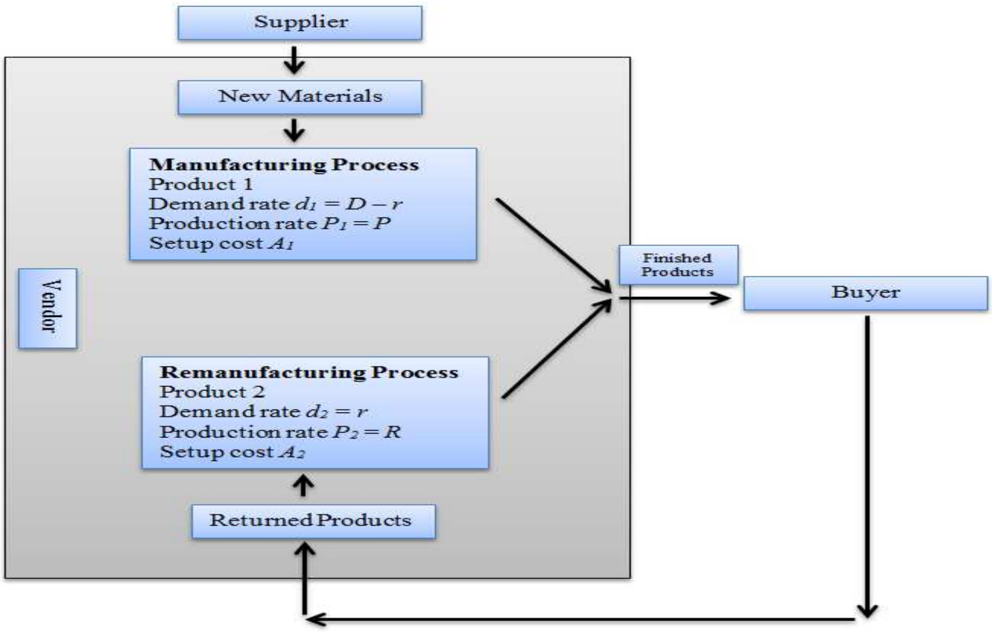

Figure 4. Closed Loop Supply Chain

The following notations are used in the remainder of this chapter. Additional notations will be introduced later when needed.

Parameters:

h_v: vendor's holding cost per unit of time

h_b: buyer's holding cost per unit of time, where $h_b > h_v$

h_r: holding cost per unit of time of the recoverable product (not yet remanufactured), where $h_v > h_r$

d_1: demand rate for product type 1, where $d_1 = D - r$

d_2: demand rate for product type 2, where $d_2 = r$

A_1: set up cost for product type 1

A_2: set up cost for product type 2

A_b: buyer's ordering cost

Decision Variables:

n_i: number of production batches for product type i, where i = 1, 2

n: total number of production runs (batches) = $n_1 + n_2$

T_{pi}: production time for product type i, where i = 1, 2

Q_i: lot size for product type i, where i = 1, 2

Given that there is no disposal of the returned quantity, the lot size and production time for product i can be determined as a function of the production cycle time and number of production batches for each product as follows.

$$Q_i = \frac{d_i}{n_i}T = q_iT \quad (9)$$

$$T_{pi} = \frac{Q_i}{P_i} = \frac{d_i}{P_in_i}T = t_{pi}T \quad (10)$$

When operating under a CS partnership, the vendor will ship manufactured and remanufactured batches as soon as they are produced in order to minimize its inventory holding cost. However, the vendor will be charged for the inventory capital cost at the buyer's premises until the product is sold. Therefore, the final product is shipped to the buyer while the manufacturer's facility is running and the last delivery is made as soon as the production is stopped (see Figures 4 and 5).

Under a closed loop supply chain with no CS partnership, most authors assumed the setup sequence is given. Koh *et al.* [46] proposed $(n_1 = 1, n_2)$ or $(n_1, n_2 = 1)$ policies, while Tuenter [44], Dobos, and Ritcher [53] assumed that n_2 setups of recovered materials are produced first before the n_1 setups of new material.

To the best of our knowledge, only Choi *et al*. [47] and Feng and Vishwanathan [48] determined the sequence of product types. However, both papers considered a single echelon system, consisting of the vendor, with no CS partnership. Moreover, Choi *et al*. [47] assumed an infinite manufacturing rate for newly purchased products. In this section, we will consider a two-echelon system under a CS partnership and with finite manufacturing and remanufacturing rates.

To illustrate the impact of the sequences, consider two different production sequences as shown in Figures 5 and 6. The sequence in Figure 4 is (1,1,2,2,1,1) while the sequence in Figure 5 is (2,1,1,1,1,2). Assume T = 1, $n_1 = 4$, n_2 = 2, D = 2000, r =

800, $P = 4000$, $R = 2000$, $h_v = 3$, $h_r = 2$, $h_b = 4$, $A_1 = 200$, $A_2 = 250$. Thus, $Q_1 = 300$, $T_{p1} = 0.075$, $Q_2 = 400$, $T_{p2} = 0.2$.

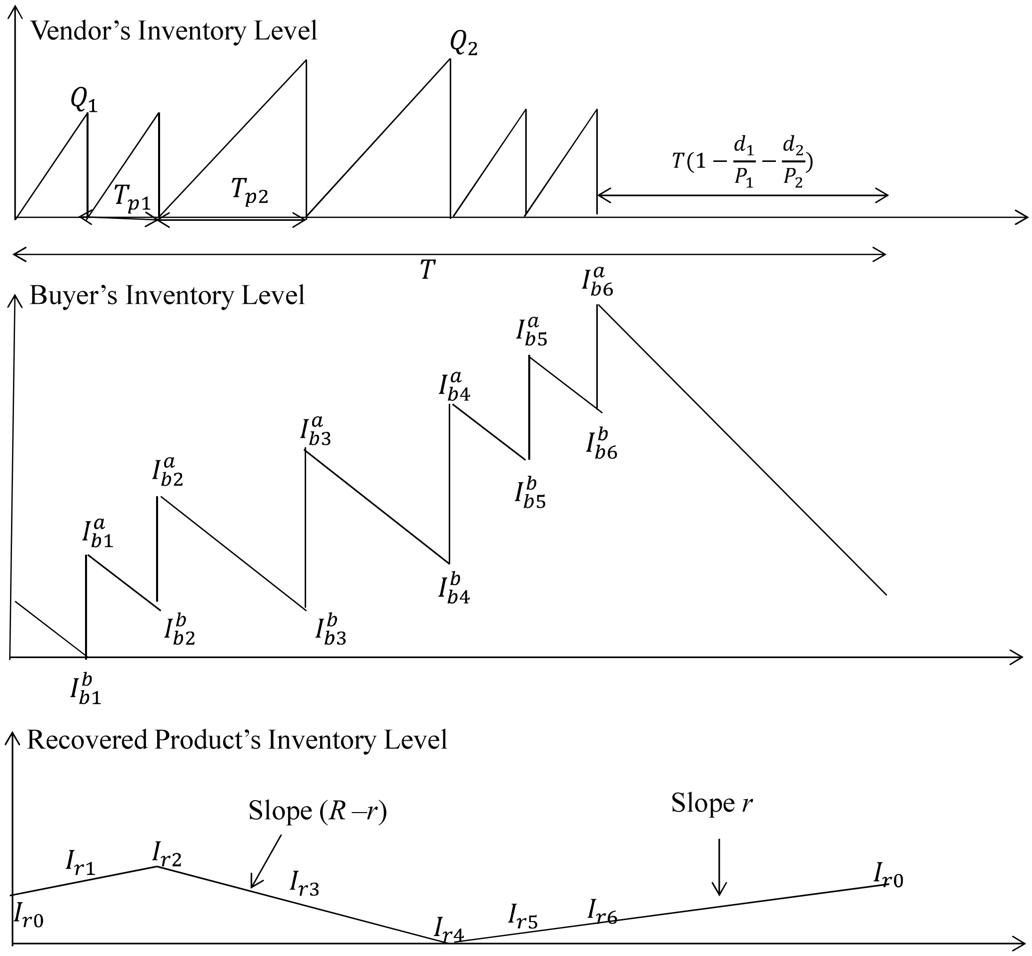

Figure 5: Inventory Levels of Vendor, Buyer, and Recovered Materials for Sequence (1,1,2,2,1,1)

The total inventory costs are computed by calculating the areas under the inventory graphs. The total inventory and setup costs for these two sequences are shown in Table 2 below.

It can be noticed from Table 2 that the total inventory and setup costs for sequence (1,1,2,2,1,1) is 2805, which is less than the costs for sequence (2,1,1,1,1,2). Therefore, it is important to find the best production sequence for new and recovered products that minimizes the integrated vendor and buyer costs in addition to recovered products costs.

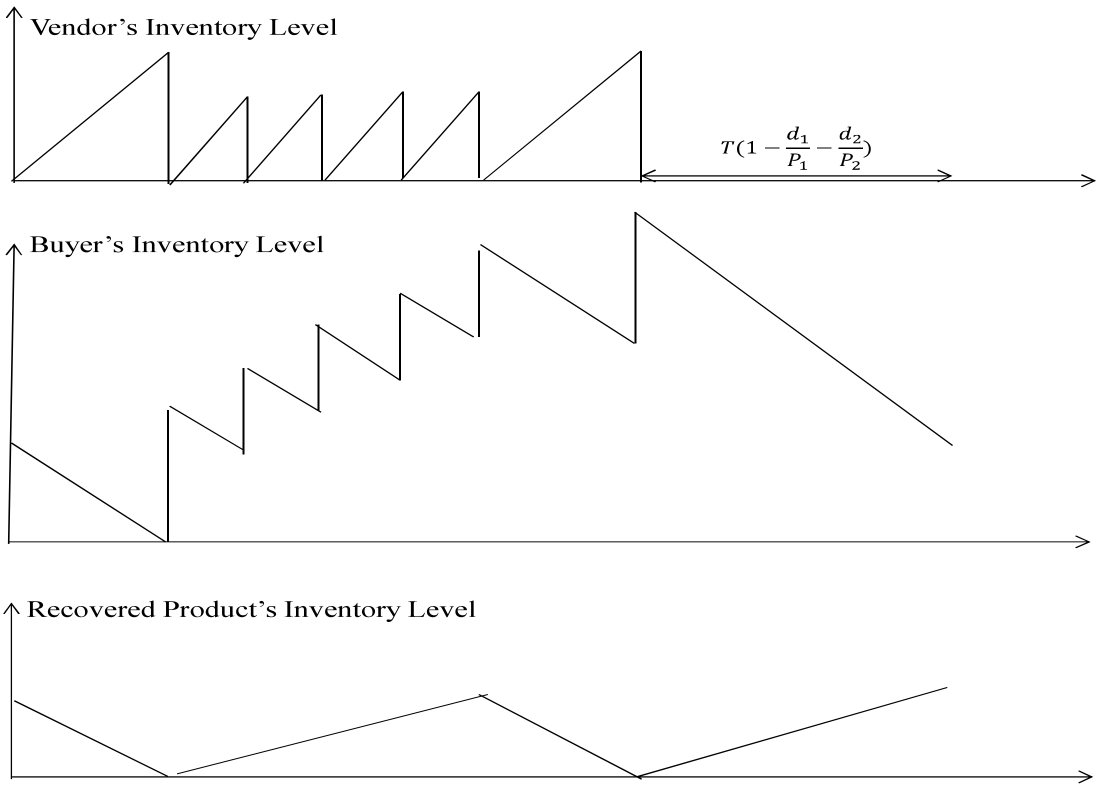

Figure 6: Inventory Levels of Vendor, Buyer, and Recovered Materials for Sequence (2,1,1,1,1,2)

Table 2: Total Inventory and Setup Costs for Two Different Setup Sequences

	Sequence(1,1,2,2,1,1)	**Sequence (2,1,1,1,1,2)**
Vendor's inventory cost	375	375
Recovered material inventory cost	480	240
Buyer's inventory cost	1300	2300
Manufacturing setup cost	2 (200) = 400	200
Remanufacturing setup cost	250	2 (250) = 500
Total inventory and setup costs	2805	3615

As can be observed from both Figures 6 and 7, the total production time over one cycle is given by:

$n_1 T_{p1} + n_2 T_{p2}$,

which becomes after substituting (10):

$(\frac{d_1}{P_1} + \frac{d_2}{P_2})T$

Therefore, the idle time per production cycle is:

$$T - \text{ production time per cycle } = T\left(1 - \frac{d_1}{P_1} - \frac{d_2}{P_2}\right) = T\rho \tag{11}$$

The objective of our model is to determine the optimal size of the manufacturing and remanufacturing batches along with the production sequence that minimize the total long-run average system-wide costs consisting of the setup costs for products 1 and 2 and the inventory holding costs for the buyer, vendor, and recovered products, and the buyer's ordering cost.

In the following we assume that the number of batches for the manufacturing and remanufacturing processes is given. The problem decision variables are then the production sequence and cycle time, *T*, as well as the initial inventories for the buyer's finished product, I_{b1}^{b}, and vendor's recovered products, I_{r0}. The production sequence is determined by the following binary variables:

These binary variables should meet the following conditions:

$$x_{ij} = \begin{cases} 1 \text{ if product } i \text{ is produced at the } j^{\text{th}} \text{ production run}, i = 1,2, \text{and } j = 1,2,\ldots,n \\ 0 \text{ otherwise} \end{cases}$$

These binary variables should meet the following conditions:

$$\sum_{j=1}^{n} x_{ij} = n_i \qquad i = 1, 2 \tag{12}$$

$$\sum_{i=1}^{2} x_{ij} = 1 \qquad j = 1, 2, \ldots . n \tag{13}$$

Constraints (12) state that the number of setups of product *i* should be equal to n_i, while constraints (13) ensure that at every setup, only one type of product is produced. In the next sub-sections, we derive the equations for inventory costs for the

vendor, the buyer, and the recovered products.

3.2.1 Derivation of the vendor's holding and setup costs.

In order to determine the holding cost per unit time for the vendor, we need to compute the total area under the inventory graph, which is independent of the production sequence. Referring to Figure 2, the vendor's inventory holding cost per unit of time is given by:

$$H_v = \frac{h_v}{T}\left[n_1 T_{p1}\frac{Q_1}{2} + n_2 T_{p2}\frac{Q_2}{2}\right],$$

which, after substituting (9) and (10), gives:

$$H_v = T\frac{h_v}{2}\sum_{i=1}^{2}\frac{{d_i}^2}{P_i n_i} = T.RH_v \tag{14}$$

where RH_v is the relative vendor's inventory cost per unit of time.

Next, to determine the cost of setting up the vendor's production facility for the manufacturing and remanufacturing processes, we need to introduce the following binary variables:

$$y_{ij} = \begin{cases} 1 \text{ if a setup for product type } i \text{ is made at the } j^{\text{th}} \text{ production run,} \\ 0 \text{ otherwise} \end{cases}$$

Note that a setup takes place only for the first batch and when two different product types are produced during two consecutive production runs. Therefore, the binary variables y_{ij} should satisfy the following conditions:

$$y_{i1} \geq x_{i1} \qquad i = 1, 2 \tag{15}$$

$$y_{ij} \geq x_{ij} - x_{i(j-1)} \quad i = 1,2 \text{ and } j = 2, 3, \dots, n \tag{16}$$

The vendor's total set up cost per production cycle is thus given by:

$$A_v = \textstyle\sum_{i=1}^{2} A_i \sum_{j=1}^{n} y_{ij} \tag{17}$$

3.2.2 Derivation of the buyer's holding cost.

The buyer's inventory holding cost per cycle is proportional to the area under the buyer's inventory graph in Figure 2. The variables in this figure are defined as follows:

I_{bj}^{b} = buyer's inventory level just before receiving the j^{th} shipment

I_{bj}^{a} = buyer's inventory level just after receiving the j^{th} shipment

Given that the length of the j^{th} production run and the size of the j^{th} shipment size depend on the product type being processed during this run, we define:

T_j = length of the j^{th} production run, which is given by

$$T_j = \sum_{i=1}^{2} T_{pi} x_{ij} = T\sum_{i=1}^{2} t_{pi} x_{ij} = T.RT_j \tag{18}$$

where RT_j is the relative length of the j^{th} production run.

We also define the j^{th} shipment size to the buyer by S_j, which is given by:

$$S_j = \sum_{i=1}^{2} Q_i x_{ij} = T \sum_{i=1}^{2} q_i x_{ij} \tag{19}$$

Referring again to Figure 2, we have:

$I_{bj}^{a} = I_{bj}^{b} + S_j \qquad \text{for } j = 1, 2, 3, \dots, n$

and

$I_{bj}^{b} = I_{b\,(j-1)}^{a} - DT_j \qquad \text{for } j = 2, 3, \dots, n$

Using the last two equations, it can be shown for *j* = 2, 3, …, *n* that

$$I_{bj}^{b} = I_{b1}^{b} + \sum_{k=1}^{j-1} S_k - D \sum_{k=2}^{j} T_k = T \left[RI_{b1}^{b} + \sum_{k=1}^{j-1} \sum_{i=1}^{2} q_i\, x_{ik} - D \sum_{k=2}^{j} \sum_{i=1}^{2} t_{pi}\, x_{ik} \right]$$

$$= T.RI_{bj}^{b} \tag{20}$$

and

$$I_{bj}^{a} = I_{b1}^{b} + \sum_{k=1}^{j} S_k - D \sum_{k=2}^{j} T_k = T \left[RI_{b1}^{b} + \sum_{k=1}^{j} \sum_{i=1}^{2} q_i\, x_{ik} - D \sum_{k=2}^{j} \sum_{i=1}^{2} t_{pi}\, x_{ik} \right]$$

$$= T.RI_{bj}^{a} \tag{21}$$

where

RI_{bj}^{b} is the relative buyer's inventory level just before the receipt of the j^{th} shipment, and

RI_{bj}^{a} is the relative buyer's inventory level just after the receipt of the j^{th} shipment.

Note that

$$RI_{b1}^{a} = RI_{b1}^{b} + \sum_{i=1}^{2} q_i\, x_{i1}$$

The area under the buyer's inventory graph is the sum of *n* trapezoids. Therefore, the buyer's inventory holding cost per unit of time is as follows:

$$H_b = T \frac{h_b}{2} \left[\sum_{j=2}^{n} (RI_{b(j-1)}^{a} + RI_{bj}^{b})\, RT_j + (RI_{bn}^{a} + RI_{b1}^{b})\, (RT_1 + \rho) \right] = T.RH_b \tag{22}$$

where RH_b is the relative buyer's inventory holding cost per unit of time, and $\rho = \left(1 - \frac{d_1}{P_1} - \frac{d_2}{P_2}\right)$

3.2.3 Derivation of the returned product's holding cost.

It can be observed from the inventory profile of the returned products in Figure 2 that the inventory of returned products increases at a rate of *r* units per unit of time when product type 1 is manufactured during the j^{th} production run. On the other hand, if product type 2 is being manufactured during the j^{th} production run, the inventory of returned products decreases at a rate of $(R - r)$. We now let:

$I_{rj} =$ inventory level of returned products just after the j^{th} shipment to the buyer

$I_{r0} =$ initial stock of returned products at the beginning of the production cycle

From Figure 6, we have

$I_{rj} = I_{r(j-1)} - (R-r)T_{p2}x_{2j} + r\,T_{p1}x_{1j}$ for $j = 1,2,..,n$

which can be rewritten as:

$$I_{rj} = T\left[RI_{ro} - (R-r)t_{p2}\sum_{k=1}^{j} x_{2k} + rt_{p1}\sum_{k=1}^{j} x_{1k}\right] = T.RI_{rj} \quad \text{for} \quad j = 1,2,\dots,n \qquad (23)$$

where RI_{r0} is the relative initial inventory for the returned products and RI_{rj}, for $j = 1,2,\dots,n$, is the relative inventory level of the returned product at the j^{th} shipment.

The area under the inventory graph for recovered products is the sum of $(n + 1)$ trapezoids. Therefore, the inventory holding cost per unit time for the returned products is given by:

$$H_r = T\frac{h_r}{2}\left[\sum_{j=1}^{n}(RI_{r(j-1)} + RI_{rj})RT_j + (RI_{rn} + RI_{r0})\rho\right] = T.RH_r \qquad (24)$$

where RH_r is the relative returned product's inventory holding cost per unit of time.

3.3 Optimization Model

As mentioned earlier, the total system cost includes the manufacturing and remanufacturing setup costs, the buyer's ordering cost, the vendor's inventory holding cost, the buyer's inventory cost, and the inventory holding cost for returned products. Using the above derived equations for these cost components, the total system cost per unit of time, K, is expressed mathematically as:

$$K\,(n_1,n_2) = \frac{A_v + nA_b}{T} + T\,(RH_v + RH_b + RH_r) \qquad (25)$$

Finally, the optimization model of the two stage closed loop supply chain with CS partnership (CLSC-CS) for given production frequencies (n_1,n_2) can be stated as follows:

CLSC-CS-1:

Min $K(n_1,n_2) = \dfrac{A_v + nA_b}{T} + T\,(RH_v + RH_b + RH_r)$

s.t.

$$\sum_{j=1}^{n} x_{ij} = n_i \qquad i = 1, 2$$

$$\sum_{i=1}^{2} x_{ij} = 1 \qquad j = 1, 2, \ldots, n$$

$$y_{i1} \geq x_{i1} \qquad i = 1, 2$$

$$y_{ij} \geq x_{ij} - x_{i(j-1)} \qquad i = 1, 2 \text{ and } j = 2, 3, \ldots, n$$

$$A_v = \sum_{i=1}^{2} A_i \sum_{j=1}^{n} y_{ij}$$

$$RT_j = \sum_{i=1}^{2} t_{pi} x_{ij} \qquad j = 1, 2, \ldots, n \tag{26}$$

$$RI_{bj}^{b} = RI_{b1}^{b} + \sum_{k=1}^{j-1} \sum_{i=1}^{2} q_i\, x_{ik} - D \sum_{k=2}^{j} \sum_{i=1}^{2} t_{pi}\, x_{ik} \qquad j = 2, 3, \ldots, n \tag{27}$$

$$RI_{b1}^{a} = \sum_{i=1}^{2} q_i x_{i1} + RI_{b1}^{b} \tag{28}$$

$$RI_{bj}^{a} = RI_{b1}^{b} + \sum_{k=1}^{j} \sum_{i=1}^{2} q_i\, x_{ik} - D \sum_{k=2}^{j} \sum_{i=1}^{2} t_{pi}\, x_{ik} \qquad j = 2, 3, \ldots, n \tag{29}$$

$$RI_{rj} = RI_{ro} - (R - r) t_{p2} \sum_{k=1}^{\mathrm{j}} x_{2k} + r t_{p1} \sum_{k=1}^{\mathrm{j}} x_{1k} \qquad j = 1, 2, \ldots, n \tag{30}$$

$$RH_v = \frac{h_v}{2} \sum_{i=1}^{2} \frac{{d_i}^2}{P_i n_i} \tag{31}$$

$$RH_b = \frac{h_b}{2} \left[\sum_{j=2}^{n} (RI_{b(j-1)}^{a} + RI_{bj}^{b})\, RT_j + (RI_{bn}^{a} + RI_{b1}^{b})\, (RT_1 + \rho) \right] \tag{32}$$

$$RH_r = \frac{h_r}{2} \left[\sum_{j=1}^{n} (RI_{r(j-1)} + RI_{rj})\, RT_j + (RI_{rn} + RI_{r0})\, \rho \right] \tag{33}$$

$$RI_{r(j-1)} \geq t_{p2}(R - r) x_{2j} \qquad j = 1,2,..,n \tag{34}$$

$$RI_{bj}^{b} \geq 0 \qquad j = 1, 2, \ldots, n \tag{35}$$

$$RI_{rn} \geq 0 \tag{36}$$

x_{ij} , $y_{ij} \epsilon \{0,1\}$ $\quad i = 1, 2$ and $j = 1, \ldots, n$ (37)

In case the returned products are remanufactured during the j^{th} production run, the set constraints (34) ensure that the initial stock of such products is enough to avoid shortages. The set constraints (35) guarantee that shortages are not allowed in the buyer's facility. Finally, constraints (34) and (36) together enforce the non-shortages restriction for the returned products.

The above optimization problem is a mixed-integer non-linear program with 4n binary variables, 4*n*+5 continuous variables, and (9*n*+7) constraints. It is also easy to see that (CLSC-CS-1) is separable in terms of the production cycle time, *T*, and the remaining decision variables. Moreover, for a given production sequence and relative initial stock at the buyer site and returned products, all the continuous variables, excluding the production cycle time can be determined using equations (26) to (33). Therefore, the optimal production cycle time, for a given production sequence T^*, minimizing *K* (n_1, n_2) is given by:

$$T^* \left(RI_{ro}, RI^b_{b1}, x_{ij} \colon i = 1{,}2 \text{ and } j = 1{,}2, \ldots, n\right) = \sqrt{\frac{A_v + nA_b}{RH_v + RH_b + RH_r}} \quad (38)$$

The optimal values for the remaining variables can be determined by solving the following optimization problem:

<u>CLSC-CS-2:</u>

$Min\ \ (A_v + nA_b)(RH_v + RH_b + RH_r)$

s.t.

All constraints of (CLSC-CS-1) problem

Despite the fact that problem CLSC-CS2 involves one less continuous variable (*T*), it still falls under the class of MINLP models where the nonlinearity is seen in the objective function as well as in constraints (32) and (33). Moreover, due to the difficulty associated with establishing the convexity of the non-differentiable objective function, it would be mathematically intractable to attain optimal solutions, especially for larger problem instances (i.e., increased numbers of batches *n*). An efficient iterative solution procedure that searches over several values of *n* is developed next

3.4 Solution Procedure

In the previous sub-sections, it was assumed that production frequencies, n_1 and n_2, are given. However, in our model, we seek to determine the optimum number of batches for both manufacturing and remanufacturing lots. Accordingly, we propose a solution procedure to solve the closed loop supply chain optimization problem under consignment partnership. First, it should be noted that for a given total number of production runs, n, say n = 3, there are 2 possible production runs (n_1 =1, n_2 = 2) and (n_1 = 2, n_2 =1,). Similarly, for n = 4 production runs, the possible batches are (n_1 =1, n_2 = 3), (n_1 =2, n_2 = 2), or (n_1 =3, n_2 = 1). The examples are further illustrated in the Tables 3 to 9. Thus, we can infer that there are (n – 1) possible number of production runs for the manufacturing and remanufacturing processes during one cycle, which are $\{(j, n-j): j = 1,2, \dots, n-1\}$.

For most of the examples we solved during our extensive numerical experimentation, we observed that $K(n_1, n_2)$ is not convex in n. Therefore, we propose the following solution procedure which is presented in an algorithm form.

Step 0.

Set K^* = + ∞, n = 2, and $v = 0$

Step 1.

$K^*(n)$= + ∞

For j = 1 to $n - 1$ Do:

Let $n_1 = j$ and $n_2 = n - j$

Solve (CLSC-CS-2)

Compute T^* and $K(n_1, n_2)$ using equations (38) and (25), respectively.

If $K(n_1, n_2) < K^*(n)$ then set $n_1^* = n_1$, $n_2^* = n_2$, and $K^*(n) = K(n_1, n_2)$

Next Do

Step 2.

If $K^*(n) < K^*$, then set $K^* = K^*(n)$, $n^* = n$, $n \rightarrow n+1$, and $v = 0$. Go to Step 1.

Else

If $K^*(n) \geq K^*(n-1)$

$v = v + 1$

If $v \leq 5, n \rightarrow n + 1,$ and Go to Step 1

If $v > 5$, then Go to Step 3

Else

$v = 0, n \rightarrow n + 1,$ and Go to Step 1

Step 3. STOP

Note that the stopping criterion for the algorithm is set by the counter variable, v. Throughout all problem instances solved, it is the stopping criterion (v=5) that takes place and terminates the algorithm rather than an upper search limit on the number of runs. Every time the value of n is increased by one and it turns out that the optimal total cost for this specific n value is greater than that for $n-1$, the value of the counter v is consequently increased by one. Due to the non-convexity of the objective function, we have opted to terminate the algorithm after 5 consecutive increases in the value of the total cost.

The above solution procedure is next illustrated with the same example used above when we showed the impact of the production sequence on the total system cost. The input parameters for this example (referred to as the base case) are: D = 2000, r = 800, P = 4000, R = 2000, A_1 = 200, A_2 = 250, A_b= 100, h_r = 2, h_v = 3 and h_b = 4. Upon employing the proposed solution procedure, the detailed model's output generated during the different iterations of varying n values is provided in Tables 3 to 9, where the numbers in boldface indicate the optimal n_1, n_2, T, and $K(n_1, n_2)$ values for each n value.

Table 3: Model's Output as a Function of the Total Number of Production Runs (n=2, 3, 4 & 5)

n	2	3		4			5			
n_1	**1**	1	**2**	1	**2**	3	1	**2**	3	4
n_2	**1**	2	**1**	3	**2**	1	4	**3**	2	1
T^*	**0.379**	0.489	**0.491**	0.502	**0.569**	0.557	0.53	**0.649**	0.726	0.587
$K(n_1, n_2)$	**3249**	3061.3	**3054.5**	3383.3	**2990.3**	3052.2	3583.8	**2928.3**	3166.7	3255
Sequence	**(1,2)**	(2,1)	**(1,2)**	(2,1)	**(2,1)**	(1,2)	(2,1)	**(2,1)**	12211	(1,2)
$K^*(n)$	**3249**	3054.5		2990.3			2928.3			

Table 4: Model's Output as a Function of the Total Number of Production Runs (n = 6 & 7)

n	**6**					**7**					
n_1	1	2	**3**	4	5	1	2	**3**	4	5	6
n_2	5	4	**3**	2	1	6	5	**4**	3	2	1
T^*	0.556	0.680	**0.698**	0.762	0.616	0.582	0.711	**0.778**	0.728	0.8	0.644
$K(n_1, n_2)$	3772.2	3087.7	**3007.7**	3282.5	3407.16	3950.8	3237.1	**2956.3**	3158.2	3373.9	3569.6
Sequence	(2,1)	(2,1)	**(2,1)**	112211	(1,2)	(2,1)	(2,1)	**(2,1)**	(2,1)	1122111	(1,2)
$K^*(n)$	3007.7					2956.3					

Table 5: Model's Output as a Function of the Total Number of Production Runs (n = 8)

n	8						
n_1	1	2	**3**	4	5	6	7
n_2	7	6	**5**	4	3	2	1
T^*	0.607	0.740	**0.809**	0.808	0.758	0.858	0.671
$K(n_1, n_2)$	4121.4	3378.9	**3088.7**	3094.4	3299.5	3380.2	3724.4
Sequence	(2,1)	(2,1)	**(2,1)**	(2,1)	(2,1)	11122111	(1,2)
$K^*(n)$	3088.7						

Table 6: Model's Output as a Function of the Total Number of Production Runs (n = 9)

n	9							
n_1	1	2	3	**4**	5	6	7	8
n_2	8	7	6	**5**	4	3	2	1
T^*	0.630	0.768	0.840	**0.875**	0.838	0.919	0.884	0.697
$K(n_1, n_2)$	4284.9	3514.3	3214.3	**3085.5**	3223.3	3374.5	3508.8	3872.8
Sequence	(2,1)	(2,1)	(2,1)	**(2,1)**	(2,1)	112221111	222211222	(1,2)
$K^*(n)$	3085.5							

Table 7: Model's Output as a Function of the Total Number of Production Runs (n = 10)

n	10								
n_1	1	2	3	**4**	5	6	7	8	9
n_2	9	8	7	**6**	5	4	3	2	1
T^*	0.653	0.796	0.870	**0.916**	0.905	0.990	0.884	0.913	0.722
$K(n_1, n_2)$	4442.2	3644.4	3334.6	**3166.1**	3205.3	3339.8	3508.8	3612.7	4015.5
Sequence	(2,1)	(2,1)	(2,1)	**(2,1)**	(2,1)	1122221111	2211122222	1111221111	(1,2)
$K^*(n)$	3166.1								

Table 8: Model's Output as a Function of the Total Number of Production Runs (n = 11)

n	11									
n_1	1	2	3	4	**5**	6	7	8	9	10
n_2	10	9	8	7	**6**	5	4	3	2	1
T^*	0.675	0.822	0.898	0.946	**0.963**	0.934	0.895	0.990	0.960	0.746
$K(n_1, n_2)$	4594.0	3769.9	3450.2	3277.1	**3218.4**	3320.2	3463.0	3534.0	3646.9	4153.1
Sequence	(2,1)	(2,1)	(2,1)	(2,1)	**(2,1)**	(2,1)	(2,1)	11122211111	111112211111	(1,2)
$K^*(n)$	3218.4									

Table 9: Model's Output as a Function of the Total Number of Production Runs (n = 12)

n	12										
n_1	1	2	3	4	**5**	6	7	8	9	10	11
n_2	11	10	9	8	7	6	5	4	3	2	1
T^*	0.696	0.848	0.926	0.975	**1.01**	0.992	0.962	1.050	1.04	0.982	0.770
$K(n_1, n_2)$	4740.9	3891.1	3561.8	3383.9	**3271.0**	3326.6	3429.7	3523.3	3560.7	3767.4	4286.3
Sequence	(2,1)	(2,1)	(2,1)	(2,1)	**(2,1)**	(2,1)	(2,1)	112222111111	111222111111	(1,2)	(1,2)
$K^*(n)$	3271.0										

This example clearly illustrates the non-convexity of the optimization model CLSP-CS-1 with respect to the total number of batches produced, n. The optimal cost figure alternates between decreasing and increasing values depending on the value of n. We can see this from Table 3 as the optimum cost $K^*(5)$ = 2928.3 which then increases to $K^*(6)$ = 3007.7 and then decreases again to $K^*(7)$ = 2956.3 until it exhibits a steady increasing trend for values of $n \geq 9$. Nevertheless, for a particular value of n, the model is convex in n_1 and accordingly n_2 since $n = n_1 + n_2$. For example, at $n = 5$, we have a convex solution at $n_1 = 2$ and $n_2 = 3$ and at $n = 6$, we have a convex solution at $n_1 = 3$ and $n_2 = 3$. Similarly, we see that for every n in our above example where n = 2 to 12, the solution is convex.

The length of the cycle time (T) is directly related to the number of batches n where T keeps on increasing as the value of n is increased. This is expected as T and n are directly proportional and the cycle time increases for higher number of batches. Concerning the optimal production sequence, it is interesting to note that for smaller values of $n \leq 3$, it is preferable to start with the newly manufactured batches first followed by remanufactured batches, denoted by sequence (1,2). However, as the total number of batches exceeds three, the optimal sequence is to start with the remanufactured batches followed by the new ones, or sequence (2,1). Note however that, in all cases, there exists only one switchover between the two product types indicating that consecutive production of (re) manufactured batches takes place. In the next section, we elaborate upon the significance of the cost parameters on the sequence of the new and returned lots.

Chapter 4: Computational Experiments

In this chapter, we conduct sensitivity analysis as we seek to analyze key factors that affect the model and demonstrate the importance of the behavior of the model as these factors change. Sensitivity analysis addresses the problem of identifying parameters that have a significant impact on the optimal solution. It is carried out by changing certain parameters and studying the effect that this change has when fixing the rest of the problem parameters. Therefore, a parameter is considered sensitive if it causes the optimum solution to change even for small changes in that parameter.

In this thesis, we select the following parameters as we expect them to have a bigger impact on the optimal cost, cycle time, sequence, or lot sizes. The non-cost related parameter, the return rate r, is studied first as it is important to consider the effect of rate of returned material on the model. Other cost-related parameters such as the holding, ordering, and setup costs, are also analyzed. Specifically, the combination of buyers' ordering and holding costs are evaluated as there is a tradeoff between these two factors. Similarly, we investigate the returned product holding cost with the remanufacturing setup cost, as well as the setup costs associated with newly and remanufactured batches.

The analysis was done by extensively running 400 different computational experiments where the values of the input parameters are taken from the base case example in Chapter 3. For convenience, the base values are presented again in Table 10. The results of the computational experiments are obtained using the optimization model developed in Section 3.4. Key results are presented below for each case of the sensitivity analysis and are discussed in the next sections of the chapter. We then show the importance and cost benefit associated with finding the optimum sequence when compared to adopting the predetermined production sequence (R,M), in which R consecutive remanufactured batches are produced followed by M newly manufactured batches, and the other sequences (M,R), (R,1), and (1,M). In case the optimum result is (M,R), we then compare it with (R,M) and vice versa. Those special sequences are obtained from the general model developed in Section 3.2 through fixing the binary variables x_{ij} and y_{ij} to either zero or one depending on the sequence. These fixed values are then used as input to the mathematical model before solving it. The results of the sensitivity analysis will also be discussed in the following sections of this chapter.

Table 10: Standard values of problem parameters

Parameter	Value	Parameter	Value
D	2000	**A_1**	200
r	800	**A_2**	250
P	4000	**h_r**	2
R	2000	**h_v**	3
A_b	100	**h_b**	4

4.1 Return Rate (*r*) Impact Analysis

We conducted the sensitivity analysis by running seven different values of the return rate, while keeping all other input parameters the same, to measure its impact on the solution. A summary of the results obtained for the seven problem instances is presented in Table 11.

Table 11: Effect of Changing Return Rate Values and Average Percentage Deviation

r	n_1	n_2	Sequence	Cycle time (*T*)	Total Cost (*TC*)	% Cost Deviation		
						(M,R)	(R,1)	(1,M)
500	3	2	(2,1)	0.6325	3004.17	7.96	9.42	4.69
600	2	2	(2,1)	0.5682	2991.7386	6.78	5.96	5.96
700	2	2	(2,1)	0.5793	2934.73	6.70	6.63	6.63
800	**2**	**3**	**(2,1)**	**0.6488**	**2928.37**	**4.23**	**8.10**	**10.95**
900	2	3	(2,1)	0.6637	2862.89	8.43	5.62	17.42
1000	2	4	(2,1)	0.7483	2806.243	9.40	2.35	23.57
1100	2	5	(2,1)	0.8252	2787.26	6.18	4.56	28.01
					Avg. Dev. %	**7.10**	**6.09**	**13.89**

From the above table, we can infer that as the percentage of r relative to the demand (D) increases from 0.25 to 0.55, fewer new batches are produced in favor of a larger number of remanufactured batches. For example, at r = 700, n_1 and n_2 = *2.* However, for r = 800, $n_1 = 3$, $n_2 = 2$. We can see from the table that for all values higher than 800, n_2 is more than n_1. This is expected as it is more economical to satisfy demand from remanufactured items with more returned materials entering the plant facility. Next, we deduce that the increasing return rate increases the cycle time. From $r = 600$ onwards, the cycle time gets larger as the total of number of batches $(n_1 + n_2)$ increases. The only case of a drop in the cycle time is when moving from $r = 500$ to

$r = 600$ which is reasonable as the total number of batches is reduced. It can also be inferred from Equation 9 that the lot sizes of new items increases as the cycle time increases. On the other hand, lot sizes of reused items increases when the number of batches remains the same for increasing return rates but decreases when there is an increase in the number of batches. This can be illustrated by considering the increase from $r = 800$ to 900 to 1000. In this case, the lot size Q_1 increases from 259.52 to 298.67 to 374.14 respectively as the number of batches stays constant at 2 while the cycle time increases. The lot size Q_2 increases from 259.2 to 365.04 for r = 800 to 900 in turn, but decreases to 187.08 at $r = 1000$. This occurs as the number of batches increases from 3 to 4 when the rate of return increases from 900 to 100. As seen in Table 9, altering the value of r has no impact on the optimal sequence where it is always preferable to start with the remanufactured batches first since it is practical to first use returned materials and minimize the holding costs. The total cost per unit time drops up to 2.2 percent for increasing r values indicating that it would be economically advantageous to satisfy the demand from the returned products as opposed to the newly purchased ones.

The benefits of optimizing the production sequence can be seen in the last three columns of Table 11. Here, costs are compared to those of the non-optimum sequences (*M,R*), (*R,1*), and (*1,M*) instead of the optimal (*R,M*). The average percentage deviation (APD) for sequence (*R,1*) is 6.09% with a slightly higher APD of 7.10% for sequence (*M,R*) and an even higher APD of 13.89% when sequence (*1,M*) is assumed. For $r =$ 900 and higher, the percentage deviation in considering sequence (*1,M*) increases significantly from 17.42 to 23.57 to 28.01. This is reasonable as there is a higher deviation in the optimum number of batches produced which results in much higher costs.

4.2 Impact of Buyer's Ordering and Holding Costs

We perform a two-way sensitivity analysis to study the existing tradeoffs between the buyer's ordering and holding costs. Different experiments are carried out by fixing the ordering cost while changing the holding cost and vice versa. All other inputs are kept constant at their base values for all the experiments. Five different levels for A_b are chosen through multiplying its standard value by 0.01, 0.10, 5, 10 and 10. For each A_b, we chose from the following five different values for h_b; $h_b = h_v$, 5 h_v, 10 h_v, 50 h_v, 100 h_v. This results in a total of 25 problems with solutions presented in Table 12.

Table 12: Effect of Changing Buyer's Ordering and Holding Costs

A_b	h_b	n_1	n_2	Sequence	T	TC	% Cost deviation			
							(M,R)	(R,M)	(R,1)	(1,M)
1	3	18	24	(2,1)	0.663	1,484.64	25.36	0	52.26	67.49
1	15	18	24	(2,1)	0.376	2,616.99	30.3	0	54.83	71.18
1	30	18	24	(2,1)	0.277	3,557.28	31.39	0	55.38	71.98
1	150	18	24	(2,1)	0.128	7,687.59	32.39	0	55.89	72.71
1	300	18	24	(2,1)	0.0909	10823.82	32.52	0	55.96	72.81
10	3	6	8	(2,1)	0.6686	1,765.00	17.68	0	31.83	43.66
10	15	6	8	(2,1)	0.3803	3,103.03	20.56	0	34.41	47.22
10	30	6	8	(2,1)	0.2799	4,215.54	20.39	0	34.97	47.98
10	150	6	8	(2,1)	0.1296	9,105.32	20.23	0	35.48	48.69
10	300	6	8	(2,1)	0.09205	12,819.00	20.2	0	35.55	48.79
500	3	1	1	(2,1)	0.6511	4,453.76	1.74	0	0	0
500	15	1	1	(2,1)	0.3614	8,023.71	0.16	0	0	0
500	**30**	**2**	**1**	**(1,2)**	**0.3566**	**10,934.99**	**0**	**0.2**	**0.2**	**0.2**
500	**150**	**2**	**1**	**(1,2)**	**0.1649**	**23,653.63**	**0**	**0.54**	**0.54**	**0.54**
500	**300**	**2**	**1**	**(1,2)**	**0.1171**	**33,307.57**	**0**	**0.58**	**0.58**	**0.58**
1000	3	1	1	(2,1)	0.8464	5,789.30	1.74	0	0	0
1000	15	1	1	(2,1)	0.4698	10,429.76	2.49	0	0	0
1000	30	1	1	(2,1)	0.344	14,242.89	2.12	0	0	0
1000	150	1	1	(2,1)	0.1585	30,911.16	1.78	0	0	0
1000	300	1	1	(2,1)	0.1125	43,546.53	1.74	0	0	0
10,000	3	1	1	(2,1)	2.4453	16,725.91	7.21	0	0	0
10,000	15	1	1	(2,1)	1.3573	30,132.71	2.67	0	0	0
10,000	30	1	1	(2,1)	0.9939	41,149.24	2.86	0	0	0
10,000	150	1	1	(2,1)	0.458	89,305.65	3.03	0	0	0
10,000	300	1	1	(2,1)	0.3251	125,810.57	3.05	0	0	0
						Avg. Dev. %	**11.26**	**0.05**	**17.92**	**23.75**

As the value of the buyer's ordering cost increases, the buyer is clearly better off making fewer orders of larger sizes, which can be seen in the reduced number of batches $(n_1 + n_2)$ produced coupled with the change in the value of the cycle time (T). For example, when comparing the problem instance of $A_b = 10$ and $h_b = 3$ with that of $A_b = 500$ and $h_b = 3$, we note that although the decrease in the value of T is small (i.e. from 0.6686 to 0.6511), the number of batches decreases from 6 to 1 for n_1 and from 8 to 1 and for n_2. Subsequently, the lot size is $Q_1 = 133.72$ and $Q_2 = 66.86$ for the first problem versus much larger batches of $Q_1 = 781.32$ and $Q_2 = 520.88$ for the second one. For extremely small values of ordering cost where A_b = 1, the number of batches of remanufactured items is 24 for remanufactured items and 18 for new items.

For A_b = 10, the number of batches decreases to n_2 = 8 and n_1 = 6. Therefore, we can infer that for smaller ordering costs it is better to produce more batches (i.e. make more frequent deliveries to the buyer). For extreme ordering cost values of $A_b =$ 1000 and 10,000, the holding cost becomes irrelevant as the number of batches, n_1 and n_2, remain one.

On the contrary, the buyer seeks to reduce lot sizes if the holding cost increases in order to minimize holding costs. This can be seen upon comparing the instance where $A_b = 10$ and $h_b = 3$ with $A_b = 10$ and $h_b = 15$, The size of those batches is smaller for the second case and is given by $Q_1 = 76.06$ and $Q_2 = 38.03$. However, the number of batches is not effected by the increase in holding cost. The table shows that in both cases, number of batches remains 6 for new items and 8 for remanufactured types. In the latter problem, this is accompanied by a large drop in the cycle time value from 0.6686 to 0.3803 implying the production of a smaller sized batch. The optimal sequence in all cases is to start with the remanufactured batches first, shown as sequence (2,1) in Table 12, with the exception of three problem instances that are listed in bold face within the table. Hence, the effect of varying the buyer's ordering and holding costs may extend beyond the ordering policy, in terms of batch size and frequency, to include the production sequence at the vendor's facility as well. As expected, the total cost per unit time increases upon fixing either the ordering or the holding cost and increasing the other one.

We now compare the cost savings with cases where the decision maker has adopted one of the non-optimum sequences. The results are summarized in the last three columns of Table 12. There is a 0.05% average percentage deviation (APD) if (*R,M*) is assumed, although (*M,R*) was optimum. However, there is a bigger difference of 11.26% observed if the vendor assumes (*M,R*). As expected, there is a much higher difference of 17.92% when the more simplified sequence (*R,1*) is adopted and a 23.75% deviation if (*1,M*) is assumed. The difference in APD is smaller for extremely large setup costs as the number of batches of both product types is one and the optimum sequence is (*R,M*) which is equivalent to the simplified case of (*R,1*) or (*1,M*). Here, the only difference is observed in the case where (*M,R*) is optimum instead of (*R,M*). Conversely, for smaller setup costs, the ADP reaches 25.102%, 44.656%, and 59.251% for the cases (*M,R*), (*R,1*), and (*1,M*), respectively, where it is optimal to use the sequence (*R,M*). In some cases, the percentage deviation might reach a maximum of almost 56% and 73%, respectively if (*R,1*) and (*1,M*) is used instead of the optimal sequence (*R,M*).

4.3 Impact of Returned Product's Holding Cost and Remanufacturing Setup Cost

In order to better understand the existing tradeoffs in the remanufacturing-related cost parameters (i.e., A_2 and h_r) and analyze their impact on the sequence obtained besides other model outputs, a two-way sensitivity analysis is carried out through experimenting with various A_2 values for a fixed h_r value and vice versa. In particular, keeping the base values of other problem parameters, we assess five different values for h_r; $h_r = 0.5, 1, 1.5, 2$ and 2.5. In addition, for each of the selected h_r values, we test six different values of A_2 obtained through multiplying its base value by 0.01, 0.05, 0.1, 0.5, 0.75 and 10, which results in a total of 30 problem instances as summarized in Table 13.

As the setup cost A_2 increases for the same h_r, the cycle time also increases. The difference in the amount of the increase is directly associated with the change in the number of batches produced of both product types. A small difference in cycle time is accompanied by no change in the batches produced while a bigger difference is seen when the batch number increases. For example, at a fixed value of h_r = 0.5, as the cycle time T increases by a negligible amount 0.66 to 0.84 when A_2 increases from 125 to 187.5, the number of batches increases by one for both new and reused products. However, when the cycle time T increments by a negligible amount from 0.6086 to 0.6129 as shown in the first two rows, the number of batches produced does not change. Therefore, the amount of increase in the cycle time is related to the number of batches produced where T increases in a larger magnitude in order to accommodate for the production of more batches.

On the other hand, increasing the value of the returned product's holding cost (h_r), while fixing A_2 value, does not have a visible impact on the number of batches produced but induces a reduction in the value of T which indicates that batches of smaller sizes are being produced. We have also tested extreme values of h_r that are as small as 0.001 and the above noticed trend prevailed.

Table 13: Effect of Remanufacturing Setup Cost and Returned Product Holding Cost

h_r	A_2	n_1	n_2	Sequence	T	TC	% Cost deviation			
							(M,R)	(R,M)	(R,1)	(1,M)
0.5	2.5	2	3	(2,1)	0.6086	2308.6	1.84	0	5.72	5.72
0.5	12.5	2	3	(2,1)	0.6129	2324.97	2.12	0	6.27	6.27
0.5	25	2	3	(2,1)	0.6183	2345.28	2.47	0	6.67	6.94
0.5	125	2	3	(2,1)	0.6595	2501.8	4.81	0	9.10	11.42
0.5	187.5	3	4	(2,1)	0.8403	2588.24	5.79	0	10.60	13.93
0.5	2500	6	8	(2,1)	1.8562	4417.69	22.80	0	35.36	48.30
1	2.5	2	3	(2,1)	0.5902	2380.51	0.89	0	4.18	4.18
1	12.5	2	3	(2,1)	0.5944	2397.39	1.18	0	4.72	4.72
1	25	2	3	(2,1)	0.5996	2418.33	1.52	0	5.38	5.38
1	125	2	3	(2,1)	0.6389	2579.73	3.83	0	7.91	9.79
1	187.5	2	3	(2,1)	0.6634	2675.66	4.88	0	9.12	11.98
1	2500	6	8	(2,1)	1.7691	4635.08	20.01	0	31.59	43.62
1.5	2.5	2	3	(2,1)	0.5734	2450.31	0.05	0	2.79	2.79
1.5	12.5	2	3	(2,1)	0.5775	2467.69	0.33	0	3.32	3.32
1.5	25	2	3	(2,1)	0.5825	2489.24	0.66	0	3.97	3.97
1.5	125	2	3	(2,1)	0.6214	2655.37	2.96	0	6.85	8.33
1.5	187.5	2	3	(2,1)	0.6445	2754.12	4.12	0	8.05	10.48
1.5	2500	5	7	(2,1)	1.6118	4839.2	17.78	0	28.45	39.70
2	**2.5**	**2**	**1**	**(1,2)**	**0.402**	**2500.22**	**0**	**0.70**	**2.26**	**2.26**
2	**12.5**	**2**	**1**	**(1,2)**	**0.4059**	**2524.98**	**0**	**0.44**	**2.51**	**2.51**
2	**25**	**2**	**1**	**(1,2)**	**0.4109**	**2555.58**	**0**	**0.10**	**2.80**	**2.80**
2	125	2	3	(2,1)	0.6046	2728.92	2.18	0	5.89	7.00
2	187.5	2	3	(2,1)	0.6271	2830.4	3.32	0	7.08	9.13
2	2500	5	7	(2,1)	1.551	5028.9	15.91	0	25.89	36.46
2.5	**2.5**	**2**	**1**	**(1,2)**	**0.3944**	**2548**	**0**	**1.04**	**1.81**	**1.81**
2.5	**12.5**	**2**	**1**	**(1,2)**	**0.3983**	**2573.23**	**0**	**0.87**	**2.06**	**2.06**
2.5	**25**	**2**	**1**	**(1,2)**	**0.4031**	**2604.42**	**0**	**0.68**	**2.35**	**2.35**
2.5	125	2	3	(2,1)	0.5892	2800.54	1.47	0	5.02	5.79
2.5	187.5	2	3	(2,1)	0.6111	2904.68	2.61	0	6.20	7.90
2.5	2500	4	5	(2,1)	1.3827	5207.23	14.36	0	23.74	33.72
						Avg. Dev. %	**4.60**	**0.13**	**9.26**	**11.82**

When it comes to the number of setups needed, the solution shows that, for all tested values of A_2 and h_r, only one setup is carried out as consecutive production of the same product type takes place before switching to the other type. This however might change while we reduce the setup cost associated with the newly manufactured batches as seen in the next subsection. The optimal sequence in all cases is to start with the remanufactured batches first, shown as sequence (2,1) in Table 13. However, it is important to note that for the six problem instances that are highlighted in bold face within the table, the optimum sequence changed to (1,2) which implied that altering the values of A_2 and h_r could possibly affect the production sequence. As expected, the total cost per unit time increases upon fixing either the remanufacturing or the holding cost and increasing the other one.

The average percentage deviation (APD) values are 0.13% and 4.60% if the decision maker assumes policy (*R,M*) and (*M,R*), respectively. Nevertheless, the deviation in total supply chain cost is 6.29% for speculating (*R,1*) and even higher for (*1,M*) with 11.82% of a deviation in the latter case. It can also be noted that the difference in cost is greater when there is a bigger difference between the holding and remanufacturing cost. For example, for values of $h_r = 0.5$ and $A_2 = 2500$, there is a 48.30% deviation for sequence (*1,M)* and for values $h_r = 1$ and $A_2 = 2500$, there is a 43.62% deviation for sequence (*1,M*).

4.4 Impact of (re)manufacturing setup costs

Like in the previous two cases, we again conduct a two-way sensitivity analysis but this time we study the impact of changing the manufacturing and remanufacturing setup costs. Among the various input parameters to the problem at hand, the setup costs for newly and remanufactured batches stand out as key parameters with an anticipated significant impact on the behavior of the model and more specifically, on the production sequence obtained. In order to provide more insights on such an impact, a two way sensitivity analysis is performed in which five different levels of the setup costs associated with newly and remanufactured batches, A_1 and A_2, are obtained through multiplying their respective base values by 0.05, 0.1, 0.5, 5 and 10 while fixing the rest of the problem parameters at their base values. This produces a total of 25 problem instances, where the results of these instances are summarized in Table 14.

As the setup cost of manufacturing new and reused items increases, we expect the cycle time to increase as well as it is more economical to reduce the set up cost per unit time. For example, for the fixed value of A_2 = 12.5, the cycle time T increases gradually from 0.234 to 0.9494 as A_1 increases from 10 1000. Similarly, T increases from 0.2341 to 0.2406 to 0.3740 when A_2 increases from 12.5 to 25 to 125. The same pattern can be seen for all values of $A_1$where A_2 is fixed and vice versa. The total number of batches either stays the same or increases.

Table 14: Impact of Altering Production Setup Related Costs

A_1	A_2	n_1	n_2	Sequence	T	TC	% Cost Deviation			
							(R,M)	(M,R)	(R,1)	(1,M)
10	12.5	1	1	(2,1)	0.2341	1900.8945	0	1.95	0	0
10	25	1	1	(2,1)	0.2406	1953.5609	0	1.95	0	0
10	**125**	**2**	**1**	**(1,2)**	**0.3740**	**2326.2416**	**0.27**	**0**	**0.27**	**0.27**
10	1250	3	4	(2,1)	1.0157	3859.5337	0	10.52	18.29	26.16
10	2500	6	4	1222211111	1.4432	4878.0324	0.53	15.77	25.61	36.00
20	12.5	1	1	(2,1)	0.2393	1943.1418	0	1.95	0	0
20	25	1	1	(2,1)	0.2457	1994.693	0	1.95	0	0
20	**125**	**2**	**1**	**(1,2)**	**0.3783**	**2352.8281**	**0.60**	**0**	**0.60**	**0.60**
20	1250	3	4	(2,1)	1.0183	3869.3669	0	10.57	18.36	26.27
20	2500	6	4	1222211111	1.4473	4891.8708	0.39	15.64	25.47	35.86
100	12.5	1	1	(2,1)	0.2774	2252.7761	0	0.56	0	0
100	25	1	1	(2,1)	0.2829	2297.3898	0	0.09	0	0
100	**125**	**2**	**1**	**(1,2)**	**0.4109**	**2555.5821**	**0.10**	**0**	**2.80**	**2.80**
100	1250	3	4	(2,1)	1.0387	3947.1509	0	10.95	18.95	27.11
100	2500	5	7	(2,1)	1.5310	4964.0072	0	15.51	25.39	35.84
1000	12.5	3	4	(2,1)	0.9494	3607.6308	0	9.06	16.07	23.00
1000	25	3	4	(2,1)	0.9528	3620.7734	0	9.15	16.20	23.19
1000	125	3	4	(2,1)	0.9801	3724.2449	0	9.77	17.16	24.56
1000	1250	4	5	(2,1)	1.3367	4713.1518	0	14.34	23.84	33.83
1000	2500	6	8	(2,1)	1.7780	5511.806	0	17.78	29.27	40.64
2000	12.5	4	5	(2,1)	1.2853	4531.9918	0	13.46	22.65	32.27
2000	25	4	5	(2,1)	1.2881	4541.707	0	13.51	22.71	32.35
2000	125	4	5	(2,1)	1.3099	4618.6903	0	13.90	23.24	33.04
2000	1250	6	8	(2,1)	1.7321	5369.3575	0	17.61	28.26	39.41
2000	2500	6	8	(2,1)	1.9510	6048.14	0	19.16	32.40	44.45
						Avg. Dev. %	**0.08**	**9.01**	**14.70**	**20.71**

For instance, when A_2 is kept fixed at 125, the total number of batches remains 3 for the first three instances of A_1, and increases to 7 and then 9 for the two instances where A_1 = 1000 and A_2 = 2000. Similarly, we can examine this growth in number of batches when A_1 is fixed while A_2 changes. This can be seen from the first five rows of the table. The number of batches of returned material is higher than that of new items except in the instances highlighted in gray or in bold. The lot size, Q, is also affected as it is a function of the cycle time and the number of batches produced as seen from Equation 9.

When the resulting number of batches n_1 and n_2 stays the same, bigger sized batches are produced with increasing setup cost values. However, when a different number of batches is produced as a result of altering either one of A_1 or A_2, the sizes of those batches might get bigger or smaller depending on the values of n_1, n_2 and T. When the ratio T/n_i increases, the lot size will also increase. This can be further illustrated by observing the first two rows of the table where A_1 is 10, the lot size Q_1 = 280.92 increases to 288.72 and Q_2 = 187.28 increases to 192.48 as the number of batches of both new and reused items is still optimum at 1. However, moving from the second to the third row leads to a decrease in lot size from Q_1 = 288.72 to 224.4 as n_1 increases from 1 to 2. As anticipated, the Q_2 = 192.48 increases to 299.2 as the optimum batch number does not change. On the contrary, moving from the third to the fourth row, we now see an increase in Q_1 = 224.4 to 406.28 although the n_1 increased from 2 to 3. This is because the ratio T/n also increased from 0.182 to 0.367. However, Q_2 = 299.2 decreases to 203.14 as T/n_i decreased from 0.374 to 0.275. The total costs also increase reasonably as the manufacturing and remanufacturing costs increase.

It is further noted that changing the setup cost parameters also effects the sequence of the batches produced. Although in 20 out of the 25 problem instances, the model is optimum when the sequence is (*R,M*), there are three instances highlighted in bold where it becomes more economical to use sequence (*M,R*). In these three instances, the optimum model shows that more batches of new items should be produced than reused ones. In both of these sequences, only one setup for each product type is conducted, regardless of the number of batches produced, and hence the total setup cost is simply $A_1 + A_2$. Furthermore, the two cases that are shaded in grey in the above table indicate that the optimal solution is neither one of the sequences (*R,M*) or (*M,R*). Here, there are two switches in sequences made between the two product types which are distinct from any other case in the experiments performed. The optimal solution in these two problems is for the decision maker to adopt the sequence (1,2,2,2,2,1,1,1,1,1,1), which means that the vendor should first produce a batch of new items followed by four batches of reused items and then there is another switch to the new products with five such batches. In both cases, the remanufacturing setup cost is much higher than the setup cost for the new products. Only one setup is required for remanufacturing of the returned items while the vendor has to alternate his setup twice for the new products. This is expected as the remanufacturing cost is much higher in these occurrences and it is justifiable to have only one setup cost for remanufacturing the products. The same sequence structure was also observed when examining different values of the buyer's holding cost, specifically when $h_b = 3,\ 15,\ 30,\ 150$ and 300 and for the problems where $A_1 = 20, A_2 = 2500.$

The chain-wide costs of the intermittent sequence are then compared with the non-optimum consecutive sequence involving the same number of batches ($R = 4, M = 6$) and with the optimal (*R*, *M*) sequence where $R = 5$ and $M = 4$ as shown in Table 15.

Table 15: A comparison of three relevant sequences

Measure	Sequence		
	1222211111 (**optimal**)	(*R* = 4, *M* = 6)	(*R* = 5, *M* = 4)
q_1	200	200	300
q_2	200	200	160
T	1.4473	1.3505	1.393
A_v	2540	2520	2520
nA_b	1000	1000	900
RH_r	480	480	480
RH_v	210	210	231
RH_b	1000	1240	1052
$K(n_1, n_2)$	**4891.87**	**5212.91**	**4910.99**

Although the optimum intermittent production sequence apparently requires a higher setup cost due to more frequent setups, this increase in the setup cost is compensated by the cost savings in some other components. As shown in the table above, the setup cost, A_v, \$2540 is the highest as the setup of the two product types are alternated twice. Therefore, it costs an additional \$20 to use the intermittent sequence. The cycle time, *T*, is also the highest for this sequence. Moreover, since the first two sequences involve the production of 10 batches versus 9 batches for the last sequence, the buyer's ordering cost (nA_b) under those two sequences incurs an additional cost of \$100. With regard to the relative inventory holding cost for the recovered materials, RH_r, this cost is \$480 under all three sequences. However, the relative inventory holding cost for the vendor, RH_v, is lower for the first two sequences by \$21. Furthermore, the biggest savings in cost emerges in the inventory holding cost at the buyer's stage, RH_b. The buyer's inventory cost is cheaper by \$240 for the intermitted sequence when compared to the second sequence and by \$52 relative to the third case. Consequently, the overall cost of the discontinuous sequence is the lowest with \$4891.87.

The average percentage deviation (APD) values are 0.08% and 9.01% if the decision maker assumes policy (*R,M*) and (*M,R*), respectively. Nevertheless, like in the previous two cases of the sensitivity analysis, the deviation in total supply chain costs are 14.70% for speculating (*R,1*) and even higher for (*1,M*) with 20.71% in the latter sequence. Therefore, the production sequence is of utmost importance and stands out as a key factor that needs to be optimized if the decision maker seeks to achieve overall system efficiency.

Chapter 5: Conclusion

Many world class companies are gearing towards optimizing their chain-wide total costs instead of focusing on individual benefits. Likewise, much research work has been conducted on integrated models under consignment stock (CS) partnerships in an attempt to minimize the total costs for all supply chain parameters. Moreover, there is ample research showing that cost savings might be achieved by efficiently managing the retrieval of products. There is only one research work developed for a closed-loop supply chain model under CS agreement. However, the sequencing of the batches was assumed to be predetermined where m remanufacturing batches are produced first followed by n manufactured batches. Thus, it is clear that there is a need for a model that takes into consideration the sequence of the returned and new materials.

The key contribution of the this research is the development of a mixed integer non-linear programming (MINLP) model that seeks to minimize the chain-wide total cost by jointly optimizing the length of the production cycle, the sequence of the newly manufactured and the remanufactured batches, as well as the inventory levels of the finished and recovered products at the beginning of the cycle.

In the thesis, a heuristic solution procedure was also developed to determine the optimum number of batches for both manufacturing and remanufacturing lots for the closed loop supply chain optimization problem under consignment partnership. The cost savings due to optimizing the production sequence is quantified through comparing it with situations where the production sequence is fixed a priori, which is an assumption made by most of the existing literature. The extensive numerical analysis was done by extensively running 400 different problem instances having different settings at the input parameters, and then comparing the optimum results with the special sequences (R,M), (R,M), $(R,1)$, and $(1,M)$. In this case, the optimum results (M,R) were then compared with (R,M) and vice versa.

The computational results showed that it is more economical to have an intermittent sequence in case of low setup costs for the newly manufactured batches and high setup costs for the remanufactured ones. However, the experiments also suggest that consecutive production of remanufactured batches followed by newly manufactured batches or the other way around with only one switchover in between still stands as the optimal sequence in most of the instances tested. The calculated cost deviations due to the departure from the optimal sequence highlight the importance of considering the sequencing decision in this sort of planning problems where the increase in the total cost due to such departure might amount to 24% on average, depending on the values of the problem parameters and the non-optimum sequence adopted.

The proposed model is an important initiative which can be further improved by incorporating additional realistic factors. For instance, the model can be extended by allowing for a stochastic versus deterministic approach for key parameters such as rate of collection and demand. The disposal rate may not be zero as it is not necessary that all returned material can be reused for production. Furthermore, the setup times for the manufacturing and remanufacturing items are not negligible and are expected to be higher than those of reused items. Another direction that requires further investigation is to address a supply chain comprised of a single vendor and multi buyers, or a more complex supply chain that extends beyond the two-stage configuration.

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Vita

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