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***Abstract***

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JEL Classification Codes: C54, C60, D10, D15, J13, J22, J24, H31, H52

Key Words: Parental investment, human capital, cognitive skills, non-cognitive skills, fertility, labor supply, optimal control

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# Parental Involvement and Children's Development: Can There Be Positive Side Effects?*

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**Abstract**

A growing body of literature indicates that meaningful time spent with parents has a significant influence on early childhood development, a future accumulation of a wide array of cognitive and non-cognitive skills, and the ultimate success of a child. The theoretical model presented in this article features endogenous fertility and labor supply while distinguishing between various types of parental time spent with children. In this model, parents are subsidized for spending publicly verifiable productive time with their children. It is shown that there are low tax-subsidy rates that would allow policymakers to stimulate the labor supply of the primary caretaker in addition to significantly enhancing children's skills. These unique by-products of human capital accumulation can have important implications both in developed countries with an ageing population and in developing countries with low female labor force participation.

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“The most valuable of all capital is that invested in human beings; and of that capital the most precious part is the result of the care and influence of the mother.” Alfred Marshall, *Principles of Economics* (1920).

“There are strong and biological evidence, strong neuroscientific evidence that suggest that early family conditions are powerful in shaping cognitive and non-cognitive skills. And those determine, to a degree we didn’t realize before, the later success of a person … We can see fundamentally how the structure of the brain even is affected by …parental interventions.” James Heckman, *Heckman Equation* (2012).

## 1. Introduction

Human capital is a powerful engine of economic and social development. It is widely accepted that the formation of human capital is not an exclusively private matter, and there is room for an efficient policy intervention.[1] There is no lack of traditional reform proposals to enhance human capital and to reduce educational and income gaps, but their efficacy and feasibility are dubious. Recent findings suggest that a positive and stimulating family environment is highly influential on the development and success of a child, and early childhood interventions have a large social rate of return.[2] As Heckman (2008b) argued, emotionally nurturing family environments lead to successful learners, and such environments are more important than direct financial resources. The prefrontal cortex is flexible until the early 20s, and the nurturing environment is conducive to non-cognitive abilities. Non-cognitive abilities, in turn, contribute to acquiring cognitive abilities. Recent economic and interdisciplinary research largely supports this view, suggesting that differences in inherited traits alone cannot explain the intergenerational persistence of poverty or the existence of income inequality and that the role of family in strengthening children’s cognitive, behavioral, and socioeconomic (non-cognitive) abilities should not be understated.[3]

[1] Numerous studies discuss a wide array of social benefits of education and human capital accumulation, their roles in alleviating income inequalities, and optimal policies when markets are not ideal. See, among others, Friedman (1955), Albrecht and Ziderman (1992), Barr (1993), Bénabou (1996), Voitchovsky (2005), Stantcheva (2017), Ward (2017), and references therein.

[2] Interested readers can refer to Gauthier et al. (2004), Guryan et al. (2008), Heckman (2008a; 2008b; 2011), Cunha et al. (2010), Ramey and Ramey (2010), Kalil et al. (2012), Attanasio (2015), Blankenaua and Youderian (2015), Yum (2015), Del Bono et al. (2016), and many references therein, among others.

[3] See, for example, Hill and Stafford (1974), Bryant and Zick (1996), Neidell (2000), Heckman and Masterov (2007), Heckman (2008a; 2008b), Heckman and Mosso (2014), Attanasio (2015), Milkie et al. (2015), Del Bono et al. (2016), Sani and Treas (2016), and Del Boca et al. (2017), among others.

This paper focuses on parental time investment in children's human capital. Unlike previous studies on parental time investment, the dynamic model presented features endogenous labor-leisure choice, consumption, saving, human capital accumulation, and fertility while distinguishing between various endogenously determined types of activities parents and their children can jointly spend time participating in. As Heckman and Mosso (2014) argued, most models of dynamic parental investment treat fertility as exogenous. One contribution of this study is that dynamic quality parenting is modelled, and the demand for children is endogenized.[4] The model is then extended to incorporate a simple tax-subsidy experiment in which parents were "rewarded" only if they spent certain amount of productive time with their children. To investigate whether encouraging parents to invest in a particular type of child quality-enhancing activities would crowd out other forms of quality-time investments, we introduce different types of parent-children activities and quantitatively assess the strength of the crowding out effect. The results contribute to the existing literature by suggesting a unique byproduct of human capital accumulation. More specifically, it is shown that despite the crowding out effect, *low* tax rates on labor income can be effectively used to subsidize productive parental time investment, balance the government budget, increase the human capital of children, and stimulate the labor supply of the primary caretaker along different margins. This finding could have far-reaching implications both for countries with high human capital and low worker-to-retiree ratios, and for those with low human capital and inactive female labor force participation.

The finding that labor supply can positively react to a proposed government policy intervention might have further implications for the status and poverty levels among women.

[4] The fertility aspect of family economics is interesting not only due its intricate link to the parental investment or even to the poverty trap. While it is true that high childbearing might not be the cause of poverty but merely reflect it, there is evidence that moderate fertility can be advantageous to the status and empowerment of women, and not necessarily always the other way around (Basu 1992; Wu et al. 2014).

There is well-documented evidence that women with children pay a wage penalty gap (Youderian 2014), partly due to low labor force participation. Given that the majority of primary caretakers are women, there is a concern that any reform that encourages a family to spend more time engaged in quality parenting might encourage women to abstain from the labor force even more. For example, Youderian (2017) developed a dynamic model with exogenous fertility, endogenous private and public education expenditures, and parental time investment. The author found that subsidizing parental investment in childcare via paid parental leave is a costly and less effective way to improve human capital, and parental labor supply (which is assumed to be elastic only in the early years of adulthood) declines due to the subsidy.[5] However, this study shows that a small tax-subsidy program used to stimulate children's human capital can have stimulating effects on primary caretakers' labor supply as well.

Introducing the quality of children in a family model with endogenous fertility has become a strong tradition since the seminal contributions of Becker (1960) and Becker and Lewis (1973). Since then, a large body of literature on demographic transitions and economic growth has shown that there is a trade-off between the quality and quantity of children.[6] Drawing an analogy with consumer durables and considering a non-linearity in the budget constraint, Becker-Lewis' framework suggests that a higher income should increase both the quantity and quality of children, but if the quality elasticity is sufficiently large, greater expenses per child associated with a higher desired quality might suppress fertility. This tradeoff, coupled with investments in human capital, has been a central argument of the theory proposed by Becker-Murphy-Tamura (1990) to explain why some countries have a better economic performance compared to others. The authors

[5] Yum (2015) treated fertility and retirement decisions as exogenous but assumed endogenous parental time investment propagates to later stages to affect the optimal choice of education of adult, independent children. The author found that the parental time investment significantly affects both the intergenerational mobility and the cross-sectional variation of early-life human capital, while parental time subsidy was shown to have a significant positive effect on human capital.

[6] See, for example, Galor and Weil (2000), Galor and Moav (2002), Moav (2005), Becker et al. (2010), Huber et al. (2010), and many references therein.

conjectured that societies with limited human capital choose to have large families and invest little in each member.[7]

Due to the groundbreaking nature of findings related to the quantity-quality trade-off, it has been the dominant theory in the economics of fertility for several decades (Doepke 2015), with the quality dimension receiving particular attention. Numerous studies have documented that issues related to a low level of human capital and a large family size are particularly endemic across the poor socio-economic strata both within developed and less-developed countries, which contributes to the vicious cycle of the poverty trap (e.g., Moav 2005). Thus, some researchers have traditionally linked poverty and its associated problems with excessive household sizes.[8]

When parents choose to enhance the quality of their children via quality parenting, this can also affect the demand for children. It is important to properly model a potentially strong, quantitative trade-off between fertility choice and quality parental time. With the endogenous labor supply factor included, we perform a simple policy experiment in which parental time directly spent on productive activities with children is subsidized. The tax policy intervention parameters were chosen so that the present value of taxes collected from a household equaled to the present value of subsidies paid to the same household. It should be noted that the model can be easily amended to include scenarios in which high-income groups should subsidize the quality parenting of disadvantaged groups.

Unlike many traditional policy reforms, the proposed policy intervention deliberately considers subsidizing a "consumer," i.e., the household. Allowing families the freedom to choose would promote competition and efficiency among both private and public service

[7] Further, the extant literature has identified the channels that link human capital accumulation to positive economic growth. When examining the role of human capital in the process of economic development, scholars have emphasized that many developing economies that have experienced rapid increases in growth have also experienced considerable increases in human capital (Attanasio 2015).

[8] Certainly, higher rates of population have their own social benefits, yet it is indeed true that a large number of births might be harmful if coupled with poorly attenuated land and property rights, and a shortage of natural resources (Stern 1989).

providers. It would also assist in tailoring to the specific needs of each child as opposed to a "one size fits all" strategy. As emphasized by Heckman (2008a, p. 314), "universal programs would be much more expensive and create the possibility of deadweight losses whereby public programs displace private investments by families."

Traditional policy reforms meant to improve human capital have incorporated a wide spectrum of mechanisms, including the government provision of public schools, the extension of student aid, the subsidization of fees, conditional cash transfers, and financing school nutrition programs. These measures have only been partly successful.[9] In addition, academics and policymakers intensely debate which socio-economic group benefits more at the expense of the other for a given educational reform as well as whether public funds should be geared towards primary, secondary or higher education or even towards science and technology development. These arguments are not discussed in this article, but it is argued that a simple, revenue-neutral reallocation of household income to subsidize a household's human capital accumulation might be less contentious. While a general income tax on labor might discourage investment in human capital (Stantcheva 2017), directing tax revenues to a targeted subsidization of parental intervention to improve the human capital of children, could more than offset the negative effects.

The rest of this paper is organized as follows. Section 2 briefly reviews the evidence on children's learning and the role of families, and provides some examples of family-based programs that can be designed to improve cognitive and personality skills and to qualify for a subsidy scheme. Section 3 introduces a theoretical model, while Section 4 calibrates it, and presents the benchmark and tax-subsidy experiment results, which are briefly discussed. Section 5 concludes the article.

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[9] For instance, Glewwe et al. (2013) painstakingly reviewed both the education and economics literature from 1990 to 2010 and found ample evidences on the negative or insignificant effects of different school and teacher variables on student learning. Heckman (2008b) argued that public job training programs, adult literacy programs and educational programs appear to have low economic returns.

## 2. Further Remarks on the Role of Families in Children's Development

### *2.1 Families as facilitators of children's development*

Numerous studies on psychology, psychiatry and family issues highlight the importance of a high-quality family environment and family cohesion on adolescents' psychological well-being (e.g., Guo et al. 2002; Carr and Francis 2009; Li ad Warner 2015). According to Neville et al. (2013, p. 12138), "decades of research indicate that children from lower ... [socioeconomic status] backgrounds are more likely to grow up in homes that are more stressful and less cognitively stimulating." The authors cited evidence, that parental nurturance is a good predictor of the size of the hippocampus, an organ located within the brain's medial temporal lobe, which is important for learning and memory, and that family-based training programs produced by far the strongest effects on cognition and educational achievement.

The economics of parental time investment and family involvement is relatively recent but is growing rapidly due to a number of important contributions. One leading expert in the area, James Heckman (2008a; 2008b; 2011), has long emphasized that genetic factors, innate abilities and IQ levels are insufficient in determining positive children outcomes, arguing that the family environments of young children strongly predict cognitive abilities, socioemotional abilities, schooling success, wage earnings, participation in crime, and health. A child's success is driven by early intervention that aims to nurture non-cognitive, life-relevant skills (conscientiousness, ability to work hard, getting along with others).

The idea that families can shape non-cognitive and cognitive skills has strong support in medical and neuroscientific circles that support the view that traits such as honesty, generosity, decency, persistence, and compassion are obtained within a home when parents and children interact in a mutually supportive, loving, and caring environment (Milteer et al. 2012). Based on a large body of development psychology, Heckman (2008a) concluded that a lack of positive family and environmental inputs interfere with normal brain development, particularly in the areas of perception, processing, interpretation, and acting on. Receiving inadequate social and intellectual stimulation during the early years causes a lack of sensory cues and a resulting delay in brain development. The author examined the effectiveness of family partnership programs, which include home visits, and concluded that the programs positively "affect the lives of the parents and create a permanent change in the home environment that supports the child after center-based interventions end" (Heckman 2008a, p. 314). Heckman (2008b) observed that disadvantaged families bear relatively more children and spend relatively less time on child development activities compared to better-educated parents. In sum, Heckman's recommendation is to develop cognition and character.

Several studies have focused on parental time invested in traditional childcare activities. Although childcare is vital, parental time shared with children is much broader, and includes activities such as learning physical and mental skills, accumulating non-cognitive skills, and understanding concepts of fair play (Bryant and Zick 1996). As emphasized by Heckman and Mosso (2014) and Francesconi and Heckman (2016), mainstream literature has mostly focused on parental investment in specific goods that can enhance the productivity of children, whereas the skills of the parent are relevant to the technology of capability formation. The authors cited considerable evidence that shows that a prime factor behind child skill formation is parental time.

Mosca et al. (2017) investigated the impact of the Marriage Bar[10] in Ireland on the probability of children's university education completion and concluded that children of mothers affected by the bar were significantly likely to complete a university degree. The authors attributed this sizeable difference to more time spent with children compared to women not affected by the Marriage Bar. Similarly, Del Boca (2017) argued that parental time is important, and observed that its efficacy on child development declined during adolescence, but increased for homework.

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[10] The Marriage Bar was the legal requirement that women leave employment when they married, and it existed in different forms between the 1920s and the 1970s in the Republic of Ireland.

Rubio-Codina et al. (2015) found considerable differences in cognitive and language development between children in the top and bottom quartiles of household wealth distribution; however, the authors concluded that this difference increases considerably over time.

In summary, existing evidence overwhelmingly illustrates that family environment and parental-child interaction are highly influential on the future success of children. The absence of these conditions are certainly not the only predictors of poor intergenerational mobility or income inequality patterns, but they are sufficiently important to constitute serious research.

### *2.2 "Verifiable" ways to support family learning and child development*

Parent-children learning can have many forms, and it is not always easy to verify the amount of family time spent on a particular activity. Both reading a book to a child before bedtime and a trip to a science museum can be equally important, but the former is more difficult for an outsider to verify due to the absence of a receipt or any easily verifiable record associated with the activity. In this section, examples are provided of activities that the government can potentially verify. Importantly, time spent on quality parenting should stimulate the development of both cognitive and non-cognitive skills. As Heckman (2011, p. 32) stated, "while important, cognitive abilities alone are not as powerful as a package of cognitive skills and social skills—defined as attentiveness, perseverance, impulse control, and sociability. In short, cognition and personality drive education and life success, with character (personality) development being an important and neglected factor."

Character development and quality learning is difficult to achieve and sustain when the environment is not stimulating, and parents lack quality parenting skills. There is a plethora of programs and activities that specifically target parents and enhance their parenting skills. Parents, guardians and extended family members can enhance children's cognitive and non-cognitive development via specialized learning centers. Adults can acquire valuable skills to become confident and efficient in encouraging, supporting and facilitating their children's life-long learning experiences.

Many present-day programs appear to target the development of cognitive abilities, yet even they can indirectly contribute to the character development of a child. For example, guided family tours to a museum to learn about prehistoric artifacts encourage patience, attentiveness, dedication, punctuality, the ability to listen and follow, and the strengthening of family bonds. Dedicating time to acquiring knowledge is an important component of character development.

Government-run, family-oriented programs are particularly widespread in the United Kingdom. For instance, the British government has launched a number of initiatives (e.g., Skillswise) that offer family learning courses that involve parents and children to improve personal literacy and numeracy skills (BBC 2018). The Ministry of Education, Children and Youth of Luxemburg, developed a family learning initiative offering "joint activities for children and their parents."[11]

A number of private providers in Australia offer sessions during which parents can acquire knowledge on children's behaviors, and can learn how to engage in their children's development and to improve family bonds (de Souza 2013). The well-known Young Men's Christian Association (YMCA) traditionally offers a variety of learning opportunities with a focus on parent-children aquatic education, play-based programs to foster healthy eating habits and nutrition, and family learning programs, such as culinary classes and physical education, which promote creativity, motor skills, and social development.

[11] It is noteworthy that along with private or public programs that tend to complement traditional learning, some providers specifically cater to homeschooling parents and advocate the freedom of families to become educated outside government-produced, mass education settings. Such programs are more widespread in the United States.

Linnemann et al. (2013) provided evidence that science museums and science centers can help parents engage their children in building inquiry-based, reflective, and critical learning skills in a collaborative environment. The authors described the *Facilitating the Engagement of Adults in Science and Technology* (FEAST) consortium that is funded by the European Union and coordinated by a European network of science centers and museums. Corresponding activities and workshops aim to strengthen parents' science content knowledge and to help them acquire greater confidence in furthering their children's development. The involved institutions disseminate educational materials and self-learning toolkits that families can complete at home, thereby fostering continuous parent-child communication.

Other examples of family learning are highlighted by the *Arts about Manchester Family Friendly Conference* (The Audience Agency 1999), which lists specific activities offered by museums and art galleries in Greater Manchester, ranging from a theatrical family learning trail to family competitions involving photography and creative writing. Similarly, based on research of The Children's Museum of Indianapolis, Wolf and Wood (2012) emphasized the importance of family learning via visits to children's museums that offer hand-on experiences and interactive learning supported via cooperation with museum educators, exhibition developers, and designers. The authors provided numerous examples of ways to facilitate cooperative family learning. For example, an exhibition based on Egypt might necessitate that parent explains the distance from Egypt to their children, which might help them complete sorting and classification activities. Another is the "Kindness Tree" activity, which reinforces parental scaffolding of children's experiences and allows children to learn important ideas such as "Let others go first" and "Sit with someone new" (Wolf and Wood, 2012, p. 34). These activities are important in developing sociability skills as well.

These examples show that there are currently many opportunities available that can enhance family learning and child development. What is less prevalent perhaps is the existence of programs that specifically target a wider array of non-cognitive abilities as well as universal price discounts for the services of many such providers; however, when a household is subsidized to freely choose from a number of diverse programs in a competitive environment, private enterprises are likely to respond by offering even more diversified initiatives that cater to a wide spectrum of needs of parents and their children. Due to competitive forces, these programs would (in the absence of excessive bureaucratic obstacles) likely become cost-effective over time.

## 3. A Theoretical Model: Benchmark Scenario

### *3.1. Basic setting and theoretical preliminaries*

The model presented is a dynamic, optimal control life-cycle model that encompasses a number of control and state variables, as well as path and inequality constraints. As analytic solutions to the model are not feasible, there are considerable computational challenges. To facilitate numerical solutions without compromising rigor, the model is as concise as possible.

Decision-making processes encompass two married adults: a *primary caretaker* (indexed by symbol $P$) and a *secondary caretaker* (indexed by symbol $S$). A secondary caretaker refers to a partner who participates in the labor market in a "traditional way," constantly working and eventually retiring. The partner is called a secondary caretaker in the sense that s/he (perhaps due to cultural and social norms) would not contemplate leaving career at some point to stay home to care for children. In contrast, a primary caretaker can potentially consider leaving a career to care for children for a brief or prolonged time.

Time is continuous and is denoted by $t$. The partners are of the same age, and they marry to begin their joint life at time $t = 0$. They both become deceased and exit the model at time $t = T > 0$. The size of the household is $X(t)$. Fertility effort is endogenous and captured by the control variable, $n(t)$. The size of the family grows according to the following differential equation and initial condition:

$$\frac{dX(t)}{dt} = n(t)X(t) \quad (1)$$

$$X(0) = 2 \quad (2)$$

Thus, the number of children at any time is $X(t) - X(0)$.

Assume each adult has his or her preferences for either private or public goods. Let the primary caretaker's utility function be

$$U^P = \int_0^T e^{-\rho t}[\log[c^P(t)] + \beta_1^P \log[l^P(t)] + \beta_2^P \log[b_{tr}^P(t)(X(t) - X(0))] + q(t)\beta_3^P \log[n(t)] + \beta_4^P \log[h(t)] + f(t)l^P(t)]dt \quad (3)$$

where parameters $\beta_1^P$, $\beta_2^P$, $\beta_3^P$, and $\beta_4^P$ are positive. The discount rate is $\rho$. Note the utility function (3) incorporates the "quantity" of children component in the form of the fertility variable, $n(t)$. Furthermore, $c^P(t)$ is the primary caretaker's life-cycle consumption profile, $l^P(t)$ is the fraction of the agent's total time endowment devoted to solely *personal* leisure (time enjoyed "alone" or with friends, i.e., not with family members), $b_{tr}^P(t)$ is the fraction of the agent's total time endowment devoted to spending some leisurely, fun, and otherwise trivial ("$tr$" for short) time with one child. Thus, $b_{tr}^P(t)(X(t) - X(0))$ is the total trivial, leisurely time spent by the primary caretaker with all children at time $t$. Parameter $\beta_1^P$ can differ from $\beta_2^P$, implying that parents could place different weights on their private leisurely time and leisurely time spent with children.

**Remark 1**. We assume that the agent also spends productive, human capital-enhancing time with a child. Part of that time ($b_{unv}^P(t)$) is impossible to verify (unverifiable, or "$unv$"). This occurs, for example, when the primary caretaker reads the child a bedtime story in the privacy of the home; part of that time ($b_{ver}^P(t)$) is possible to verify (verifiable, or "$ver$"). This happens, for example, when the primary caretaker spends time with the child at a local museum or library. Both $b_{ver}^P(t)$ and $b_{unv}^P(t)$ are chosen endogenously.

The "quality" of children component is incorporated by assuming the agent derives enjoyment from the family's per capita human capital, $h(t)$ (more details in Section 3.2). Adding human capital to the family's utility function is consistent with the literature (e.g., Galor and Weil 2000; Galor and Moav 2002; Barro and Sala-i-Martin 2004).

**Remark 2**. The fourth and the sixth terms in the integrand of (3) are multiplied by continuously differentiable functions $q(t)$ and $f(t)$, respectively. Function $q(t)$ takes the value of unity before a certain time and suddenly drops to near a nil value thereafter. This is done to prevent the household from bearing children too late in life. Similarly, function $f(t)$ takes the value of near zero before a certain time, and then rises rapidly to unity thereafter. This was done to increase the marginal utility of leisure at old age, prompting the agent to retire and leave the employment sector. The exact functional forms are specified in Section 4.1.

The secondary caretaker's utility function is similar to that of the primary caretaker:

$$U^S = \int_0^T e^{-\rho t}[\beta^S \log[c^S(t)] + \beta_1^S l^S(t) + \beta_2^S \log[\lambda b_{tr}^P(t)(X(t) - X(0))] + q(t)\beta_3^S \log[n(t)] + \beta_4^S \log[h(t)] + f(t) l^S(t)]dt \tag{4}$$

where parameters $\beta^S$, $\beta_1^S$, $\beta_2^S$, $\beta_3^S$, and $\beta_4^S$ are positive. Note in the case of the primary caretaker, the weight parameter on consumption is unity, to reduce the number of parameters and to ease computational burden. Yet, the value of $\beta^S$ can be chosen in a way to produce potentially different consumption profiles for the two adults.

**Remark 3**. We assume that the secondary caretaker does not spend any productive, human capital-enhancing time with children, but that s/he does spend leisurely/fun time with them. The agent follows the other partner in "deciding" how much time to spend participating in leisurely/trivial activities with the children. If his/her partner spends $b_{tr}^{P}(t)(X(t) - X(0))$ amount of leisurely time with the children, the secondary caretaker spends up to that amount as well (hence, we assume $0 < \lambda \leq 1$ in the third term of the integrant of (4)).

We also assume that personal leisure enters the secondary caretaker's function in a linear form. Doing so generally helps to facilitate the numerical solutions and to obtain more reasonable life-cycle paths for the model's choice variables. Finally, because the model contains many control variables, the analysis is simplified by assuming that the labor supply of the secondary caretaker is predetermined and fixed. Section 3.3 discusses this further.

*3.2 Human capital formation*

The total time spent with all children at time $t$, $\xi(t)$, is:

$$\xi(t) = (X(t) - X(0))((1 + \lambda)b_{tr}^{P}(t) + b_{ver}^{P}(t) + b_{unv}^{P}(t)) \qquad (5)$$

At time $t = 0$, the total human capital of the family, $H(t)$, is $H(0) = 2$, so that $H(0)/X(0) = 1$. Let us assume $H(t)$ would increase, partly because parents and children interact productively. Thus, $H(t)$ increases at the rate that depends on function $\chi(t)$ and some constant $\bar{h} > 0$, where

$$\chi(t) = (X(t) - X(0))[\sigma_1 b_{ver}^{P}(t) + \sigma_2 b_{unv}^{P}(t)] \qquad (6)$$

Positive parameters $\sigma_1$ and $\sigma_2$ capture the contribution of different types of activities involving the primary caretaker and children that enhance the family's human capital stock.

We assume $\frac{dH(t)}{dt} = y(t)\chi(t)H(t) + \bar{h}$, where $\bar{h}$ is an exogenous ("outside") factor behind the formation of human capital.[12]

Note that the part of the human capital equation is multiplied by $y(t)$. The latter is a continuously differentiable function that takes the value of unity up to a certain time, and then abruptly drops to nil thereafter. The function is needed to prevent the primary caretaker from attempting to enhance a child's skills when the latter is old enough as doing so is unlikely to be effective. In other words, late-life investment is less productive, and this assumption that has solid empirical support (e.g., Heckman 2008b).

The family's per capita human capital variable, $h(t) \equiv \frac{H(t)}{X(t)}$, is now introduced. Differentiating $h(t)$ with respect to time, noting that $\frac{dH(t)}{dt}/X(t) = y(t)\chi(t)H(t)/X(t) + \bar{h}/X(t)$, the evolution equation for the per family member human capital is:

$$\frac{dh(t)}{dt} = (y(t)\chi(t) - n(t))h(t) + \frac{\bar{h}}{X(t)} \qquad (7)$$

Since $H(0) = X(0) = 2$,

$$h(0) = 1 \qquad (8)$$

We will assume that the per capita human capital is what parents would attempt to consider in their utility maximization exercise; hence, in (3) and (4), there are respective utility terms.

*3.3 Labor supply and wage earnings*

As everyone is endowed with one unit of time, the primary caretaker's labor supply is:

$$L^P(t) = 1 - l^P(t) - \xi^P(t) \qquad (9)$$

[12] Note that the growth rate of the total human capital is given by $\frac{dH(t)}{dt}/H(t) = y(t)\chi(t) + \bar{h}/H(t)$. The human capital return from outside factors diminishes as the existing knowledge stock becomes larger over time.

Here, the total time the primary caretaker spends with all children, $\xi^P(t)$, is given by:

$$\xi^P(t) = (X(t) - X(0))(b_{tr}^P(t) + b_{ver}^P(t) + b_{unv}^P(t)) \tag{10}$$

which is the same as the right-hand-side of (5), but $\lambda = 0$.

The secondary caretaker spends time with children equal to:

$$\xi^S(t) = (X(t) - X(0))\lambda b_{tr}^P(t) \tag{11}$$

and clearly, $\xi^S(t) + \xi^P(t) = \xi(t)$.

**Remark 4**. As mentioned earlier, we assume the secondary caretaker's labor supply behavior is fixed. Let it given by $\bar{L}^S(t)$. In general, it does not necessarily have to be a constant number; rather it must be a given function. It is continuously differentiable, taking stable, non-zero values throughout working life before approaching zero value at a senior (retirement) age. The exact functional form for $\bar{L}^S(t)$ is specified in Section 4.1.

Hence, the secondary caretaker's personal leisure consumption is given by:

$$l^S(t) = 1 - \xi^S(t) - \bar{L}^S(t) \tag{12}$$

Let $\epsilon(t)$ be the efficiency unit supplied to market production for every time unit worked by any parent, while $\omega$ is the wage per efficiency unit. For the sake of simplicity, both partners are assumed to be endowed with the same efficiency profile. Thus, the gross labor income of the family is $\omega\epsilon(t)(L^P(t) + \bar{L}^S(t))$.

*3.4. Monetary costs associated with children*

Let constant $\alpha$ capture the family's explicit monetary costs of raising a child at time $t$ incurred until the child is old enough. Assume the costs are not associated with skill-inducing, productive *parental* time. They capture, among others, the costs of clothing and food. So, $(X(t) - X(0))\alpha$ represents the total child-rearing outlays at time $t$.

In addition, note that spending time with children entails two costs. The first are foregone labor market earnings. The second are explicit payments associated with spending that time. For example, to read a book to a child, the parent is likely to have to buy the book. To spend time with the child at a museum, the parent has to buy the entrance ticket. Thus, $\psi_1$, $\psi_2$, and $\psi_3$ represent corresponding explicit costs per unit of time per child associated with verifiable, unverifiable, and trivial activities, respectively.

### 3.5. *Family's budget constraint*

Let the total assets of the family ("$Assets$") earn a market rate of return $r$. Total income received by the household is $r(Assets) + \omega\epsilon(t)(L^P(t) + \bar{L}^S(t))$. Hence:

$$\begin{aligned} \frac{d(Assets)}{dt} &= r(Assets) + \omega\epsilon(t)(L^P(t) + \bar{L}^S(t)) - c^P(t) - c^S(t) \\ &\quad - (X(t) - X(0))(m(t)\alpha + \psi_1 b^P_{ver}(t) + \psi_2 b^P_{unv}(t) \\ &\quad + \psi_3(1+\lambda)b^P_{tr}(t)) \end{aligned} \tag{13}$$

From (13):

$$\begin{aligned} \frac{\frac{d(Assets)}{dt}}{X(t)} &= \frac{r(Assets)}{X(t)} + \frac{\omega\epsilon(t)(L^P(t) + \bar{L}^S(t))}{X(t)} - \frac{c^P(t)}{X(t)} - \frac{c^S(t)}{X(t)} \\ &\quad - \frac{X(t) - X(0)}{X(t)}(m(t)\alpha + \varphi_1 b^P_{ver}(t) + \varphi_2 b^P_{unv}(t) \\ &\quad + \varphi_3(1+\lambda)b^P_{tr}(t)) \end{aligned} \tag{14}$$

**Remark 5**. On the right-hand-side of (13) and (14), we include the $m(t)$ function, which is multiplied by the term capturing basic child-rearing expenses. Again, this continuously differentiable function, which abruptly decreases from unity to nil at some point, is consistent with the assumption that once children are old enough, they do not need to be raised, so there will not be associated costs.

Now, let us define $k(t) \equiv \frac{Assets}{X(t)}$. This leads to

$$\frac{dk(t)}{dt} = \frac{\frac{d(Assets)}{dt}}{X(t)} - n(t)k(t) \tag{15}$$

Using (14), the change in assets per person is given by:

$$\begin{aligned}\frac{dk(t)}{dt} = (r - n(t))k(t) + \frac{\omega\epsilon(t)(L^P(t) + \bar{L}^S(t))}{X(t)} - \frac{c^P(t)}{X(t)} - \frac{c^S(t)}{X(t)} \\ - \frac{X(t) - X(0)}{X(t)}(m(t)\alpha + \varphi_1 b_{ver}^P(t) + \varphi_2 b_{unv}^P(t) \\ + \varphi_3(1 + \lambda)b_{tr}^P(t))\end{aligned} \tag{16}$$

Let

$$k(0) = 0 \tag{17}$$

$$k(T) = 0 \tag{18}$$

*3.6. Household optimization*

We assume a household utility function exists that respects individual preferences. For a constant $0 < \mu < 1$, the household utility function is:

$$U = \mu U^P + (1 - \mu)U^S \tag{19}$$

Joint decision-making leads to the maximization of (19) subject to (1)-(2), (7)-(8), and (16)-(18) and the following constraints:

$$c^P(t) \geq 0 \tag{20}$$

$$c^S(t) \geq 0 \tag{21}$$

$$0 \leq l^P(t) + \xi^P(t) \leq 1 \tag{22}$$

$$0 \leq l^P(t) \leq 1 \tag{23}$$

$$0 \leq b_{ver}^P(t) \leq 1 \tag{24}$$

$$0 \leq b_{unv}^P(t) \leq 1 \tag{25}$$

$$0 \leq b_{tr}^P(t) \leq 1 \tag{26}$$

$$0 \leq \bar{L}^S(t) + \xi^S(t) \leq 1 \tag{27}$$

$$n(t) \geq 0 \tag{28}$$

**Remark 5**. Note $X(t) - X(0) \geq 0$ due to (28) and (2). Conditions (24)-(26) would then imply that $\xi^P(t) \geq 0$. Therefore, (22) and (23) ensure that $0 \leq \xi^P(t) \leq 1$. Thus, the primary caretaker cannot spend more than the total time endowment with the children. Further, note that because $0 \leq \bar{L}^S(t) \leq 1$ by construction and $\xi^S(t)$ is nonnegative, condition (27) also ensures that $l^S(t)$ does not exceed unity.

## 3. Departing from the Benchmark Scenario: A Tax-Subsidy Experiment

In this section, a simple policy scheme is introduced. The government imposes a proportional income tax and subsidizes the cost of spending productive verifiable time with children. In the benchmark scenario considered in Section 2, there were no tax-subsidy parameters. We assume that the government's objective is to choose the tax-subsidy parameters in a way that satisfies its own budget constraint, and given the government's knowledge of the best response function of the households to arbitrary tax-subsidy rates.

With the tax subsidy program, the budget constraint (16) of the household becomes:

$$\frac{dk(t)}{dt} = (r - n(t))k(t) + \frac{(1-\tau)\omega\epsilon(t)(L^P(t) + \bar{L}^S(t))}{X(t)} - \frac{c^P(t)}{X(t)}$$
$$- \frac{c^S(t)}{X(t)} - \frac{X(t) - X(0)}{X(t)}(m(t)\alpha + \varphi_1(1-\delta)b^P_{ver}(t) \qquad (29)$$
$$+ \varphi_2 b^P_{unv}(t) + \varphi_3(1+\lambda)b^P_{tr}(t))$$

where $0 \leq \tau \leq 1$ is the income tax rate, and $0 \leq \delta \leq 1$ is the subsidy parameter. The budget constraint of the government is satisfied if the present value of income taxes collected does not fall short of the present value of the subsidies paid to the household.

$$\int_0^T e^{-rt}\,\tau\omega\epsilon(t)(L^P(t) + \bar{L}^S(t))dt - \int_0^T e^{-rt}\,\delta(X(t)$$
$$- X(0))\psi_1 b^P_{ver}(t)dt \geq 0 \qquad (30)$$

The government problem is as follows. For any given income tax rate $0 \leq \tau \leq 1$, the government searches for the subsidy rate $\delta > 0$, such that (30) is satisfied with equality, and assuming the agent would maximize (19), subject to (29) and (20)-(28).

## 4. Calibration and Solution Strategy

### *4.1. Benchmark parameterization and targets in the data*

In this section, some functional forms and parameters are introduced. The model has many unobservable parameters that have to be chosen to make the results of the decentralized household optimization as realistic as possible. In this section, no tax-subsidy parameters are introduced. Once the exogenous parameters for the benchmark parameterization are fixed, a tax-subsidy experiment will be introduced in section 4.2.

We assume the model time $t = 0$ corresponds to the actual age 25. For example, $t = 35$ corresponds to the actual age 60. Thus we set $T = 55$ implying death at age 80. Bagchi (2015) estimated the household age-dependent labor productivity profile for the agents who are at least 25 years old. Thus the following quartic polynomial was fit to the Bagchi's data:

$$\begin{aligned} \epsilon(t) = 1.002114527773819 + 0.011811948225355988t \\ - 0.00018076482451962419t^2 \\ - 1.4134457090567158 \times 10^{-6}t^3 \\ + 1.1000740602147595 \times 10^{-8}t^4 \end{aligned} \tag{31}$$

Logistic functions that are advantageous due to continuous differentiability are used, and thus the "indicator-like" variables are:

$$q(t) = 1 - \frac{1}{1 + e^{-q_1(t-q_2)}} \tag{32}$$

$$y(t) = 1 - \frac{1}{1 + e^{-y_1(t-y_2)}} \tag{33}$$

$$m(t) = 1 - \frac{1}{1 + e^{-m_1(t-m_2)}} \tag{34}$$

$$f(t) = z_1 + z_2 z_3^t \tag{35}$$

We set $q_1 = y_1 = m_1 = 1000$ because doing so assures that the corresponding functions very abruptly drop from the value one to nil without causing much computational difficulty. Function (32) helps ensure the family size does not grow after a certain age. Setting $q_2 = 11.3$ leads to the last incidence of birth occuring at the actual age 36.3. This value is based on the American Time Use Survey (ATUS) dataset (Bureau of Labor Statistics 2003-2016). The distribution of the age of a household's youngest child among parents at different ages was analyzed to estimate the age when the last birth occurs. For example, in the majority of households where the age of the youngest child is 17, the average age of the parents tends to be around 54. This leads to the conclusion that the age when the parents have their last child is around 37. Similarly, the age of the youngest child in the household of 50-year-old parents tends to be approximately 14, suggesting that the last birth occurs at the age of 36. We follow these calculations for all age cohorts. The average of these numbers, estimated at 36.3, is then used as the average age of the last child in the dataset.

Consequently, we set $m_2$ = 29.3 (because 18 + $q_2$ = 18 + 11.3 = 29.3), i.e., parents will not incur ordinary child-rearing expenses when 18 years pass after a family's last birth occurrence. This assumption is reasonable because most parents who are in their mid-fifties do not have younger child requiring child rearing and related expenses. The form of function (35) is based on Gahramanov and Tang (2016), which helps achieve reasonable and irreversible retirement.

Furthermore, evidence suggests that early childhood is a critical period for cognitive and non-cognitive development, while early parental interventions are being more productive than later investments (Heckman 2008a, 2008b; Francesconi and Heckman 2016). Cunha et al. (2010) considered 13-14 years old as still being a stage of development, and we assume likewise. Thus, as a rough approximation, we set $y_2$ equal to 26, which would prevent the human capital stock from growing after more than fourteen years pass since the last birth incidence.

Next, assume the labor supply of the secondary caretaker is constant before the age of retirement and then abruptly drops to a near zero value. Let

$$\bar{L}^S(t) = d_0 - \frac{d_0}{1 + e^{-d_1(t-d_2)}} \tag{36}$$

where $d_0$ = 41.22/168 = 0.2454, $d_1$ = 1000, $d_2$ = 38.72. Hence, the secondary caretaker is expected to retire at age 63.72 (= 25 + 38.72). The labor supply values are based on a 2015 Panel Study on Income Dynamics (PSID) dataset. This dataset is also used to generate average hours worked per week for both the household head and the spouse. In all these calculations, families with a child aged 14 or below are used. We assumed that in a seven-day period, there are 24*7 = 168 hours available in total. The secondary caretaker works 41.22 hours per week, or 0.2454 of the time endowment ($d_0$ = 0.2454).

For the benchmark scenario, the aim is to ensure that the primary caretaker finds it optimal to permanently leave the labor force and retire at time $t = T_{ret}^P < T$ because the secondary caretaker permanently leaves the labor force and retires at time $t = T_{ret}^S$. As $T_{ret}^S$ is set exogenously to 38.72, and the aim is to generate an optimal retirement age for the primary caretaker that is very close to this value as well. Doing so allows for introducing $AI$ ("average income") parameter, which is a proxy for the "total per period labor income" of the family:

$$\begin{aligned} AI &= \frac{\int_0^{T_{ret}^P} \omega\epsilon(t)L^P(t)dt}{T_{ret}^P} + \frac{\int_0^{T_{ret}^S} \omega\epsilon(t)\bar{L}^S(t)dt}{T_{ret}^S} \\ &\approx \frac{\int_0^{38.72} \omega\epsilon(t)0.1248dt}{38.72} + \frac{\int_0^{38.72} \omega\epsilon(t)0.2454dt}{38.72} \end{aligned} \tag{37}$$

Shortly in this section, the way numerical values were specified in the integrands of (37) is explained (see explanations for Targets 2 and 3). Note from (37) that $AI$ is a constant number. Knowing the value of $AI$ is important because it is a nominal indicator, based on which some exogenous model parameters can be approximated. Because an aim of the model is to generate labor supply data for the parents that are very close to the labor supply and retirement values in (37), such a priori $AI$ approximation is reasonable. Next, we set $\omega = 1$. An interest rate of 3.5% is the standard target in life-cycle macro models, so we set $r$ = 0.035. The remaining exogenous parameters are chosen so that the model replicates some of the salient features observed in the PSID and ATUS datasets. Based on the above data, targets are summarized as follows.

*Target 1*. The number of children, $X(T) - X(0)$, is 1.99. This is consistent with the ATUS data, where households with primary caretaker outside the labor force tend to have 1.99 children on average.

*Target 2*. The age of last employment for the primary caretaker, $T_{ret}^{P}$, is targeted at the value of 38.72, i.e., the actual age of 63.72. We use secondary caretaker as a proxy to compute this value. Similarly, these targets are set to replicate the average retirement age derived from the PSID dataset.

*Target 3*. The measure of hours worked (since entering the model) on average before the agent permanently retires ("average work intensity") is determined to be:

$$\Phi = \frac{\int_0^{T_{ret}^P} L^P(t)dt}{T_{ret}^P} \tag{38}$$

According to the PSID data, the average hours worked for primary caretaker was 19.91 in 2014 and 22.02 in 2013 per week. We use the average of these two years (20.97) as a proxy for primary caretakers' lifetime average hours for the period in the labor force. Thus, $\Phi = 20.97/168 = 12.48\%$ is targeted, which explains why $L^P(t)$ is approximated as being equal to 0.1248 in (37).

*Target 4*. The measure of average hours spent (since entering the model) with children on productive, verifiable activities ("average intensity of verifiable time investment") is determined to be:

$$\Gamma_1 = \frac{\int_0^{y_2}(X(t) - X(0))b_{ver}^P(t)dt}{y_2} \tag{39}$$

Based on the ATUS data, primary caretakers spend 1.55 hours per week with their children on corresponding activities. These include activities such as attending child's events, assisting a child with home schooling, visiting museums, etc. Thus, $\Gamma_1$ equal to 1.55/168 = 0.92% is targeted.

*Target 5*. The measure of average hours spent (since entering the model) with children engaged in productive, unverifiable activities ("average intensity of unverifiable time investment") is determined to be:

$$\Gamma_2 = \frac{\int_0^{y_2} (X(t) - X(0)) b_{unv}^P(t) dt}{y_2} \quad (40)$$

Based on the ATUS data, primary caretakers spend 9.48 hours per week with their children engaged in corresponding activities. Activities such as assisting children with their homework, playing games with a child, reading books to a child and similar activities are classified under non-verifiable productive activities. Thus, $\Gamma_2$ equal to 9.48/168 = 5.64% is targeted.

*Target 6*. The measure of average hours spent (since entering the model) with children engaged in trivial activities ("average intensity of trivial time spent") is determined to be:

$$\Gamma_3 = \frac{\int_0^{y_2} (X(t) - X(0)) b_{tr}^P(t) dt}{y_2} \quad (41)$$

Based on the ATUS data, primary caretakers spend 12.86 hours per week with their children in corresponding activities. Activities that are not classified either as verifiable productive time or non-verifiable productive time are classified under trivial activities, including transporting children, providing physical and health care to children, etc. Thus, $\Gamma_3$ equal to 12.86/168 = 7.65% is targeted.

*Target 7*. Note per capita human capital, $h(t)$, might change up to the terminal age, beginning at $h(0) = 1$. To estimate the annual growth of human capital, we consider several data sources, including the OECD dataset (Liu 2011), Barro and Lee (2013), and Cohen and Leker (2014). The former estimates the stock of human capital using the Jorgenson-Fraumeni lifetime income approach, while the latter two use a measure based on years of schooling and assumed rate of return on education. The Penn World Table combines the Barro and Lee (2013) approach with the approach proposed by Cohen and Leker (2014) to suggest an alternative method. Overall, the annual growth rate of human capital ranges between 0.2% and 1% per year depending on the time horizon chosen.

The ATUS data reveal that secondary caretakers spend about 8.61 hours weekly on trivial activities with their children, and thus we set $\lambda$ equal to 8.61/12.86 = 66.95%. This number was calculated using the same methodology discussed for the primary caretaker.

The next task is to approximate the price parameters, $\psi_1$, $\psi_2$, and $\psi_3$. To do so, the average annual labor income of secondary caretakers is set at about $66,712, while their partners earn about $21,010 per annum. Thus, the combined labor income on average is $87,722.

Next, we proceed with estimating the costs associated with child rearing. For this purpose, we use *The Cost of Raising a Child* report by the US Department of Agriculture. The 2017 report concludes that the annual child-rearing expense estimate ranges between $12,350 and $13,900 for a child in a two-child, married-couple family in the middle-income group. We use the average of these figures as a proxy for an annual expenditure per child ($13,125). This figure reflects expenditures in several categories. We investigate these categories and allocate some of the expenses under child-rearing expenditures ($\alpha$, which includes expenses related to housing, food, etc.) with the remaining expenses shared between verifiable, unverifiable, and trivial expenditures. Based on the allocation, the total yearly child-rearing expenditure is $10,605 with the remaining $2,520 allocated to the cost of time spent with children during various activities. Based on the average number of children set at 1.99, on average, families spend $21,104 for child rearing, which comprises roughly 24.1% of total household income. Thus, the $(X(t) - X(0))\alpha$ component of the budget constraint should correspond to 24.1% of $AI$. Given that the number of children in the model would ranges from 0 to 1.99, the average of the values (0.995) is taken, and thus proxy $\alpha$ equals to 24.1%*$AI$/0.995.

Next, we allocate \$2,520 among verifiable, unverifiable, and trivial time spent with children. Based on a notional split, we calculated that the yearly monetary expenditures associated with these activities are \$901.3, \$621.3, and \$997.5, respectively. By multiplying these numbers by the average number of children in a family (1.99), we get \$1,793.59 for verifiable productive time, \$1,236.39 for unverifiable productive time, and \$1,985.03 for trivial time spent with all children.

Thus, in the context of the model, $(X(t) - X(0))\psi_3(1 + \lambda)b_{tr}^P(t)$ should comprise roughly \$1,985.03/\$87,722 (or, 2.26%) of average household income, $AI$. Because $\Gamma_3$ is targeted to be equal to 7.65%, and $\lambda$ is 66.95%, we proxy $\psi_3$ as 2.26%*$AI$/((1 + 0.6695)*7.65%). Similarly, the $(X(t) - X(0))\varphi_1 b_{ver}^P(t)$ and $(X(t) - X(0))\varphi_2 b_{unv}^P(t)$ components of the budget constraint should equal \$1,793.59/\$87,722 (or 2.04%) of $AI$, and \$1,236.39/\$87,722 (or 1.41%) of $AI$, respectively. Given the target values for $\Gamma_1$ and $\Gamma_2$, we proxy $\varphi_1$ as 2.04%* $AI$ /((1 + 0.6695)*0.92%) and $\varphi_2$ as 1.41%*$AI$/((1 + 0.6695)*5.64%).

Using GPOPS-II software (Patterson and Rao 2014), and programming multiple loops in MATLAB, the solutions of the model are simulated for a large number of parameter combinations, which is extremely burdensome computationally. To lessen the burden, we set wage rate $\omega = 1$, while assuming that some of the preference parameters of the parents are aligned. Following several experiments and trials-and-errors, we isolate a set of parameter spaces that result in a very small gap between the solutions of the benchmark model and out targets. The model solution results and the corresponding targets are presented in Table 1.

[TABLE 1 HERE]

The model parameters that produced the results presented in Table 1 are as follows: $\rho = 0.05$, $\sigma_1 = 0.2738$, $\sigma_2 = 0.20552$, $\bar{h} = 0.05$, $\mu = 0.5$, $z_1 \approx 9.979\times10^{-5}$, $z_2 \approx 2.140\times10^{-8}$, $z_3 = 1.64146$,

$\beta_1^S = \beta_1^P = 7.63$, $\beta_2^S = \beta_2^P = 0.7035$, $\beta_3^S = \beta_3^P = 3.505$, $\beta_4^S = \beta_4^P = 1.6$, and $\beta^S = 0.8$. As illustrated by Table 1, the model fits the data targets accurately.

In Figure 1, the life-cycle profiles of the model's control variables are plotted under benchmark parameterization. For example, Figure 1a) shows that the private consumption of the primary caretaker exceeds that of the secondary caretaker, implying some sort of "compensation" for the efforts the former devotes to child care. Figure 1b) shows that due to fertility efforts, the size of the family indeed increases until about age 36.3 and then stabilizes. On average, the parents have roughly 1.99 children over the entire life cycle. In Figures 1c)-1f), we plot hours per day that parents spend on various activities, including paid employment. From Figure 1f), it is clear that the target age of retirement for the primary caretaker is reached as well as the average amount of daily hours spent at work (since entering the model); however, it also shows that the optimal hours profile is such that the representative primary caretaker abstains from the labor force perhaps longer than most early primary caretakers would. Figures 1c)-1e) show that the longest number of years is devoted to spending trivial time with children because both partners derive leisurely enjoyment from it. Time spent on publically verifiable productive activities is relatively short-lived, while that on unverifiable, private activities lasts until about the parent is about 41-42 years old.

[FIGURE 1a)-1f) HERE]

In Figure 2, the evolution of the family's financial asset account and per capita human capital as state variables is plotted. The family, which is not borrowing-constrained, borrows early in life, steadily repaying debt by the end of the life-cycle. Per capita human capital drops initially because the newly born children have few skills; however, parental time investment and exogenous factors that are conducive to learning ultimately increase the human capital per person. The human capital stock reaches 1.455, averaging roughly a 0.68% increase per year.

[FIGURES 2a), 2b) HERE]

*4.2. Introducing a tax-subsidy experiment*

In this section, we introduce a simple tax-subsidy experiment and focus on its behavioral implications. The focus is not on household utility, social welfare, or growth, because the model is not a general-equilibrium model with or without market imperfections that might have justified interventions simply because a given decentralized equilibrium is inefficient. Rather, we are interested in the impact of a policy intervention on people's behaviors and human capital accumulation within the family. Because the environment in question is friction-free and in partial equilibrium, the policy-driven distortions would lower household utility, so the optimal tax-subsidy rate is not a focus. With real-life frictions that would justify increasing human capital, the tax-subsidy policy proposal might be even more desirable to implement. Even in the present setting, we derive some interesting results.

Assume that the wage earnings of the family are taxed at the rate $\tau$, while the cost of spending productive parental time on verifiable activities is reduced based on the rate $\delta$. During numerical simulations, we vary the tax rate from 0% to 100% at a 0.01 increment, assuming that the family budget equation is (29). For all tax rates, the government considers the best response of the household and computes the subsidy rate such that the government budget constraint (30) is satisfied with equality. All other parameters are fixed at their benchmark values. Figure 3 shows the tax-subsidy combinations. For example, when the tax rate is in the range of 48%-49%, $\delta$ is close to unity, and doing so balances the government budget. Note that in Figure 3, the the tax rates are reported up to about 72%, because higher tax-subsidy rates steadily reduce the optimal number of children to below unity (see Figure 4a), which is not a practically interesting case to consider for a country that is not overpopulated.

[FIGURE 3 HERE]

Figure 3) shows, that the budget-balancing subsidy rate monotonically increases with the income tax rate, which is intuitively sensible; however, for each increase in the tax rate, the rate of the subsidy increase becomes smaller.

[FIGURE 4a), 4b) HERE]

As higher taxes/subsidies stimulate human capital investment, Figure 4a) reflects the negative relationship between the quality and the quantity of children in the model. As can be clearly observed in Figure 4b), when the tax rate increases, increased subsidies significantly increase parental time investment on productive verifiable activities with children, but crowd out the unverifiable productive time investment. For example, as the tax rate rises to about $\tau$ = 4%, the time spent on productive unverifiable activities drops sharply, and then remains roughly stable with further increases in taxes and subsidies. Recall that with no tax-subsidy program in place, the family would spend roughly 9.5 hours weekly (a little over 81 minutes per day) on unverifiable productive activities, but when the tax rate increases to 4% (and the corresponding subsidy $\delta$ = 24.9%), the time spent drops to just a couple of minutes per day. Even with such a strong crowding out effect, the family's maximum human capital per person increases by about 44% compared to the benchmark case because productive verifiable time increases from the benchmark value of about 1.55 hours per week (about 13 minutes per day) to about 20.8 hours per week (or about 178 minutes per day). Trivial parental time investment falls as well but does so much more moderately. Even when an extreme case (tax rate is 72%) is considered, the trivial time spent drops from about 12.9 hours per week (about 111 minutes per day) to about 7.7 hours per week (about 66 minutes per day).

When the tax rate is even lower (e.g., $\tau$ = 2%, and the corresponding subsidy $\delta$ = 16.2%), the family spends about 131 minutes per day on productive verifiable activities, which is about 118 minutes more compared to the benchmark case. Consequently, the per capita human capital is greater by about 24%.

Clearly, per capita human capital increases as the tax and subsidy rates increase, but the effect of higher taxes and subsidies on overall work efforts is ambiguous. When the government announces a higher tax and subsidy rate combination, *ceteris paribus*, income and substitution effects influence parents. A higher tax causes a negative income effect (i.e., less purchase of leisure), but a higher subsidy causes a positive income effect (i.e., more purchase of leisure). If time spent on productive verifiable activities has been low, the reduction in the corresponding price due to the subsidy is likely to carry an insignificant income effect; however, a higher income tax also implies a lower opportunity cost of leisure (more purchase of leisure), while a reduction in the price of verifiable time spent with children similarly encourages the agent to substitute away from personal leisure and other types of productive time spent with children. Thus, the net effect on the labor supply must to be quantitatively determined.

Therefore, the average labor supply intensity (approximated by parameter $\Phi$) initially increases (Figure 4c)). The extensive margin of the labor supply increases when the tax rate rises significantly from lower rates (Figure 4d)); however, note that primary caretakers decide to enter the labor force significantly late in life (not shown in the figures). We compute that when the tax rate is moderate (such as $\tau$ = 4%), the primary caretaker *delays* his/her retirement by about 4.6 months compared to the benchmark retirement age, and enters the labor force about 6.5 months *later*. As shown in Table 2, when the tax rate is 4%, the fraction of the lifetime spent in employment is only 40.8% (lower than in the benchmark case, which is 41.1%). The corresponding weekly hours of work, averaged from the youngest age of the primary caretaker until the retirement age, increases from about 20.9 hours per week (benchmark case) to about 25.3 hours per week; however, when the tax rate is even lower (e.g., $\tau$ = 2%), the primary caretaker *postpones* his/her retirement age by about 3.2 months and enters the labor force even *earlier* (by about 3.7 months compared to the benchmark case). Table 2 shows that the corresponding fraction of a lifetime spent in employment is 42.2%, which exceeds that in the benchmark case. Weekly hours worked increase to about 24.5 hours on average. The corresponding increase in human capital stock compared to the benchmark case is still impressive and roughly equals to 24%.

The results indicate that policymakers could introduce a small, but efficient tax-subsidy program that targets enhancing productive, publically verifiable time investments in children and could reap "double dividends" from the intervention.

## 5. Conclusions

Parental intervention and a stimulating family environment play decisive roles in developing a child's cognitive and non-cognitive abilities, which jointly and strongly predict the future success of a child. A theoretical model has been developed that features endogenous fertility, labor supply, consumption/saving and human capital accumulation, when parents invest their time in their children using different approaches. We find that a small labor income tax rate (up to 2%) can be used to subsidize parental time investment in publicly verifiable productive activities involving children, to remain revenue-neutral, and to simultaneously increase children's human capital and primary caretakers' labor supply.

The model introduced is flexible enough to incorporate various market imperfections, highly heterogeneous households, and general-equilibrium characteristics and to assess policymakers' attempts to improve children's human capital via subsidizing parental involvement. Doing so would be computationally time-consuming, but it would be highly worthwhile because reducing alarming educational and earnings gaps and increasing women's participation in the labor force, can have high social returns. These extension should be examined in future research.

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**Table 1**. Data Targets and the Model Characteristics under Benchmark Parameterization

| *Target* | *U.S. Data* | *Model Results* |
|---|---|---|
| The number of children, $X(T)-2$ | 1.99 | 1.989693 |
| Retirement age (primary), $T_{ret}^{P}+25$ | 63.72 | 63.72017 |
| Average work intensity (primary), $\Phi$ | 12.48% | 12.439006% |
| Average intensity of verifiable time investment, $\Gamma_1$ | 0.92% | 0.920427% |
| Average intensity of unverifiable time investment, $\Gamma_2$ | 5.64% | 5.666505% |
| Average intensity of trivial time spent (primary), $\Gamma_3$ | 7.65% | 7.675535% |
| Annual growth rate of human capital | 0.2%-1.0% | 0.684678% |

**Figure 1**. Lifetime Paths of Consumption, Time Allocation, and the Number of Children under Benchmark Parameterization

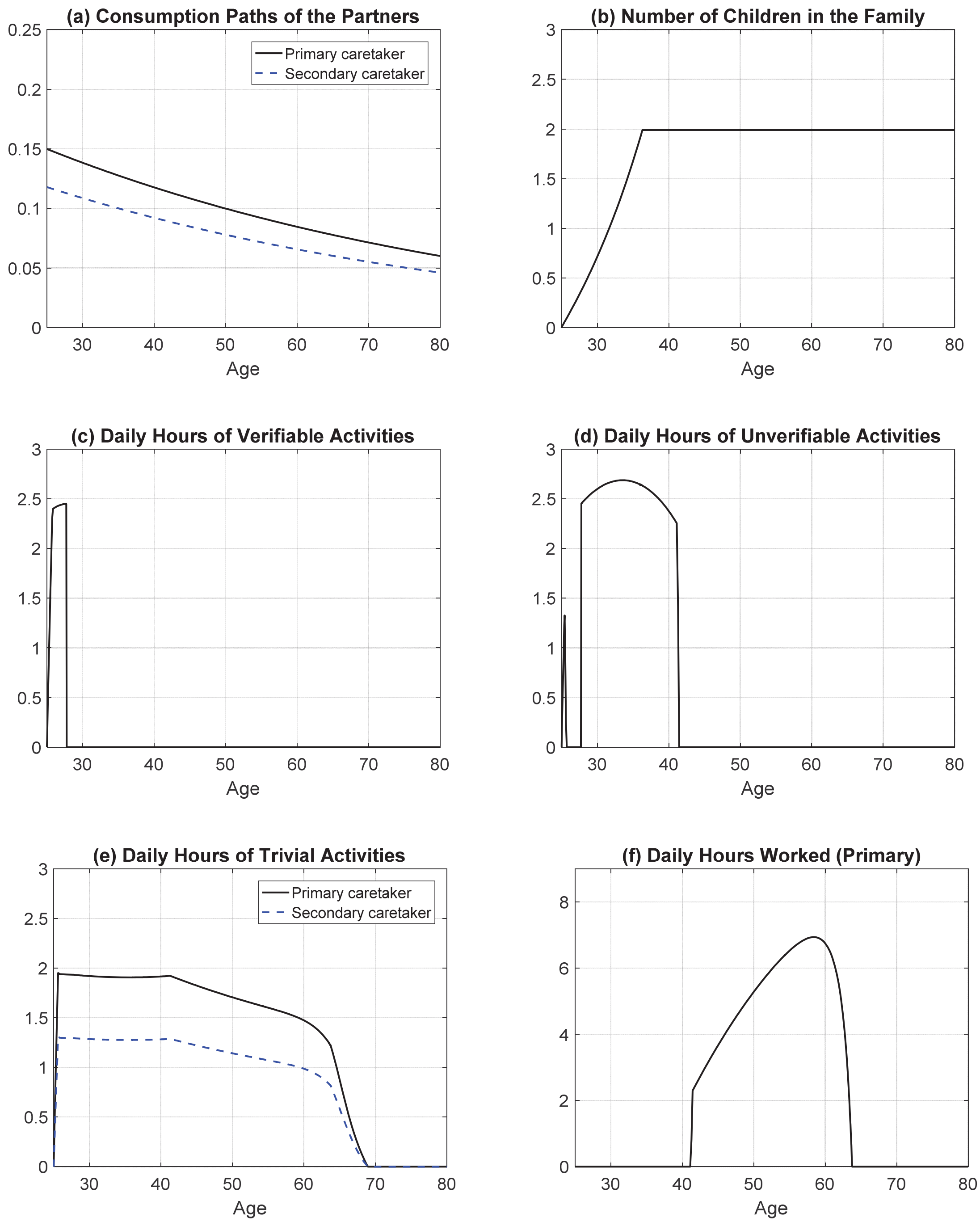


**Figure 2**. Lifetime Paths of Savings Demand and Human Capital under Benchmark Parameterization

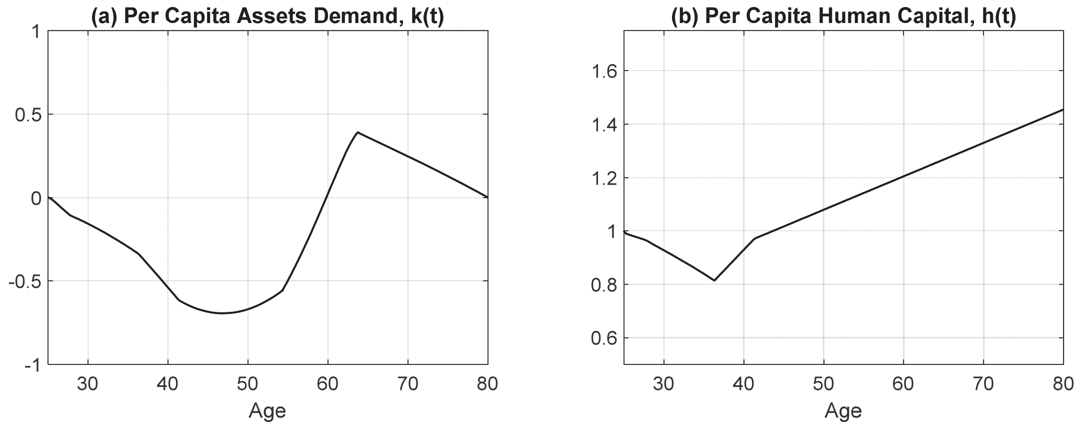
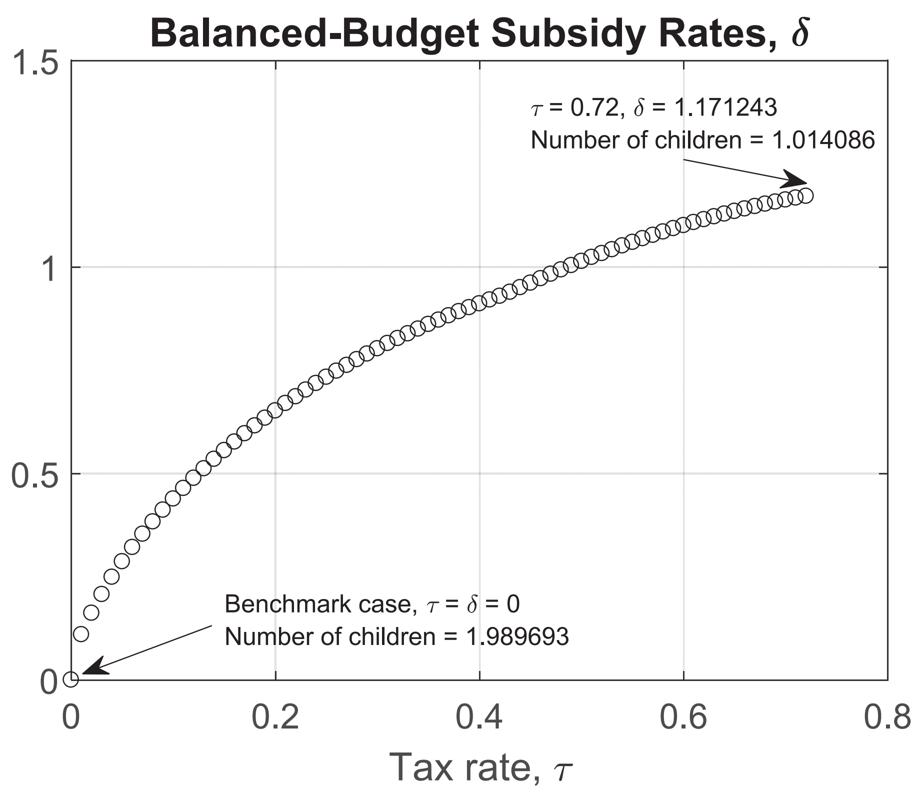
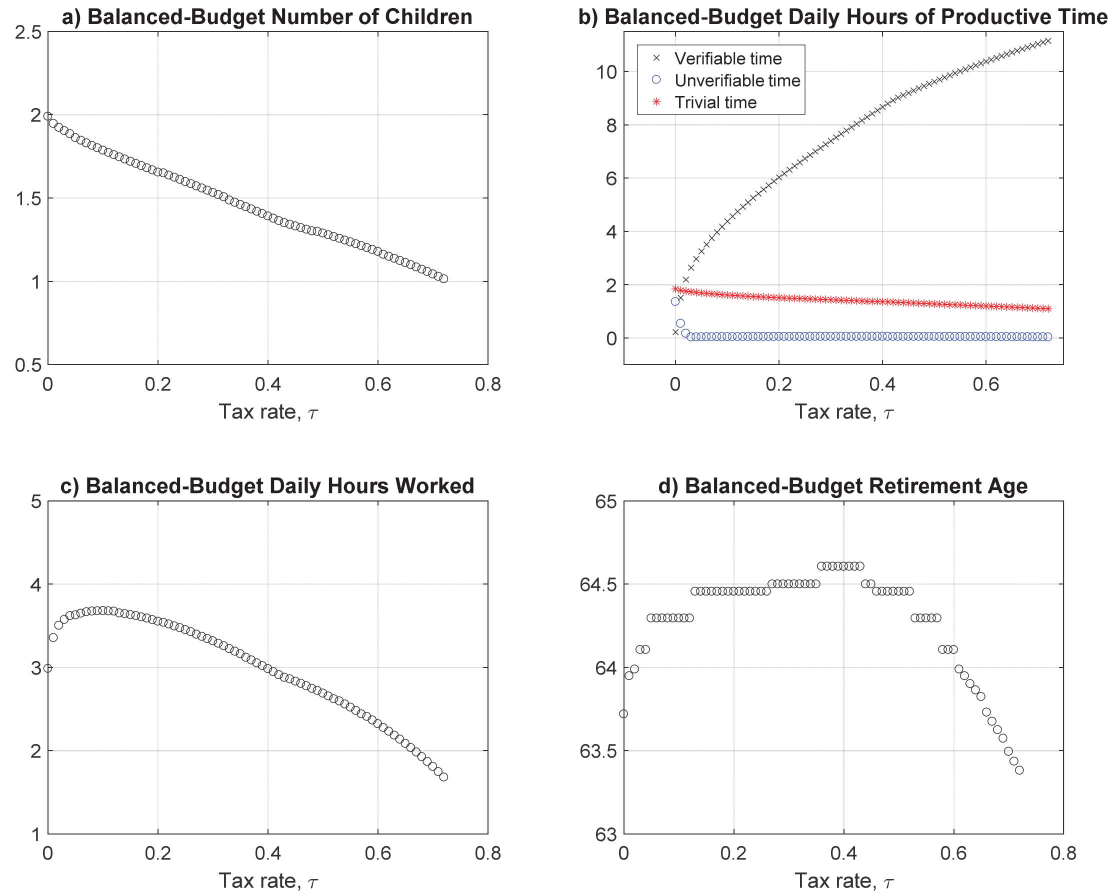



**Figure 3**. Balanced-Budget Tax-Subsidy Combinations




**Note**: The heights of the circles indicate the subsidy values that balance the budget for the corresponding tax rate. For the tax values equal to 0.73 or above, the number of children born drop below unity, so those policy parameters are not reported.

**Figure 4**. Optimal Responses of the Household to the Balanced-Budget Tax-Subsidy Experiment




**Note**: For a given balanced-budget tax-subsidy combination, the corresponding heights of the circles indicate the number of children in the family (Figure 4(a)), the hours spent at paid employment per day (Figure 4(c)), and the retirement ages (Figure 4(d)). Figure 4(b) shows the hours spent on different productive activities per day.

**Figure 5**. Annual Growth Rates of Human Capital under the Balanced-Budget Tax-Subsidy Experiment

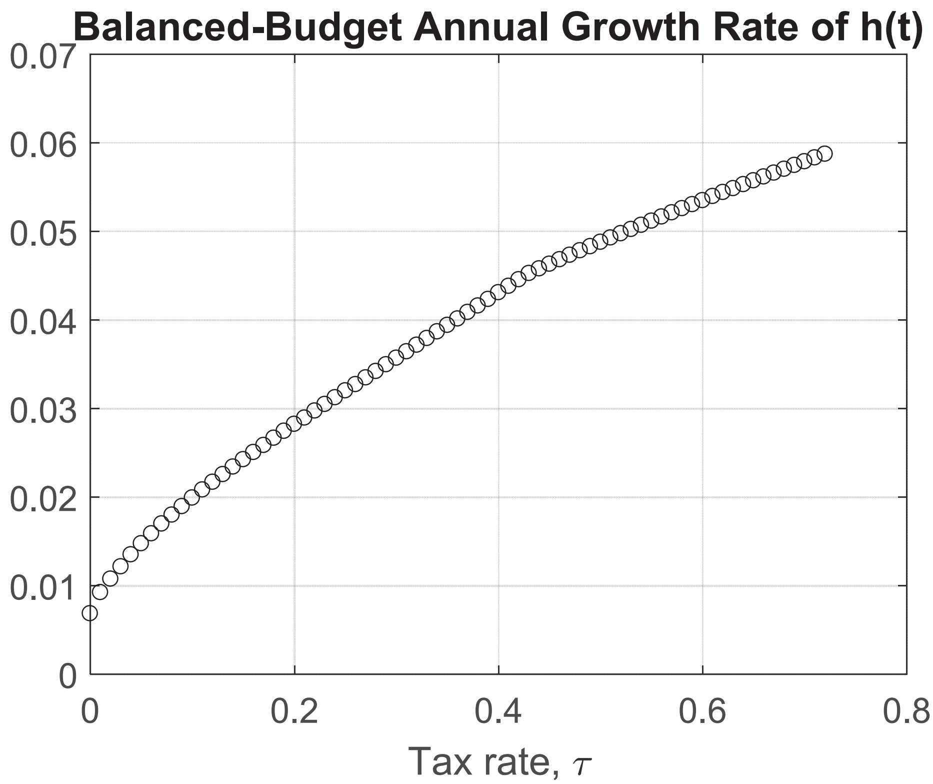


**Table 2**. Effects of Higher Taxes (and Subsidies) on Various Margins of Labor Supply.

| *Tax Rate* | *Fraction of Lifetime Spent at Employment, %* | *Weekly average Hours Worked* | *Starts Working Earlier than in the Benchmark Case?* |
|---|---|---|---|
| 0% | 41.1 | 20.9 | - |
| 1% | 42.1 | 23.5 | Yes |
| 2% | 42.2 | 24.5 | Yes |
| 3% | 41.9 | 25.0 | Yes |
| 4% | 40.8 | 25.3 | No |
| 5% | 40.4 | 25.4 | No |
| 6% | 39.6 | 25.5 | No |
| 7% | 38.9 | 25.7 | No |
| 8% | 38.3 | 25.7 | No |
| 9% | 37.8 | 25.7 | No |
| 10% | 37.1 | 25.8 | No |
| 11% | 36.8 | 25.7 | No |
| 12% | 36.2 | 25.7 | No |
| 13% | 36.0 | 25.6 | No |
| 14% | 35.5 | 25.5 | No |
| 15% | 35.1 | 25.4 | No |