ESTIMATING VEHICLE STATE BY GPS/IMU FUSION WITH VEHICLE
DYNAMICS

by

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ABSTRACT

In this thesis, an implementation of a low-cost inertial navigation system with high integrity and reliability is proposed. The proposed system can be utilized for warning drivers in land vehicle applications of an approaching dangerous situation. The solution uses Inertial Measurement Unit (IMU) to provide the vehicle's state by position, velocity and attitude readings. However, these readings quickly diverge from the correct state of the vehicle in a fashion that makes them unreliable to use. To solve this issue, a GPS is fused with the Inertial Navigation System (INS). This fusion reduces the estimation errors considerably. Two approaches are proposed for sensor fusion. In the first approach, the extended Kalman filter (EKF) is used to optimally fuse the sensor readings. In order to reduce the computational complexity of the EKF algorithm, the extended information filter (EIF) is used as an alternative state estimator. The performance of these two approaches is analyzed and compared in this thesis. Given that the proposed system is intended for use in land vehicle applications, nonholonomic velocity constraints and encoder velocity readings are fused with the algorithm to enhance the accuracy of the estimates. The filter uses a fifteen-element state. This state is composed of the errors in position, velocity, quaternions, accelerometers' biases and gyroscopes' biases. To enhance the integrity of the estimated state, the Limited Memory Noise Estimation (LMNE) algorithm is developed and proposed to detect possible bias in the encoder measurement. Extensive experimental tests were conducted to verify the accuracy of the proposed algorithms. The obtained results were compared with a commercial off-the shelf (COTS) MIDG solution. The results obtained from the EKF and the EIF were comparable in accuracy, but EIF needed less processing time compared to the EKF. The estimation accuracy was improved when the nonholonomic velocity constraints were used. The experimental results also showed that the LMNE was effective in detecting and identifying biases that were intentionally added on the encoder's velocity readings.

Search Terms: Kalman filter, information filter, velocity constraints, nonholonomic constraints, fault detection, noise estimation.

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Nomenclature

i	ECI frame
e	ECEF frame
n	Navigation (NED) frame
b	Body frame
ψ	Yaw angle
θ	Pitch angle
ϕ	Roll angle
C_x^y	Transformation matrix from a frame, x , to a frame, y .
Φ	Geodetic latitude
Λ	Geodetic longitude
R	Range from the Earth's center of gravity to vehicle's center of gravity
Ω	Earth's angular velocity ($7.292115 \times 10^{-5} \text{ rad/s}$)
μ	The Earth's gravitational constant ($3986004.418 \times 10^8 \text{ m}^3/\text{s}^2$)
J_2	Coefficient of second zonal harmonics of Earth's potential function ($1.082629989051944 \times 10^{-3}$)
R_e	Semi major axis of the Earth (6378137.0 m)
$\hat{x}$	Estimated states vector
$\bar{x}$	Estimated value, x
$(k k-1)$	Denotes a quantity at time k just before the measurement update
$(k k)$	Denotes a quantity at time k just after the measurement update
AI	Artificial Intelligence
ANFIS	Adaptive Neuro-Fuzzy Inference System
COTS	Commercial-off-the-Shelf
DAF	Distribution Approximation Filter
DCM	Direction Cosine Matrix
DGPS	Differential Global Positioning System

ECEF	Earth-Centered Earth-Fixed
ECI	Earth-Centered Inertial
EIF	Extended Information Filter
EKF	Extended Kalman Filter
ENU	East-North-Up
GLONASS	Global Navigation Satellite System
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
IDNN	Input-Delayed Neural Network
IF	Information Filter
IMU	Inertial Measurement Unit
INS	Inertial Navigation Solution
KF	Kalman Filter
LMNE	Limited Memory Noise Estimation
MEMS	Micro-Electro-Mechanical System
NED	North-East-Down
NN	Neural Network
PND	Personal Navigation Device
UKF	Unscented Kalman Filter

1. INTRODUCTION

1.1 BACKGROUND

Thousands of years ago, people used to navigate their way in long travels by looking at stars, like the star Polaris (Pole Star), which currently points almost exactly to the celestial North Pole. However, as the people's need for travelling (mainly for trading) increased dramatically, it became crucial to develop new methods and devices for navigation. So, some sources claim that the first preliminary compass was made of loadstone over 2 millenniums ago [1]. Nowadays, with the boom in information technology and computerization, new navigation systems have been developed, like the Global Positioning System (GPS) [2-3], differential GPS (DGPS) [4-7], magnetometers [6, 8-9], solid state electronic compasses [10], Inertial Navigation Systems (INS) [11-15], gyroscopes [16], encoders [17] and more.

GPS standalone solutions have been used in many land vehicle applications. However, the use of a GPS alone for providing a navigation solution can have some inadequacies under some conditions. For example, when a land vehicle passes through a natural canyon, a human-made tunnel or in an urban region with many high-rise skyscrapers, a GPS signal gets blocked. Also, sometimes, although a GPS signal is present, the receiver's signal is deteriorated due to multipath errors. Moreover, GPS usually gets updated at a very slow rate compared to update rates that an INS can reach to. More about errors that are involved with GPS is discussed in Chapter 2.

So, due to the problems accompanied with GPS standalone solutions, it can be argued that the use of a strapdown inertial navigation system (INS) is a robust solution for GPS problems. This is because it is a self-contained system. This means that the position and velocity of a vehicle can be found from its specific force and angular velocity provided by accelerometers and rate gyros, respectively. However, the errors obtained from these sensors propagate and increase in time due to drifts. That makes the use of a gimbaled INS a better solution in terms of error reduction. However, gimbaled systems are usually very expensive to be used commercially.

Due to these reasons, it becomes important and crucial to fuse strapdown INS with GPS in order to have a compact, low-cost and high-accuracy INS/GPS

navigation solution that estimates accurately the position, velocity and attitude of a vehicle. Moreover, as the solution used in this thesis is specialized with land vehicle applications, nonholonomic velocity constraints and encoder velocity readings are fused with the solution to further reduce the errors of the navigation solution [18-19]. The algorithms used to fuse the different sensors and estimate errors are explained in the following chapters.

Furthermore, since in some cases navigation sensors might get faulty and biased from true navigation information, a fault detection algorithm is developed and tested to detect simulated faults in the encoder. The algorithm should detect biases present in the sensor and take it out from the solution, thus returning the navigation solution errors to around zero. The algorithm developed is explained in detail in Chapter 4.

1.2 MOTIVATION

One major challenge facing people worldwide is how to minimize and control the growing death toll caused by vehicle accidents. Too many researches and huge amounts of money were spent attempting to tackle this issue. Hundreds of thousands are killed on the roads every year and it is becoming the responsibility of everyone to face this challenge. Death and injury tolls due to road accidents are mostly caused by unawareness of incoming danger by drivers. Also, these tolls are increasing day by day. Research is going on worldwide to try reducing the possibility of accidents. This work falls into this category.

1.3 THESIS OBJECTIVES

In this work, a low-cost navigation system is proposed for enhancing highway traffic safety even in situations such as fog or rain. The navigation system used in this work can be implemented in an instrumentation system installed in each vehicle. This system may consist of a microcontroller, a Zigbee transceiver, a GPS receiver, an IMU and encoders. The microcontroller system should sample the measurements of the IMU at a rate of 50 Hz, and those of the GPS at 5 Hz fused with the vehicle's speedometer. The accelerometers would measure acceleration and rate gyros would measure angular velocity. These measurements would be integrated to find the position and velocity [11-15]. The position, velocity, and attitude obtained for a vehicle would then be transmitted to other vehicles within a close range using the

Zigbee network, so that the driver can be warned if there is a dangerous situation [20-22]. Two sensor fusion algorithms are used in this thesis, which are: extended Kalman filter (EKF) [11-12] and extended information filter (EIF) [12]. When a filter is described as extended, it means that its state matrix is linearized. These two methods are used to obtain a navigation solution by fusing INS, GPS, and the odometer with nonholonomic velocity constraints. The two methods are compared with one another. Moreover, a fault detection algorithm is applied to detect any fault present in the encoder and correct the solution accordingly.

Figure 1 Collision Forecast Using Vehicles' Relative Positions and Velocities [23]

1.4 CONTRIBUTIONS

This thesis is part of a wider project that aims at enhancing drivers' awareness of possible conditions, especially in foggy or rainy weather, where vision systems are not feasible. To achieve this goal, this thesis has the following contributions:

- 1) A low-cost hardware composed of an IMU, a GPS antenna, an encoder, a microprocessor and a Zigbee transceiver is proposed.
- 2) The measurements are fused using two approaches
 - a) An EKF
 - b) AN EIF
- 3) The performance of the two approaches is analyzed and compared.
- 4) Velocity constraints are used to enhance state estimation accuracy.

- 5) To enhance the integrity of the navigation algorithm, a fault detection algorithm is proposed to detect possible bias in the encoder measurements.
- 6) Extensive experimental tests were conducted to compare and verify the accuracy of the proposed algorithms.

Next, the thesis structure is summarized.

1.5 THESIS STRUCTURE

This thesis consists of the following 5 chapters:

- Chapter 1 is an introduction about the theoretical background behind inertial navigation systems in order to give a better understanding of the topic. This would allow the reader to cope more easily with the materials in the following chapters. Also, the objectives and contributions of this thesis are explained in this chapter. This chapter also consists of a literature review of some external sources that have been published about topics in the same field.
- Chapter 2 describes the inertial navigation system (INS) and explains the algorithms of EKF and EIF that were used in this thesis. It also describes the nonholonomic velocity constraints of a land vehicle and how to fuse them in the navigation solution along with odometer readings.
- Chapter 3 shows a set of results obtained experimentally. It compares the results obtained using EKF algorithm and EIF algorithm. It also discusses the effects of nonholonomic velocity constraints fusion on both algorithms.
- Chapter 4 explains the Limited Memory Noise Estimation (LMNE), which is the fault detection algorithm that was developed and used in this work. Also, the effect of the algorithm is demonstrated by applying it to experimental data that was used in Chapter 3.
- Finally, Chapter 5 is the conclusion of this thesis and includes proposed future work that can be performed in this field.

The next section summarizes the proposed system.

1.6 SUMMARY OF PROPOSED SYSTEM

The following diagram shows the layout of the whole system that can be used for accident avoidance purposes. In order to know our position, velocity and attitude, three navigation sensors are being used, which are: encoders, IMU and GPS. The

readings obtained from these sensors are fused and estimated through a Kalman or an information filter, to give one pose for all of these sensors. Then the position, velocity and attitude of each vehicle are communicated to other vehicles to be analyzed by the microcontroller. Based on this, a warning can be released in case of a dangerous situation.

Figure 2 Whole System Flowchart [23-24]

The next section consists of a brief literature review of the work that was performed and published about the field of navigation.

1.7 LITERATURE REVIEW

Many research works have been made to obtain a reliable positioning method for different applications of land vehicle navigation. In one work, GPS and encoders, fused with a digital map, which is a layer of General Information System (GIS), are used to estimate a vehicle's state [25]. However, the implementation of such a system would be too expensive and not applicable in places where a precise digital map, built up from small 2-D curves, is not available.

Another work discusses the use of radar and vision based systems to predict traffic accidents [26]. Although this approach has an advantage of preventing collisions with objects in addition to vehicles, they usually do not provide a 360

degree view. To get a 360 degree view, multiple vision sensors have to be used and these are quite expensive.

Recently, some interesting developments have been made for the EKF to develop a collision prevention system and to make a more efficient nonlinear estimator [27-28]. Some of these studies went to the developments of the unscented Kalman filter (UKF) and second-order divided difference filter (DDF2). A comparison between EKF, UKF and DDF2 was performed by J. Ali and M. Mirza, in order to demonstrate DDF2 is more accurate in attitude error estimation than EKF and UKF [29]. The DDF uses a linearization approach known as the divided difference approach. This approach is well explained in [30-34]. This approach is based on expanding the nonlinear functions to higher order terms easily by replacing derivatives with functional evaluations. Although the DDF might seem a bit similar to that of the UKF, we do not have to go through finding the Jacobian for the nonlinear system dynamics. Also, the UKF development is too complicated for the slight improvement that it provides when compared to EKF, which is used in this thesis.

In another work presented in [35], the implementation of car to infrastructure communication is suggested. This infrastructure communication can provide vehicle state. This approach is suitable for applications aided with real-time traffic data or internet connectivity. However, collision prediction communication is only required between vehicles present within the vicinity of each other. Vehicle-to-Vehicle communication has the advantage of not requiring an infrastructure. It can work in any area where all vehicles are provided with a collision avoidance system along with a compatible communication protocol. Different issues faced with the implementation of this system are discussed in [36].

In [37], measurements are obtained using a gyroscope, GPS and a speedometer. In the solution proposed in [37], a nonlinear observer which consists of three subfilters is used in developing a vehicle navigation solution. One of the subfilters has two gains and estimates yaw angle and gyro bias. Another subfilter has one gain and is responsible for estimating the speedometer's scaling. The third one also has one gain and gives estimates of the vehicle's velocity and position. In the filtering stage proposed in [37], nonholonomic constraints are taken into consideration to further enhance the estimates. When tested, the algorithm gave reliable results.

However, the algorithm does not take into consideration the accelerometer bias and it also does not use the quaternion approach which is known for its stability in finding a vehicle's attitude.

Constraints are also fused in a navigation solution for land vehicles proposed in [18]. These constraints depend on the orientation of the vehicle relative to the earth and the relation between the attitude of the vehicle with its velocity, and are mainly used to enhance the estimates, especially the estimates of velocity and position.

Velocity constraints are also fused with an INS and GPS in [19] for a navigation solution consisting of nine states. The results of this fusion of the velocity constraints along with the vehicle's speed showed that the observability and attitude of the vehicle are guaranteed. This leads to decreases in the errors of these states.

Also, several navigation techniques were developed recently for navigation using artificial intelligence (AI) rather than using Kalman filter (KF) to avoid inadequacies present in the approach of KF [38-44]. Some of the main inadequacies in the KF approach for GPS/INS integration are a stochastic error model for every sensor used, which should be predefined, and values like correlation time, variance and covariance should be known accurately. However, these AI-based techniques relate INS errors to INS output at specific time intervals only. They do not take into consideration the error dependency on past INS readings. To overcome this problem, a study proposes the use of Input-Delayed Neural Networks (IDNN) to model the INS errors based on current and some of the past INS data [45].

Nowadays, the issue of GPS outages is becoming more frequent, since for example, in the UAE, life is becoming more urbanized, which means having more buildings and urban canyons (tunnels). These man-made structures contribute in GPS outages, which degrade the state estimate of a navigation solution. The effectiveness of these AI algorithms during GPS outages is also demonstrated in [45-47].

Dynamic networks, which use the method of a sliding window that moves with every time step, are more reliable than static networks, which do not use sliding windows [48]. This is because they can be trained to learn patterns that are time-varying. However, the drawbacks of these networks are mainly the increase in complexity and training time [48].

In [49], uncompensated INS measurements are fused with measurements from a differential global positioning system (DGPS) using two different architectures that involve multilayer feed-forward artificial neural networks (ANN). One of the two architectures deals with updating the position and the other architecture deals with updating the position and velocity. The results obtained using the two methods were compared and are shown to be better than conventional KF fusion method [49].

Neural networks are also used with wavelet multi-resolution analysis (WMRA) in [46] for fusion of strapdown INS, GPS and magnetometer in order to enhance results when having GPS outages. This is done by training NN while having a GPS signal and using WMRA for de-noising errors.

In [50], the authors suggest two algorithms to find the location of a mobile using received signal strength (RSS) based on artificial intelligent (AI) approaches. The study used data from Global System for Mobile communications (GSM) to obtain data. One of them is the focused time delay neural networks (FTDNN), which uses tapped delay lines (TDL) memory in the input layer, and the other uses distributed time delay neural networks (DTDNN), which uses a temporal back-propagation learning method to distribute TDL memory throughout the network.

In [47], ANFIS is used to estimate errors in INS position using the position of the INS and the time with a non-overlapping fixed size shifting window. Also, a collision-free navigation system that uses six different AI approaches is developed to control a car-like robot [51]. One of the approaches used in [51] uses the neuro-fuzzy system, where the distance and angle are used as the input to the neuro-fuzzy system, while the deviation and acceleration are obtained as the output of the system. Also, some of the other approaches used in [51] involve a fuzzy logic controller (FLC) and the use of a genetics algorithm (GA) to tune ANFIS. These approaches are then compared and analyzed.

So, due to the inadequacies of using ordinary Kalman filtering for fusing GPS and INS only, in this work, vehicle constraints are also fused with the GPS/INS to enhance the error estimation. Also, other than EKF, an EIF, which has a lighter computational load compared to EKF, is going to be demonstrated and compared with the results of the EKF algorithm.

Moreover, a fault detection algorithm is applied in this work in order to detect any fault readings of encoders. In cases of GPS outages or multipath errors, which are frequent in urban places, the functionality of the EKF is degraded drastically. To reduce this problem, a two-filter smoother (TFS) algorithm has been developed in [52] and compared with a previously developed algorithm in [53], called the Rauch-Tung-Striebel smoother (RTSS). Like TFS, RTSS is also used to improve land vehicle navigation in such cases. RTSS does not require information of full-scale backward Kalman filter (BKF). This is due to the reason that it uses all information available in forward Kalman filter (FKF), while TFS combines results from FKF and BKF by minimizing the smoother error covariance [52]. There were similar results in terms of error improvement; however, TFS is more suitable for nonlinear applications compared to RTSS.

In [54], faults in a navigation system that uses KF for sensor fusion, are detected using an algorithm that calculates the spectral norm of matrix $A(k)$ and the statistic of spectral norm arithmetic mean $\overline{\|A(k)\|_2}$. So, if measurements exceed a predefined threshold of $\overline{\|A(k)\|_2}$, they are considered faulty.

In [55], four algorithms were implemented on an aircraft to verify the covariance matrix of KF innovation sequence. These algorithms are compared with respect to detected minimum fault rate. They are also compared with respect to detection time. The algorithms were tested on longitudinal aircraft dynamics. A KF was used to estimate aircraft control system state vector. It was discovered that the algorithm that verifies the covariance matrix of the innovation sequence using the sum of all elements of a matrix calculated in [55] was the best in fault detection as it was able to detect very small changes rapidly. An optimal eigenvector of the product of that matrix and the covariance matrix is used to calculate the sum of these elements.

In another research, which was about developing a fault detection algorithm for a spacecraft, data from sensors were fused using a linearized KF to estimate attitude and gyroscope bias [56]. The fault isolation in the algorithm proposed is achieved by monitoring the innovation sequences of the EKF. This isolation is done through two phases. In the first phase, the fault sources are verified by using two EKFs which provide state estimates and residuals. In the second phase, faulty components are identified through testing of multiple hypotheses.

In the last few years, some researchers started using artificial intelligence for fault detection. For example, in [57], techniques based on fuzzy logic are used for this purpose by adjusting the measurement covariance based on its relative quantity. The proposed algorithm in [57] was compared to other fault detection algorithms and it showed its capability of detecting both soft and hard sensor failures.

In [58], a hybrid intelligent system is used to detect faulty sensor readings. The approach used by the authors consists of two phases. The Fuzzy C-Means (FCM) algorithm is used in the first phase for separating signals. Then, a single fuser is produced by fusion engine depending on input sensor signals. In the second phase, which is the fault detection phase, an artificial immune system (AIS) is used to detect faults.

In [59], a method based on the Bayesian belief network (BBN) is used for fault detection and identification. A single-sensor BBN is used for applications with transient processes. Then, a multi-stage BBN is used to avoid direct linkage in the BBN network. A model-driven and a data-driven multi-stage BBNs were used. The difference between lower and upper bounds of fault detection is usually less for data-driven multi-stage BBN than model-driven multi-stage BBN. This is because not all state combination of parent nodes are included in a multi-stage BBN.

In the next chapter, the methodology of the solution's algorithms will be explained.

2. INERTIAL NAVIGATION SYSTEMS

2.1 INTRODUCTION

To determine the pose of a vehicle, we can use the principle which implies that an object would keep a steady uniform motion, unless subjected to an external force, which generates acceleration. In order to measure this acceleration, an accelerometer is needed. Then, this acceleration is integrated once to get the velocity, and twice to get the position with respect to an initial condition. However, the acceleration that we measure with an accelerometer is in fact a combination of acceleration due to gravity and other external forces. The gravity component needs to be removed from the acceleration. In order to do so, the accelerometer's attitude with respect to the local vertical needs to be measured, and here comes the role of a gyroscope. The gyroscope measures the angular velocity, which when integrated, gives us the angle change with respect to an initial condition.

2.2 COORDINATE FRAMES

In order to have a good understanding of how INS and their algorithms work, and before we go into more detail, the coordinate frames used in inertial navigation systems should be understood. So, listed below are the main coordinate frames used in inertial navigation systems. More about the inertial coordinate frames can be found in the Appendix.

- Earth-Centered Inertial (ECI) (i-frame):

This frame's origin is fixed at the center of the Earth. The axes of this frame do not rotate with the Earth and always point to reference stars. This frame is considered a reference to other coordinate frames.

- Earth-Centered Earth-Fixed (ECEF) (e-frame):

In this coordinate frame, we have the origin fixed to the center of the Earth. The x-axis points to the North Pole of the Earth, while the y-axis points to the intersection point between Greenwich meridian and the equator. The z-axis is orthogonal to both, the x and y axes, and it is found using the right hand rule. Unlike ECI frame, this frame moves with the Earth.

- Navigation Frame (n-frame):

In this coordinate frame, the origin is at the location of inertial navigation system. The x and y axes are tangent to the Earth ellipsoid, with the x-axis pointing to

the North Pole of the Earth, and the y-axis pointing towards the east. The z-axis points perpendicularly down to the ellipsoid, which means that this axis does not have to pass through the center of the Earth. That is why sometimes this frame is referred to as NED frame, meaning North East Down frame. This frame is also known as the geocentric frame.

- Body Frame (b-frame):

This frame is fixed to the vehicle. The x-axis of this frame is aligned with the x-axis of the vehicle, pointing forward. The y-axis points perpendicularly to the right, while the z-axis points perpendicularly down. The frame moves with the vehicle. The following figure shows the axes of the three coordinate frames: ECI, ECEF and body frames.

Figure 3 ECI, ECEF and Body Frames

- WGS-84 Frame:

GPS uses another coordinate frame, called the World Geodetic System (WGS-84). This frame is an ECEF frame. Its origin is at the center of the Earth. The x-axis

points to the intersection point between Greenwich meridian and the mean astronomic equator. This equator is the reference meridian which is parallel to the zero-meridian that is defined by the Bureau International de L'Heure (BIH), based on adopted longitudes for its stations. The y-axis is measured 90 degrees to the east of the x-axis and completes the right hand rule in the mean astronomic equator plane. The z-axis is parallel to the conventional international origin for a polar motion based on the adopted latitudes for BIH stations.

The different types of INS are illustrated in the next section.

2.3 INS TYPES

Mainly, inertial navigation systems fall under two categories:

1. Gimbaled (Platform) Systems.
2. Strapdown Systems.

In gimbaled systems, accelerometers and gyroscopes are fixed on an inner stationary gimbal in a certain direction and pose relative to inertial or earth navigation frame. This state does not change even if the vehicle's pose and orientation change. This is because the inner gimbal is isolated from being affected by the motions of the surrounding vehicle. For the inner gimbal to be stable, torque actuators are used along with servo feedback. These systems usually have high accuracy and cost due to the high complexity of their control system and design that can only measure very small changes in deviations. One main drawback of such systems is that they sometimes suffer from gimbal lock. This lock can be fixed by adding a fourth gimbal to the already present three-axis gimbals. However, this solution is usually a very expensive one.

Figure 4 A Gimbaled System

On the other hand, strapdown inertial navigation systems are fixed to the body of the vehicle and are not stable in space as they move with the body's motion. The system's angular motion is continuously measured with a set of three perpendicular rate gyros, and the acceleration is measured with a set of three perpendicular accelerometers. Such systems are cheaper, mechanically simpler and stronger than gimbaled ones. However, it is more complex mathematically, especially if pose in earth and navigation frames are needed.

Figure 5 A Strapdown System [60]

2.4 WIRELESS COMMUNICATION

For communication between sensors in different vehicles, Zigbee technology is going to be used. Zigbee is a low-power and low-cost wireless mesh networking standard. It uses small digital radios for high level communication protocols. This is based on IEEE 802.15.4-2003 standard for Low-Rate Wireless Personal Area Networks (LR-WPANs). It is targeted at radio-frequency (RF) applications. These applications require a long-lasting battery, low data rate and secure networking. Zigbee usually operates in industrial, scientific and medical (ISM) radio of 2.4 GHz worldwide. Zigbee can support a maximum of 65 thousand nodes. This is enough for the low range that these devices cover. Since sensors need low latency and low energy consumption for large device arrays and longer battery life; Zigbee is a very suitable way to communicate wirelessly between different endpoints and routers. Moreover, Zigbee is a low-cost solution compared to other alternatives available in the market [22].

Figure 6 Model of Standard Zigbee Network [22]

2.5 SENSORS ERRORS

These sensors, accelerometers and gyroscopes, are usually known together as inertial measurement systems (IMU). Mainly, there are two advantages of using IMU, which are:

1. The position and velocity of an object can be evaluated in any frame.
2. The readings are not affected by wheel slip.

However, the main errors in such sensors come from scale factor, bias and non-linearity in readings. Consequently, any misalignment in the unit's axes with respect to local navigation axes would cause inaccurate results in the component of acceleration due to gravity, which in turn, affects the position and velocity. These errors are carried on for the next time steps. So, they keep propagating in time.

2.5.1 ALIGNMENT AND INITIALIZATION ERRORS

One of the most important issues in any navigation solution is the initialization of position, velocity and attitude, and the alignment of the IMU unit. The initialization of velocity can be set to zero thus assuming a stationary vehicle initially. However, the position can be initialized from an external source like a GPS antenna or by manually entering initial position values. Also, the initialization of the attitude angles is very important as they are used in the initialization of the quaternions by initializing

cosine rotation matrices. Therefore, it should be ensured that the IMU unit is aligned accurately such that the axes of the unit's body frame coincide with the axes of the vehicle's body frame. There are several techniques which were developed for alignment purposes. Some of these are [60]:

- Estimating alignment variables by matching the position with a GPS.
 - The usage of a star tracker or an optical line of sight reference to a ground station for estimating alignment.
 - Using the gyrocompass alignment approach, where acceleration readings are used to rotation directions and local vertical thus estimating north direction.
- The next section talks about the characteristics and errors involved with GPS.

2.6 GPS FUNDAMENTALS

A satellite navigation system that has a worldwide coverage is known as Global Navigation Satellite Systems (GNSS). Currently there are four GNSSs, which are:

- Navigation and Satellite Timing and Ranging (NAVSTAR) that is commonly known as the Global Positioning System (GPS)
- Global Orbit Navigation Satellite System (GLONASS)
- Galileo
- Compass

Since GPS is well established and low-cost GPS receivers are highly available, it was decided to use GPS in this work.

GPS consists of a set of 24 satellites orbiting the Earth unevenly in 6 symmetrical orbital planes. These satellite locations in space can be known on Earth through GPS receivers. GPS uses triangulation method to know a certain location on Earth. Four or more satellites have to be in sight of a GPS receiver to be able to know the location of that receiver. Three distances to available satellites are used to specify the three dimensional position of a GPS receiver. The forth satellite is used to know the GPS clock time.

There are two types of positioning involved with a GPS: static and kinematic positioning. Static positioning involves the use of a GPS receiver of a fixed location to know its position. This type of positioning is usually used for applications, like detecting changes of position in Earth's tectonic plates. However, the kinematic

positioning, which is used in this thesis, involves finding the position of a GPS receiver fixed on a moving platform, like a vehicle.

2.6.1 GPS ERRORS

A GPS signal consists of a carrier sinusoidal wave modulated in phase by binary code. A GPS signal can be represented as:

$$s(t) = ac(t)d(t)\cos(2\pi ft) \quad (1)$$

where a is the signal's amplitude, c is the step function with values between -1 and 1, d is a data message, f is the frequency of the wave and t is the time period.

GPS satellites transmit two types of codes: coarse acquisition (C/A) code and precision code (P-code). P-code has a wavelength that is 10 times higher and chipping rate than the C/A code. GPS signals are transmitted as L1 signal and L2 signal. L1 signal has a carrier frequency of 1545.72 MHz and a wavelength of 0.1903 m, while the L2 signal has a carrier of 1227.6 MHz and a wavelength of 0.2442 m.

This GPS signal suffers from various sources of errors. The following table shows an overview of these errors [61].

Table 1 GPS Error Sources

Error Source	Typical Magnitude
Receiver clock error (synchronized)	1 μs (300 m)
Ionospheric delay	≤ 5 m
Selective availability (not after 2001)	100 m
Satellite synchronization to UTC	100 ns (30 m)
Tropospheric delay	≤ 0.5 m
Residual satellite clock error	100 ns (6 m)
Multi-path	≤ 1 m (P-code), ≤ 5 cm (phase)
Receiver noise	1 m (C/A code), 0.1 m (P-code), 0.2 mm (L1 phase)
Orbit error (precise, IGS)	20 cm

One of the crucial GPS errors is the delays that occur in the Ionospheric and Tropospheric layers of the earth's atmosphere. The Ionosphere lies in an altitude of 80 to 400 km, while troposphere lies up to about 20 km above sea level. The delays in the Ionosphere layer happen due to the large quantities of free electrons and positively charged ions in that layer. Delays in the Tropospheric layer occur due to different water vapor concentrations present in the layer. However, the Tropospheric delays are less than delays in the Ionosphere, because the amount of free electrons is less.

Due to the reasons mentioned, the GPS signals, which travel with the speed of light in outer space, get refracted when they enter these atmospheric layers. This causes delays in the signal. The errors caused in the Ionosphere are usually compensated by COTS GPS receivers. However, these receivers cannot compensate for errors that are caused due to runtime changes in the Ionosphere, like strong solar winds. Both high and low frequency signals, L1 and L2, respectively, are used to calculate the difference in their delays. This is because a signal with a low frequency is delayed more than that of a high frequency. This is due to the fact that a signal is slowed down while passing the Ionosphere inversely proportional to its frequency. On the other hand, the errors that occur in the Troposphere cannot be compensated, as they change with different weather conditions. Eventually, sometimes a general calculation model is used to try compensating for these errors as much as possible.

Figure 7 GPS Atmospheric Delays

Also, a very common source of errors associated with GPS is what is known as multi-path errors. This type of error occurs when a GPS receiver is placed in a partially open space such as between buildings and trees. This happens due to the reflections of the GPS signal from the surfaces of the obstacles that surround the GPS receiver. This results in a distorted position reading obtained by the GPS receiver as shown in the following figure. Even if a GPS receiver has access to a full hemisphere of sky, multi-path errors do still occur. However, they are reduced drastically.

Figure 8 Multi-Path Error

2.6.2 DGPS

DGPS is an enhancement to the conventional GPS with an accuracy which can reach about 10 cm. The main principle behind this technique is that, unlike GPS, it has a ground reference station which enhances the accuracy of the result. It does so by finding the difference between the position obtained through satellites and the distance from that position to a ground reference station of a previously known location. Due to its high cost, this technique was not implemented in this work. The following figure illustrates how this system works.

Figure 9 Functionality of DGPS

2.7 VEHICLE'S DYNAMICS

2.7.1 VELOCITY AND POSITION

The vehicle's dynamics are obtained from the accelerometers' and gyroscopes' measurements. The time rate of change in the vehicle ECEF velocity can be shown to be given as [14]:

$$\dot{V}^e = C_B^E F^b - 2\Omega V^e - \Omega^2 P^e + G^e \quad (2)$$

where $2\Omega V^e$ represents the Coriolis acceleration and $\Omega^2 P^e$ is the Centrifugal acceleration. After obtaining the velocity of the vehicle, we can integrate the velocity to find the vehicle's position, that is

$$\dot{P}^e = V^e \quad (3)$$

As the errors that accumulate due to bias that we have in the accelerometer's specific force, F^b , the results obtained from equations 2 and 3 quickly diverge from the correct readings, making it inapplicable to depend on them. This makes it necessary to fuse GPS position and velocity readings using a sensor fusion algorithm that is explained later.

2.7.2 ATTITUDE

The pitch, roll and yaw are included in the transformation matrix from the body frame to the navigation frame, C_b^n , that is updated each time step. Angular rates that are obtained from rate gyros are used in the process of attitude calculations. There

are three algorithms to do so: the Euler, Direct Cosine Matrix (DCM) and Quaternions algorithms.

- Euler Algorithm:

In this approach, the transformation matrix, C_B^E , that transforms body frame to ECEF frame, is calculated as:

$$C_B^E = C_N^E C_B^N \quad (4)$$

C_N^E is the transformation matrix from ECEF frame to navigation frame, and C_B^N is the transformation matrix from body frame to navigation frame.

$$C_N^E = \begin{bmatrix} -\sin\Phi \cos\Lambda & -\sin\Phi \sin\Lambda & \cos\Phi \\ -\sin\Lambda & \cos\Lambda & 0 \\ -\sin\Phi \cos\Lambda & -\cos\Phi \sin\Lambda & -\sin\Phi \end{bmatrix} \quad (5)$$

$$C_B^N = \begin{bmatrix} \cos\theta \cos\psi & \sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi & \cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi \\ \cos\theta \sin\psi & \sin\phi \sin\theta \sin\psi + \cos\phi \cos\psi & \cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi \\ -\sin\theta & \sin\phi \cos\theta & \cos\phi \cos\theta \end{bmatrix} \quad (6)$$

Note that Φ and Λ are the latitude and longitude of the vehicle, respectively, and ϕ, θ and ψ are the roll, pitch and yaw angle of the vehicle, respectively. As the body frame rotates, the angles are updated as follows:

$$\dot{\phi} = (\omega_y \sin \phi + \omega_z \cos \phi) \tan \theta + \omega_x \quad (7a)$$

$$\dot{\theta} = \omega_y \cos \phi - \omega_z \sin \phi \quad (7b)$$

$$\dot{\psi} = (\omega_y \sin \phi + \omega_x \cos \phi) \sec \theta \quad (7c)$$

From these equations, we might notice that there are singularities in ϕ and θ at $\pi/2$. However, this is not likely to cause problems for land vehicles. This singularity can be seen as a weakness for other applications, such as those which are used for aerial purposes. Another weak point of this approach is its high demand in terms of computation. Therefore, it needs powerful processing in case of high update rates. The only point that can be seen an advantage is its easiness in terms of simplicity and the assumptions that can be made for small angles [12].

- Direction Cosine Matrix (DCM) Algorithm:

In this algorithm, the cosine rotation matrix, C_b^n , that is used for the conversion from body frame to navigation frame, is propagated by:

$$\dot{C}_b^n = C_b^n \Omega_{nb}^b \quad (8a)$$

where C_b^n is initialized using equation 6 and Ω_{nb}^b is skew symmetric matrix of the angular velocity of the body relative to the navigation frame represented in the body frame. This algorithm is commonly used for land navigation applications and needs the update of nine variables iteratively [12].

- Quaternion Algorithm:

Usually the quaternion approach is used to avoid the singularity problem that is faced with the Euler approach. In [62], the quaternion algorithm is used in avoiding uncertainty in initial attitude. This algorithm is computed using the computer frame. Also, for fusion of nonlinear data, the distribution approximation filter (DAF) is used in [61]. The DAF has the advantage of avoiding the complications that are usually accompanied with calculating Jacobians in EKF algorithm. It does this by approximating a distribution rather than approximating an arbitrary nonlinear function, as the case is in EKF. More is explained about DAF algorithm in [63-64].

In [65], researchers use acceleration and orientation from GPS velocity to determine angles using a personal navigation device (PND). Three methods are proposed by the researchers to determine yaw from roll and pitch. Taking the road angles also into account, the researchers are able to obtain angles within a 2 degrees error. The only concern about such a system is the time it takes to be initialized.

The quaternion algorithm has the strength of getting rid of the singularity problem that is present in the Euler algorithm. It also needs the update of four variables only every iteration. In the quaternion algorithm, we can represent the attitude as quaternions that can be initially calculated from C_B^E . In C_{ij} , the subscript, i , is the i -th row, and the subscript, j is the j -th column. The initial attitude is calculated as

$$q_0 = \frac{1}{2} \sqrt{1 + C_{11} + C_{22} + C_{33}} \quad (9a)$$

$$q_1 = \frac{c_{32}-c_{23}}{4q_0} \quad (9b)$$

$$q_2 = \frac{c_{13}-c_{31}}{4q_0} \quad (9c)$$

$$q_3 = \frac{c_{21}-c_{12}}{4q_0} \quad (9d)$$

These quaternions are assembled in one matrix as follows.

$$q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (10)$$

The change in the attitude of the vehicle is measured by the angular velocity of the vehicle measured by the three-axis gyroscopes. The time rate of change of the attitude of the vehicle is given as:

$$\dot{q}^e = \frac{1}{2} \Omega_{eb}^b q^e \quad (11)$$

where Ω_{eb}^b is skew symmetric matrix of the angular velocity of the body relative to the ECEF frame represented in the body frame. To update the quaternions, we use the equation below.

$$\hat{q}^e = \delta \hat{q}^e \oplus \bar{q}^e = \begin{bmatrix} \bar{q}_0 & -\bar{q}_1 & -\bar{q}_2 & -\bar{q}_3 \\ \bar{q}_1 & \bar{q}_0 & \bar{q}_3 & -\bar{q}_2 \\ \bar{q}_2 & -\bar{q}_3 & \bar{q}_0 & \bar{q}_1 \\ \bar{q}_3 & \bar{q}_2 & -\bar{q}_1 & \bar{q}_0 \end{bmatrix} \begin{bmatrix} 1 \\ \delta \hat{q}_1 \\ \delta \hat{q}_2 \\ \delta \hat{q}_3 \end{bmatrix} \quad (12)$$

The angular velocity, ω_{ib} , is provided by the gyroscope relative to the inertial frame. We subtract the angular velocity of the Earth relative to the inertial frame from that of the vehicle body relative to the inertial frame to get the angular velocity of the body relative to ECEF frame.

$$\omega_{eb} = \omega_{ib} - \omega_{ie} \quad (13)$$

This equation can be represented as:

$$\omega_{eb}^b = \omega_{ib}^b - C_E^B \omega_{ie}^e = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (14)$$

$$\Omega_{eb}^b = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \quad (15)$$

When a small rotation over a time interval δt is considered, the quaternions that are associated with such a small amount of displacement can be represented as the following:

$$\begin{bmatrix} \delta\alpha \\ \delta\beta \end{bmatrix} = \begin{bmatrix} \cos(\|\omega\|\delta t/2) \\ \sin\left(\frac{\|\omega\|\delta t}{2}\right) \omega/\|\omega\| \end{bmatrix} \quad (16)$$

Here, ω represents ω_{eb}^b , α represents the scalar quaternion q_0 , and β represents the vector quaternions $[q_1 \ q_2 \ q_3]^T$.

So, as δt approaches zero, when the sampling time is very small, the previous equation can be represented as follows:

$$\lim_{\delta t \rightarrow 0} \frac{1}{\delta t} (\delta\alpha - 1) = \lim_{\delta t \rightarrow 0} \frac{1}{\delta t} \left[\cos\left(\frac{\|\omega\|\delta t}{2}\right) - 1 \right] = 0 \quad (17a)$$

$$\lim_{\delta t \rightarrow 0} \frac{1}{\delta t} \delta\beta = \lim_{\delta t \rightarrow 0} \frac{1}{\delta t} \left[\sin\left(\frac{\|\omega\|\delta t}{2}\right) \omega/\|\omega\| \right] = \frac{1}{2} \omega \quad (17b)$$

Therefore, after a small displacement, the four quaternions can be updated as:

$$q = \begin{bmatrix} \delta\alpha & -\delta\beta^T \\ \delta\beta & \delta\alpha I + \delta\beta^\times \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (18)$$

where, I is a 3×3 identity matrix.

To update DCM, $C_B^{\bar{E}}$ becomes

$$C_B^{\bar{E}} = C_E^{\bar{E}} C_B^E \quad (19a)$$

where $C_E^{\bar{E}}$ is the error in C_B^E computation and is represented, assuming small error, as

$$C_E^{\bar{E}} = (I - 2[\delta\beta]^\times) \quad (19b)$$

So,

$$C_B^{\bar{E}} = (I - 2[\delta q^e]^\times) C_B^E \quad (19c)$$

Also, C_B^E can be updated as the following representation.

$$C_B^E = (I - \omega^\times) C_B^E \quad (19d)$$

Rearranging equation 4, the transformation matrix from body to navigation frame can be found as:

$$C_B^N = C_E^N C_B^E \quad (20)$$

To get the Euler angles from the quaternions are used as follows. Here, C_{ij} represents C_B^N . The subscript, i , is the i -th row, and the subscript, j is the j -th column.

$$\tan\phi = C_{32}/C_{33} \quad (21a)$$

$$\sin\theta = -C_{31} \quad (21b)$$

$$\tan\psi = C_{21}/C_{11} \quad (21c)$$

2.8 ERROR ANALYSIS USING EXTENDED KALMAN FILTER APPROACH

For the state space representation [23-24] for the error vector between the INS and GPS readings,

$$A = \begin{bmatrix} 0 & I & 0 & 0 & 0 \\ \nabla \bar{G}^e - \Omega^2 & -2\Omega & -2[C_B^{\bar{E}} \bar{F}^b]^\times & C_B^{\bar{E}} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} C_B^{\bar{E}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ C_B^{\bar{E}} & 0 & 0 & 0 \\ 0 & \frac{1}{2} C_B^{\bar{E}} & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \quad (23)$$

$$x = [\delta P^e \quad \delta V^e \quad \delta q^e \quad \delta b_a^b \quad \delta b_g^b]^T \quad (24)$$

$$\hat{x}(k|k-1) = Ax(k-1|k-1) + B(k)w(k) \quad (25)$$

where $\nabla \bar{G}^e$ is the gravity tensor matrix, which is

$$\nabla \bar{G}^e = \begin{bmatrix} \frac{\partial g_x}{\partial x_e} & \frac{\partial g_x}{\partial y_e} & \frac{\partial g_x}{\partial z_e} \\ \frac{\partial g_y}{\partial x_e} & \frac{\partial g_y}{\partial y_e} & \frac{\partial g_y}{\partial z_e} \\ \frac{\partial g_z}{\partial x_e} & \frac{\partial g_z}{\partial y_e} & \frac{\partial g_z}{\partial z_e} \end{bmatrix} \quad (26)$$

Neglecting the second zonal harmonic term in gravitational force, we come to have

$$\frac{\partial g_x}{\partial x_e} = -\frac{\mu}{\bar{R}^3} \left[1 - 3 \left(\frac{\bar{x}_e}{\bar{R}} \right)^2 \right] \quad (27a)$$

$$\frac{\partial g_y}{\partial y_e} = -\frac{\mu}{\bar{R}^3} \left[1 - 3 \left(\frac{\bar{y}_e}{\bar{R}} \right)^2 \right] \quad (27b)$$

$$\frac{\partial g_z}{\partial z_e} = -\frac{\mu}{\bar{R}^3} \left[1 - 3 \left(\frac{\bar{z}_e}{\bar{R}} \right)^2 \right] \quad (27c)$$

$$\frac{\partial g_x}{\partial y_e} = \frac{3\mu}{\bar{R}^3} \left(\frac{\bar{x}_e \bar{y}_e}{\bar{R}^2} \right) \quad (27d)$$

$$\frac{\partial g_x}{\partial z_e} = \frac{3\mu}{\bar{R}^3} \left(\frac{\bar{x}_e \bar{z}_e}{\bar{R}^2} \right) \quad (27e)$$

$$\frac{\partial g_y}{\partial z_e} = \frac{3\mu}{\bar{R}^3} \left(\frac{\bar{y}_e \bar{z}_e}{\bar{R}^2} \right) \quad (27f)$$

$$\frac{\partial g_y}{\partial x_e} = \frac{\partial g_x}{\partial y_e} \quad (27g)$$

$$\frac{\partial g_z}{\partial x_e} = \frac{\partial g_x}{\partial z_e} \quad (27h)$$

$$\frac{\partial g_z}{\partial y_e} = \frac{\partial g_y}{\partial z_e} \quad (27i)$$

As $x(1|0)$ is set to zero, as it is assumed that all errors are removed in initial calibration, any update for these errors would be also zero. However, due to drift in INS, the uncertainty in the states grows and this causes the covariance matrix, $P(k|k-1)$ to change every time step.

$$P(k|k-1) = A(k)P(k-1|k-1)A^T(k) + Q(k) \quad (28)$$

$Q(k)$ is the process noise matrix and is evaluated as:

$$Q(k) = \frac{1}{2} [A(k)B(k)Q_c(k)B^T(k)A^T(k) + B(k)Q_c(k)B^T(k)]\Delta t \quad (29)$$

$Q_c(k)$ is the process model uncertainty and is evaluated as:

$$Q_c(k) = \begin{bmatrix} \delta\omega_a & 0 & 0 & 0 \\ 0 & \delta\omega_g & 0 & 0 \\ 0 & 0 & \delta\omega_b^a & 0 \\ 0 & 0 & 0 & \delta\omega_b^g \end{bmatrix} \quad (30)$$

$\delta\omega_a$ is the noise of acceleration error due to uncertainty in accelerometers. While, $\delta\omega_g$ is the noise of acceleration error due to uncertainty in rate gyros. Uncertainties of acceleration and angular velocities, $\delta\omega_a$ and $\delta\omega_g$, respectively, can be obtained from the manufacturer's manual or by experimentation if the manual is not available. $\delta\omega_b^a$ is the noise of uncertainty error in accelerometers biases, while $\delta\omega_b^g$ is the noise of uncertainty error in rate gyros biases. Also, these two can be obtained experimentally.

The error observation is updated using EKF and then sent to INS.

$$z(k) = \begin{bmatrix} z_p^{INS}(k) - z_p^{GPS}(k) \\ z_v^{INS}(k) - z_v^{GPS}(k) \\ z_v^{INS}(k) - z_v^{velocity}(k) \end{bmatrix} \quad (31)$$

Using an EKF, the error estimate is updated every time step through the following equation.

$$\hat{x}(k|k) = \hat{x}(k|k-1) + W(k)v(k) \quad (32)$$

$W(k)$ is a Kalman gain matrix and $v(k)$ is called the innovation matrix. $\hat{x}(k|k-1)$ is always assumed to be zero, since we are assuming accurate initial alignment of the unit with the local coordinates.

$$v(k) = z(k) - H(k)\hat{x}(k|k-1) \quad (33)$$

$$W(k) = P(k|k-1)H^T(k)S^{-1}(k) \quad (34)$$

The matrix $S(k)$ is called the innovation covariance.

$$S(k) = H(k)P(k|k-1)H^T(k) + R(k) \quad (35)$$

Then, the covariance matrix is updated every time step, k .

$$P(k|k) = (I - W(k)H(k))P(k|k-1)(I - W(k)H(k))^T + W(k)R(k)W^T(k) \quad (36)$$

$$H(k) = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \end{bmatrix} \quad (37)$$

$$R(k) = \begin{bmatrix} \sigma_{P_{GPS}^e}^2 & 0 & 0 \\ 0 & \sigma_{V_{GPS}^e}^2 & 0 \\ 0 & 0 & \sigma_{V_{velocity}^e}^2 \end{bmatrix} \quad (38)$$

where I is a 3×3 identity matrix and 0 is a 3×3 matrix of zeros.

2.9 VELOCITY CONSTRAINTS FUSION

Since the application is for a land vehicle, we can assume accurately the values of V_y and V_z in body frame to be very close to zero, so we can reduce the error growth rate. We assume that

$$V_y = \eta_y \quad (39a)$$

$$V_z = \eta_z \quad (39b)$$

where η_y and η_z are white noises with zero mean, and have variances of σ_y and σ_z , respectively. Note that V_f is the forward velocity of the vehicle measured from the encoder along its path, and η_x is white noise with zero mean.

In land vehicles applications, the velocity in the y and z axes (body frame), v_y^b and v_z^b , are constrained to zero mean with an additive white noise process. The fusion of these constraints helps in improving the estimated states. The velocity in the x axis (body frame), v_x^b , is obtained from wheel encoders. Note that the measurement vector of these readings should be multiplied by C_B^E to convert these readings into ECEF frame. This is because the ECEF frame is used as a basis frame in the filtering stage for this thesis. The measurement equation is represented as:

$$z_v^{velocity}(k) = C_B^E \left(\begin{bmatrix} v_x^b(k) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix} \right) \quad (40)$$

The error observation vector between the INS readings and the velocity constraints is

$$z(k) = z_v^{INS}(k) - z_v^{velocity}(k) \quad (41)$$

$$H_v^{velocity}(k) = [0 \quad I \quad 0 \quad 0 \quad 0] \quad (42)$$

$$R_v^{velocity}(k) = C_B^E \left(\begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix} \right) \quad (43)$$

The use of velocity constraints in the measurement equation will enhance the accuracy of the state estimate as long as the encoder measurement is not biased due to problems, such as a malfunction in the encoder or due to improperly inflated vehicle's wheels. Also, if the navigation unit is not installed at the center of the mass of the vehicle, then the velocity constraint measurement needs to be adjusted. Therefore, in this thesis, a bias estimation method is proposed to estimate any possible bias in the encoder measurement.

Note that equations 42 and 43 are parts of equation 37 and 38. Also, note that $R_v^{velocity}$ in equation 43 is represented as $\sigma_{V_{velocity}^e}$ in equation 38, respectively, and it is multiplied by C_B^E to change it from body frame to ECEF frame. In the next section, the algorithm of the EIF is explained.

2.10 EXTENDED INFORMATION FILTER APPROACH

The EIF gives the same result mathematically as that of an EKF. However, it is easier to apply in cases where we have readings from multiple sensors, as it uses information space rather than state space. Therefore, we do not have to worry about the innovation sequence if we are using this approach as in the EKF. The readings we are taking in this work are position and velocity from GPS and IMU, and velocity from encoders and nonholonomic constraints. Moreover, the EIF algorithm is not computationally heavy for a simple processor to do, so it is very suitable for real time applications [12, 24].

The main components of the information filter [12, 24] are the information state vector, y , and the information state matrix, Y , which is the inverse of the covariance matrix, that is

$$Y(k) = P^{-1}(k) \quad (44a)$$

and the information state vector is

$$y(k) = Y(k)x(k) \quad (44b)$$

The predicted information state matrix is

$$\hat{Y}(k|k-1) = [A(k)\hat{Y}(k-1|k-1)A^T(k) + Q(k)]^{-1} \quad (45a)$$

While the predicted information vector is evaluated as

$$\hat{y}(k|k-1) = \hat{Y}(k|k-1)A(k)\hat{Y}^{-1}(k|k-1) \quad (45b)$$

The error estimation is fed back into the INS for correction. Then, the information observation vector, $i(k)$, and the information observation matrix, $I(k)$ are evaluated when an observation $z(k)$ is made. The vector, $i(k)$, represents the contribution of the observation, while the matrix, $I(k)$ represents the certainty.

$$i(k) = H^T(k)R^{-1}(k)z(k) \quad (46a)$$

$$I(k) = H^T(k)R^{-1}(k)H(k) \quad (46b)$$

When the observations are obtained, the estimates are updated as

$$\hat{y}(k|k) = \hat{y}(k|k-1) + i(k) \quad (47a)$$

$$\hat{Y}(k|k) = \hat{Y}(k|k-1) + I(k) \quad (47b)$$

For more than one sensor used, we can easily update the estimates by adding the summation of the information observation vectors and matrices of the multiple sensors used, that is

$$\hat{y}(k|k) = \hat{y}(k|k-1) + \sum i(k) \quad (48a)$$

$$\hat{Y}(k|k) = \hat{Y}(k|k-1) + \sum I(k) \quad (48b)$$

2.11 ALGORITHM SUMMARY

The following figure summarizes the algorithm used in this work to estimate the position, velocity and attitude.

As the position and velocity obtained from integrating IMU measurements, the vehicle's attitude, which is obtained by integrating the angular velocity of the vehicle measured by the gyroscope, will diverge quickly from its correct attitude due to bias error in the gyroscope's measurements. EKF or EIF is used to fuse the measurements

of the GPS and the *a priori* vehicle's estimates obtained from IMU to eliminate this error.

Figure 10 Filter Algorithm

The GPS position and velocity readings are subtracted from the INS position and velocity whenever a GPS signal is available. Also, the velocity constraints and encoder velocity readings are subtracted from the INS velocity. These observation errors are fed into the filter to be used. Then the estimated errors obtained from that filter are used to correct the INS position and velocity, and the loop goes on.

3. EXPERIMENTAL WORK

3.1 INTRODUCTION

The algorithm proposed was implemented and tested using a vehicle driving on a path with an IMU, a GPS antenna and an encoder. After getting the data, a bias was simulated and added on the encoder velocity to verify the method of LMNE that is explained in the next chapter. In the following results, the estimation using the solution proposed (green line) is validated against a commercial off-the shelf (COTS) MIDG solution (blue line).

The results shown in this chapter are for a static test, a single dynamic vehicle test and three multiple vehicles' dynamic tests that validate the algorithms in real life scenarios. The reason behind including a static test is that the truth of zero velocities in all axes of the unit should be zero if km/h is known. For these tests, the algorithms of EIF and EKF with and without the fusion of nonholonomic velocity constraints are tested and validated. The main results shown for dynamic tests are the velocity, position and attitude of the vehicles. Before going to the results, it is important to note that small deviations from the expected values of zero degrees in pitch and roll in dynamic tests results are mainly due to tilts that might be present in roads used in the tests, bumps, turns and traces of experimental errors. The sensors used are described in the following section.

3.2 SENSORS USED

- MIDG IIC

The MIDG IIC is an aided inertial navigation system (INS) for use in applications requiring attitude, position, velocity, acceleration, and angular rates for navigation or control. An internal GPS receiver measures position and velocity and passes this information to the data fusion processor to be combined with the inertial data to generate an optimal solution. An internal three-axis magnetometer provides a magnetic heading reference when needed. The frequency at which readings can be obtained, range from 1 to 50 Hz. For the application in this work, the sensor is used at 50 Hz. The specifications of the MIDG unit are shown in Table 2.

Figure 11 MIDG II Unit [66]

Table 2 MIDG Specifications

Power Requirements	
Input Voltage	10 VDC-32 VDC
Power	1.2 W max
Measurements	
Angular Rate	
Range	$\pm 300^\circ/sec$
Non-Linearity	0.1% of FS
Noise Density	$0.05^\circ/sec/\sqrt{Hz}$
3 dB Bandwidth	20 Hz
Acceleration	
Range	$\pm 6 g$
Non-Linearity	0.3% of FS
Noise Density	$150 \mu g/\sqrt{Hz}$
3 dB Bandwidth	20 Hz
Attitude Accuracy (Tilt)	$0.4^\circ (1\sigma)$
Position Accuracy	2 m CEP, WAAS/EGNOS available
Physical	
Size	$1.50" W \times 0.87" H \times 1.67" D$
Weight	55 grams

- GPS Antenna

A GPS antenna is used to provide us with the position and velocity of the vehicle in ECEF frame at 5 Hz. The GPS works properly when at least four of 24 satellites orbiting the Earth are in line of sight of the receiver antenna. GPS readings are fused with those obtained from IMU to correct the estimate of our solution as GPS usually gives more accurate readings than IMU but is updated at a slower rate. Also, a GPS signal might suffer from multipath errors or jamming, or might be totally blocked due to several reasons discussed in Chapter 2.

Figure 12 GPS Antenna

- Encoders

The vehicle's wheel encoders (odometers) are read and fused with the solution along with velocity constraints of land vehicle in the y and z directions in body frame to provide a more accurate estimate. The speedometer gives the forward velocity of the vehicle, or in other words, it provides us with the velocity in the x direction in body frame. As the solution used in this thesis uses position and velocity coordinates in ECEF frame, the velocity obtained from encoders are converted from body frame to ECEF frame for fusing the encoder readings with the solution.

Figure 13 A Vehicle's Speedometer

3.3 STATIC TEST

First, a static test was performed. Results of this experiment should give zero velocity and constant position. This can be used for validating the accuracy of the used sensors and the proposed estimation filter. This is because a partial truth state, zero velocity and constant position is known.

3.3.1 STATIC TEST USING EKF WITHOUT VELOCITY CONSTRAINTS

In Figures 14 and 15, the position and velocity, respectively, of a static test are shown. The data was treated using an EKF for INS and GPS fusion only. So, in these figures, the results of the solution are not constrained with nonholonomic velocity constraints and only INS and GPS are fused together.

Figure 14 Velocity Estimate, Static Test Using EKF without Velocity Constraints

Figure 15 Position Estimate, Static Test Using EKF without Velocity Constraints

In the previous two figures, we can see that oscillations in y and z directions of velocities away from zero are present. This also reduces the accuracy of position readings. In these figures, the position is given in ECEF frame, while the velocity is given in body frame.

3.3.2 STATIC TEST USING EKF WITH VELOCITY CONSTRAINTS

In Figures 16 and 17, the position and velocity, respectively, of the same static test are shown. The data was also treated using the same EKF. However, in these figures, the results of the solution are constrained with nonholonomic velocity constraints and not only are INS and GPS fused together, but also forward velocity from an encoder with constraints are fused to the solution. In these figures, position is given in ECEF frame, while velocity is given in body frame.

Figure 16 Velocity Estimate, Static Test Using EKF with Velocity Constraints

Figure 17 Position Estimate, Static Test Using EKF with Velocity Constraints

From Figure 16, we see that the estimate solution is closer to zero than that of MIDG in x, y and z directions, since the encoder and velocity constraints are zero.

3.3.3 STATIC TEST USING EIF WITHOUT VELOCITY CONSTRAINTS

In Figures 18 and 19, the position and velocities, respectively, of the same static test are shown. However, here the data was treated using the EIF described in

Chapter 2 for INS and GPS fusion only. So, in these figures, the results of the solution are not constrained with nonholonomic velocity constraints and only INS and GPS are fused together. Therefore, we can see that oscillations in y and z directions of velocities away from zero are present. This also reduces the accuracy of position readings. In these figures, the position is given in ECEF frame, while the velocity is given in body frame.

Figure 18 Velocity Estimate, Static Test Using EIF without Velocity Constraints

Figure 19 Position Estimate, Static Test Using EIF without Velocity Constraints

In Figures 18 and 19, we can see that velocities in y and z directions of oscillate away from zero. This also reduces the accuracy of position readings. In these figures, the position is given in ECEF frame, while the velocity is given in body frame.

3.3.4 STATIC TEST USING EIF WITH VELOCITY CONSTRAINTS

The static test data is now fused with the GPS and encoder data using the EIF described in Chapter 2. The estimation error resulting from this integration should be bounded by the GPS measurement error and the fact that our encoder velocity reading with nonholonomic constraints is zero.

Figure 20 Velocity Estimate, Static Test Using EIF with Velocity Constraints

Figure 21 Position Estimate, Static Test Using EIF with Velocity Constraints

In Figure 20, we notice that the estimate solution is closer to zero than that of MIDG in all directions, because the unit is static. Both encoder and velocity constraints are at zero.

3.4 SINGLE VEHICLE DYNAMIC TEST

To verify the algorithms explained in Chapter 2 practically a single vehicle dynamic test was performed and the experimental data obtained was tested using the EKF and EIF.

3.4.1 SINGLE VEHICLE DYNAMIC TEST USING EKF WITHOUT VELOCITY CONSTRAINTS

The figures in this section show the results of a single vehicle dynamic test, where the vehicle was driven around a three quarters complete roundabout during the first 20 seconds of the test. The vehicle had a 90 degrees right turn after 65 seconds from the start of the test. Then the vehicle took a sharper 90 degrees turn to its left after 90 seconds of the test. Finally, the test was concluded when the vehicle started turning left to enter a parking lot. The results shown in this section were treated using EKF with no velocity constraints. As seen, overall the results from the solution closely follow the results from MIDG, but since no velocity constraints are implemented here, they also follow velocities deviations of MIDG from zero in y and z directions.

Figure 22 Path Estimate of the Vehicle, Dynamic Test Using EKF without Velocity Constraints

Figure 23 Velocity Estimate, Dynamic Using EKF without Velocity Constraints

Figure 24 Velocity Error in Body Frame, Dynamic Test Using EKF without Velocity Constraints

Figure 24 shows the velocity error normal distribution which is around -0.04 km/h most of the time in the x-axis, -0.72 km/h in the y-axis and -0.32 km/h in the z-axis of the vehicle's body frame. In this section, velocity constraints were not implemented. From Figure 23, it can be noticed that the velocity of the vehicle in the y and z axes sometimes move away from zero km/h. When travelling with a land vehicle, the velocity in the y and z axes should be precisely close to zero km/h in the vehicle's body frame. At some instances in this test, this is not the case. This is improved by using nonholonomic velocity constraints in the next section. The four glitches shown on the velocity z-axis shown in Figure 23 indicate the presence of four bumps on the road travelled on.

Figure 25 Position Estimate, Dynamic Test Using EKF without Velocity Constraints

Figure 26 Position Error in ECEF Frame, Dynamic Test Using EKF without Velocity Constraints

Figure 25 shows the ECEF position of the vehicle. Figure 26 illustrates a normal Gaussian distribution of position error in ECEF frame. This is the difference of positions from MIDG and the estimated solution. Figure 26 shows the position error normal distribution. From Figure 26, it can be noted that the position mean errors in the ECEF frame are -0.34 meters for x direction, 0.20 meters for y direction and 0.69 meters for z direction. The following two figures show the attitude of the vehicle and the errors in the attitude. The change of the yaw angle in the first 20 seconds shown in figure 27 indicates that the vehicle was travelling on the roundabout. It is clear that the estimate of the yaw angle provided by the solution follows the yaw angle provided by the MIDG unit despite the highly involved dynamics of the vehicle in the first 20 seconds and the two turns at around the 70th and 90th seconds of the test. Figure 28 shows that the errors in yaw, pitch and roll have means of 0.65, -0.14 and 0.16 degrees.

Figure 27 Attitude Estimate, Dynamic Test Using EKF without Velocity Constraints

Figure 28 Attitude Error, Dynamic Test Using EKF without Velocity Constraints

3.4.2 SINGLE VEHICLE DYNAMIC TEST USING EKF WITH VELOCITY CONSTRAINTS

In this section, the velocity constraints were fused into the EKF used in order to enhance the estimates. The following three figures, the path driven during the test, the velocity (in body frame) estimates of the vehicle and the error in this velocity are shown and compared against the MIDG solution using a EKF with velocity constraints.

Figure 29 Path Estimate of the Vehicle, Dynamic Test Using EKF with Velocity Constraints

Figure 30 Velocity Estimate, Dynamic Test Using EKF with Velocity Constraints

Figure 31 Velocity Error, Dynamic Test Using EKF with Velocity Constraints

As noted in the figures above, we see that implementing the velocity constraints had improved the estimation accuracy. Given the application in consideration, it is known that a vehicle driving on a road would have zero body-frame velocities in y and z directions. Therefore, especially when looking at Figure 30, it can be noted that the y and z components of the velocity estimate are closer to zero than those of MIDG. Figure 31 shows a considerable enhancement in terms of velocity errors reduction in all the three axes. The following two figures show the position and the position errors in ECEF frame.

Figure 32 Position Estimate, Dynamic Test Using EKF with Velocity Constraints

Figure 33 Position Error, Dynamic Test Using EKF with Velocity Constraints

From Figure 33, it can be noted that most position errors in ECEF frame are around -0.18 meter for x direction, -0.76 meters for y direction and 0.90 meters in z axis. This shows an improvement in the error compared to results obtained using EKF without velocity constraints. Figure 34 shows the attitude estimation of the vehicle. The peaks in the pitch angle show that the vehicle passed over 4 bumps at seconds 30, 40, 78 and 98. The attitude errors' normal distribution in Figure 35 show that the algorithm used is quite reliable as the errors are small and close to zero degrees in yaw, pitch and roll. The next section discusses results for the same test but using EIF without velocity constraints.

Figure 34 Attitude Estimate, Dynamic Test Using EKF with Velocity Constraints

Figure 35 Attitude Error, Dynamic Test Using EKF with Velocity Constraints

3.4.3 SINGLE VEHICLE DYNAMIC TEST USING EIF WITHOUT VELOCITY CONSTRAINTS

Figures in this section show the results of the same single vehicle dynamic test. The results in the following figures were treated using EIF with no velocity constraints. Figure 36 shows the path driven by the vehicle in the same test discussed in the previous sections. The blue line shows the path derived from MIDG unit and the green line shows the path derived from the solution.

Figure 36 Path Estimate of the Vehicle, Dynamic Test Using EIF without Velocity Constraints

Figure 37 Velocity Estimate, Dynamic Test Using EIF without Velocity Constraints

Figure 38 Velocity Error, Dynamic Test Using EIF without Velocity Constraints

It can be noted that, overall, the results from the EIF closely follows the results from MIDG. Nevertheless, since no velocity constraints are implemented here, they also follow velocities' deviations of the MIDG from zero in y and z directions. The results of velocity estimates and its errors are very close to those results provided by the solution that uses EKF without velocity constraints demonstrated in section 3.3.1. The next two figures show the position and the normal distribution of the position error in ECEF frame.

Figure 39 Position Estimate, Dynamic Test Using EIF without Velocity Constraints

Figure 40 Position Error, Dynamic Test Using EIF without Velocity Constraints

As in EKF, overall the results from the solution closely follow the results from MIDG but since no velocity constraints are implemented here, they also follow velocities deviations of MIDG from zero in y and z directions. Figure 40 shows the position error normal distribution which has a mean around -0.34 m in the x-axis, 0.21 m in the y-axis and 0.70 m in the z-axis (ECEF frame). The following two figures show the attitude estimates and errors and prove that the quaternion algorithm used in the solution is reliable for estimating attitude angles.

Figure 41 Attitude Estimate, Dynamic Test Using EIF without Velocity Constraints

Figure 42 Attitude Error, Dynamic Test Using EIF without Velocity Constraints

3.4.4 SINGLE VEHICLE DYNAMIC TEST USING EIF WITH VELOCITY CONSTRAINTS

The same dynamic test data is now fused with the GPS and encoder data using the EIF described in the previous section. The estimation error resulting from this integration should be bounded by the GPS measurement error and the fact that our nonholonomic constraints are zero with a white noise of low magnitude.

Figure 43 Path Estimate of the Vehicle, Dynamic Test Using EIF with Velocity Constraints

Figure 44 Velocity Estimate, Dynamic Test Using EIF with Velocity Constraints

Figure 45 Velocity Error, Dynamic Test Using EIF with Velocity Constraints

It can be noted from Figure 44 that, since the velocity constraints are applied, velocities closer to zero are obtained in the body frame x and y axes. This is not the case for the results reported by the MIDG solution. Also, Figure 45 shows the velocity error normal distribution which is around 0.004 km/h in mean in the body frame x-axis, -0.03 km/h in the body frame y-axis and -0.04 km/h in the body frame z-axis. This shows an improvement in the error compared to results obtained using EIF without velocity constraints. The following two figures demonstrate the position estimates and normal distribution errors in the ECEF frame.

Figure 46 Position Estimate, Dynamic Test Using EIF with Velocity Constraints

Figure 47 Position Error, Dynamic Test Using EIF with Velocity Constraints

It can be noted from Figure 44 that since the velocity constraints are applied, velocities closer to zero in the ECEF frame x and y axes are obtained from the proposed solution than those obtained from MIDG. It can be also noted that the estimate of positions in ECEF frame in x direction in ECEF frame obtained from the algorithm with the used low-cost solution closely matches the results obtained from the commercial standalone MIDG unit. Figure 47 shows the position error normal distribution which is around -0.18 m in the x-axis, -0.76 m in the y-axis and 0.90 m in the z-axis. This shows an improvement in the error compared to results obtained using EIF without velocity constraints, especially in the x-axis. Also, the following two figures show the reliability of the quaternion algorithm used in the solution as the mean errors in the attitude angles are close to zero degrees.

Figure 48 Attitude Estimate, Dynamic Test Using EIF with Velocity Constraints

Figure 49 Attitude Error, Dynamic Test Using EIF with Velocity Constraints

Next, three multiple vehicle tests results are shown and discussed. In the first test, two vehicles were following each other. The second test shows an overtaking process, while the third test shows two vehicles driving in opposite directions.

3.5 TWO-VEHICLES DYNAMIC TESTS

To test the algorithms in real life driving scenarios, like overtaking, two vehicles dynamic tests were performed. In these tests, the algorithms were fused with nonholonomic velocity constraints and encoder velocity readings.

3.5.1 TWO VEHICLES FOLLOWING EACH OTHER (TEST 1) USING EIF WITH VELOCITY CONSTRAINTS

Figures in this section show the results for a two-vehicle dynamic test that was performed by two vehicles with each one having the proposed solution. EIF with velocity constraints was used to analyze the data. Figure 50 shows the path estimates of the two vehicles using MIDG and EIF.

Figure 50 Path Estimates of the Vehicles, Two-Vehicle Dynamic Test 1 Using EIF with Velocity Constraints

Figures 51 and 52 show the velocities of both vehicles in body frame, and the velocity of vehicle 2 in body frame relative to that of vehicle 1, respectively. From the two figures, it can be noted that at the beginning, the forward velocity of vehicle 1 was more than that of vehicle 2. At around the seventh second of the test, the forward velocity of vehicle 2 exceeds that of vehicle 1. At the end of test 1, the forward velocity of vehicle 2 becomes about 9 km/h (2.5 m/s) more than that of vehicle 1 as shown in Figure 52.

Figure 51 Velocity Estimate, Two-Vehicle Dynamic Test 1 Using EIF with Velocity Constraints

Figure 52 Vehicle 2 Velocity Relative to Vehicle 1 Velocity in Test 1 Using EIF with Velocity Constraints

The next figure shows the position of the two vehicles in ECEF frame. It shows that vehicle 2 was following vehicle 1 throughout the whole test. Figure 54 shows that at the beginning of the test, vehicle 2 was around 49 m behind vehicle 1. At the seventh second of the test, vehicle 2 moved to about 60 m behind vehicle 1. This is due to the fact that the forward velocity of vehicle 1 was more than that of vehicle 2 in the first seven seconds of the test as shown in Figures 51 and 52. At the end of the test, since after the seventh second of vehicle 2 became more than that of vehicle 1, vehicle 2 moved to 36 m behind vehicle 1. To know how much vehicle 2 is lagging behind vehicle 1 in seconds, we take the distance between the two vehicles in meters from Figure 54 and divide it by the forward velocity of the lagging vehicle, which is obtained from Figure 51. This is represented in the equation below.

$$\text{Difference in seconds} = \frac{P_1^{bx} - P_2^{bx}}{V_2^{bx}} \quad (49)$$

Figure 53 Position Estimate, Two-Vehicle Dynamic Test 1 Using EIF with Velocity Constraints

Figure 54 Vehicle 2 Position Relative to Vehicle 1 Position in Test 1 Using EIF with Velocity Constraints

Figure 55 Lagging of Vehicle 2 behind Vehicle 1 in Seconds Using EIF with Velocity Constraints

From Figure 55, it is noted that at the beginning of the test, vehicle 2 was almost 3.6 seconds behind vehicle 1. At the fifth second, vehicle 2 was about 4.2 seconds behind vehicle 1. At the end of the test, vehicle 2 was only 2.5 seconds behind vehicle 1, passing the warning line of 3 seconds in the fourteenth second of the test. The next two figures show the attitude of the two vehicles and the relative attitude of vehicle 2 relative to vehicle 1, respectively. It is noted from Figure 57 that the difference in yaw is close to zero degrees, which shows that the two vehicles were travelling in the same direction for the whole test. The next section shows the results of the second test using EIF with velocity constraints.

Figure 56 Attitude Estimate, Two-Vehicle Dynamic Test 1 Using EIF with Velocity Constraints

Figure 57 Vehicle 2 Attitude Relative to Vehicle 1 Attitude in Test 1 Using EIF with Velocity Constraints

3.5.2 TWO VEHICLES FOLLOWING EACH OTHER (TEST 1) USING EKF WITH VELOCITY CONSTRAINTS

In this section, the results for the same previous two-vehicle dynamic test are demonstrated. However, this time the results were processed with a EKF rather than an EIF. Figure 58 shows the path estimates of the two vehicles using MIDG and EKF. The figures in this section show a similar behavior of the EKF and EIF used.

Figure 58 Path Estimates of the Vehicles, Two-Vehicle Dynamic Test 1 Using EKF with Velocity Constraints

Figures 59 and 60 show the velocities of both vehicles in body frame, and the velocity of vehicle 2 in body frame relative to that of vehicle 1, respectively. As discussed previously, it can be noted that during the first 7 seconds of the test, the forward velocity of vehicle 1 was more than that of vehicle 2. After that, the forward velocity of vehicle 2 exceeds that of vehicle 1 to become about 9 km/h (2.5 m/s) more than that of vehicle 1 as shown in Figure 60.

Figure 59 Velocity Estimate, Two-Vehicle Dynamic Test 1 Using EKF with Velocity Constraints

Figure 60 Vehicle 2 Velocity Relative to Vehicle 1 Velocity in Test 1 Using EKF with Velocity Constraints

The next figure shows the position of the two vehicles in ECEF frame. Figure 62 shows that vehicle 2 was lagging behind vehicle 1 with around 49 m at the beginning, 60 m at the seventh second and 36 m at the end of the test.

Figure 61 Position Estimate, Two-Vehicle Dynamic Test 1 Using EKF with Velocity Constraints

Figure 62 Vehicle 2 Position Relative to Vehicle 1 Position in Test 1 Using EKF with Velocity Constraints

Figure 63 Difference between the Two Vehicles in Seconds Using EKF with Velocity Constraints

As shown in the previous figure, at the beginning of the test, vehicle 2 was lagging behind vehicle 1 with about 3.6 seconds at the beginning, 4.2 seconds at the fifth second and 2.5 seconds at the end of the test. At the fourteenth second of the test, a warning line of 3 seconds was crossed. As a result, a warning can be issued to the driver using an LCD screen for example. The next two figures show the attitude of the two vehicles and the attitude of vehicle 2 relative to vehicle 1, respectively. From Figure 65, it is noted that the difference in yaw is close to zero degrees, which shows that the two vehicles were travelling in the same direction. The warning has to be also conditioned with this fact keeping in mind that the distance between the two vehicles in the y-axis is less than 2 meters, as shown in Figure 62, which also shows that the two vehicles are on the same lane. As in the previous section, we see the results of the test where vehicle 1 was following vehicle 2. The results using EKF were very close to those which were obtained in the previous section, using EIF. The next section shows the results of the second test using EIF with velocity constraints.

Figure 64 Attitude Estimate, Two-Vehicle Dynamic Test 1 Using EKF with Velocity Constraints

Figure 65 Vehicle 2 Attitude Relative to Vehicle 1 Attitude in Test 1 Using EKF with Velocity Constraints

3.5.3 ONE VEHICLE OVERTAKING ANOTHER VEHICLE (TEST 2) USING EIF WITH VELOCITY CONSTRAINTS

The test that this section discusses involves one vehicle overtaking the other vehicle during the test. Figure 66 shows the path estimates of the two vehicles. In this test, vehicle 2 overtook vehicle 1 after 2 seconds of the start of the test. From the path estimate in Figure 66, we can see that vehicle 2 is overtaking vehicle 1 from the left side of the road.

Figure 66 Path Estimates of the Vehicles, Two-Vehicle Dynamic Test 2 Using EIF with Velocity Constraints

Figures 67 and 68 show the velocities of both vehicles in body frame and the forward velocity of vehicle 2 relative to vehicle 1, respectively. From Figure 67, we see that vehicle 2 starts increasing its velocity in the x axis in order to overtake vehicle 1 which was driving at around 50 km/h. Vehicle 2 increases its velocity to reach about 70 km/h in the x-axis. The negative values in the x-axis difference in velocities between the two vehicles, shown in Figure 68, indicate that vehicle 2 had a velocity higher than that of vehicle 1.

Figure 67 Velocity Estimates, Two-Vehicle Dynamic Test 2 Using EIF with Velocity Constraints

Figure 68 Vehicle 2 Velocity Relative to Vehicle 1 Velocity in Test 2 Using EIF with Velocity Constraints

Figure 69 shows the positions of both vehicles in ECEF frame and Figure 70 shows the distance between the two vehicles in the x, y and z axes in body frame relative to vehicle 1. As shown in Figure 70, vehicle 2 crossed vehicle 1 at the fifth second of the test. This fact is also shown in Figure 71. When vehicle 2 moved to the left lane the distance between the two vehicles in the y-axis became close to 5 meters. The negative values of the second difference between the two vehicles indicates that vehicle 2 has crossed vehicle 1, and vehicle 1 is the one lagging behind after the fifth second.

Figure 69 Position Estimates, Two-Vehicle Dynamic Test 2 Using EIF with Velocity Constraints

Figure 70 Vehicle 2 Position Relative to Vehicle 1 Velocity in Test 2 Using EIF with Velocity Constraints

Figure 71 Difference between the Two Vehicles in Seconds for Test 2 Using EIF with Velocity Constraints

The next figure shows the yaw, pitch and roll of the two vehicles in degrees and the difference between them is shown in Figure 73. It can be noted that when vehicle 2 was overtaking vehicle 1 the difference in the yaw angle was more than 10 degrees at the start of the test. This shows that vehicle 2 was overtaking vehicle 1 sharply. Then, when vehicle 2 settled in the left lane, the change in yaw angle dropped close to zero. The next section shows the results obtained for the same test using EKF instead of EIF.

Figure 72 Attitude Estimates, Two-Vehicle Dynamic Test 2 Using EIF with Velocity Constraints

Figure 73 Vehicle 2 Attitude Relative to Vehicle 1 Velocity in Test 2 Using EIF with Velocity Constraints

3.5.4 ONE VEHICLE OVERTAKING ANOTHER VEHICLE (TEST 2) USING EKF WITH VELOCITY CONSTRAINTS

Data from the same test were analyzed using an EKF this time. Figure 74 shows the path estimates of the two vehicles. Again, in this section, it is shown that the EKF and the EIF have very similar results.

Figure 74 Path Estimates of the Vehicles, Two-Vehicle Dynamic Test 2 Using EKF with Velocity Constraints

Figure 75 Velocity Estimates, Two-Vehicle Dynamic Test 2 Using EKF with Velocity Constraints

Figure 76 Vehicle 2 Velocity Relative to Vehicle 1 Velocity in Test 2 Using EKF with Velocity Constraints

Figures 75 and 76 show the velocities of both vehicles in body frame, and the difference between them, respectively. From the figures in this section, we can see that the results are very close to those obtained in the previous section. Again, from Figure 75, we see that vehicle 2 increases its velocity to around 70 km/h to overtake vehicle 1.

Figure 77 Position Estimates, Two-Vehicle Dynamic Test 2 Using EKF with Velocity Constraints

Figure 78 Vehicle 2 Position Relative to Vehicle 1 Velocity in Test 2 Using EKF with Velocity Constraints

Figure 79 Difference between the Two Vehicles in Seconds for Test 2 Using EKF with Velocity Constraints

Figure 77 shows the positions of both vehicles in ECEF frame and the distance between them in body frame relative to vehicle 1. As shown in Figures 78 and 79, vehicle 2 crosses vehicle 1 in the fifth second. This is where the difference in x direction (body frame) becomes more than zero meters and the difference in seconds between the two vehicles becomes less than zero seconds. When vehicle 2 got settled in the left lane of the road, the distance between the two vehicles in y direction (body frame) became around 5 meters. In the next two figures, it is shown that when vehicle 2 was overtaking vehicle 1 at the beginning of the test, the heading of vehicle 2 was more than 10 degrees less than that of vehicle 1 to get settled at 35° northeast along with vehicle 1. This is because both the vehicles started travelling on different lanes of the road and in the same direction shortly before vehicle 2 crossed vehicle 1. The results in this section show that the EKF and EIF are operating similarly in this test as the case was in the first multiple vehicles test. The next two sections show a comparison between EIF and EKF is performed for a third multiple vehicles test.

Figure 80 Attitude Estimates, Two-Vehicle Dynamic Test 2 Using EKF with Velocity Constraints

Figure 81 Vehicle 2 Attitude Relative to Vehicle 1 Velocity in Test 2 Using EKF with Velocity Constraints

3.5.5 TWO VEHICLES HEADING TOWARDS EACH OTHER (TEST 3) USING EIF WITH VELOCITY CONSTRAINTS

The figures in this section show the results of another test using EIF to analyze information of position and velocity given for two vehicles. Figure 82 shows the path estimates of the two vehicles. As shown in the next figure, the two vehicles are driving in opposite directions on two sides of the road.

Figure 82 Path Estimates of the Vehicles, Two-Vehicle Dynamic Test 3 Using EIF with Velocity Constraints

Figure 83 Velocity Estimates, Two-Vehicle Dynamic Test 3 Using EIF with Velocity Constraints

Figure 84 Vehicle 2 Velocity Relative to Vehicle 1 Attitude in Test 3 Using EIF with Velocity Constraints

This test was performed using two vehicles heading towards each other but on two different sides of the road. The test took 12 seconds. Figure 83 shows the velocities of both vehicles in body frame, and Figure 84 shows the difference between the velocities of the two vehicles. In this test, having knowledge of the velocities of the two vehicles does not affect the safety from collision avoidance perspective as the vehicles are travelling on two separate sides of the road.

Figure 85 Position Estimates, Two-Vehicle Dynamic Test 3 Using EIF with Velocity Constraints

Figure 86 Vehicle 2 Position Relative to Vehicle 1 Attitude in Test 3 Using EIF with Velocity Constraints

Figure 87 Difference between the Two Vehicles in Seconds for Test 3 Using EIF with Velocity Constraints

Figure 85 shows the positions of both vehicles in ECEF frame, while Figure 86 shows the distance between them in body frame relative to vehicle 1. The following two figures show that vehicle 1 was travelling southeast at an angle close to 123° and vehicle 2 was travelling northwest at an angle close to 303° with an absolute difference of about 180° degrees between their heading angles. This indicates that the two vehicles are heading in opposite directions. From Figures 86 and 87, it is shown that the two vehicles cross each other at 5.4 seconds from the start of the test. In this case, a warning is not issued, since Figure 86 shows that there is a distance of 10 meters between the two vehicles in the y-axis (body frame). This shows that the two vehicles are not driving head to head on the same lane but at two different sides of the road. In the following section, the same test results are shown using EKF instead of EIF.

Figure 88 Attitude Estimates, Two-Vehicle Dynamic Test 3 Using EIF with Velocity Constraints

Figure 89 Vehicle 2 Attitude Relative to Vehicle 1 Attitude in Test 3 Using EIF with Velocity Constraints

3.5.6 TWO VEHICLES HEADING TOWARDS EACH OTHER (TEST 3) USING EKF WITH VELOCITY CONSTRAINTS

The following figures in this section show the results of the same test, which was discussed in the previous section, but this time, the data was analyzed using an EKF rather than an EIF. Figure 90 shows the path estimates of the two vehicles, Figure 91 shows the velocities of both vehicles in body frame, and Figure 92 shows the difference of the two vehicles velocities, respectively.

Figure 90 Path Estimates of the Vehicles, Two-Vehicle Dynamic Test 3 with Velocity Constraints Using EKF

Figure 91 Velocity Estimates, Two-Vehicle Dynamic Test 3 Using EKF with Velocity Constraints

Figure 92 Vehicle 2 Velocity Relative to Vehicle 1 Attitude in Test 3 Using EKF with Velocity Constraints

From Figure 91, it is clear that the velocities estimates of both the vehicles obtained using EKF are similar to those which are obtained using EIF in the previous section. It is also clear that the velocities in the y and z directions (body frame) of both the vehicles are very close to zero km/h due to the fusion of the velocity constraints in the navigation algorithm used. Figure 92 shows the difference of velocities of both vehicles relative to vehicle 1. It is worth noticing that throughout most of the trip vehicle 1 was increasing its speed, while vehicle 2 was decreasing its speed. Also, it is worth noticing that the difference in y and z velocities of both vehicles (body frame) is almost zero km/h, which shows that both the vehicles velocities in these two directions were very close to each other.

Figure 93 Position Estimates, Two-Vehicle Dynamic Test 3 Using EKF with Velocity Constraints

Figure 94 Vehicle 2 Position Relative to Vehicle 1 Attitude in Test 3 Using EKF with Velocity Constraints

Figure 95 Difference between the Two Vehicles in Seconds for Test 2 Using EKF with Velocity Constraints

Figure 93 shows the positions of both the vehicles in ECEF frame. It is important to keep in mind that the intersection between both vehicles in the x-axis shown in this Figure 93 does not necessarily mean that the vehicles were passing by each other at that instance, as the coordinates in this figure are shown in ECEF frame. The instance at which both the vehicles passed by each other can be seen in Figure 94 that shows the difference in positions of the two vehicles relative to vehicle 1 (body frame). The difference between the two vehicles in seconds can be also seen in Figure 95. From Figure 96, it can be seen that vehicle 1 was travelling southeast and vehicle 2 was travelling in the opposite direction, to northwest. This can also be noted in Figure 97 from the absolute difference of 180° in the heading of both vehicles. The next section compares the time taken by a processor to implement the different algorithms implemented in this work.

Figure 96 Attitude Estimates, Two-Vehicle Dynamic Test 3 Using EKF with Velocity Constraints

Figure 97 Vehicle 2 Attitude Relative to Vehicle 1 Attitude in Test 3 Using EKF with Velocity Constraints

3.6 PROCESSING SPEED OF THE TWO FILTERS

As already discussed, the information filter has been developed mainly for two reasons, which are:

1. Its simplicity compared to KF, especially when fusing multiple sensors, and
2. Its low computational cost compared to KF.

The low computational burden of an EIF on a processor compared to an EKF comes from the fact that, unlike EKF, the measurement covariance matrices of the sensors fused are added sequentially in EIF. This means that a processor with lower processing speed can be used to implement EIF than EKF, thus reducing the production cost of the system.

The next figure shows the time taken by a 2.33 GHz dual core processor to compute one cycle of the two filters, once with velocity constraints fused and once with no velocity constraints fused.

Figure 98 Time Taken for the Two Filters to be Processed with and without Velocity Constraints by a 2.33 GHz Dual Core Processor

From Figure 98, it can be noted that the EIF without constraints has the lowest processing time, and the EKF with velocity constraints has the highest processing time. With no constraints fused the EIF is a little lower in terms of average computational time with 1.4581×10^{-6} seconds for a single iteration, compared to EKF without velocity constraints, which took 1.4613×10^{-6} seconds for a single iteration. The difference in both the times taken is not that considerable since it is only the GPS which is fused with the INS here. However, when the number of sensors increase and an encoder with velocity constraints on the y and z axes of the vehicle are fused in the algorithm, the difference in the time taken becomes more considerable and noticeable. With the constraints fused, the EKF takes 1.84×10^{-6} seconds for one cycle on average; while the EIF takes 1.535×10^{-6} seconds per cycle on average. This means that in this case, using EIF reduces computational time by about 16.58% per cycle.

The next section summarizes the important points that can be concluded from the results demonstrated in this chapter.

3.7 CONCLUSION

In this chapter the EKF and EIF were implemented on experimental data that was obtained from a static test, a single vehicle dynamic test and three multiple vehicles dynamic tests to mimic real life driving scenarios. The EKF and EIF were shown to have very similar results. The average processing cycle time for EIF was shown to be less than that of EKF. Also nonholonomic velocity constraints along with encoder velocity readings were fused with the algorithms. This fusion showed an enhancement in the estimates of both algorithms. Both the filters used proved to be reliable solutions as they were compared and validated against a high-cost COTS MIDG unit.

4. HIGH INTEGRITY GPS/IMU INTEGRATION THROUGH FAULT DETECTION

4.1 INTRODUCTION

In some cases, the navigation estimate of an object can be inaccurate and misleading, because a sensor is providing faulty readings. For example, a GPS antenna mounted on a vehicle would provide faulty readings when that vehicle enters a tunnel, or an encoder might give biased velocity readings due to malfunctioning errors. Therefore, in order to avoid deviations in the estimate, a fault detection algorithm should be incorporated in the solution. Here, an algorithm to detect faults is used and its results are verified. The algorithm used here works on detecting the source of fault and correcting the estimate on information acquired about this fault.

4.2 LIMITED MEMORY NOISE ESTIMATION (LMNE)

In the EKF formulation, the assumption of the *a priori* knowledge of the statistical distribution of the measurements noise is usually made (see references [15, 65, 67]). If these *a priori* noise statistics are wrong or change with time under certain operational environments, then the accuracy of the state estimation will degrade. Therefore, it becomes crucial to fuse a noise estimation approach with the estimation algorithm to compensate for any change in measurement noise statistics such as an increased measurement noise covariance or a bias in the measurements.

As discussed earlier, nonholonomic velocity constraints are used to enhance the accuracy of the vehicle state estimate. For the ground vehicle application at hand, the velocity constraints assume that a vehicle moves along its longitudinal axis and that the navigation unit is at the center of mass of the vehicle. If the longitudinal velocity measurement from the encoder acquires a bias due to malfunctioning or aging, then the accuracy of the state estimate will degrade. Also, since the navigation unit cannot be placed exactly at the center mass of the vehicle, then the velocity of the vehicle can have a velocity component, although small in magnitude, in the body y and z axes. Therefore, in this section, the limited memory noise estimation (LMNE) [67], is used to estimate a possible bias in the velocity constraint measurement.

In references [15, 67-68], the authors proposed techniques to obtain a high accuracy state estimate in an environment where the measurement noise covariance is unknown. In this paper, the LMNE, which is a statistical measurement matching technique, is used to estimate the mean of the velocity constraints measurements to

account for a possible measurement bias or modeling error. In the LMNE, the statistical mean and covariance of the residual are matched to the mean and covariance of the EKF model. The LMNE assumes that the measurement residual is independent and identically distributed. The formulation of the algorithm is described next.

Given the measurement residual:

$$r(k) = \begin{bmatrix} P_{INS} - P_{GPS} \\ V_{INS} - V_{GPS} \\ V_{INS} - V_{velocity} \end{bmatrix} \quad (50)$$

If the statistical sample variance of the residuals for a sample size of N , is calculated as

$$\bar{R}_r = \frac{1}{N-1} \sum_{k=1}^N (r_k - \bar{r})(r_k - \bar{r})^T \quad (51)$$

where the sample mean of the residuals, $\bar{r}$, is

$$\bar{r} = \frac{1}{N} \sum_{k=1}^N r_k \quad (52)$$

Then, this sample covariance is matched to the covariance of the residual which is calculated as:

$$E[(r - \bar{r})(r - \bar{r})^T] = E[(H(x - \bar{x}) + v)(H(x - \bar{x}) + v)^T] \quad (53)$$

Since the measurement noise is independent from the error in the state [36], the above covariance can be evaluated as:

$$E[(r - \bar{r})(r - \bar{r})^T] = H M H^T + R \quad (54)$$

A statistical value of the term $H M H^T$ can be obtained as:

$$H M H^T = \frac{1}{N} \sum_{k=1}^N H_k M_k H_k^T \quad (55)$$

Then, the measurement covariance, R_N , can be estimated from N measurements as:

$$R_N = \frac{1}{N-1} \sum_{k=1}^N (r_k - \bar{r}_N)(r_k - \bar{r}_N)^T - \frac{N-1}{N} H_k M_k H_k^T \quad (56)$$

When $t < t_N$, the EKF has a zero mean and a predefined covariance. However, when $t = t_N$, equations 52 and 56 are used to find the sample mean and sample covariance. When $t > t_N$, the sample mean and sample covariance can be calculated recursively as:

$$\bar{r}_i = \bar{r}_{i-1} + \frac{1}{N} (r_i - r_{i-N}) \quad (57)$$

$$R_i = R_{i-1} + \frac{1}{N-1} \left[(r_i - \bar{r}_i)(r_i - \bar{r}_i)^T - (r_{i-N} - \bar{r}_{i-N})(r_{i-N} - \bar{r}_{i-N})^T + \frac{1}{N} (r_i - r_{i-N})(r_i - r_{i-N})^T - \frac{N-1}{N} (H_i M_i H_i^T - H_{i-N} M_{i-N} H_{i-N}^T) \right] \quad (58)$$

Thus the sample mean is removed in the EKF error states estimate is updated as shown in the following equation.

$$\hat{x}_i = \bar{x}_i + K_i [z_i - h(\bar{x}_i) - \bar{r}_i] \quad (59)$$

By using equation 59 and by using the sample covariance in place of the measurements covariance, any biases present in the measurements are removed.

To test the integrity of the proposed navigation solution, experimental results are presented in the next section. Also, biases of different magnitudes were simulated on the encoder measurement and the accuracy of the state estimate was checked against a commercial stand-alone MIDG solution.

4.3 EXPERIMENTAL RESULTS

The results obtained experimentally from the single vehicle dynamic test shown in section 3.3 were tested with LMNE to demonstrate its effectiveness in detecting a fault in an encoder. First, LMNE was tested without adding any bias to the encoder readings. Then, LMNE was tested with added biases of 10 and 20 km/h on the encoder readings.

4.3.1 NO BIAS ON ENCODER

Figure 99 shows that most of the means are around the value of zero. Talking more specifically, it can be seen that the seventh element which should detect a bias in the encoder is around zero km/h, as we do not have a bias in the encoder.

Figure 99 Sample Means with No Encoder Bias

The bias of around 3 to 5 meters in the third element is due to biased readings from the GPS in the z direction. This problem is common in low cost GPS due to distortion of GPS signals in the ionosphere. The next three figures show the velocity error in time, and the normal distributions of velocity and position errors.

Figure 100 Velocity Error in Time with No Bias

Figure 101 Normal Distribution of Velocity Error with No Bias

Figure 102 Normal Distribution of Position Error with No Bias

From Figures 100 and 101, it can be seen that throughout the journey, velocity error in the x axis (body frame) is around zero km/h. Figure 102 shows that the mean of the position errors normal distributions in x, y and z (ECEF frame) are close to zero meters. The next figure shows the path obtained by the solution and compared to standalone MIDG solution.

Figure 103 Path Estimate of the Vehicle with No Bias

From Figure 103 shows that the solution is following the path precisely. At this stage, this proves that the EKF algorithm implemented in this paper, regardless of the LMNE, is functioning properly. In the next section, a bias of 10 km/h is added on the encoder to validate the functionality of the LMNE.

4.3.2 A BIAS OF 10 KM/H ON ENCODER

In this section, a bias of 10 km/h is added on the encoder velocity readings to simulate a faulty encoder. The LMNE should detect this bias and get rid of it. The next figure shows the nine elements of the sample means vector $\bar{v}$.

Figure 104 Sample Means with an Encoder Bias of 10 km/h

From the figure above, it can be noted that the seventh element of the mean (circled in red) detected the encoder bias of 10 km/h. This is because the seventh element is the element responsible for detecting any bias in the encoder. The biases in the other elements are close to zero. This is expected as the bias is only imposed on the forward velocity of the encoder by the user. The next three figures show the velocity and position errors in body frame and ECEF frame, respectively.

Figure 105 Velocity Error in Time with a Bias of 10 km/h

Figure 106 Normal Distribution of Velocity Error with a Bias of 10 km/h

Figure 107 Normal Distribution of Position Error with a Bias of 10 km/h

Figure 105 shows the x axis velocity error in time. The first two seconds marked in the figure show the bias close to 9 km/h obtained by the solution. This is when the LMNE is not yet used. The LMNE starts functioning after the window of two seconds is over. Figure 106 shows the normal Gaussian distribution of the velocity error in the x axis. It can be noted that there are a few velocity errors around 9 km/h (marked with a circle). These errors are the ones which are obtained by the solution before applying the fault detection algorithm. Figure 106 shows that the velocity error in the x axis has a mean of 0.25 km/h. Also, Figure 107 shows that the means in all position axes are close to zero meters. The following figure shows the path driven by the vehicle and compares the result of the solution proposed with that of the MIDG.

Figure 108 Path Estimate of the Vehicle with a Bias of 10 km/h

From the figure above, it can be seen that the solution is following the path precisely, despite the bias present on the encoder. In the next section, a bias of 20 km/h is simulated on the encoder.

4.3.3 A BIAS OF 20 KM/H ON ENCODER

Now, a more extreme condition is tested. A bias of 20 km/h (5.56 m/s) is simulated on the encoder and the fault detection algorithm is tested to check its effectiveness in removing this bias. Figure 109 shows the biases of the different sensors used. The graph circled in red shows the bias detected on the encoder. The bias is given in m/s.

Figure 109 Sample Means with an Encoder Bias of 20 km/h

The bias of 5.56 m/s that was simulated on the encoder was detected by the algorithm proposed. This bias detected is incorporated in equation 52 to get rid of it. Figure 110 shows the velocity error in time, while Figures 111 and 112 show the normal distribution of velocity and position errors, respectively.

Figure 110 Velocity Error in Time with a Bias of 20 km/h

Figure 111 Normal Distribution of Velocity Error with a Bias of 20 km/h

Figure 112 Normal Distribution of Position Error with a Bias of 20 km/h

Figure 111 shows the normal Gaussian distribution of the velocity error in the x axis of the body frame. It is worth noting the few velocity errors that are around 19 km/h (marked with a circle). These errors are the ones that were in the solution before applying the fault detection algorithm. These errors can be also seen in the first two seconds of the journey in Figure 110. After the first two seconds the velocity

error drops close to zero km/h. The bias in the velocity which was in the first two seconds of the journey is also marked in the same figure. Figure 111 shows that velocity error in the x axis has a mean of 0.53 km/h. Moreover, Figure 112 shows that the means in all position axes (ECEF frame) are close to zero meters. The following figure shows the path driven by the vehicle.

Figure 113 Path Estimate of the Vehicle with a Bias of 20 km/h

From the figure above, it can be seen that the solution follows the path precisely, despite of the bias 20 km/h present on the encoder. The next chapter concludes this thesis, and sheds some light on recommendations for future work.

5. CONCLUSION AND FUTURE WORK

5.1 SUMMARY AND CONCLUSION

In this work, an INS/GPS system was implemented successfully. The system showed accurate results. The thesis explained different navigation methods in the field. It also explained the three coordinate frames used in this work: ECEF, NED and body frames. In this thesis, information in any coordinate frame is converted to the ECEF frame to be used in the main integration algorithm using cosine rotation matrices.

The thesis discussed in detail two INS/GPS integration and sensor fusion algorithms that can be utilized in low-cost, highly accurate and reliable navigation solutions. The first of these algorithms explained is an EKF with its space model and two main stages: the prediction and update stages. KF is an algorithm that is a least-squares recursive error estimation method. It is known in theory that it is an optimal data processing method when the system's model perfectly matches the real system. Also, the noise entering the filter is white and the noise covariance is precisely known. The main difference between an EKF and an ordinary KF is that the process model is linearized when using an EKF. In this work, the linearization is done by discretization of the process model using low order terms in Taylor series. Taylor series is approximately equal and is derived from the Jacobian method of linearization. The EKF used the available information of accelerations and angular rates from three accelerometers and three rate gyros, aided with position and velocity readings from a GPS antenna, to estimate 15 states for the navigation solution. These states were the position in x, y and z axes, velocity in x, y and z axes, quaternions in x, y and z axes, accelerometers biases in x, y and z axes, and finally, rate gyro in x, y and z axes. All of these 15 states were used in ECEF frame. The EKF used was a loosely-coupled EKF due to simplicity in implementation.

Also, the EIF was employed and explained in this thesis for navigation solution development. The IF is another method of estimation and sensor fusion, which theoretically provides similar results to a KF. However, the main advantage of an IF is its low computational cost compared to that of a KF. This is due to the fact that the measurements of multiple sensors can be fused by sequentially adding their information vectors and matrices, while this is not the case with a KF. This makes it easier for a processor to handle the algorithm for multiple sensors in a shorter time

compared to a KF. This leads to reducing the cost of the processor used to implement the navigation solution. The information filter algorithm works like a KF algorithm, except that the states vector is converted to an information vector, and the covariance is converted to an information matrix. Also, the states and covariance are updated by sequentially adding the information vectors and matrices of the sensors used. Both of the algorithms, EKF and EIF, were compared and were found to be very close to each other in terms of their results.

Moreover, due to the fact that the applications for the developed navigation algorithms in this work are for land vehicles, the solutions were also aided with readings of forward velocity provided by an odometer (encoder) and nonholonomic velocity constraints. These constraints come from the fact that, unlike aerial vehicles, a typical land vehicle does not have velocity components in its y and z axes. All of its velocity is considered to be in the vehicle's x axis. So, in these constraints, the velocities in the y and z axes are modeled as zero with some white noise to account for any slippage that might occur. Also, encoders are known for their precise velocity measurement, which enhances velocity results. The fusion of these constraints and the forward velocity from encoder with the navigation solutions made them even better in terms of errors reduction.

Also, a fault detection algorithm called the limited memory noise estimation (LMNE) was implemented to detect any bias in the solution due to a faulty sensor. The algorithm was implemented and tested on an encoder with a number of simulated biases imposed on its velocity readings. The LMNE showed its effectiveness in detecting biases that reached even up to a value of 20 km/h. This would ensure that the navigation solution gets rid of the undesirable effects present from biased readings by faulty sensors.

A Collision prevention system was proposed to be implemented using the algorithms discussed in the thesis. This system consists (for each vehicle) of a low-cost IMU that measures accelerations and angular rates of the vehicle, GPS receiver, wheel encoders. The sensors measure position and velocities. To implement the algorithm, an MCU, like MPC 555, is needed. Also, a wireless communication protocol, like Zigbee transceiver, is needed to send and receive position, velocity and attitude information to and from other vehicles.

5.2 FUTURE WORK

This thesis work represents some contributions in the ocean of knowledge especially in the navigation field. So, as a continuation of this work, it is advised that more effort should be put into improving the algorithm performance and implementation. This can be achieved by applying more effort on the tuning of the process and the measurements covariance. The use of AI methods, like NN and ANFIS, to set these covariance parameters is highly advisable to be considered. Also, AI can be considered instead of mathematical approaches for states estimation, or a hybrid system that combines AI and EKF/EIF methods can be considered.

The accuracy of attitude angles, especially the yaw angle, should be considered more. This applies especially for the estimation of the yaw angle as yaw is not an observable state in this thesis. The enhancement of the estimation of this state can be considered by the fusion of a magnetometer or a digital compass to make yaw an observable state.

Also, the implementation of the algorithm and the code on the collision prevention system proposed in this work should be considered in order to be developed into a commercial product. These issues include the choice of wireless protocol to be used and a microcontroller that can handle an adequate processing speed for these algorithms.

References

- [1] Grant, A. George and J. Klinkert, *The Ship's Compass*, 2nd ed., 1970.
- [2] M. Wright, D. Stallings and D. Dunn, "The effectiveness of global positioning system electronic navigation," in *2003 Proceedings IEEE SoutheastCon*, pp. 62-67, 2003.
- [3] A. Theiss, D. C. Yen and C. Ku, "Global Positioning Systems: an analysis of applications, current development and future implementations," *Computer Standards & Interfaces*, vol. 27, pp. 89-100, 2005.
- [4] O. Anderson, "Experience with kinematic GPS during aerial photography in Norway," in *Proceedings of the 42nd Photogrammetric Week, Stuttgart Univ., Stuttgart, Germany*, pp. 299-312, 1989.
- [5] K. P. Schwartz, "An integrated INS/GPS approach to the georeferencing of remotely sensed data," *Photogramm. Eng. Remote Sens.*, vol. 59 (11), pp. 1667-1674, 1993.
- [6] Y. Yang and J. A. Farrell, "Magnetometer and Differential Carrier Phase GPS-Aided INS for Advanced Vehicle Control," *IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION*, vol. 19 (2), pp. 269-282, 2003.
- [7] T. Bell, "Automatic tractor guidance using carrier-phase differential GPS," *Computers and Electronics in Agriculture*, vol. 25, pp. 53-66, 2000.
- [8] S. Han and J. Wang, "A novel method to integrate IMU and magnetometers in attitude and heading reference systems," *Journal of Navigation*, vol. 64 (4), pp. 727-738, 2011.
- [9] L. Di, R. Landry and P. Lavoie, "Low-cost MEMS sensor-based attitude determination system by integration of magnetometers and GPS: A real-data test and performance evaluation," in *Position, Location and Navigation Symposium, 2008 IEEE/ION*, pp. 1190-1198, 2008.
- [10] M. Greene, "A solid state attitude heading reference system for general aviation," in *1996 IEEE Conference on Emerging Technologies and Factory Automation, 1996. EFTA '96. Proceedings*, pp. 413-417 vol. 2, 1996.
- [11] L. R. Sahawneh et al., "Real-Time Implementation of GPS Aided Low Cost Strapdown Inertial Navigation System," *Journal of Intelligent & Robotic Systems*, vol. 61 (1-4), pp. 527-544, 2011.
- [12] S. Sukkarieh, "Low Cost, High Integrity, Aided Inertial Navigation Systems for Autonomous Land Vehicles," Ph.D. dissertation, Mechanical and Mechatronic Engineering, Australian Centre for Field Robotics, The University of Sydney, Sydney, Australia, March, 2000.
- [13] A. Noureldin et al., "Performance Enhancement of MEMS-Based INS/GPS Integration for Low-Cost Navigation Applications," *IEEE Transactions on Vehicular Technology*, vol. 58 (3), pp. 1077-1096, March, 2009.
- [14] R. Schelling, "A Low-Cost Angular Rate Sensor for Automotive Applications in Surface Micromachining Technology," in *Third Annual International Conference on Advanced Microsystems for Automotive Applications Proceedings*, March, 1999.
- [15] M. F. Abdel-Hafez. "The Autocovariance Least Squares Technique for GPS Measurement Noise Estimation," *IEEE Transactions on Vehicular Technology*, vol. 59 (2), pp. 574-588, January, 2010.

- [16] S. Zhen et al., "Angular velocity estimation in gyroscope-free inertial measurement system based on unscented kalman filter," in *2010 8th World Congress on Intelligent Control and Automation (WCICA)*, pp. 2031-2034, 2010.
- [17] T. Katagiri, T. Momose and H. Mizumoto, "An integrated single-output signal encoder for both multi-turn absolute encoder use and incremental encoder use," in *Industry Applications Society Annual Meeting, 1994., Conference Record of the 1994 IEEE*, pp. 347-351 vol. 1, 1994.
- [18] A. Brandt and J. F. Gardner, "Constrained Navigation Algorithms for Strapdown Inertial Navigation Systems with Reduced Set of Sensors," in *Proceedings of the American Control Conference*, pp. 1848-1852, June, 1998.
- [19] G. Dissanayake and S. Sukkariyeh, "The Aiding of a Low-Cost Strapdown Inertial Measurement Unit Using Vehicle Model Constraints for Land Vehicle Applications," *IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION*, vol. 17 (5), pp. 731-747, October, 2001.
- [20] S. Biswas, R. Tatchikou and F. Dion. "Vehicle-to-Vehicle Wireless Communication Protocols for Enhancing Highway Traffic Safety," *IEEE Communications Magazine*, vol. 44 (1), pp. 74-82, 2006.
- [21] P. Belanović et al., "On Wireless Links for Vehicle-to-Infrastructure Communications," *IEEE Transactions on Vehicular Technology*, vol. 59 (1), pp. 269-282, 2010.
- [22] P. Kinney, "ZigBee Technology: Wireless Control that Simply Works," in *Communications Design Conference*, October, 2003.
- [23] M. F. Abdel-Hafez, I. Muhammad, K. M. Saadeddin and A. A. Al-Radaideh, "Low-cost, Non-centralized, Vehicle Collision Prevention System," in *18th Annual International Conference on Mechatronics and Machine Vision in Practice 2011*, pp. 121-137, October, 2011.
- [24] K. Saadeddin, M. F. Abdel-Hafez and M. A. Jarrah, "Low-cost, high-accuracy, state estimation for vehicle collision prevention system," in *2012 8th International Symposium on Mechatronics and its Applications (ISMA)*, pp. 1-11, 2012.
- [25] A. Lahrech, C. Boucher and J. Noyer, "Accurate vehicle positioning in urban areas," in *IMTC 2007 - IEEE Instrumentation and Measurement Technology Conference*, Warsaw, Poland, pp. 486-490, May, 1-3, 2007.
- [26] L. Che-Chung, L. Chi-Wei, H. Dau-Chen and C. Yung-Hsin, "Design a Support Vector Machine-based Intelligent System for Vehicle Driving Safety Warning," in *Intelligent Transportation Systems, 2008. ITSC 2008. 11th International IEEE Conference on Intelligent Transportation Systems*, pp. 938-943, 12-15 Oct. 2008.
- [27] B. W. Parkinson and J. J. Spilker. *Global Positioning System: Theory and Applications*, vol. 1 and 2, Washington D.C.: American Institute of Aeronautics and Astronautics, Inc., 1996.
- [28] B. Hoffman-Wellenhof, H. Lichtenegger and J. Collins. *GPS Theory and Practice*, 4th Ed., New York: Springer-Verlag/Wien, 1997.
- [29] Ali and M. R. Ullah Baig Mirza, "Initial orientation of inertial navigation system realized through nonlinear modeling and filtering," *Measurement*, vol. 44, pp. 793-801, 2011.
- [30] N. El-Sheimy, K. Chiang and A. Noureldin, "Developing a low cost MEMS IMU/ GPS integration scheme using constructive neural networks," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44 (2), pp. 582-594, 2008.

- [31] M. M. Reda Taha, A. Noureldin and N. El-Sheimy, "Improving INS/GPS positioning accuracy during GPS outages using fuzzy Logic," in *Proceedings of the 16th International Technical Meeting of the Satellite Division of the Institute of Navigation ION GPS/GNSS 2003*, Oregon Convention Center, Portland, Oregon, pp. 499-508, September 9-12, 2003.
- [32] A. Hiliuta, R. Landry, F. Gagnon, "Fuzzy corrections in a GPS/INS hybrid navigation system," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 2 (2), pp. 591-600, 2004.
- [33] A. Noureldin, A. Osman and N. El-Sheimy, "A neuro-wavelet method for multisensor system integration for vehicular navigation," *J. Meas. Sci. Technol.*, Lond., UK, vol. 15 (2), pp. 404-412, 2004.
- [34] R. Sharaf et al., "Online INS/GPS integration with a radial basis function neural network," *IEEE Aerosp. Electron. Syst.*, vol. 20 (3), pp. 8-14, 2005.
- [35] D. Hightower. "Wireless Technology Advances Crash Avoidance," *Microwaves & RF*, pp. 22, 2010.
- [36] J. Santaet et al., "An analysis of communication and navigation issues in collision avoidance support systems," *Transportation Research Part C: Emerging Technologies*, vol. 18 (3), pp. 351-366, 2010.
- [37] S. Bonnabel and E. Salaün, "Design and prototyping of a low-cost vehicle localization system with guaranteed convergence properties," *Control Engineering Practice*, vol. 19, pp. 591-601, 2011.
- [38] S. Sadhu et al., "Central difference formulation of risk-sensitive filter," *IEEE Signal Processing Letters*, vol. 14 (6), pp. 421-424, 2007.
- [39] P. Setoodeh, A. R. Khayatian and E. Farjah, "Attitude estimation by divided difference filter-based sensor fusion," *The Journal of Navigation*, vol. 60, pp. 119-128, 2007.
- [40] R. G. Brown and P. Y. C. Hwang. *Introduction to Random Signals and Applied Kalman Filtering*, New York: John Wiley & Sons, Inc., 1992.
- [41] J. Rezaie et al., "GPS/INS Integration using Nonlinear Blending Filters," in *SICE Annual Conference*, pp. 1674-1680, 17-20 September, 2007.
- [42] A. Gelb. *Applied Optimal Estimation*, Massachusetts: The MIT Press, 1974.
- [43] Y. L. Zhang, F. Gao and L. Tian, "INS/GPS integrated navigation for wheeled agricultural robot based on sigma-point Kalman filter," in *7th International Conference on System Simulation and Scientific Computing*, pp. 1425-1431, 10-12 October, 2008.
- [44] J. S. Kim, S. K. Yoon and D. R. Shin, "A state-space approach to multiuser parameter estimation using central difference filter for CDMA systems," *Wireless Personal Communication*, vol. 40, pp. 457-478, 2007.
- [45] A. Noureldin, A. El-Shafie and M. Bayoumi, "GPS/INS integration utilizing dynamic neural networks for vehicular navigation," *Information Fusion*, vol. 12, pp. 48-57, 2011.
- [46] T. Zhang and X. Xu, "A new method of seamless land navigation for GPS/INS integrated system," *Measurement*, vol. 45, pp. 691-701, 2012.
- [47] A. Noureldin, A. El-Shafie and M. R. Taha, "Optimizing neuro-fuzzy modules for data fusion of vehicular navigation systems using temporal cross-validation," *Engineering Applications of Artificial Intelligence*, vol. 20, pp. 49-61, 2007.
- [48] C. M. Bishop. *Neural Networks for Pattern Recognition*. New York: Oxford University Press, 1995.

- [49] N. El-Sheimy, K. Chiang and A. Noureldin, "The Utilization of Artificial Neural Networks for Multisensor System Integration in Navigation and Positioning Instruments," *IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT*, vol. 55 (5), pp. 1606-1615, October, 2006.
- [50] S. Fang, B. Lu and Y. Hsu, "Learning Location From Sequential Signal Strength Based on GSM Experimental Data," *IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY*, vol. 61 (2), pp. 726-736, February, 2012.
- [51] N. Hui, V. Mahendar and D. K. Pratihar, "Time-optimal, collision-free navigation of a car-like mobile robot using neuro-fuzzy approaches," *Fuzzy Sets and Systems*, vol. 157, pp. 2171-2204, 2006.
- [52] H. Liu, S. Nassar and N. El-Sheimy, "Two-Filter Smoothing for Accurate INS/GPS Land-Vehicle Navigation in Urban Centers," *IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY*, vol. 59 (9), pp. 4256-4267, November, 2010.
- [53] H. Rauch, F. Tung and C. Striebel, "Maximum likelihood estimates of linear dynamic systems," *AIAA J.*, vol. 3 (8), pp. 1445-1450, Aug., 1965.
- [54] A. Okatan, C. Hajiyeve and U. Hajiyeve, "Fault detection in sensor information fusion Kalman filter," *Int. J. Electron. Commun. (AEÜ)*, vol. 63, pp. 762-768, 2009.
- [55] F. Caliskan and C. Hajiyeve, "Innovation sequence application to aircraft sensor fault detection: comparison of checking covariance matrix algorithms," *ISA Transactions*, vol. 39, pp. 47-56, 2000.
- [56] F. N. Pirmoradi, F. Sassani and C. W. de Silva, "Fault detection and diagnosis in a spacecraft attitude determination system," *Acta Astronautica*, vol. 65, pp. 710-729, 2009.
- [57] X. Li and W. Zhang, "An Adaptive Fault-Tolerant Multisensor Navigation Strategy for Automated Vehicles," *IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY*, vol. 59 (6), pp. 2815-2829, July, 2010.
- [58] M. A. Jaradat and R. Langari, "A hybrid intelligent system for fault detection and sensor fusion," *Applied Soft Computing*, vol. 9, pp. 415-422, 2009.
- [59] N. Mehranbod, M. Soroush and C. Panjapornpon, "A method of sensor fault detection and identification," *Journal of Process Control*, vol. 15, pp. 321-339, 2005.
- [60] M. S. Grewal, L. R. Weill and A. P. Andrews. *Global Positioning Systems, Inertial Navigation, and Integration*. New Jersey: Wiley Interscience, 2007.
- [61] L. R. Sahawneh, "Real-Time Implementation of GPS aided Low Cost Strapdown Inertial Navigation System," Ph.D. dissertation, Mechatronics Engineering, College of Engineering, The American University of Sharjah, UAE, April, 2009.
- [62] X. Kong, "INS algorithm using quaternion model for low cost IMU," *Robotics and Autonomous Systems*, vol. 46, pp. 221-246, 2004.
- [63] S. J. Julier, J. K. Uhlmann and H. F. Durrant-Whyte, "A new approach for filtering nonlinear systems," in *Proceedings of the 1995 American Control Conference, Seattle, WA*, pp. 1628-1632, 1995.
- [64] S. J. Julier, "Process models for the navigation of high speed land vehicles," Ph.D. thesis, University of Oxford, UK, 1997.
- [65] E. Vinande, P. Axelard and D. Akkos, "Mounting-Angle Estimation for Personal Navigation Devices," *IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY*, vol. 59 (3), pp. 1129-1138, March, 2010.
- [66] Microbotic, Inc. MIDG II Display and Configuration Program. *Microbotic, Inc.*, March, 2007.

- [67] M. F. Abdel-Hafez and J. L. Speyer, "GPS Measurements' Noise-Estimation for Operation in Non-Ideal Environments," *Journal of the Institute of Navigation*, vol. 55 (1), pp. 55-66, 2008.
- [68] T. S. AbuHashim, M. F. Abdel-Hafez and M. A. Al-Jarrah, "Building a robust integrity monitoring algorithm for a low cost GPS-aided-INS system," *International Journal of Control, Automation, and Systems*, vol. 8 (5), pp. 1108-1122, Oct., 2010.

APPENDIX: NAVIGATION COORDINATE FRAMES

A.1 EARTH CENTERED INERTIAL FRAME (ECI)

The Earth Centered Inertial (ECI) reference frame is a frame which is fixed to celestial objects with its origin at the center of the earth. That means as the Earth rotates, the ECI frame stays fixed. Although it is called inertial, it is not inertial in fact. This is due to the acceleration of the Earth while orbiting around the Sun. However, in many cases this has a small effect.

The orientation of the ECI frame is defined based on the Earth's orbit plane around the Sun, which is called ecliptic. The angle between this plane and the Earth's equatorial plane (known as the celestial equatorial plane) is around 23.4° . The ecliptic and the celestial equator do intersect at two points. A point of intersection between the two planes is known as a vernal equinox. The two planes, the angle between them and one of the two vernal equinox points are shown in the figure below.

Figure 114 Ecliptic and Celestial Equator

The x-axis of the ECI frame points towards the vernal equinox. In other words, the x-axis can be thought of as an axis extended from the center of the Earth to the center of the Sun at the beginning of spring season. The z-axis of the ECI frame points towards the North Pole along the rotational axis of the Earth. Finally the y-axis lies on the celestial equator of the Earth thus completing a right-handed orthogonal system. The table below gives a summary about the origin and axes of the ECI frame.

Table 3 ECI Frame Summary

Origin	At the geometric center of the Earth
x-axis	Points to vernal equinox
y-axis	On celestial equator completing right-handed orthogonal system
Z axis	Points to the North Celestial Pole

Figure 115 The ECI Frame

Figure 115 shows the three axes of the ECI frame. The next section talks about the frame usually used by the global positioning system (GPS).

A.2 WORLD GEODETIC SYSTEM 1984 (WGS-84)

The Earth's surface is assumed to be an ellipsoid. The distance between the center of the Earth and this ellipsoid is considered as the mean distance of the different topographies on the surface of the Earth. This makes the reference to any altitude above the Earth's surface. Figure 116 shows a schematic diagram of the geoid ellipsoidal orthometric height. This frame is an ECEF frame used by GPS with its origin located at the center of the Earth. The x-axis points to the intersection point between Greenwich meridian and the mean celestial equator. The celestial equator is the reference meridian which is parallel to the zero-meridian. The zero-meridian is defined by the Bureau International de L'Heure (BIH) based on adopted longitudes for its stations. The y-axis is 90 degrees to the east of the x-axis. It completes the right hand orthogonal rule in the mean astronomic (celestial) equator plane. The z-axis is parallel to the conventional international origin for a polar motion based on the adopted latitudes for BIH stations. The table below lists the main parameters defined in this frame.

Table 4 WGS-84 Parameters

Parameter	Value	Unit
Length of semi-major axis	6378137.0	m
Length of semi-minor axis	6356752.314245	m
Inverse flattening	298.257223563	-
Eccentricity of the Earth's ellipsoid	0.0818191908426	-
Earth's angular velocity	7.292115×10^{-5}	rad/s
Earth's gravitational constant	3986004.418×10^8	m^3/s^2
Earth's mass	5.9733328×10^{24}	kg
Pi (π)	3.145926535898	-

Figure 116 A Schematic Diagram of the Geoid

A.3 EARTH-CENTERED EARTH-FIXEDFRAME (ECEF)

One of the most commonly coordinate systems used as a basis in navigation solutions, like in this thesis, is the Earth-Centered Earth-Fixed (ECEF) frame. It has its x axis pointing to the intersection between the Equator and Prime Meridian as shown in the next figure. The y axis points 90° to the east and the z axis points to the celestial North Pole of the Earth. This coordinate system rotates with the Earth and its origin is at the Earth's center of gravity, hence its name. The next figure shows the ECEF frame axes.

Figure 117 ECEF Frame Axes

A.4 NORTH-EAST-DOWN FRAME (NED)

This coordinate system is commonly used in aviation applications. There are three axes in this coordinate system. The x axis points tangentially to the celestial North Pole of the Earth. The y axis points to the East direction, while the z-axis completes the right-handed orthogonal rule by pointing down as shown in the following figure. This coordinate system has its origin at the vehicle's center of gravity. The following figure shows the axes directions in NED frame.

Figure 118 NED Frame Axes (in Green)

A.5 EAST-NORTH-UP FRAME (ENU)

This frame's axes' directions are exactly like the previously explained NED frame except that the x axis points east, the y axis points North and the z axis direction is chosen to complete the right-handed orthogonal rule by pointing up instead of pointing down to the Earth's center of gravity. The origin is still considered to be the vehicle's center of gravity. The next figure shows the axes of the ENU frame.

Figure 119 ENU Frame Axes (in Green)

A.6 BODY FRAME

In this frame, the origin is considered to be at the vehicle's center of gravity. It has three axes. The x axis points to the forward direction of the vehicle, the y axis points 90° to the right hand side of the vehicle and the z axis points directly down. The attitude of a vehicle is calculated around these axes. The pitch angle is around the y axis, roll is around the x axis and yaw is around the z axis. The axes of this frame are shown in the following figure. Usually, to have an accurate navigation system the inertial measurement units' axes are aligned with this frame's axes.

Figure 120 Body Frame Axes and Euler Angles

VITA

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