CARGO-DELIVERY-BOX SYSTEM FOR MODELING THE LAST MILE OF BUSINESS-TO-CONSUMER LOGISTICS

by

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Dedication

To my dear family and future wife

Abstract

The current business trends suggest that the e-commerce business in the UAE is increasing rapidly, which means that the supply chain facilities must be improved to sustain efficiency and profitability. The last mile is highlighted to be the most expensive part of the supply chain. Therefore, optimizing the last mile can increase cost savings for the logistics companies and may even translate into reduced charges for the end consumer. The traditional approach of house to house delivery not only poses high costs but it also contributes to road congestions, delays in deliveries, fragmentation of deliveries and higher carbon emissions. The research showed a real gap in providing a solution that considers all aspects of delivery; the parcel routing, delivery mechanism but also the reception method for the convenience of the customer and the company alike. In this research, we propose a new solution, which calls for consolidating the deliveries from e-retailers at the urban consolidation centers. The urban consolidation center then groups the shipments according to their destinations, loads them on to the cargo delivery boxes. The cargo delivery boxes are then shipped to the corresponding locations using commercially economical vehicles. These potential delivery locations can be either restaurants, grocery stores or retail shops that are spread across the city and are easily in reach to the customers of that area. The customers would be able to collect their deliveries at their conveniences, and the delivery box is collected by the delivery vehicle at night. We formulated a two-stage approach solution and a consolidated model. We validated the models on many scenarios before finally testing the final model (Two-stage Solution) on a data set of five hundred customers, spread across two hundred and fifty-kilometer squares. Finally, the sensitivity analysis on the model showed that the customer footprint across a given area and the size range options of the cargo delivery boxes were the most sensitive parameters to the total last-mile cost.

Keywords: ***Last-mile; e-commerce; logistics collaboration; cargo delivery box model; cluster delivery; missed delivery***

Table of Contents

List of Figures

List of Tables

Chapter 1. Introduction

In this chapter, we provide a brief overview about e-commerce, followed by a description of the last mile. Then we present the issues, their factors and our solution approach and its significance. Finally, a general organization of the thesis is presented.

1.1. Background

In the last decade, the growth of the e-commerce industry has been substantial and is still growing despite the recent drop in oil prices. The estimates show that the world-wide Business to Consumer (B2C) e-commerce sales have risen to 1.915 trillion dollars in 2016, although the majority of this statistic is due to the US, European and Asia-pacific markets. An emerging percentage of this statistics would also be from the emerging markets like Middle-east [1].

One of the significant contributors to the e-commerce is the growth of the smartphone industry and the ease of access to the internet that has offered numerous channels for the consumers to opt through all kinds of products and services. A study by ATKearney on the Middle East shows that smartphone penetration was 65%, while the market size is expected to grow from \$5.3 billion in 2015 to \$19.8 billion by 2020 [2]. Other retail companies are also realizing the significance of the e-retail channels and are transforming their business models to include e-services where the customers can browse and order the products online. Some companies operating in the UAE and the Middle East work entirely through e-services like souq.com, cobone.com, and groupon.com; however, the e-business is not restricted to these companies only, people also tend to order goods internationally as well through websites like Amazon, eBay, Bonanza, and several others.

It is essential to optimize the last part of the supply chain to maximize profits while maintaining the best service levels to achieve customer satisfaction. Customer service are characterized by lower delivery costs, timely delivery, and delivery time window (time window in which the customer expects the delivery), including several other factors like frequency of delivery and allowing the customer to return the goods also characterize customer service levels [3]. To understand the flow of the process that undergo in a B2C e-commerce fulfillment environment we can refer to the diagram in Figure 1.1, which gives a brief overview of the entire process.

1.2. The Last Mile of Supply Chain

The supply chain in the simplest terms can be described as a sequence of events and processes that occur in the entire product life cycle from the starting phase of a product i.e. from raw material to processing, producing finished good, packaging and shipping to the warehouses, and then either to the retail shops or directly to the consumer [4].

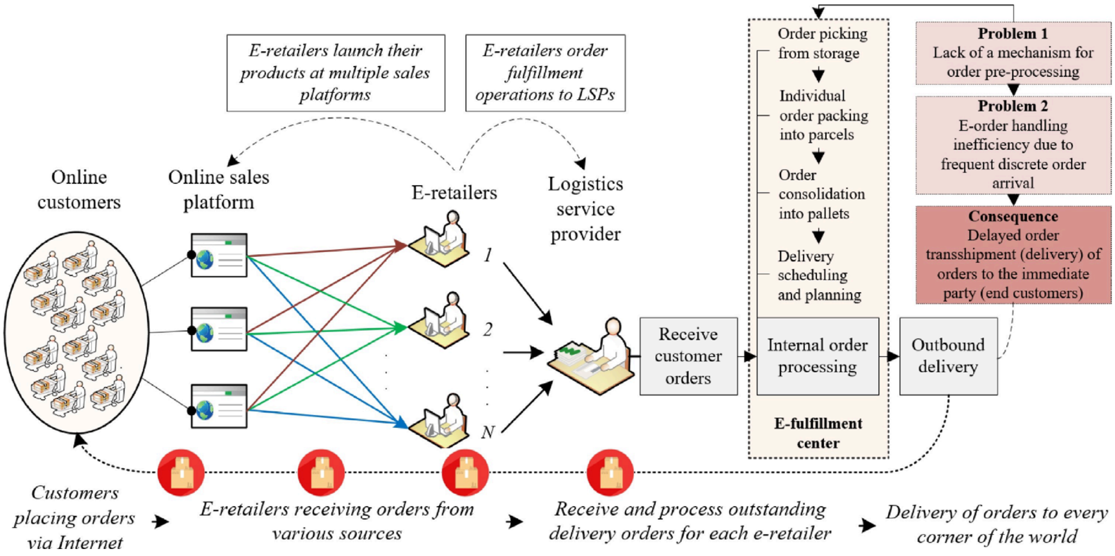

Figure 1.1: E-fulfillment process in a standard B2C environment [5]

The first part of the supply chain, that is related to the inbound logistics of raw materials to the processing and production is the first mile, likewise, the last part of the supply chain which is related to the outbound logistics, in which the good reaches the customer, is referred to as the last mile. In this research, however, our focus is only on the latter. We describe the Last mile as the last leg of the supply chain in which the product customer receives the delivery. The delivery goes from either the hub, warehouse or a distribution center, or it can be from retail shop directly to the customer as well, as can also be seen from Figure 1 further details about the typologies of the last mile are described in the next section. In this thesis, our focus is on the business to consumer sector, the words “Business to Consumer (B2C)” imply that a transaction occurs between a company and a consumer, while a transaction between two distinct companies is “Business to Business (B2B)” scenario. The last mile is typically known as a problematic area in the city logistics, resulting in high transportation costs for the logistics providers on top of various other challenges that are discussed in detail in the upcoming sections [6].

1.2.1. Typologies of the last mile. Several typologies have been mentioned in the previous years to summarize the sub-flows in the last mile. It is necessary to define the topologies because it helps in describing the different types of delivery methods that exist in the last mile. Figure 1.2 illustrates a very common typology to represent the last mile.

Figure 1.2: A typical Supply-chain network depicting the first and last mile [3].

The last mile is most commonly divided into four quadrants, direct delivery, indirect delivery, Store-based order delivery or distribution center-based delivery as show in Figure 1.3. In case of the semi-extended supply chain, the order is delivered from a store to a nearby location like a post office. The fully extended supply chain is also similar since the delivery is made from the store directly to the customer’s home. Likewise, in the decoupled supply chain delivery is from the DC to a nearby location, and lastly, in a centralized extended supply chain, the goods are delivered from the DC directly to the customer [7]. Although this version is quite popular amongst the last mile related researches, however, a relatively better depiction is described in Figure 1.4. The topology depicted in Figure 1.3, does not completely cover all the reception methods; however, it covers most of them and gives a good overview of the types of delivery. The starting point is the storage location which can be either a DC or a shop. The customer has a choice to either collect the goods directly from there or order delivery. In the case of delivery, either clustering can be used or home deliveries. In clustering, usually a delivery box or a reception box are fitted in a populated location, the customer can travel to these boxes and collect their items.

Figure 1.3: Topology to describe the last mile [7]

In the case of home deliveries, there are two types, attended in which the customer must be home, or unattended, where the delivery is done even if the customer is not home.

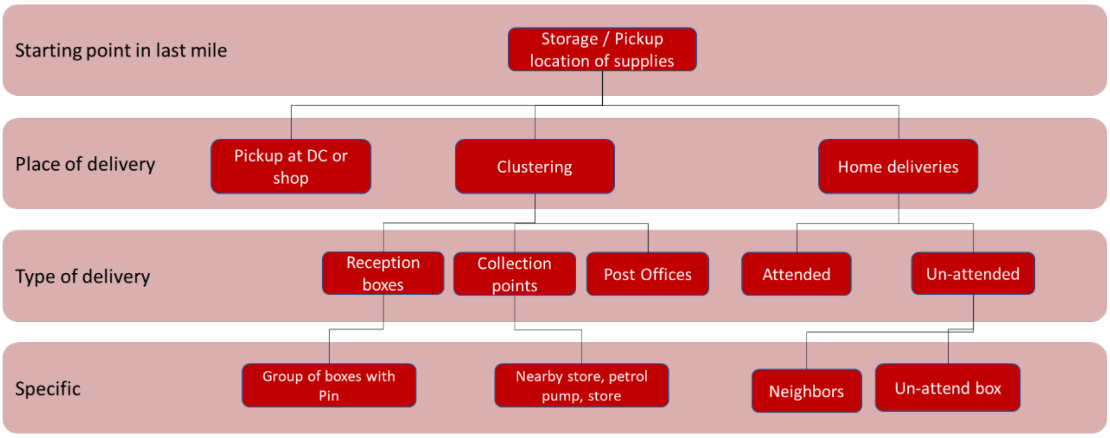

Figure 1.4: Reception methods of the last mile [3]

1.2.2. Challenges in the last mile. In general, the last mile is the most inefficient area of the supply chain, in some cases, it was reported to cost up to 75 % of the total logistics cost [7]. The Challenges that arise are a function of several factors:

The Population of the city: When considering a town with a considerable population, issues like traffic congestions are sure to occur daily, especially around office hours (example Dubai and Sharjah traffic). Long hours of traffic congestion, followed by parking issues could mean that the delivery vans reach their respective destinations late, resulting in late deliveries or missed deliveries, consequently, affecting the customer satisfaction and the transportation costs. Another issue that is bound to arise in a highly populated city is the fragmentation of deliveries (example Seoul, Korea). The fragmentation usually occurs for instance when multiple companies are delivering to the same building or an area. Also, each company individually would have too many deliveries taking place each day. On the other hand, when we consider areas of lower population, each delivering company will encounter issues of low orders. That means lower utilization of trucks and vans, delivery to remote areas, longer traveling distances for a small number of deliveries, in turn translates into higher transportation costs for the carrier.

Reception methods: The type of reception method that a company chooses for its delivery mechanism is essential due to its effect on the resultant customer Satisfaction and cost. The customer would have a tradeoff between traveling to collect delivery or paying a higher fee. In the case of attended home delivery, complications like missed deliveries can arise, for example, if the customer is not available, while in the case of unattended home deliveries, the companies would have to deliver to the fixed delivery boxes. In this case, the company would have to consider high initial costs for establishing the delivery boxes in addition to the delivery costs. If the company finds employing the reception method where the customer must collect the deliveries from the store or directly from the warehouse, it will reduce the transportation cost, but hurt the customer satisfaction.

Characterization of products: Food items and non-food items categorize the delivery products; food items can be further broken down into refrigerated food items and non-refrigerated food items. Now when we consider the food items, the customers will expect same day deliveries or next day deliveries which means added pressure on the delivering entity for on-time delivery, in addition to the food-related additional costs, for example, refrigerated trucks, refrigerated delivery boxes, etc. In case of non-food items, we will have parcels (e.g., letters, documents), or light packages (e.g., small electronics items, clothing items, etc.) or heavy items (e.g., heavy appliances, machinery, furniture, etc.). In this area, challenges may arise concerning heavy objects, as they will require personalized deliveries and large trucks.

Companies often fail to overcome these challenges and consequently go bankrupt; one such example is that of Webvan. It started as a last mile service provider for the e-grocer companies in the USA. Webvan introduced the concept of home delivery in 30-minute time windows during 1999, but they couldn't maintain profitability and compete with other companies, hence they tried increasing the time window to 60-minutes to reduce cost of operations, as a result the customer service level was negatively affected, finally the company failed and filed for bankruptcy later on by 2004 [8].

1.3. Problem Statement

We are modeling the delivery system as "un-attended cluster delivery." However, as the customers must travel a long distance to collect their deliveries, we must bring the parcels as close as possible to the customers

The fundamental problem is to allow the customers to collect the parcel from the closest pickup locations, and then the parcels of that location are aggregated into the corresponding delivery box and delivered to the relevant area. Each delivery box is constrained to carry up to a certain number of parcels at a time. Every customer is required to be within a range of 500m from the pickup location. The use of every delivery box at the pickup location incurs a cost. Also, the potential sites have a limit concerning the number of delivery boxes they can store. Hence, the total number of delivery boxes at each location will determine the maximum capacity of that site. The goal is to serve all the customers with a minimum number of resources, to reduce the overall cost while improving the customer satisfaction. The underlying idea is to avoid underutilization of the delivery boxes. We assumed that the cargo boxes are on lease; therefore, there is also a specific cost of using/leasing the delivery box, and hence the problem is to determine the minimum number of delivery boxes to satisfy the total requirements.

Another issue is to determine the routing of the parcels, from the companies to the pickup locations, taking into the account, which urban consolidation they go through to reach the destination. Besides, the delivery boxes that go to the delivery locations are assumed to be delivered by a vehicle. This vehicle also needs to have proper routing based on which Urban consolidation center (UCC) it belongs to, the load of the vehicle, and the number of deliveries to be made from each UCC and the number of locations. This model should also account for the missed deliveries that may or may not occur. Since some customers may not be able to pick up their shipments on the first day altogether.

The model is to be run each day, to plan the deliveries throughout the chain. Therefore, the runtime for the model should not exceed a couple of hours so that there is ample time to execute the plan.

1.4. Research Objective

In this research, our primary focus is the optimization of the last mile in the logistics of B2C deliveries in an urban environment. We propose a last mile solution that will help reduce the negative impacts of the last mile (increased transportation costs, traffic congestion during business hours, and delivery issues). We used consolidation of the last mile as a tool to overcome fragmentation of deliveries. Therefore, an alliance can help reduce the costs collectively for each of them. In this research, we consider a possible area where the delivery companies collaborate to make the deliveries. Hence if multiple companies must deliver to the same area, they can be served by the same warehouse/ Urban consolidation center (UCC).

In the proposed model, firstly, the logistics companies will deliver the goods to the transshipment areas that are in the city, where the deliveries from these companies will be consolidated and sorted according to the delivery locations. Secondly, from the transshipment areas, the goods are going to be delivered to the sites using the cargo delivery box (CDB). We assumed that a fuel-efficient vehicle would make the delivery of the cargo delivery boxes. The delivery person will take the cargo delivery box to the appropriate delivery location. The delivery collection points can be retail shops, grocery stores or restaurants that are located all over the city. We will have pre-arranged agreements with the owners of those shops, to leave the delivery boxes there at a certain rent per day. This agreement will be good for the shop owners since it will increase visits to their shops. In return, they will serve as a security measure for our delivery boxes.

The idea of the CDB is that the customers will be able to take the deliveries, return the deliveries and even pay for the deliveries directly into the CDB with ample time on their hands. Next day, the delivery person will have new deliveries and will deliver the CDB to a new location, after providing the fresh CDB he can collect the old

CDB on the way back, thus completing the cycle. We will equip the CDB with lockers that are carrying the deliveries of individual customers of that area. When the Cargo delivery box goes to its location, then customers will be notified on their registered phones to collect their deliveries. The delivery company will provide each customer with a code to unlock the locker.

This model (as described in Figure 1.5) will enable the deliveries to be made once in a day to each of the locations, at the time in the day when the traffic condition is better than the rush hour times. The deliveries can be dispatched simultaneously to most of the locations based on the available number of vehicles. The single trip of vehicles will simplify routing and eliminate the need of time windows in determining the delivery schedule. The same vehicle will collect it back and bring it to the transshipment area for next day.

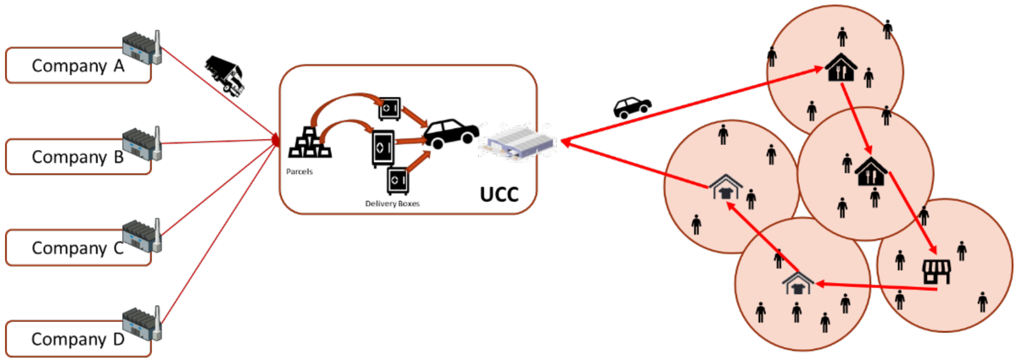

Figure 1.5: Consolidation and Cargo Delivery Box logistics model

We will use a fuel-efficient vehicle for the delivery from the transshipment area like for example, a cargo delivery bike. The advantage of cargo bikes is that they have a smaller footprint compared to the conventional delivery vans. Hence, they require less maintenance, reduced emissions, and reduced fuel consumption thus low transportation costs. Since we are just delivering the CDBs to the delivery points, the delivery person doesn’t have to visit each of the customers separately thus conserving overall time and costs. Also, we don’t have to worry about the vehicle utilization rate since we are considering deliveries for the companies to an area. As such, it is bound to have a considerable amount of shipments compared to the individual delivery case.

For the proposed solution, we constructed a mathematical model, with randomly generated orders from around the city. The model identifies the optimum routing of parcels, vehicles, trucks, and routes the customer the correct location. A realistic scenario is modeled from the base model to consider how we manage the missed deliveries, and situations where we also take into consideration the capacities of the Trucks, the transshipment areas in addition to the delivery vehicles.

1.5. Research Methodology

The steps that we followed in this research are defined here:

- An extensive review of the literature on, transshipment areas, the fleet of vehicles, vehicle routing, consolidation, reception methods and new methods of delivery is conducted.
- A comprehensive analysis of solution approaches, algorithms, and heuristics approaches that are related to the research is carried out.
- Develop mathematical models to optimize the locations of the cargo box delivery points, and the size and the number of cargo delivery boxes required for parcel shipments.
- Develop mathematical models to optimize the parcel route from the source to destination while reducing overall transportation costs and streamlining the solution for the customers and the companies the same.
- Perform sensitivity analysis on critical parameters to test the sensitivity of the model.

1.6. The Significance of the Research

The main contribution of this research is as follows:

- Propose a solution approach that that will optimize the last mile collectively for the e-retailers, courier companies and improve the delivery service for the customers as well.
- Reduce the traffic congestion by freight vehicles movement in the inner-city areas.
- Provide a mathematical formulation for optimizing the locations of the delivery points, the routing of parcels from their sources, the routing of delivery boxes and the required number of cargo delivery boxes and their types.

- Ultimately the model will also help reduce the number of missed deliveries for the companies and increase the cost savings for the involved parties.
- Our research opens areas for further analysis on the last mile, green logistics, and reception methods

1.7. Thesis Organization

The rest of this thesis is organized as follows. I present the literature review, followed by the literature gaps in Chapter 2. Chapter 3 presents the formulation of the model and each additional iteration of the model and its subsequent validation, leading up to the final model. Chapter 4 presents the analysis on the final model of a sample case study area for a relatively large customer base, followed by sensitivity analysis on the same model. Finally, Chapter 5 presents the conclusion, recommendations and areas of future research.

Chapter 2. Literature Review

This chapter provides a review of the methodologies and solution approaches undertook in the literature to face the challenges mentioned earlier. Firstly, the research concerning the vehicle routing problems is discussed because it is the most crucial part of the literature and covers a vast majority of research, on the last mile.

2.1. Vehicle Routing Problems Based Approaches

Vehicle routing problems (VRP) is one of the most extensively researched areas since it was first formulated as a mathematical programming model in 1958 by Dantzig and Ramser [9]. However, in the last decade there has been a considerable amount of research done on enhancing the VRP models and solutions for a variety of applications like for city logistics' problems of traffic congestion, noise and air pollution, fuel consumption including a range of other issues faced by the stakeholders such as carriers, shippers and the residents [10].

The VRP is a derivation of the traveling salesman problem (TSP), where several vehicles with a given capacity are routed from a depot or multiple depots to several customers optimally. VRP is considered np-hard problem because of its difficulty level in the formulation and modeling [10]. Over the years, several variations of the VRP have been developed after the capacitated vehicle routing problem (CVRP), starting initially with vehicle routing problems with time windows(VRPTW) [11], in which they route a homogenous group of vehicles, while the delivery has to follow the time windows allotted for each of the customers. Other examples of VRP include routing of a heterogeneous fleet of vehicles [12], which considers a bit more practical scenario by routing a group of vehicles with different capacities. Other variants of the heterogeneous VRP also exist like the site-dependent vehicle routing problem (SDVRP), in which a limited fleet of different vehicles are available for routing and the type of vehicles that can visit each of the customers are restricted [13]. The older researches, however, were too impractical when considering the assumptions that they made, like for example, neglecting the dynamic behavior of the traffic on the streets whereas the objectives were also short-sighted since most of them overlooked the time factors from the studies and focused only on goals like minimizing the total travel distance. One of the recent researches extends the VRPTW to solve the problem of the distribution of fruits and vegetable using a Genetic Algorithm to solve the problem, in the cold chain context [14]. The routing research also focuses on the delivery of goods in a different setting, for example in disaster relief operations. Rabta *et al.* [15] proposes a MILP model for routing a fleet of drone vehicles through the depot, recharging stations to required areas and back, while carrying relief aid.

Nevertheless, the recent research into the VRPs is evolving to tackle different objectives such as reducing total travel cost, travel time, fleet size of the vehicle, even scenarios considering the real time-traffic congestion problems in the formulation. Therefore, many recent papers aim at optimizing the last mile use dynamic VRP approaches, rather than static approaches considering the continuously changing status of the traffic congestion on the city roads during different times on each day, including added complexities like stochastic demands, loading constraints, canceled deliveries, etc. [10]. Kim *et al.* [16] proposes using dynamic programming to calculate optimal routing policy using real-time based varying traffic data. Similarly, Cao *et al.* [17] solve the stochastic vehicle routing problem, which considers the uncertainties in the traffic flow, by formulating it as a mixed integer linear programming model (MILP) and then solve it using the partial Lagrangian multiplier method. Ghannadpour *et al.* [18] addresses the dynamic vehicle routing problem (DVRP) with fuzzy travel times and fuzzy time windows. Schneider *et al.*[19] proposes an e-VRPTW, electric vehicle routing solution where they optimally route a group of vehicles through recharging stations during delivery tours. They use a variable neighborhood search algorithm combined with a tabu search heuristic as a solution method. Sitek and Wikarek [20] proposes a capacitated vehicle routing problem with pickup and delivery, they categorize the set of nodes by either pickup or delivery or both and depot. The solution method is a hybrid approach between CLP and MP to solve the problem.

Some studies specifically focusing on the last mile have also proposed the idea of collaborative delivery amongst the shippers with the aim of reducing the overall transportation costs. For example, Villamizar *et al.* [21], in a B2B setting compare the non-collaborative and collaborative scenarios amongst the shippers in the last mile, by modeling the non-collaborative as a CVRP model and solving it using MILP programming. They model the collaborative scenario as a multi-depot capacitated vehicle routing problem (MDCVRP) where any company can deliver to any store with global optimization. They solve the model in 2 phases; firstly, the delivery points are allocated to the depots and then they use MILP to solve the routing. Similarly, Park *et al.* [22] focus on the courier express and parcel delivery in Korea, analyzing the effects of the collaboration during the distribution in the apartment complexes in a metropolitan area. They calculate the impact of collaboration and non-collaboration on the total distance traveled and entire time taken, where they use CVRP to model both of cases. For the collaborative scenario they propose CEP delivery problem vertical routing and horizontal routing (CDPVR and CDPHR), with the objective of minimizing distance and time. Souza *et al.* [23] propose coordinating a multi-party collaboration to optimize the resource utilization of the respective parties through resource planning and allocation of resources to improve overall efficiency.

We categorize the remaining literature by the reception methods that they use in their studies. Reception methods are basically, how the customer receives the delivery. Typically, three main reception methods can appear which are, attended reception, un-attended reception or clustering. In the attended reception, the customer should receive the delivery at home. Hence, it usually involves consideration of the time windows in the modeling; as a result, it can typically be harder than the unattended one. In the context of attended reception, Ehmke and Mattfeld [24] propose using time-dependent travel time data sets to model time-dependent vehicle routing models. They use historical floating car data (FCD) which is probe vehicle data indicating the state of traffic congestion at different times around the city. The FCD data is aggregated using arithmetic means and medians, and then further cluster analysis approach is employed to aggregate the data further. They use a Time-dependent vehicle routing problem (TDVRP) as the optimization model. In a recent study, Kaewpuang *et al.* [25] propose a cooperative environment between small shippers where they allow sharing of vehicles to build a vehicle pool for delivery, using the VRPTW framework to minimize the overall cost. A study by Han *et al.* [26] focus on the missed delivery issue by incorporating appointment scheduling with soft time-windows. The integrative model is then solved using a heuristic dynamic programming approach, a hybrid approach where the result from appointment scheduling is fed into tabu search to produce the final solution.

Furthermore, Rais *et al.* [27] propose a MIP model for the pickup and delivery problem with transshipment (PDPT), which is a generalization of the pick-up and delivery model (PDP) where the vehicles perform service of pick and delivery for the customer. The transshipment option provides an improvement in routing options for the carrier. Moreover, they test the model with and without the inclusion of time windows. However, the model is solved for 10 and 14 nodes only thus indicating an area of further research for a larger dataset.

Murray and Chu [28] propose a flying side-kick traveling salesman problem, where a remotely controlled Unmanned aerial vehicle (UAV) or a drone assist the normal van deliveries. They provide two mathematical formulations as MILP models for two different scenarios with the aim of reducing the returning time to the depot, for the shipments made close to the depot, they provide (PDSTSP) parallel drone scheduling problem and for deliveries made further from the depot (FSTSP) flying side kick traveling salesman problem. They also give an FSTSP heuristic and PDSTSP heuristics for practical utilization of the model. The concept is highly innovative but may face problems in the face of FAA regulations concerning drone flights, besides several other issues need to be addressed such as malfunction or crash scenario. Bányai *et al.* [29] propose a first and last mile (FMLM), real-time smart scheduling of delivery solution, followed by a black hole optimization-based algorithm for the multi-objective mode l.

The researches in VRP are not specific only to the last mile but to many other fields of study as well, which is why there is such huge literature focused on it. Several innovative ideas and methods were introduced to optimize the last mile problems, However, the most significant issue in most of these researches is that many of them do not consider the customer's point of view in the study, which should be given paramount importance since without the customer's approval the company could eventually fail, hence maintaining the customer satisfaction level should also be one of the core objectives in the studies — a study was done by Chen *et al.* [30] research specifically on customer behavior towards one of the approaches which are self-service portal to see what factors may influence them in to using or not using the approach. The overall research does present a gap in similar studies as well.

2.2. Other Last Mile Approaches

The reception method is very an important factor in the last mile because firstly, it takes into consideration the service point of view of the customer as to how and when they receive the delivery, secondly it may help the logistics service provider or the shipper in better routing options as well.

2.2.1. Un-attended reception and clustering. In the previous section, we discussed the attended reception in the context of VRPs. However, we found different approaches in the case of unattended delivery. Kämäräinen [31] compares the idea of shared boxes (set near the customer's home and shared by others), reception box (placed outside customer's home), or attended reception (delivery directly to the customer). The comparison between the attended reception and unattended (using reception box set outside the house) is compared using simulation. Punakivi *et al.* [8] analyzes the solutions of a delivery box to a reception box using simulation. Both have the same concept of being placed outside of the customer's home thus following the unattended reception terminology. However, the only difference amongst them is that reception boxes are refrigerated and can store products for more extended periods as compared to the delivery box which as the name suggests is just a box. Moreover, Punakivi and Tanskanen [32] further research on increasing the cost efficiency of shared reception boxes, performing simulation using point of sales data (POS) from the stores, to compare shared reception box, delivery box, and reception box concepts based on cost efficiency. Moreover, Punakivi and Saranen [33] investigates the home delivery service concepts, comparing the manned and reception concept with home delivery for e-grocers case, where he concluded that manned reception would cost 2.5 times higher than unmanned reception.

The concept of shared reception boxes is a clustering approach, which in simple terms means that the delivery is to a cluster of customers in an area. A similar idea amidst the unattended and cluster approaches is of a parcel locker, which entails placing several reception boxes in a highly public area. The Inpost company currently uses parcel locker, since this allows for better routing for the company while providing better service level as well concerning security and convenience for the customer [34]. A comparison between attended delivery, reception box, controlled access system, locker banks, and collection points is made concerning several criteria including delivery window, delivery cost, number of failed deliveries, initial investment, etc. [35]. Dell'amico and Hadjidimitriou [36] propose an inventive logistics model in, with a two-fold solution for improving logistics and decreasing the emissions. The freight trucks carry several load units, sending the parcels of sorts, from the depot, and then at the transshipment area, the load units become the body of a smaller delivery van, which in turn delivers the load unit to a Bento-Box system, from where the customer can collect the parcel easily. They test different locations for the transshipment area based on distance traveled and total emissions. Frank *et al.* [37] propose framework for un-attended home delivery based on the customer, logistics companies' preferences. Duin *et al.* [38] propose using principles of address intelligence using multiple regression to predict the improvement or potential rework based on zip codes.

Zhang and Zhang [39] compare the environmental impact of attended and unattended deliveries for book deliveries in the retail sector. The attended deliveries are where the courier distributes directly to each customer one by one. While, on the other hand in, unattended deliveries the deliveries are made to a pickup point for the reader to collect the parcel. The main advantage in the un-attended and clustering solutions is that the routing becomes easier and most importantly the time variable is overcome since the delivery is on the company's convenience and the customer collects the shipment at their convenience. Nevertheless, it is also worth mentioning that usually, these solutions involve placing equipment at the delivery point such as a delivery box, reception box or a locker. As a result, the distributor would bear these additional costs. Usually, these costs can be very high and may result in the infeasibility of the solutions. They also assume the cooperation of the customers, which might not be the case if they are not happy about covering the distance themselves.

2.2.2. Crowd-sourcing based approaches. The study of unattended and clustering-based approaches improves the routing, decreases emissions and increases service levels based on how far the customer must travel to collect the delivery. Nonetheless, all if not most of these solutions fail regarding financial considerations since the initial cost of the systems is too high for implementation unless the government or an external investment incentivize the delivery.

Wang *et al.* [40], propose a combination of two solutions to handle the problem of high initial investment in the pick-own-parcel stations or POP stations which can be of any nature. In, addition to the pop stations, crowdsourcing is proposed to make the delivery from the pop stations to the customers instead of the customers picking the parcel themselves, hence, reducing the number of required pop stations while improving the customer service and satisfaction. They solve the model as a minimum network cost flow problem, which uses a large pool of workers to make the delivery and solved it with the simplex algorithm with the aim of reducing the total cost. Furthermore, they perform cost-based pruning using a greedy algorithm, capacity-based pruning and frequency-based pruning to reduce the network size drastically to a much more manageable size. However, the proposed model fails to consider the initial issue of the time constraints for delivering to the customer and may result in missed deliveries eventually. Furthermore, Chen and Pan [41] propose a taxi crowd shipping system, using a crowd of taxis as the crowd to make the deliveries. The concept is that suppose a parcel is going to the same area, as the passenger, the taxi delivers that item to a package pickup station, in that area, at no additional cost. Firstly, offline trajectory mining is done to compute the traffic flow. Then in online package routing and taxi scheduling, the taxis are scheduled and routed optimally. Furthermore, this study was further continued in Chen *et al.* [42] where they propose algorithms for the previous methodological research and compare the solution to other algorithms, and furthermore perform a case study-based analysis to indicate favorable results. Arslan *et al.* [43] propose using ad-hoc drivers with excess capacity to make the deliveries or using backup drivers in case no ad-hoc driver is available. They use a rolling horizon framework and an exact approach as a solution method. Paskalathis and Azhari [44] use ant colony optimization on the crowdsource delivery trip consolidation, using greedy heuristics as a solution method. Lee *et al.* [45] proposes an integrated decision-making framework for on-demand delivery services that also takes in to account the environmental emissions and aims at reducing them along with the cost. Markov decision making combined with dynamic programming algorithm is used to process delivery requests and route scheduling.

2.2.3. Cargo bikes. In general, the literature on cargo bikes is scarce. The concept of utilizing cargo delivery bikes is analyzed in King [46], where they make the comparison between the usage of cargo bikes as opposed to the delivery vans. They give an example of "Outspoken delivery," a cycle courier company which operates one of the largest courier operations in the country. However, the paper only describes the idea as implemented by the company and doesn't present any mathematical analysis on how it affects the last mile as compared to the other systems.

Nocerino *et al.* [47] analyze the implementation of green scooters and bicycles that the Pro-E-bikes sue for delivering goods. Pro-E-bike is an Italian bike project, spanning thirty-seven companies and seven European countries. The analysis demonstrates the emission reductions and energy savings achieved through the project. Schliwa *et al.* [48], develops typologies for cycle logistics, identifies the social and regulatory barriers that exist in the implementation of the cargo bikes in the UK and finally propose a sustainable logistics framework for the successful implementation of the cargo cycle logistics. Gruber *et al.* [49], explores the possibility of implementing the electric vehicles as opposed to the combustion engine driven counterparts as potential replacements. Even though the article shows a potential of technical substitution of combustion engine vehicles of around 19-48 %, regardless of the benefits of electric vehicles, the conclusion is that the costs are still a barrier to implementation. Furthermore, Gruber *et al.* [50] explores the potential using E-CBs (electric cargo bikes) for urban courier services, where they test the acceptance of E-CBs amongst the messengers and a general assessment of the usability of E-CBs is done, which reflects the attitudes and the awareness level of the messengers towards electric cargo bikes.

2.3. Research Gaps in the Literature Review

This section summarizes the literature gaps for each of the areas discussed in the previous sections.

2.3.1. Gaps in VRP literature. The VRP literature focuses mostly on providing optimum routing solutions for the delivery vehicles. The central hole in the VRP literature is that the solutions do not focus on the problem of traffic congestion, while the issues of order cancellation, late delivery, and missed deliveries are still un-answered in these researches since they all assume that the customer will be available during the delivery, for example [9], [11], [12], [16], [17], [18], [21] and [22]. The extensive time windows used in the researches will decrease the chance of delivery reception by the customer, while smaller time windows will make it more expensive and complicated concerning computational time requirements.

In addition, the deliveries are still carried out using the delivery vans, who must visit each customer one by one, hence considerable time is spent by the vans on the roads, resulting in a high amount of emissions and traffic congestions once considering deliveries from all companies that are taking place simultaneously.

2.3.2. Gaps in reception, delivery boxes and attended delivery literature. The common issues for literature around reception boxes are that the delivery company or logistics provider fix the reception boxes at a single location for the customers to collect their delivery, this causes inflexibility in the delivery structure since it is targeted towards selected customers only. For a larger area, a lot of reception boxes or delivery boxes would need to be fixed, which would come at a significant cost. The delivery boxes are only useful if the utilization on average is enough. The second most prominent issue is the security concern. In all these cases, the reception, delivery boxes, parcel lockers are all left unattended or guarded, increasing the vulnerability to thefts and robberies. In addition to this, the proposed solutions propose making the deliveries through Vans however the delivery routing or optimization is usually absent.

2.3.3. Gaps in crowd delivery literature. In Crowd delivery solutions for example [40], the underlying assumption of the availability of a large crowd to make the deliveries seems implausible. In some cases, for example [41] and [42], the reception method from the customer is missing. There are chances that the customer might not be available during the delivery and this can lead to missed deliveries. There is a no fail-safe solution in the case of a missed delivery. Also, the concept of returns is also not addressed.

2.3.4. Gaps in cargo bike literature. The Cargo bike is also a greener replacement for the typical delivery van as suggested in [47], compares different types of cargo bikes and cycles based on price, capacity, and payload amongst several other factors. However, no model is presented to test or compare them through a mathematical model. The research focus is mainly on promoting cargo bikes as an environmentally friendly solution as opposed to the vans; no research could be found to try to optimize the cargo bike solution through any modeling or detailed logistical analysis for example [49] and [50].

Chapter 3. Developing Last Mile B2C Logistics Models

To get an overview of the problems that could be faced, a basic model was created, consisting of Companies, urban consolidation centers, potential locations, and end customers. The basic idea of the model is that in a city a series of pre-identified possible delivery locations exist. For each delivery, the model allocates the customer to a nearby site. After the allocation, all the shipments of that customer must reach that location. The parcels are then routed from the companies to appropriate urban consolidation centers (UCCs). We use UCCs to consolidate shipments into delivery boxes and then dispatch to corresponding locations for the customer to pick up their deliveries, as described in Figure 3.1. In later models, we cover all the problems and issues identified in the basic model (model 1).

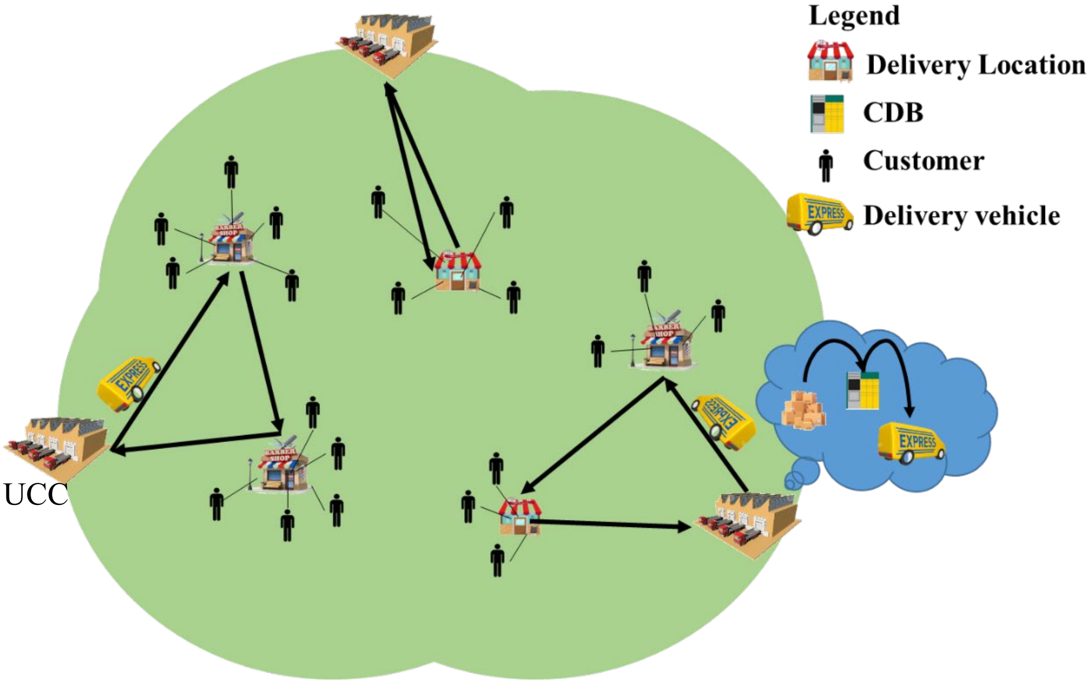

Figure 3.1: One-time delivery model using delivery boxes overview

3.1. Model Assumptions

The Mathematical formulations are subject to the following set of assumptions:

- Each customer should be within a range of 500 m of the potential location where their deliveries arrive.

- A customer should be served only from a single delivery point.
- We outsource all the delivery boxes.
- All parcels are of the same size.
- The delivery box is delivered in the morning and collected at night to refill for new orders for the next day.
- The customers collect all parcels by the end of the day.
- There is a fixed cost associated with each potential location that must be accounted for if the location is open.
- Traffic conditions are calmest at the time when we dispatch the shipment.
- The cost of collecting the delivery box is part of the delivery cost
- The vehicles used for deliveries are environmentally friendly and very economical.
- We neglected the operating cost of the urban consolidation centers.
- We have overlooked all the cost of personnel or labor involved in executing the deliveries and in the operations at the UCCs.
- All customers are assumed to be okay with collecting their parcels themselves.
- The capacities of Trucks, Vans, UCCs, are also neglected (In the base model 1 and 2 only).
- The total cost of shipment is shared fairly amongst the companies involved.
- The environmental effects are not accounted for in the formulation but are assumed to improve compared to average delivery models.
- Each delivery is made separately, where the truck making the delivery to the UCC delivers and returns, while the van delivering from the UCC delivers to each location and returns. (All models except models 4 and 5)

3.2. Mathematical Formulations

In this section, the notations and the nomenclature are explained followed by the description of each of the models, leading up to the final model and finally each of the models is validated over small data sets.

Nomenclature

i	Company from a set of I companies.

u	Urban Consolidation Center (UCC), from a set of U, UCCs.
j	Potential locations, where customer can pick up their deliveries, from a set of J locations.
k	Delivery box type, indicating the number of parcels that it can carry.
l	Customer from the set of L customers.
m	Vehicle number
a	Set of nodes that includes the UCC and the locations.
b	Alias of a.
e	Alias of a.
O	Constant that denotes a very large number.
F_j	Fixed cost of using location j, where j=1….J.
D_{jl}	Distance between arc j-l, , where j=1….J, l = 1…L.
P_{il}	Demand of customer l from company i, in terms of number of parcels, where i=1…I and l = 1…L.
C_k	Cost of leasing a delivery box, where k =1….K.
S_k	Maximum capacity of each type of delivery box, in terms of the number of parcels, where k =1….K.
E_{iu}	Average parcel delivery cost, from company i to the UCC u, per each Km, where i=1…I, and u = 1….U.
G_{uj}	Average Delivery box, delivery cost, between u and j, per each Km.
H_{uj}	Distance from UCC u to location j in Km, where u=1…U, j=1…J.
R_{iu}	Distance from company i to UCC u in Km, where i=1…I, and u = 1….U..
$MaxDB_j$	Maximum number of delivery boxes that can be delivered to a location
X_{ujk}	Number of delivery boxes of size k delivered from UCC u to location j, where u=1…U, j=1…J, k=1…K.
Y_j	Decision variable that chooses if a location is open or not, where j = 1…J.
Q_{jl}	Decision variable that decides the allocation of customer to the delivery location, where j=1….J, l = 1…L.
W_{iujl}	Number of parcels delivered from company i to UCC u, to location j, for customer l, where i=1…I, and u = 1….U, where j=1….J, l = 1…L.

T_{uj}	Decision variable that allocates the location to the UCC, where u=1…U, j=1…J.
N_{ulm}	Scenario where the customer l either picks up the delivery coming from u on day m, or not.
TR_{iu}	Cost per Km, of each truck going from i to u, where i=1…I, and u = 1….U..
CP_{uj}	Cost per Km, of each van carrying delivery boxes from u to j, where u=1…U, j=1…J.
$MaxV_k$	Maximum capacity of each van in terms of type of delivery box k, where k = 1…K.
CAP_u	Maximum capacity of each UCC in terms of the number of parcels it can handle.
MTr	Max capacity of each truck in terms of the number of parcels it can carry.
ρ_j	Demand at location j, in terms of delivery boxes, where j= 1…J.
D_{ab}	Cost of travelling between the nodes a and b, where a, b ∈ U ∪ J.
F_r	Fixed cost of using vehicle r, where r= 1…R.
$MaxV_r$	Max capacity of each vehicle r, where r= 1…R.
T_{uj}	Allocation of u to j, where u=1…U, j=1…J.
N_{ur}	Allocation of vehicle r to u, r=1…. R, and u=1…U.
V_{abr}	Decision variable that decides if arc a-b is visited by vehicle r, where a, b ∈ U ∪ J, and r=1…. R.
CU_{ar}	Rank of node a in the routing of vehicle r, where a ∈ U ∪ J and r=1- R.

3.2.1. Model 1 (basic / un-capacitated model

3.2.1.1. *Mathematical formulation.* In this section, we describe the key decision variables, followed by the objective function and finally explain each of the constraints for the basic model. Finally, we validate the model against a small sample data set.

Decision variables:

X_{ujk} : Number of delivery boxes of size k, delivered from UCC u to location j.

$$Y_j : \begin{cases} 1\text{, if location j is open} \\ 0\text{, otherwise} \end{cases}$$

$$Q_{jl}: \begin{cases} 1\text{ , if customer l is allocated to locaiton j} \\ 0\text{, otherwise} \end{cases}$$

W_{iujl}: Number of parcels delivered from company i to UCC u, to location j, for customer l.

$$T_{uj}: \begin{cases} 1\text{ , if the parcel should go from U to J} \\ 0\text{, otherwise} \end{cases}$$

Objective Function:

$$\text{Min} = \sum_{j=1}^{J} F_j Y_j + \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{u=1}^{U} C_k X_{ujk} + \sum_{i=1}^{I} \sum_{u=1}^{U} \sum_{j=1}^{J} \sum_{l=1}^{L} E_{iu} R_{iu} W_{iujl} + \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{u=1}^{U} G_{uj} H_{uj} X_{ujk} \quad (1)$$

The objective function seeks to minimize the sum of the delivery box leasing cost, fixed location cost, and finally the cost of shipping to the UCCs, and the cost of delivering from the UCCs to location (j)

Subject to:

- Distance Constraint: Each customer must be at a maximum distance of 500m from its assigned pickup location (j).

$$Q_{jl}.D_{jl} \leq 500.Y_j \qquad \forall \text{j and l} \quad (2)$$

- Capacity Constraint: The number of parcels at each location cannot exceed the total capacity at that location. The total capacity is based on the combination of delivery boxes present at that location.

$$\sum_{i=1}^{I} \sum_{l=1}^{L} P_{il}.Q_{jl} \leq \sum_{u=1}^{U} \sum_{k=1}^{K} S_k.X_{ujk} \quad \forall j \quad (3)$$

- Each customer can only be allocated to a single location. Therefore, if they have multiple parcels to be collected, then they must be delivered to a single location to make it convenient for the customer.

$$\sum_{j=1}^{J} Q_{jl} = 1, \quad \forall l \quad (4)$$

- The total number of parcels for each customer is the sum of parcels delivered to that customer location, from all the companies and UCCs.

$$\sum_{i=1}^{I} \sum_{u=1}^{U} W_{iujl} = \sum_{i=1}^{I} P_{il}.Q_{jl} \quad \forall \text{j and l,} \quad (5)$$

- The parcel for a customer should be routed to the location to which the corresponding customer is allocated.

$$W_{iujl} \leq O.Q_{jl}, \qquad \forall \text{i, u, j and l} \quad (6)$$

- The total number of delivery boxes present at a UCC should be more than the total number of parcels arriving at a UCC.

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} \mathrm{W}_{\mathrm{iujl}} \leq \sum_{j=1}^{J}\sum_{k=1}^{K} \mathrm{S}_{\mathrm{k}}\mathrm{X}_{\mathrm{ujk}} \qquad \forall \mathrm{u} \qquad (7)$$

- At each location, the maximum capacity of the total number of boxes denoted by "MaxDB$_j$".

$$\sum_{u=1}^{U} \mathrm{X}_{\mathrm{ujk}} \leq \mathrm{MaxDB}_{\mathrm{j}} \quad \forall \mathrm{j} \text{ and } \mathrm{k} \qquad (8)$$

- Only the company, with whom the order was placed, can fulfill the demand of each customer.

$$\sum_{u=1}^{U}\sum_{j=1}^{J} \mathrm{W}_{\mathrm{iujl}} \leq \mathrm{P}_{\mathrm{il}} \qquad \forall \mathrm{i} \text{ and } \mathrm{l} \qquad (9)$$

- If the parcel is going from u to j according to W$_{iujl}$, then T$_{uj}$ will become a 1.

$$\mathrm{W}_{\mathrm{iujl}} \leq 0.\mathrm{T}_{\mathrm{uj}} \qquad \forall \mathrm{i}, \mathrm{u}\ \mathrm{j} \text{ and } \mathrm{l} \qquad (10)$$

- If T$_{uj}$ is one, then X$_{ujk}$ will have to be positive and the delivery box should also go from u to j accordingly.

$$\sum_{k=1}^{K} \mathrm{X}_{\mathrm{ujk}} \leq 0.\mathrm{T}_{\mathrm{uj}} \qquad \forall \mathrm{j} \text{ and } \mathrm{l} \qquad (11)$$

3.2.1.2. ***Model 1 validation***. We tested each formulation over sample instances and then analyzed the results. For each formulation, we describe the data followed by the validation and the results discussion.

Tables 3.1-3.8 show the data used for each of the parameters. Table 3.1, shows the distances from each customer to location, based on which they are allocated. Table 3.2 shows the demand of each of the customers from each of the companies. Table 3.3 shows us the fixed cost for choosing a location. Therefore, having more locations open, would mean a higher fixed price. Tables 3.4 and 3.5 gives us the average transportation cost per route between the company and UCC and then between UCC and the delivery location. The cost is different for each route if there is always a slight difference in fuel consumption for each route. And then we have the distance matrices in Table 3.6 and 3.7. Table 3.8 gives the capacity of each type of the delivery box. Each delivery box type can hold up to a certain number of parcels; each has a separate cost for being leased from an external entity.

Table 3.1: Distance matrix (in meters) from potential Location to customers

	Customer			
Location	**1**	**2**	**3**	**4**
1	600	300	1800	700
2	400	7000	820	900
3	2700	550	270	850
4	3000	1200	400	250

Table 3.2: Demand matrix of each customer from each company

	Customer			
Company	**1**	**2**	**3**	**4**
1	2	1	3	4
2	1	2	4	3
3	2	4	4	2

Table 3.3: Fixed cost of using each location

Location	**Cost**
1	$ 7.00
2	$ 7.00
3	$ 7.00
4	$ 7.00

Table 3.4: Avg. transportation cost per Km per parcel from Company i to UCC u

	UCC	
Company	**1**	**2**
1	$ 1.50	$ 1.45
2	$ 1.65	$ 1.75
3	$ 1.45	$ 1.25

Table 3.5: Avg. transportation cost ($) per Km per DB from UCC u to Location j

	Location			
UCC	**1**	**2**	**3**	**4**
1	$ 0.85	$ 0.80	$ 0.83	$ 0.75
2	$ 0.90	$ 0.79	$ 0.77	$ 0.80

Table 3.6: Distance matrix from u to j (in Km)

	Location			
UCC	**1**	**2**	**3**	**4**
1	2	8	4	3
2	6	3	5	2

Table 3.7: Distance matrix from company 'i' to UCC 'u (in Km)

	UCC	
Company	**1**	**2**
1	5	15
2	30	20
3	15	45

Table 3.8: Capacity of each type of delivery box 'k', in terms of parcels.

Type	Capacity of delivery box
1	5
2	10
3	15

Results tabulation for model 1: According to the Table 3.9, the number of parcels that each customer demanded is precisely the number of packages sent from the company. For example, customer 2 ordered from company 1, one parcel, from company 2, two parcels and finally from company 3, four parcels. Correspondingly, from the above Table, we can see that W_{1112} is equal to one, W_{2212} is equal two, and W_{3112} is equal to four. In addition, we will later use this result to verify if it matches with the routing of delivery boxes according to the X_{ujk}.

Table 3.9: W_{iujl}, Number of parcels routed for each customer

i	u	j	l	Number of parcels
1	1	1	2	1
1	1	2	1	2
1	1	3	3	3
1	1	4	4	4
2	2	4	4	3
2	2	1	2	2
2	2	2	1	1
2	2	3	3	4
3	1	1	2	4
3	1	2	1	2
3	1	3	3	4
3	1	4	4	2

Table 3.10 shows the number of delivery boxes going from each UCC to a potential location with appropriate sizes. For example, let's take UCC 1 and location 2; we know that the type of delivery box is type 1 which means a maximum of five parcels can go from UCC 1 to location 2. From W_{iujl}, we can see that W_{1121} is equal to two parcels and W_{3121} is equal to two parcels as well. That means a total of four parcels are

going and hence a delivery box of type 1 is enough to fulfill the capacity requirement. Similarly, considering UCC 1 and location 4 the kind of delivery box selected by the model is type 2, which can carry up to ten parcels. Examining W_{iujl}, W_{1144} is equal to 4, and W_{3144} is equal to 2. Therefore the total number of parcels going from UCC 1 to location 4 is six parcels which makes type 2 of the delivery box the appropriate size.

Table 3.10: X_{ujk}, number of delivery boxes sent from UCC 'u' to Location 'j' of delivery box 'k' size

u	j	k	x - Number of Del. Boxes
1	1	1	1
1	2	1	1
1	3	1	2
1	4	2	1
2	1	1	1
2	2	1	1
2	3	1	1

Table 3.11 depicts the allocation of each of the customers to an appropriate location that is within their range. The model allocates the customer to any of the location with in the range. For example, customer 3 has two locations in its range; however, the model assigns it to location 3 since it is closer.

Table 3.11: Q_{jl}, Allocation of a customer 'l' to a close by location 'j'

j	l
1	2
2	1
3	3
4	4

As we can see, where possible the model tries to save the cost, since there is no requirement to send a delivery box from UCC 2 to location 1, the transportation cost is saved there.

Figure 3.2 shows us clearly that delivery boxes are being sent from both UCCs to each of the potential locations, according to the variable X_{ujk}. Variable Q_{jl} depicts the allocation of the customers to corresponding locations. From the diagram, it is apparent that UCC '1' delivers to all locations while UCC '2' delivers to only the first three locations. However, this diagram doesn't show clearly if all the parcels are reaching the appropriate customers, through the indicated routes of delivery boxes.

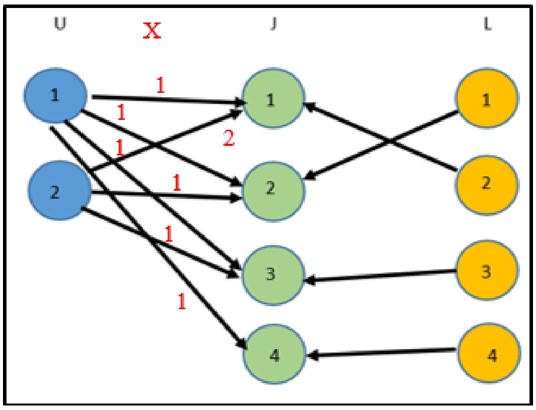

Figure 3.2: Routing of delivery boxes from 'u' to 'j' and the assignment of customers

To ensure the capacity constraints, at each location are satisfied, Table 3.12, shows the total delivery boxes and their types for to each location. Firstly, we know the total demand at each location from Table 3.2, by summing the orders of each customer from all companies. The combination of delivery boxes chosen should reduce the total cost while fulfilling the demand requirement. Figure 3.3. Illustrates the routes of the parcels from each company to each of the customers. The route from each company is color defined, blue is for company '1', green is for company '2' and red for company '3'.

Table 3.12: Comparison of capacities and demand at each of the Locations j

j	Total Number of D. Bs	Delivery box type	Total capacity at j	Total demand at j
1	2	1	10	7
2	2	1	10	5
3	3	1	15	11
4	1	2	10	9

Let's consider the two sets of nodes I and U first, the model shows that each company delivers to the same UCC, regardless of the customer, with the exception in the case of customer '4'. For customer '4', company '2' is delivering to UCC '1', while in other cases it delivers to UCC '2'. Moreover, it also shows that the packages reach the correct location for each customer. The route from company to UCC is the same for most customers, which is again economical since here the deliveries can be consolidated from each company for all customers to the appropriate UCCs. The assignments based on W_{iujl}, match with the assignment derived from Q_{jl}. For example, for customer 1 all deliveries are routed towards location 2.

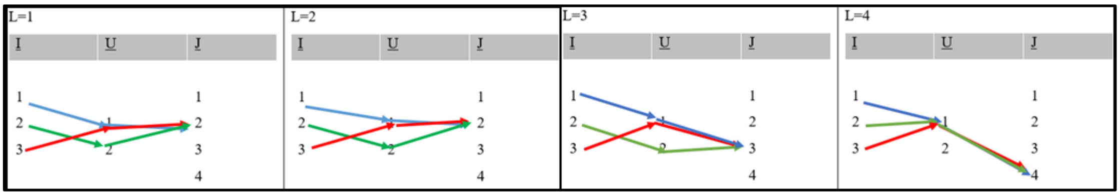

Figure 3.3: Routing of parcels for each of the customers "l", and from company “I” to corresponding Location “j”

Issues with model 1: The central assumption in model 1 is that all the customers collect all deliveries by the end of the day. However, in practically that might not be the case. There is of course some probability that the customers do not collect some deliveries. Hence, to model this, we follow a scenario where we assume that there will be some missed deliveries. Thus, those deliveries are returned to the UCC along with the delivery boxes are added to the pool of deliveries for the next day. To model this approach, index “M” to represent the day of the delivery, was added, as can be seen in model 2. Moreover, the capacities of the UCC, delivery vans and trucks are also neglected in the above scenario.

3.2.2. Model 2 (un-capacitated model for missed deliveries)

3.2.2.1. ***Mathematical formulation.*** In this model, we tackle, the issue of missed deliveries. We start by considering the shipments of more than one day while assuming a scenario where some missed deliveries on the first day. Those missed deliveries return to the UCC along with the delivery box, and then the model adds them to the rest of the deliveries for the next day. However, in this case, the underlying assumption is that the delivery, if missed on the first day will be collected on the next or the third delivery. The scenario where the shipment is canceled altogether, by the customer, is not accounted for in this formulation.

Decision variables:

X_{ujkm} : Number of delivery boxes of size k, delivered from UCC u to location j.

$$Y_{jm} : \begin{cases} 1, \text{if db of size k is used at location j} \\ 0, \text{otherwise} \end{cases}$$

$Q_{jlm}: \begin{cases} 1\text{, if customer l is allocated to locaiton j} \\ 0\text{, otherwise} \end{cases}$

W_{iujlm}: Number of parcels delivered from company i to UCC u, to location j, for customer l.

$T_{ujm}: \begin{cases} 1\text{, if the parcel should go from U to J on Day m.} \\ 0\text{, otherwise} \end{cases}$

Objective Function:

$$\min = \sum_{j=1}^{J}\sum_{m=1}^{M} F_{jm}Y_{jm} + \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{u=1}^{U} C_kX_{ujkm} + \sum_{i=1}^{I}\sum_{u=1}^{U}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{m=1}^{M} E_{iu}R_{iu}W_{iujlm} + \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{u=1}^{U} G_{uj}H_{uj}X_{ujkm} \quad (12)$$

Minimizing the sum of the delivery box leasing cost, fixed location cost, and finally the cost shipping to the UCCs, and the cost of delivering from the UCCs to location j, everyday m.

Subject to:

- Distance Constraint: Each customer must be at a maximum distance of 500m from its assigned pickup location (potential location j).

$$Q_{jlm}.D_{jlm} \leq 500.Y_jm \qquad \forall j \text{ and } l \quad (13)$$

- Capacity Constraint: The number of parcels at each location cannot be more than the total capacity at that location. The total capacity is based on the combination of delivery boxes present at that location.

$$\sum_{i=1}^{I}\sum_{l=1}^{L} P_{ilm}.Q_{jlm} \leq \sum_{u=1}^{U}\sum_{k=1}^{K} S_k.X_{ujkm} \;\forall j \text{ and } m \quad (14)$$

- Each customer can only be allocated to a single location. Therefore, if they have multiple parcels to be collected, then they must be delivered to a single location to make it easier for the customer.

$$\sum_{j=1}^{J} Q_{jlm} = 1, \qquad \forall l \text{ and } m \quad (15)$$

- The total number of parcels for each customer is the sum of parcels delivered to that customer location, from all the companies and UCCs.

$$\sum_{i=1}^{I}\sum_{u=1}^{U} W_{iujlm} = \sum_{i=1}^{I} P_{ilm}.Q_{jlm} \;\forall j, l \text{ and } m \quad (16)$$

- The parcel should be routed to the location to which the corresponding customer has to arrive to or in other words to a location which is in the range of the customer.

$$W_{\mathrm{iujlm}} \le O.\,Q_{\mathrm{jlm}}, \qquad \forall \mathrm{i, u, j, l\ and\ m} \tag{17}$$

- The total number of delivery boxes present at a UCC should be more than the total number of parcels arriving at a UCC.

$$\sum_{i=1}^{I}\sum_{l=1}^{L} W_{\mathrm{iujlm}} \le \sum_{k=1}^{K} S_k X_{\mathrm{ujkm}} \qquad \forall \mathrm{j, m\ and\ u} \tag{18}$$

- At each location, not more than "MaxDB$_j$" number of each sizes of the delivery boxes can be delivered.

$$\sum_{u=1}^{U} X_{\mathrm{ujkm}} \le \mathrm{MaxDB_j} \qquad \forall \mathrm{j, k\ and\ m} \tag{19}$$

- The specific company with whom the order was placed must fulfill the demand of each customer.

$$\sum_{u=1}^{U}\sum_{j=1}^{J} W_{\mathrm{iujlm}} = P_{\mathrm{ilm}} \qquad \forall \mathrm{i,\ l\ and\ m} \tag{20}$$

- If the parcel is going from u to j according to W_{iujl} then the T_{uj} will become a 1.

$$W_{\mathrm{iujlm}} \le O.\,T_{\mathrm{ujm}}, \qquad \forall \mathrm{i,\ u,\ j, l\ and\ m} \tag{21}$$

- If the T_{uj} is one, then the X_{ujk} will have to be positive and the delivery box should go from u to j accordingly.

$$\sum_{k=1}^{K} X_{\mathrm{ujkm}} \le O.\,T_{\mathrm{ujm}}, \qquad \forall \mathrm{u,\ j\ and\ m} \tag{22}$$

- If a customer does not pick up their delivery, then the parcel should return to the UCC to be re-added to the pool of deliveries for the next day, and hence the parcel would then be redelivered the next day.

$$\sum_{i=1}^{I}\sum_{l=1}^{L} N_{\mathrm{ulm-1}} P_{\mathrm{ilm-1}} + \sum_{i=1}^{I}\sum_{l=1}^{L} W_{\mathrm{iujlm}} \le \sum_{j=1}^{J}\sum_{k=1}^{K} S_k X_{\mathrm{ujkm}} \qquad \forall \mathrm{u, j\ and\ m} \tag{23}$$

3.2.2.2. *Model 2 validation.* This model has an extra index "m", which sets it apart from other formulations and because of that the results are also different.

Tables 3.13-3.21 show the data used for each of the parameters. Table 3.13, shows the distances from each customer to location, based on which the allocations are done, for both days that are under consideration. Although the distances are not going to change the next day, still are required to simplify formulation. Table 3.14 shows the demand of each of the customers from each of the companies, for both days. Table 3.15 shows us the fixed cost for choosing a location, for both days. The fixed cost might vary for different days however we are considering a constant cost for each day. Therefore, having more locations open, would mean a higher fixed cost. Tables 3.16

and 3.17 give us the average transportation cost per route between the company and UCC and then between UCC and location. The cost is different for each route; we assumed that there is always a slight difference in fuel consumption for each route. And then we have the distance matrices in Table 3.18 and 3.19. Table 3.20 gives the capacity of each type of the delivery box. Each delivery box type can hold up to a certain number of parcels; each has a separate cost for being leased from an external entity. Table 3.21 is what differentiates this model from other models. The Parameter "N_{ulm}" allows us to set a scenario where some customers do not receive their deliveries. This shows us how the model reacts to such scenarios.

Table 3.13: Distance matrix (in meters) from potential Location to customers

	Customers							
	Day 1				**Day 2**			
Locations	1	2	3	4	1	2	3	4
1	600	300	1800	700	600	300	1800	700
2	400	7000	820	900	400	7000	820	900
3	2700	550	270	850	2700	550	270	850
4	3000	1200	400	250	3000	1200	400	250

Table 3.14: Demand of each customer l from each company i

	Customers							
	Day 1				**Day 2**			
Company	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
1	2	1	3	4	3	1	0	2
2	1	2	4	3	0	2	1	2
3	2	4	4	2	1	1	1	2

Table 3.15: Fixed cost of using each Location j

Location	**Day 1**	**Day 2**
1	$ 7.00	$ 7.00
2	$ 7.00	$ 7.00
3	$ 7.00	$ 7.00
4	$ 7.00	$ 7.00

Table 3.16: Avg. transportation cost ($) per kilometer from company i to UCC u

	UCC	
Company	**1**	**2**
1	$ 1.50	$ 1.45
2	$ 1.65	$ 1.75
3	$ 1.45	$ 1.25

Table 3.17: Avg. transportation cost ($) per kilometer from u to j

	Location			
UCC	**1**	**2**	**3**	**4**
1	$ 0.85	$ 0.8	$ 0.83	$ 0.75
2	$ 0.9	$ 0.79	$ 0.77	$ 0.8

Table 3.18: Distance matrix from u to u (in Km)

	Location			
UCC	**1**	**2**	**3**	**4**
1	2	8	4	3
2	6	3	5	2

Table 3.19: Distance matrix from i to u (in Km)

	UCC	
Company	**1**	**2**
1	5	15
2	30	20
3	15	45

Table 3.20: Capacity of delivery box sizes

K	**Max capacity of each size**
1	5
2	10
3	15

In Table 3.21, the ones in the table represent that the parcels of the customer has been returned, while the zeroes represent successful deliveries.

Table 3.21: N_{ulm} Scenario model (assumption where deliveries are not picked up)

	Customers							
	Day 1				**Day 2**			
UCC	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
1	0	1	1	0	0	0	0	0
2	0	1	1	0	0	0	0	0

Results tabulation for model 2: Table 3.22 shows the number of parcels that are going through a company, for each customer and which UCC it goes through to reach the corresponding pick up location. The packages transported for each customer is the same as the number demanded by that customer, from that company. For example, Customer 1’s order from company ‘3’ is two parcels, on day ‘1’. According to W_{31211}, from Table 3.22, the number of parcels reaching location ‘2’ from company ‘3’ are two, for customer ‘1’.

Table 3.22: W_{iujlm}, number of parcels going from company ‘i’ through UCC ‘u’ to location ‘j’ for each customer ‘l’ on day ‘m’

i	u	j	l	m	w - Number of parcels
1	1	1	2	1	1
1	1	1	2	2	1
1	1	2	1	1	2
1	1	2	1	2	3
1	1	4	3	1	3
1	1	4	4	1	4
1	1	4	4	2	2
2	2	1	2	1	2
2	2	1	2	2	2
2	2	2	1	1	1
2	2	4	3	1	4
2	2	4	3	2	1
2	2	4	4	1	3
2	2	4	4	2	2
3	1	1	2	1	4
3	1	1	2	2	1
3	1	2	1	1	2
3	1	2	1	2	1
3	1	4	3	1	4
3	1	4	3	2	1
3	1	4	4	1	2
3	1	4	4	2	2
1	1	1	2	1	1
1	1	1	2	2	1

To validate the results from Table 3.23, take, for example, UCC “1” and location “4” on day “2”, and we can see if the type and number of delivery boxes are appropriate or not. According to the solution, a delivery box of type “3” goes from UCC “1” to location “4”, which means a capacity of maximum fifteen parcels, then another delivery box is sent to location “4” from UCC “2” of type “2”, which increases the maximum capacity at that location to twenty-five parcels. Now we see how many parcels are going to that location, according to Table 3.22. $W_{11442} = 2$, $W_{22442}=2$, $W_{22432}=1$, $W_{31432}=1$, $W_{31442}=2$, also we need to consider the deliveries of the customer “3” on the first day again since according to our scenario case, they were not collected by the customer on day “1”, therefore, $W_{11431}=3$, $W_{22431}=4$, $W_{31431}=4$. The total number of parcels going to location “4” on day “2” are nineteen. A more detailed comparison of supply and demand is also showcased in Table 3.26. To validate the results from Table 3.23, take, for example, UCC “1” and location “4” on day “2”, and we can see if the type and number of delivery boxes is appropriate or not. According to the Table, a delivery box “3” goes from UCC “1” to location “4”, which means a capacity of maximum fifteen parcels, then another delivery box is sent to location “4” from UCC “2” of type “2”, which increases the maximum capacity at that location to twenty-five parcels.

Table 3.23: X_{ujkm}, Number of delivery boxes going from UCC “u” to location “j” of type “k”, on day “m”

u	j	k	m	x - Number of Delivery boxes
1	1	1	1	1
1	1	2	2	1
1	2	1	1	1
1	2	1	2	1
1	4	3	1	1
1	4	3	2	1
2	1	1	1	1
2	1	1	2	1
2	2	1	1	1
2	4	2	1	1
2	4	2	2	1

. Now we see how many parcels are shipped to that location, according to Table 3.22. $W_{11442} = 2$, $W_{22442}=2$, $W_{22432}=1$, $W_{31432}=1$, $W_{31442}=2$, also we need to consider the deliveries of the customer “3” on the first day again since according to our scenario case, they were not collected by the customer on day “1”, therefore, $W_{11431}=3$, $W_{22431}=4$, $W_{31431}=4$. The total number of parcels going to location “4” on the day “2” are nineteen. Table 3.26 shows a more detailed comparison of capacity and demand. The Customer-Pickup Location allocations in Table 3.24 are based on the distance matrix, where each customer is allocated to a location, only if it is within five-hundred meters.

Table 3.24: Q_{jlm}, Customer “l” allocation to location “j”, on day “m”

j	l	M
1	2	1
1	2	2
2	1	1
2	1	2
4	3	1
4	3	2
4	4	1
4	4	2

Also, since the distance is not changing, the allocations are the same for both days. According to Table 3.25, except for location “3”, all the locations are open on both day “1” and day “2”. For the model to be validated, it should not assign any customer or route any parcel to a closed location. From the results in Tables 3.23 and 3.24, it is verified that no delivery box goes to location “3”. Figure 3.4 is showing the delivery box distribution and customer allocation, according to Q_{jlm} and X_{ujkm}. We can see that all locations are open, except location “3”. Both the UCCs are delivering to both locations.

Table 3.25: Y_{jm}, location “j” that is open on a day m”

j	m	Location Status
1	1	Open
1	2	Open
2	1	Open
2	2	Open
3	1	Close
3	2	Close
4	1	Open
4	2	Open

Further validation requires analysis of results from W_{iujlm}, to find out if the supply and demand at each location are being satisfied, by the delivery boxes that go there. Table 3.26 summarizes the results from the variables W, X, Y and shows that the total capacities at each location j are satisfied by the most appropriate combination of the delivery boxes.

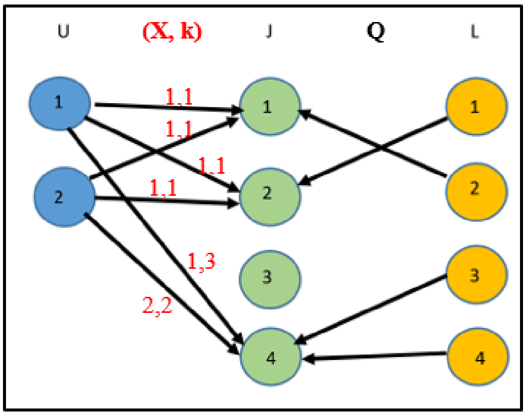

Figure 3.4: Path of delivery boxes form u to j and allocation of l to j on day 1

Considering the location “2”, and in other cases as well we can see that the total capacity is double which is usually because each UCC is sending a delivery box. And since the smallest size, type “1”, can carry five parcels, two of those bring the capacity to ten.

Table 3.26: Comparison of capacities and demand at each of the locations, on day 1

j	Total Number of D.B. s	D.B. type	Total capacity at j	Total demand at j
1	2	2,1	15	7
2	2	1	10	5
3	0	-	0	0
4	2	2,3	25	20

Figure 3.5 depicts the routing of parcels to the final location where the customers will pick up the deliveries. In general, we can see that for each customer, the parcels converge to a single location, which is always the location assigned to the customer. If we consider Customer “2”, from Figure 3.4, we can see that all parcels for customer “2” go through UCC “1”, where they would be batched up into a delivery box and sent to location “1”. Where according to Figure 3, customer “2” is arriving.

On the day “2”, the demands were different for example, customer “1” did not have a demand from company “2”, while, customer “3” did not have a demand from the company “1”. Therefore, we can now see that location “3” is not open and customer “3” and “4” are both allocated to the same location to save costs. While UCC “2” delivers only to location “1” and “4”, UCC “1” sends delivery boxes only to locations “1” to “3”.

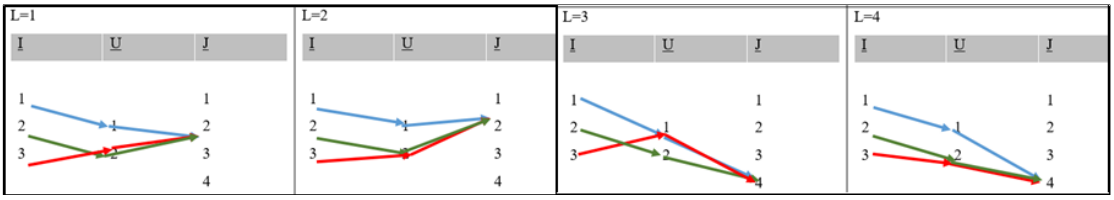

Figure 3.5: Path of parcels from i to u to j for each customer l on day 1 according to the Results for W_{iujlm}

We have summarized a detailed analysis concerning total capacity, total demand at locations, in Table 3.27. For location “1”, “2” delivery boxes go from each UCC.

While, for the location “2”, delivery boxes of the same size are sent from each UCC. For location “4” two delivery boxes of type “2” and “3” are selected. The total capacities and demands are satisfied according to the Table. However, we mapped the path from the company to the destination in Figure 3.5. While Figure 3.6, shows the routing of delivery boxes and customer allocations.

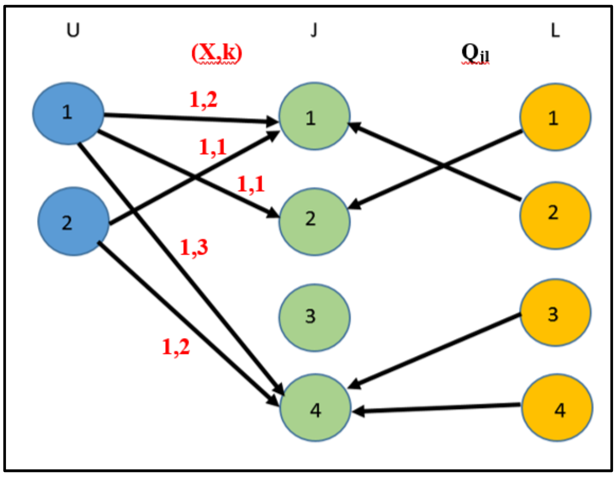

Figure 3.6: Path of delivery boxes form u to j and allocation of l to j on day 2

Table 3.27 summarizes the detailed analysis concerning total capacity, total demand at locations. For location “1”, “2” and “4”, the delivery boxes go from each UCC, for location “2”, both UCCs send one delivery box of type”1”. For location “4”, the solution shows that two delivery boxes of type “2” and “3” are selected. The total capacities and demands are satisfied according to the Table. However, to understand the path taken by each parcel, from the company to its final location, we must refer to Figure 3.7. Under the previous results, the model routes the parcels for each customer to the appropriate locations.

Table 3.27: Comparison of capacities and demand at each location, on day 2

J	Total Number of D. Bs	D.B type	Total Capacity	Total Demand
1	2	1,2	15	11
2	2	1	10	4
3	0	-	0	0
4	2	2,3	25	19

Secondly, the paths from the UCC to the pick-up location match the result in Figure 3.5 from the variable X_{ujkm}, concerning both routing and capacity. For customers

“1” and “3”, no parcels are coming from company “2” and “1” respectively since there were no demands from those companies. This solution validates the model, based on the given data. Furthermore, the paths from each company to the UCC are also the same as it was for day 1, which shows that consolidation is taking place during deliveries.

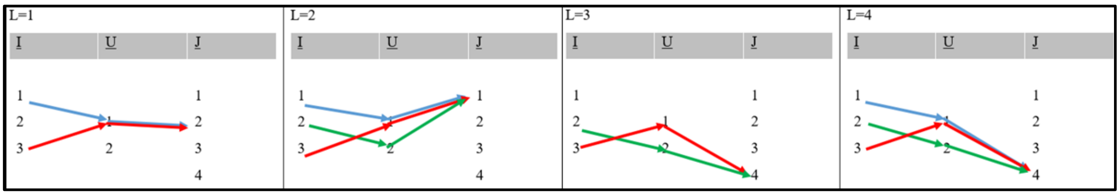

Figure 3.7: Path of parcels for each customer l on day 2, according to the results for W_{iujlm}

3.2.3. Model 3 (Capacitated model)

3.2.3.1. *Mathematical formulation*. The capacitated model tackles another one of the assumptions that we neglected in the previous models. In this formulation, we consider the capacities of the urban consolidation centers, the trucks delivering from the companies and the vans delivering from the companies.

Decision variables:

TR_{iu}: Number of trucks going from i to u

$T_{uj}: \begin{cases} 1, \text{location j is served from UCC u} \\ 0, \text{otherwise} \end{cases}$

$YT_{iu}: \begin{cases} 1, \text{if UCC u is served by company i} \\ 0, \text{otherwise} \end{cases}$

VAN_{uj}: Number of vans going from u to j.

Objective function:

$$\text{Min} = \sum_{u=1}^{U} \sum_{j=1}^{J} VAN_{uj}.CP_{uj}.H_{uj} + \sum_{i=1}^{I} \sum_{u=1}^{U} TR_{iu}.CT_{iu}.R_{iu} + \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{u=1}^{U} C_k.X_{ujk} + \sum_{j=1}^{J} F_j Y_j \tag{24}$$

The objective is to minimize the total cost of vans, trucks, delivery boxes and locations.

Subject to:

- Equations 2-11.

- The number of parcels going from i to u cannot exceed the total capacity of the trucks going from i to u.

$$\sum_{j=1}^{J}\sum_{l=1}^{L} \mathrm{W_{iujl}} \leq \mathrm{MTr.TR_{iu}}, \qquad \forall \mathrm{i\ and\ u} \tag{25}$$

- The total number of parcels going to a UCC cannot exceed the total capacity of the UCC.

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} \mathrm{W_{iujl}} \leq \mathrm{CAP_u} \qquad \forall \mathrm{u}, \tag{26}$$

- The total parcels going from u to j cannot exceed the total capacity of vans going from u to j.

$$\sum_{k=1}^{K} \mathrm{X_{ujk}} \leq \sum_{k=1}^{K} \mathrm{MaxV_k.VAN_{uj}} \qquad \forall \mathrm{u\ and\ j} \tag{27}$$

- Trucks can only be sent from a company to a UCC if a route exists for that route.

$$\mathrm{TR_{iu}} \leq \mathrm{O.YT_{iu}} \qquad \forall \mathrm{i\ and\ u} \tag{28}$$

- Each company must deliver to at most one UCC.

$$\sum_{u=1}^{U} \mathrm{YT_{iu}} \leq 1 \qquad \forall \mathrm{I} \tag{29}$$

3.2.3.2. ***Model 3 validation.*** Tables 3.28-3.38 show the data for model 3 test. In this case, we have some extra parameters since model 3 accounts for the trucks delivering from company to UCC and vans delivering from UCC to Delivery Locations. Tables 3.28, 3.29 are the costs of the vans and trucks respectively, per km. Table 3.30, gives the maximum van capacity, which we kept as constant. And the truck capacity was also held constant as a scalar value. Tables 3.31 and 3.32 represent the demand and distance matrix for the customers, respectively. Table 3.33 is the fixed cost for each location which is kept constant in this case. Table 3.34 and 3.35 are distance matrices between u-j and i-u respectively. Tables 3.36 and 3.37 are the capacities and costs of each of the delivery box types, respectively. Finally, we have the capacities of the UCC in the last Table. We varied the capacities of the UCC, to see how the model would react to this variation.

Table 3.288: Cost of vans per km from UCC "u" to location "j"

	Location		
UCC	**1**	**2**	**3**
1	$ 1.34	$ 1.44	$ 1.35
2	$ 1.60	$ 1.20	$ 1.80

Table 3.299: Avg. Cost of truck per km from Company "i" to UCC "u"

	UCC	
Company	**1**	**2**
1	$ 2.45	$ 2.50
2	$ 2.40	$ 2.55
3	$ 2.35	$ 2.50
4	$ 2.50	$ 2.45

Table 3.30: Maximum Van capacity in terms of delivery box type

Type	Capacity
1	6
2	5
3	4

Table 3.31: Demand of each customer "l" from each company "I"

Company	Customer			
	1	**2**	**3**	**4**
1	2	2	1	2
2	1	1	3	0
3	1	1	0	4
4	2	3	4	4

Table 3.32: Distance matrix of location "j" and customers "I" in Km

Location	Customers			
	1	**2**	**3**	**4**
1	2	0.45	7	13
2	0.3	1	25	0.2
3	10	0.4	0.5	14

Table 3.33: Fixed cost of using location "j"

Location	Cost
1	$ 6.00
2	$ 6.00
3	$ 6.00

Table 3.34: Distance of UCC from locations (Km)

UCC	Location		
	1	**2**	**3**
1	12	15	30
2	40	25	15

Table 3.35: Distance of Company "I" to UCC "u" (Km)

Company	UCC	
	1	2
1	20	45
2	130	145
3	70	85
4	30	40

Table 3.36: Capacity of delivery box each delivery box type "k", in terms of number of parcels

Type	Capacity
1	20
2	25
3	30

Table 3.37: Cost of each delivery box type "k"

Type	Cost
1	$ 1.00
2	$ 2.00
3	$ 3.00

Table 3.38: Capacity of each UCC "u" in terms of the number of parcels, it can manage

u	Capacity
1	10
2	30

Results for model 3 (Instance 1): For model 3 we increased the number of companies to 4 and reduced the number of locations by 1 to test the model differently compared to previous models. We also have several new results that were not present previously like the number of trucks or the number of vans and their routes. As shown below Table 3.39, not only shows the number of trucks going from the 'i' to 'u' but also the allocation of the company to a specific UCC.

Table 3.39 shows two things; it shows the allocation of the company to UCC, and the number of trucks dispatched from each company. Since we limited the capacity of UCC to only ten parcels, the remainder of the trucks go to the second UCC. Table 3.40 shows the number of parcels that are going from a company, for each customer and which UCC it goes through, to reach the corresponding pick up location, for customer L.

Table 3.39: Number of Trucks dispatched from each company

i	u	TR - Number of Trucks
1	1	1
2	2	1
3	2	1
4	2	1

The number of parcels going through, for each customer, is the same as the number demanded by the customer, from that company, for example, from Table 31, Customer 2's order from company "4" is three parcels. According to W_{4232}, from Table 40, the number of parcels reaching location "3" from company "4", are four, for customer "2" is three parcels.

Table 3.40: W_{iujl}, Number of parcels routed for each Customer

i	**u**	**j**	**l**	**Number of parcels**
1	1	2	1	2
1	1	2	4	2
1	1	3	2	2
1	1	3	3	1
2	2	2	1	1
2	2	3	2	1
2	2	3	3	3
3	2	2	1	1
3	2	2	4	4
3	2	3	2	1
4	2	2	1	2
4	2	2	4	4
4	2	3	2	3
4	2	3	3	4

According to Table 3.41, location 1 gets no shipments, which matches the result from the result from Table 3.40 for W_{iujl}. Table 41 shows that each UCC delivers to locations 2 and 3 and delivers only type "1" delivery boxes.

Table 3.41: X_{ujk}, Number of delivery boxes going from UCC "u" to location "j", of type "k"

u	**j**	**K**	**x - Number of Delivery boxes**
1	2	1	1
1	3	1	1
2	2	1	1
2	3	1	1

From the customer to location allocation in Table 3.42, we can again see no assignment to location 1, which matches the previous results. Customers “1” and “4” visit location “2” and the rest visit location “3”. Based on the distance matrix in Table 32, it is consistent that customers “1” and “4” are within a five-hundred-meter range of location “2”, while customer “2” and “3” are in the same range of location “3”. Each van going from UCC to a location must follow the same path as the delivery boxes since they will be carrying the latter. Table 3.43 gives us the allocation of UCC to delivery locations, and the number of trucks dispatched, as well. As we can see the distribution in Table 3.43 is consistent with the one in Table 3.41, for X_{ujk}, we can conclude that the result is consistent.

Table 3.42: Q_{jl}, Allocation of customer “l” to potential locations “j”

Location	Customer
2	1
2	4
3	2
3	3

Table 3.43: VAN_{uj}, Number of Vans going from UCC “u” to Location “j”

u	j	VAN - Number of Vans
1	2	1
1	3	1
2	2	1
2	3	1

Table 3.44, confirms the status of the locations, showing that only location “2” and “3” are open.

Table 3.44: Y_j, indicates whether Location “j” is open or closed

j	Location status
1	Closed
2	Open
3	Open

Figure 3.8 gives the depiction of the routes of delivery for parcels from companies to UCCs, the routes of delivery boxes and delivery VANs from UCCs to locations, and finally the allocation of customers to the appropriate locations. From

Table 3.38, we can see that the UCC “1” capacity is only 10 parcels, hence accordingly the rest of the parcels are routed to UCC “2”.

From Table 3.45, the capacity at each of the locations is satisfied with the type “1” of the delivery box. Since each UCC is sending one delivery box to each location, the total capacity adds up to a maximum of 40 parcels at each location. In a different scenario, if we choose to decrease the capacity of the delivery box type “1”, consequently the model might opt for type “2” delivery box if needed.

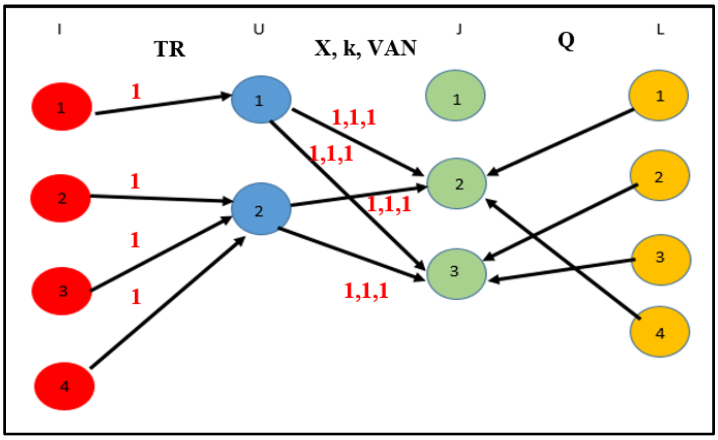

Figure 3.8: Path of Trucks, Allocation of Delivery boxes and Customers

Table 3.45: Comparison of capacities and demand at j

j	Total Number of D.Bs	D.B type	Total capacity at j	Total demand at j
1	0	-	0	0
2	2	1	40	16
3	2	1	40	15

Table 3.46 shows, UCC “1”, has a capacity of processing only ten parcels in this test scenario hence the rest of the parcels go to UCC “2”. Even though UCC “2” is much further than all the locations, but due to the capacity restriction, the model has no choice but to route the parcels from there. Also notice that the model doesn’t send partial loads to separate UCCs instead it sends a truck to each UCC, which is more economical for the concerned parties. Each truck has a capacity of carrying 21 parcels, in this test scenario. Hence, only 1 truck Is sent to the UCCs, from each company. “YT_{iu}” is the binary variable that decides which UCC is to be visited by each company. The model constrains the truck to visit a single UCC only, to save on transportation

costs. The path of the parcels shown in Figure. 3.9 is according to the results from the variable W_{iujl}, the routes are the same as seen in Figure. 3.8 and the route from the UCC to the pick-up for each customer also matches.

Table 3.46: Comparison of capacities and demand at u

u	Total Number of Parcels	Parcels Company	Total capacity at the UCC
1	7	1	10
2	24	2,3,4	30

For example, the model assigns customer “1” to location “2”; hence the deliveries are routed appropriately, to location “2”. From the demand Table, no order was made by customer 3, from company “3”, hence no route exists from that company, for that customer.

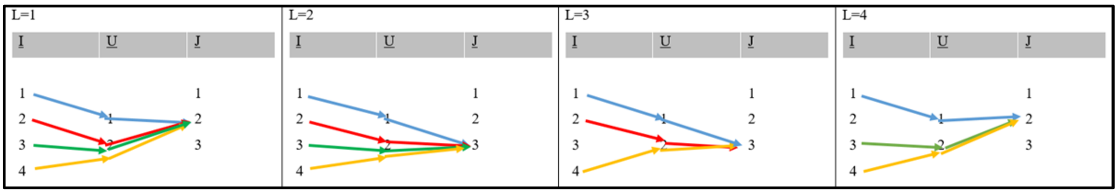

Figure 3.9: Path of parcels from i to u to j for each customer l, according to the results for W_{iujl}

Results for model 3 (Instance 2): To test the model further, the following changes were made in the data to see how the model would react.

1. Capacity of UCC “1” increased to 30 parcels.
2. Maximum truck capacity decreased to 10 parcels.
3. Capacity of delivery boxes reduced for type 1, 2 and 3, to, 5, 10 and 15 parcels respectively.

Table 3.47 shows the number of trucks delivering to each UCC, where companies 1, 2 and 3 only deliver to UCC 1, since it is closer and a cheaper option, while company 4 delivers to UCC 2. From Table 3.48, the Results from W_{iujl} show that the parcels from companies “2” and “3” now go through UCC “1” instead of UCC “2”. The number of parcels delivered is the same as demanded by each of the customers and the delivery location is according to the customer's location allocations. (refer to Figures 3.9 and 3.10).

Table 3.47: Number of Trucks going from company "i" to UCC "u"

I	U	Number of trucks
1	1	1
2	1	1
3	1	1
4	2	2

Table 3.48: W_{iujl}, number of parcels going from company "i" through UCC "u" to location "j" for each customer "l"

i	u	j	l	Number of parcels
1	1	2	1	2
1	1	2	4	2
1	1	3	2	2
1	1	3	3	1
2	1	2	1	1
2	1	3	2	1
2	1	3	3	3
3	1	2	1	1
3	1	2	4	4
3	1	3	2	1
4	2	2	1	2
4	2	2	4	4
4	2	3	2	3
4	2	3	3	4

Table 3.49: X_{ujk}, Number of delivery boxes going from UCC "u" to location "j"

u	j	k	Number of Delivery boxes
1	2	2	1
1	3	2	1
2	2	2	1
2	3	2	1

Since we decreased the capacity of each type "k" of delivery box, the model now chooses type "2" of the delivery box to accommodate for the parcels. The result in the rest of the variables is the same as before. Figure 3.9 shows that companies "1", "2" and "3" send trucks to UCC "1", since it is closer than the second UCC. It is also worth noting that if we increased the capacity of UCC "1', all companies would route through UCC "1" only. The model is constrained not to have double deliveries from one company. However, as a result, both UCCs are delivering to the same locations since all customers have demand from all companies. We were able to achieve the best solution, when the capacity of the UCC “1” was increased high enough to manage all demand.

Meanwhile, no routes exist to or from UCC “2”. Another thing to note is that since we reduced the capacity of the Trucks, there are now two trucks dispatched from company “4”. As we can see in Figure 3.10, only the routes between the companies and the UCCs have been altered while the rest is all the same, as compared to Figure 3.8. Table 3.50 summarizes that now each location has two delivery boxes of type “2”, delivered.

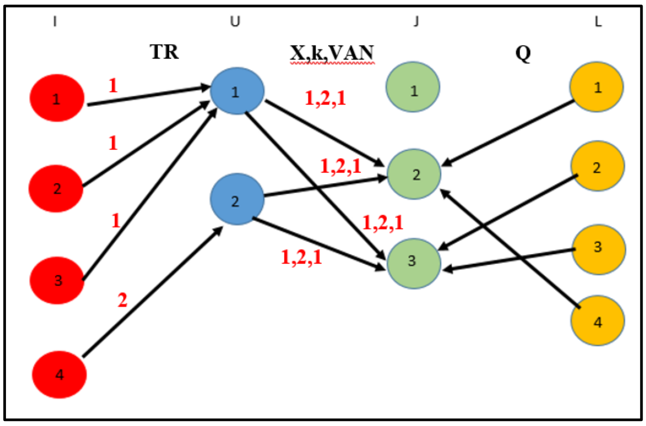

Figure 3.10: Path of Trucks, Allocation of Delivery boxes and Customers

Table 3.50: Comparison of Capacities and Demand at each location

j	Total Number of D.Bs	D.B type	Total Capacity at j	Total Demand at j
1	0	-	0	0
2	2	2	20	16
3	2	2	20	15

Table 3.51 shows that the capacity increment for UCC “1” allows it to handle deliveries of companies “1”, “2” and “3” only, while the 4th company is allocated to UCC “2”. The routing of parcels in W_{iujl} also reflects a similar change in Figure 3.11, where the parcels of companies 2 and 3 now go through UCC 1.

Table 3.51: Comparison of capacities and demand at each of the UCC

u	Total Number of Parcels	Company	UCC Max Capacity
1	18	1,2,3	30
2	13	4	30

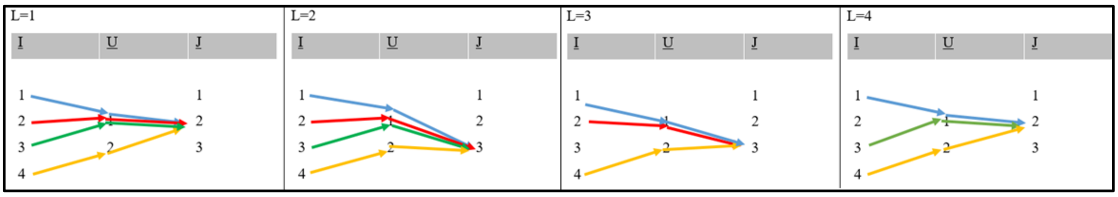

Figure 3.11: Path of parcels from i to u to j for each customer l on day 2, according to the Results for W_{iujl}

3.2.4. Model 4 (two-phase capacitated model with vehicle routing)

The capacitated model is realistic in the fact that it accounts for capacities of the delivering trucks, vans, and even the urban consolidation centers. Up until now, we were considering one to one delivery between UCC and potential locations. However, in a realistic scenario, one to one delivery between UCCs and pickup locations is not a viable solution since it implies that the van delivers to each location and then return to the UCC for the second delivery and so on. Therefore, a better solution would require proper vehicle routing between the UCCs and the locations to minimize the total transportation cost. In other words, the van would leave the UCC with full capacity and deliver to as many locations as possible according to its capacity, before returning to the depot for subsequent deliveries. The Vehicle routing model that we modeled was a type of multi-depot homogenous fleet routing since we are considering having multiple UCCs. In this case, each UCC would have its own set of vehicles, delivering and returning from their corresponding UCCs.

3.2.4.1. ***Mathematical formulation***. In this section we first describe the model for phase 1, followed by phase and then we test the model on a small sample dataset.

Phase 1 (Capacitated model)

Objective function:

$$\text{Min} = \sum_{j=1}^{J}\sum_{u=1}^{U}\mathrm{T_{uj}}.\mathrm{CP_{uj}}.\mathrm{H_{uj}} + \sum_{i=1}^{I}\sum_{u=1}^{U}\mathrm{TR_{iu}}.\mathrm{CT_{iu}}.\mathrm{R_{iu}} + \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{u=1}^{U}\mathrm{C_k}.\mathrm{X_{ujk}} + \sum_{j=1}^{J}\mathrm{F_j Y_j} \tag{30}$$

Minimize the Total costs of Truck deliveries from companies, cost of allocation of UCC to location j, cost of leasing delivery boxes, fixed cost of using a location.

Subject to:

- Refer to Constraints: Eq. 2-11, Eq. 25, 26, 28 and 29, while Eq. 27, from Model 3, is replaced by Eq. 31 and Eq. 32 (below).
- The total parcels going from u to j cannot exceed the total capacity of vans going from u to j.

$$\sum_{k=1}^{K} \mathrm{X}_{\mathrm{ujk}} \leq \sum_{k=1}^{K} \mathrm{MaxV}_{\mathrm{k}} \, . \, \mathrm{T}_{\mathrm{uj}} \qquad \forall \mathrm{u \ and \ j} \tag{31}$$

- Total number of parcels going from u to j cannot exceed the total capacity of delivery boxes going from u to j.

$$\sum_{i=1}^{I} \sum_{l=1}^{L} \mathrm{W}_{\mathrm{iujl}} \leq \ \sum_{k=1}^{K} \mathrm{S}_{\mathrm{k}} \mathrm{X}_{\mathrm{ujk}} \qquad \forall \mathrm{j \ and \ u} \tag{32}$$

- No parcel can be routed to a location that is closed.

$$W_{iujl} \leq O. Y_j \qquad \forall \mathrm{i, u, j \ andl} \tag{33}$$

Phase 1 of model 4 is an improvement of the capacitated model described in model 3 with a slight change. We have omitted the variable "$\mathrm{VAN}_{\mathrm{uj}}$", since now, we manage the routing of vans in phase 2. We use the allocations of "u" to "j" from the result of the variable T_{uj}, to do the routing in phase 2.

Phase 2 (Vehicle Routing Between UCC and Delivery Location)

Decision Variables:

$$\mathrm{V}_{\mathrm{abr}}: \begin{cases} \mathrm{1, if \ arc \ ab \ is \ to \ be \ served \ by \ vehicle \ r} \\ \mathrm{0, otherwise} \end{cases}$$

CU_{ar}: The accumulated load on the vehicle r at node a

Objective function: minimize the total cost (Z)

$$\mathrm{Z} = \sum_{a=1}^{A} \sum_{b=1}^{B} \sum_{r=1}^{R} \mathrm{D}_{\mathrm{ab}} \mathrm{V}_{\mathrm{abr}} \tag{34}$$

Subject to:

- Each location can be visited by vehicles from multiple UCCs.

$$\sum_{\mathrm{u}} \mathrm{T}_{\mathrm{uj}} \leq \sum_{a=1}^{A} \sum_{r=1}^{R} \mathrm{V}_{\mathrm{abr}} \qquad \mathrm{where \ a} \in \mathrm{u} \cup \mathrm{j, b} = \mathrm{j.} \tag{35}$$

- Sub-tour elimination constraint

$$\mathrm{CU}_{\mathrm{br}} \ \geq \ \ \mathrm{CU}_{\mathrm{ar}} \ + \ \ \sum_{u=1}^{U} \sum_{k=1}^{K} X_{ujk} V_{abr} - \mathrm{O} * \left(1 - \mathrm{V}_{\mathrm{ajr}}\right) \qquad \forall \mathrm{a, r, b} = \mathrm{j} \tag{36}$$

- Flow constraint

$$\sum_{b=1}^{B} \mathrm{V_{abr}} = \sum_{b=1}^{B} \mathrm{V_{bar}} \qquad \mathrm{a} \neq \mathrm{b} \in \mathrm{u} \cup \mathrm{j}, \forall \mathrm{r} \tag{37}$$

- Routing can only be assigned from a UCC to a location if the location is allocated to that UCC.

$$\sum_{e=j}^{J} (\mathrm{V_{uer}} + \mathrm{V_{ejr}}) \leq 1 + \mathrm{T_{uj}} \qquad \forall \mathrm{u}, \mathrm{j} \text{ and } \mathrm{r} \tag{38}$$

- Routing of vehicle “r” can only be done from UCC u, if vehicle “r” is allocated to UCC “u”.

$$\sum_{b=1}^{B} \mathrm{V_{abr}} \leq \mathrm{N_{ur}} \qquad \text{where}, \mathrm{a} = \mathrm{u}, \mathrm{b} \in \mathrm{u} \cup \mathrm{j}, \forall \mathrm{u} \text{ and } \mathrm{r} \tag{39}$$

- The Vehicle cannot visit a location from itself

$$\mathrm{V_{aar}} = 0 \qquad \forall \mathrm{a} \text{ and } \mathrm{r} \tag{40}$$

- The sum of the demand carried by each vehicle cannot be more than the cumulated load on a vehicle at each a location j.

$$\sum_{k=1}^{K} X_{ujk} \leq \mathrm{CU_{br}} \qquad \forall \mathrm{u}, \mathrm{r}, \mathrm{b} = \mathrm{j} \tag{41}$$

- The cumulated load on a vehicle cannot exceed the maximum capacity of the vehicle.

$$\mathrm{CU_{br}} \leq \sum_{k=1}^{K} MaxV_k \qquad \forall \mathrm{r}, \mathrm{b} = \mathrm{j}, \tag{42}$$

- If $\mathrm{T_{uj}}$ is positive then vehicle r must be routed to location j, from the corresponding UCC u.

$$T_{uj} \leq \sum_{a=1}^{A} \sum_{r=1}^{R} \mathrm{V_{abr}} \qquad \text{where } \forall \mathrm{a} \in \mathrm{u} \cup \mathrm{j}, \mathrm{b} = \mathrm{j}, \mathrm{u} \text{ and } \mathrm{j}. \tag{43}$$

3.2.4.2 ***Model 4 validation.*** Model 4 is divided into two phases hence the result is also described in two phases.

Phase 1: We modeled phase 1 as the capacitated model with a few changes. In model 3, we were assuming that the delivery van delivers directly from the UCC to each location one by one, which is very impractical in real life, since, there are several hundred locations to be serviced, by each UCC. Hence, a more appropriate solution requires a proper vehicle routing from UCCs to Locations, which we now manage in phase 2 of model 4. From Phase 1, we used the result from Variables T and X, for the routing in phase 2. The Data for validating model 4 is the same as the one used in the first trial of model 3 earlier.

Table 3.52 and 3.53 show the results of $\mathrm{X_{ujk}}$ and $\mathrm{T_{uj}}$, Now $\mathrm{X_{ujk}}$ tells us the number of delivery boxes to be delivered from a “u” to each “j,” in other words, it serves as the demand of each "j" in this model. T_{uj}, is a parameter which is a solution from phase 1, it allocates each location to a UCC, so if the location is visited from a UCC, the T_{uj} would be 1.

Table 3.52: X_{ujk}, Number of Delivery boxes

U	J	K	Number of Delivery boxes
1	2	2	1
1	3	2	1
2	2	1	2
2	3	2	1

Table 3.53: T_{uj}, Allocation of j to be serviced by u

U	J
1	2
1	3
2	2
2	3

We use the solution from phase 1 as data for Phase 2. In Phase 2, the data refers the locations "1", "2" and "3" as nodes "3", "4" and "5" respectively. Table 3.54 – 3.56, describe the vehicle capacities, fixed costs, and their allocated UCCs respectively.

Table 3.54: Max delivery vehicle capacity for each vehicle

Vehicle	Max capacity
1	10
2	10

Table 3.55: Fixed cost incurred per vehicle

Vehicle	Fixed cost
1	$ 5.00
2	$ 5.00

Table 3.56: T_{uj}, allocation of u to j, according to the result derived from Phase 1

	Locations		
UCCs	**3**	**4**	**5**
1	0	1	1
2	0	1	1

We got T_{uj} from Phase 1, where it was a variable that decides which UCC will serve which locations. In Phase 2, it serves as a parameter for the routing. Table 3.57

assigns the set of vehicles to the UCCs, which means that the vehicles assigned to a UCC can only delivery from that UCC and must return to the same UCC.

Table 3.57: Assignment of delivery vehicles to the UCCs from where they deliver and return to

	Vehicles	
UCCs	**1**	**2**
1	1	0
2	0	1

Phase 2: *We found t***wo** routes, one from UCC "1" and the second from UCC "2".

Routing for vehicle 1: 1 – 4 - 5 – 1.

Routing for vehicle 2: 2 – 5 – 4 – 2.

Nodes 1 and 2 represent the UCCs, while the nodes 3 to 5 represent the locations. Both vehicles start an end the journey at their home depot (UCCs).

3.2.5. Model 5 (Integrated capacitated model with vehicle routing between UCC and location)

3.2.5.1. ***Mathematical formulation***. In this section, the final consolidated model that incorporates parcel allocation, DB allocation, customer to location allocation and vehicle routing is presented, where we describe first the objective function followed by its constraints. Finally, the model is validated over a sample data set.

Objective function: Minimize the total cost (Z)

$$Z = \sum_a \sum_b \sum_r V_{abr}.H_{ab} + \sum_i \sum_u TR_{iu}.CT_{iu}.R_{iu} + \sum_j \sum_k \sum_u C_k.X_{ujk} + \sum_j F_j Y_j \quad (44)$$

Subject to:

- Refer to equations 2-11, 25, 26, 28, 29, 32.
- The total parcels going from u to j cannot exceed the total capacity of vans going from u to j.

$$X_{ujk} \leq MaxV * T_{uj} \quad \forall \text{u, j, and k} \quad (45)$$

- Refer to Eq. 35 – 43.
- A route can only be allocated to j if, the location j is assigned to a customer l.

$$\sum_{u=1}^{U} T_{uj} \leq O.\sum_{l=1}^{L} Q_{jl} \qquad \forall j \tag{46}$$

3.2.5.2. ***Model 5 validation***. Tables 3.58-3.69 show the data for model 5 test. Table 3.58 shows the cost of travelling of the trucks from the companies to UCCs. Table 3.59 shows the demand of each customer from each company, while Table 3.60, shows the distance of the customer from the potential locations. Table 3.61 and 3.62 define the fixed costs of locations and vehicles respectively. For testing, we have kept the fixed costs the same. Table 3.63 describes the distance between the companies and the UCCs. It is worth noting that all companies are closer to UCC 1 compared to UCC 2. Tables 3.64 and 3.65 give the capacity and cost of each of the delivery box types. Table 3.66 outlines the capacity of the UCCs. It is also worth noting that the capacities are currently kept equal between the two UCCs, but this parameter will be varied to test the model. Table 3.67 gives the capacities of trucks and delivery vehicles concerning the parcels and delivery boxes, respectively. Table 3.68 shows which UCC the vehicles belong to, and Lastly, Table 3.69, gives the distance matrix between nodes a and b where $a, b \in U \cup J$.

Table 3.58: Avg. Cost of truck per km from Company "i" to UCC "u"

	UCC	
Company	**1**	**2**
1	$ 2.45	$ 2.50
2	$ 2.40	$ 2.55
3	$ 2.35	$ 2.50
4	$ 2.50	$ 2.45

Table 3.59: Demand of each customer "l" from each company "I"

Company	**Customer**			
	1	**2**	**3**	**4**
1	2	2	1	2
2	1	1	3	0
3	1	1	0	4
4	2	3	4	4

Table 3.60: Fixed cost of using location "j"

Location	**Cost**
3	$ 6.00
4	$ 6.00
5	$ 6.00

Table 3.61: Distance matrix of location “j” and customers “I” (Km)

Location	Customers			
	1	**2**	**3**	**4**
1	2	0.45	7	13
2	0.3	1	25	0.2
3	10	0.4	0.5	14

Table 3.62: Distance of Company “I” to UCC “u” (Km)

Company	UCC	
	1	**2**
1	20	45
2	130	145
3	70	85
4	30	40

Table 3.63: Capacity of delivery box each delivery box type “k”, in terms of number of parcels

Type	Capacity
1	5
2	10
3	15

Table 3.64: Cost of each delivery box type “k”

Type	Cost
1	$ 1.00
2	$ 2.00
3	$ 3.00

Table 3.65: Capacity of each UCC\ “u” in terms of the number of parcels, it can manage

UCC	Capacity
1	30
2	30

Table 3.66: Maximum capacity of vans and trucks in the respective units

Vehicle	Capacity
Van	5 delivery boxes
Truck	10 Parcels

Table 3.67: N_{ur}, Allocation of vehicles to UCCs

	Vehicles	
UCC	**1**	**2**
1	1	0
2	0	1

Table 3.68: Distance (in Km) between the node a and b, where nodes 1 and 2 are UCC and the rest are depot

	Nodes b				
Nodes a	**1**	**2**	**3**	**4**	**5**
1	0	4	3	5	7
2	4	0	6	2	4
3	3	6	0	3	5
4	5	2	3	0	8
5	7	4	5	8	0

Validation for model 5 (Instance 1). This model is different from all the previous ones in the sense that it is an integrated version all models, hence, showing that it can solve the entire model plus doing the vehicle routing between the UCCs and locations. For the first instance, we test the model on the above data.

Table 3.69: Number of trucks going from Company "i" to UCC "u"

i	**u**	**Number of trucks**
1	2	1
2	1	1
3	1	1
4	1	2

Table 3.70 shows the number of parcels that are going from a company, for each customer and which UCC they go through, to reach the corresponding pick up location, for the relevant customer. The number of packages going through, for each customer, is the same as the number demanded by the customer, from that company. This result is comparable to model 3 outcome since we tested both the models on the same data. From Table 3.71, no deliveries go to location "3". The model has distributed the delivery boxes between locations "4" and "5" only. While Table 3.72, shows that only two locations are open, customers "1" and "4", visit location "4", while the other two visit location "5".

Table 3.70: W_{iujl}, Number of parcels delivered for each Customer

i	u	j	l	Number of parcels
1	2	4	1	2
1	2	4	4	2
1	2	5	2	2
1	2	5	3	1
2	1	4	1	1
2	1	5	2	1
2	1	5	3	3
3	1	4	1	1
3	1	4	4	4
3	1	5	2	1
4	1	4	1	2
4	1	4	4	4
4	1	5	2	3
4	1	5	3	4

Table 3.71: X_{ujk}, Number of delivery boxes going from UCC "u" to location "j"

u	j	k	x- Number of Delivery boxes
1	4	1	2
1	4	2	1
1	5	1	1
1	5	2	1
2	4	1	1
2	5	1	1

Table 3.72: Q_{jl}, Allocation of customer "L" to potential locations "j"

j	l
4	1
4	4
5	2
5	3

Figure 3.12 illustrates the routes of delivery from companies to UCCs. We can see that the first three companies deliver to UCC "1" while the last one delivers to UCC "2". Customers "1" and "4" are allocated to location "4", because it is the only one in their range. While customers "2" and "3" go to location "5".

We can see the analysis of capacities at each the location in Table 3.73. Location "3" has zero capacity since no delivery box going to that location. Location "4" is to receive four delivery boxes; the total capacity is twenty-five parcels. Total demand at location "4", for customer "1" and "4" is sixteen parcels. For location "5", the capacity is twenty parcels and the total demand for customers "2," and "3" is fifteen parcels. As we can see that for both cases, the capacities are within bounds.

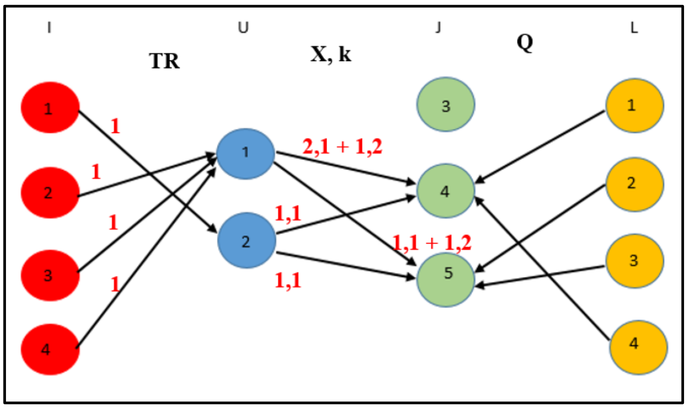

Figure 3.12: Path of trucks, allocation of delivery boxes and customers

Table 3.73: Comparison of capacities and demand at each of the locations

j	Total Number of D.Bs	D.B Type	Total capacity at j	Total demand at j
3	0	-	0	0
4	4	1,1,1,2	25	16
5	3	1,1,2	20	15

We fixed the capacities of both UCCs at thirty parcels. Hence, if the number of parcels exceeds the capacity of one UCC, then the second UCC must be utilized for the remainder. In this case, UCC “1” is closer to the companies hence all of them deliver there. However, company “1’ cannot route to UCC “1” because of the capacity restriction. Since split deliveries are not allowed, it only delivers to UCC “2”.

Table 3.74: Comparison of capacities and demand at each of the UCC

u	Total Parcels	Company	UCC Max Capacity
1	24	2,3,4	30
2	7	1	30

The path of the parcels shown in Figure. 3.14, are according to the results from the variable $W_{iujl,}$ the routes from nodes “I” to “u” are the same as seen in Figure. 3.12. and the assignment of “u” to “j” also matches. However, in this model we provide routing between the nodes since, in a realistic situation, each UCC will be serving hundreds of customers. Therefore, a proper vehicle routing system was required, while the one on one delivery was deemed unfeasible. Figure. 3.13, Shows the path of vehicles from their respective UCCs.

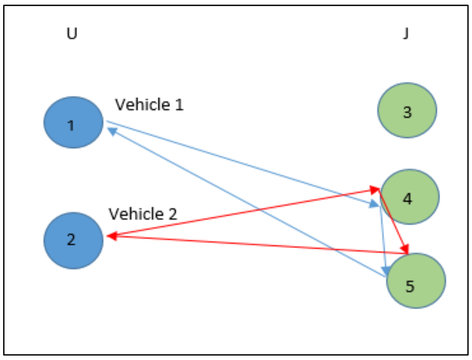

Figure 3.13: Routing of vehicles from UCCs to locations

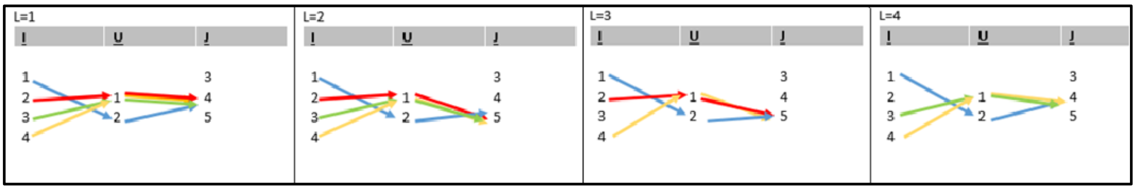

Figure 3.14: Path of parcels according to the results for W_{iujl}

Validation For model 5 (Instance 2). In this instance, we increased the capacity of UCC "1" is increased to forty parcels, while the capacity of UCC "2" remains the same. Therefore, UCC "1" is now capable of handling parcels from all the companies, given that the demand of customers remains the same. Table 75 shows that all companies deliver to UCC 1 since it is closer and has enough capacity.

Table 3.75: Number of Trucks going from the company "i" to UCC "u"

i	u	TR - Number of Trucks
1	1	1
2	1	1
3	1	1
4	1	2

The results of W_{iujl} from table 3.76, show that all parcels now go through UCC "1" only. The number of parcels transported for each customer and the final location of delivery remains the same in comparison to the previous instance. Table 3.77 shows that only UCC "1" is managing all the deliveries.

Figure 3.15 depicts the overview of the allocations between companies and UCCs showing that all companies deliver to UCC "1". The distribution of customers remains the same since we made no changes in their respective distance matrices. For the model to work, the capacities at each of the locations must also be enough to cater to the demands of the incoming parcels.

Table 3.76: W_{iujl}, Number of parcels going from company "i" through UCC "u" to location "j" for each customer "l"

i	u	j	l	w - Number of parcels
1	1	4	1	2
1	1	4	4	2
1	1	5	2	2
1	1	5	3	1
2	1	4	1	1
2	1	5	2	1
2	1	5	3	3
3	1	4	1	1
3	1	4	4	4
3	1	5	2	1
4	1	4	1	2
4	1	4	4	4
4	1	5	2	3
4	1	5	3	4

Table 3.77: X_{ujk}, Number of delivery boxes going from UCC "u" to location "j"

u	j	k	x - Number of Delivery boxes
1	4	1	1
1	4	3	1
1	5	3	1

The analysis in Table 3.78 shows demand and capacities at the receiving locations. While Table 3.79 shows the respective capacities and delivery loads of each of the UCCs.

Table 3.78: Comparison of Capacities and Demand at each of the locations

J	Total Number of D.Bs	D.B Type	Total Capacity	Total Demand
3	0	-	0	0
4	2	1,3	20	16
5	1	3	15	15

From Figure 3.15, Location "4" receives two delivery boxes of types "1" and "3" while location "5" receives only one delivery box of size "3". In both, cases the capacities are just enough to be able to handle all the parcels. According to Table 3.79, UCC “1” has enough capacity, it can manage all deliveries unlike instance 1. While UCC “2” remains closed thus saving on the operational (not accounted for) and the transportation costs. Figure 3.16 shows that all deliveries go through UCC “1” only, hence confirming the stance of all the previous results.

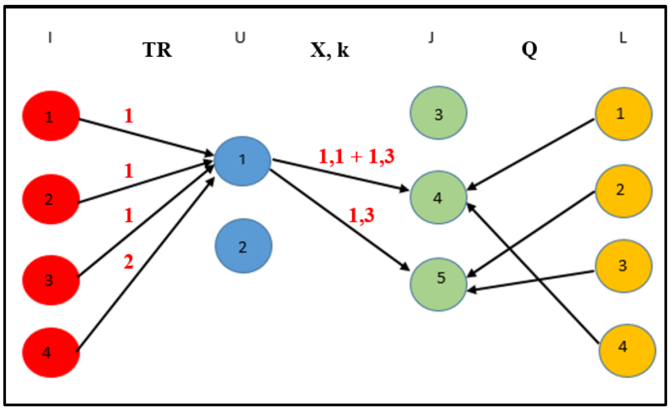

Figure 3.15: Path of trucks, allocation of delivery boxes and customers

Table 3.79: Comparison of capacities and demand at each of the UCC

U	Total Parcels	Company	Total Capacity at UCC
1	31	1,2,3,4	40
2	0	-	30

The routing is accordingly, only one vehicle is required since it will have enough capacity, and it will operate from UCC “1”. Figure 3.17 shows that vehicle “2” visits the locations from UCC “1” and returns.

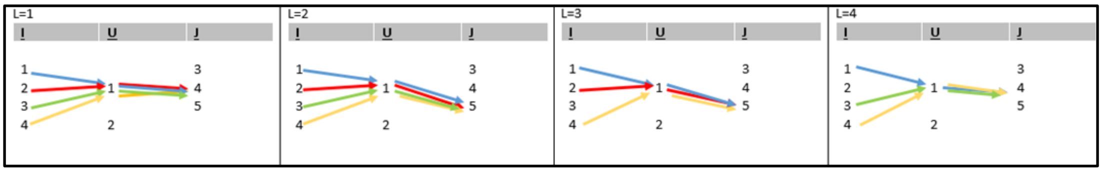

Figure 3.16: Path of parcels from i to u to j for each customer l on day 2, according to the results for W_{iujl}

3.3. Comparison of Model 4 and Model 5 (4 Customers)

Since model 5 is an integrated model, in theory, it should present a more optimized result as compared to model 4 which we divided into two separate parts. We examine the two models on the same data sets. The data used to validate model 4, and the first instance of model 5 are the same. Hence, we compare both cases below. Table 3.80 compares the variable TR_{iu}, where we can see that the result is quite similar. Table 3.81 compares the variable W_{iujl}. There is a difference in notation, in model 4 (phase 1), where we renamed the locations (1, 2, 3) while in model 4 (phase 2) and model 5, we renamed the same locations as (3, 4, 5) respectively. Other than that, the result is identical.

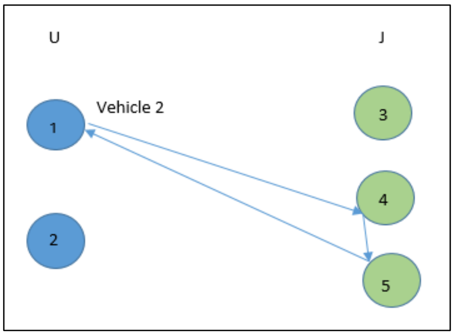

Figure 3.17: Routing of vehicles from UCCs to locations

Table 3.80: Comparison of TR_{iu}

Model 4 (phase 1)			Model 5		
i	**u**	**Number of Trucks**	**i**	**u**	**Number of Trucks**
1	1	1	1	2	1
2	2	1	2	1	1
3	1	1	3	1	1
4	1	2	4	1	2

Table 3.82 shows the comparison of the results of the variable X_{ujk}. The results of Variables T_{uj} and Q_{jl} are also the same for each of the cases. However, the difference arises in the matter of vehicle routing, in the sense that in model 4 the vehicle routing is done separately in phase 2. While in model 5 it is part of the same solution. The total for model 4 is calculated by adding the objective value of phase 1 and 2 and deducting the dummy cost added in phase 1 ($\sum_j \sum_u CP_{uj}T_{uj}$). Then, we see the main difference when we compare the objective functions (the total costs) from both cases.

Table 3.84 shows that the integrated model 5 has lower costs compared to model 4. The net difference in the total costs from Table 86 is only seventy-two. It is worth noting that this is a difference in the cases of 4 customers, however, in large scenarios, the difference will be larger.

Table 3.81: Comparison of W_{iujl}

Model 4 (phase 1)					Model 5				
i	u	j	l	Number of parcels	i	u	j	l	Number of parcels
1	1	2	1	2	1	2	4	1	2
1	1	2	4	2	1	2	4	4	2
1	1	3	2	2	1	2	5	2	2
1	1	3	3	1	1	2	5	3	1
2	2	2	1	1	2	1	4	1	1
2	2	3	2	1	2	1	5	2	1
2	2	3	3	3	2	1	5	3	3
3	1	2	1	1	3	1	4	1	1
3	1	2	4	4	3	1	4	4	4
3	1	3	2	1	3	1	5	2	1
4	1	2	1	2	4	1	4	1	2
4	1	2	4	4	4	1	4	4	4
4	1	3	2	3	4	1	5	2	3
4	1	3	3	4	4	1	5	3	4

Table 3.82: Comparison of the number of the delivery boxes

Model 4 (phase 1)				Model 5			
u	j	k	Number of Delivery boxes	u	j	k	Number of Delivery boxes
1	2	3	1	1	4	1	2
1	3	3	1	1	4	2	1
2	2	1	1	1	5	1	1
2	3	1	1	1	5	2	1
				2	4	1	1
				2	5	1	1

Table 3.83: Comparison of vehicle routings travelling between the UCCs and locations

Model 4 (phase 2)	Model 5
Vehicle 3: 1-5-4-1 Vehicle 1: 2-4-5-2	Vehicle 2: 1-4-5-1 Vehicle 3: 2-4-5-2

Table 3.84: Comparison of the objective functions (the total costs)

Model 4	Model 5
Obj. of Phase 1 + Obj. of Phase 2 = 838 + 34 = 872 – 4= 868	Objective function = 794

3.4. Comparison of Model 4 and Model 5 (20 Customers)

For this analysis, the 2 final models are tested on a test region with a slightly bigger data. In this case, we are assuming sample test region with 20 customers and 7 potential locations.

Data:

We tested both models over the same data set for the comparison of the objective value.

1. **Demand:**

We assumed a random Demand distribution from zero to four parcels, for each of the twenty customers ordering from the three companies.

2. **The Distance between Potential Location and Customer:**

In the Test Scenario, we assumed a potential location to be available within every Kilometer, with a total of seven Potential Locations, we randomly distributed the customers around these nine Kilometers, and hence, each customer had a nearby pick-up location.

3. **The Distance between Company and Urban Consolidation Center (UCC):**

In a realistic environment, the companies could be far away from the UCCs, since the UCCs are usually located closer to the Cities, while the companies could either be in the industrial areas or other cities. Therefore, we assumed distances in the range of 100 – 300 Km for each case.

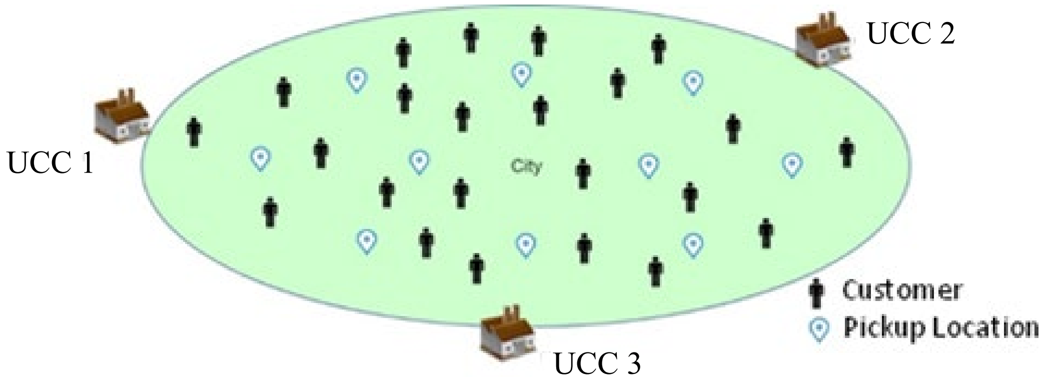

Figure 3.18: Case study area depiction

4. The Capacity of Cargo Delivery Boxes:

We assumed the Cargo Delivery Boxes to be of three types/sizes, each with a different leasing cost. We assumed that the smallest CDB could hold up to fifteen parcels, the medium CDB twenty parcels and the largest CDB can hold up to twenty-five parcels.

5. The Capacity of the Urban Consolidation Centers (UCC):

For the model to work, the sum of the capacities of all the UCCs must be more than the total demand hence we kept the capacity of each of the 2 UCCs to be able to manage 200 parcels at max.

6. Allocation of Vehicles to UCCs:

Each UCC has a vehicle. While the nature of the vehicles has not been taken into consideration. We have assumed that each vehicle can carry up to fifteen delivery boxes (of any size) at a time.

Test Scenario Result Discussion

Table 3.85 shows that companies “1”, “3”, and “4” send the trucks to UCC “1” while company “2’ sends only to UCC “3”. For each case, a single truck is being sent only. The results are identical for both the models. Based on the result of the variable Q_{jl}, Table 3.86 shows the mapping of each of the end customers to a location, and hence the total demand at each location. This result is the same for both models.

Table 3.85: Number of trucks going from company “i” to UCC “u”

Model 4 (phase 1)			Model 5		
i	u	Number of Trucks	i	u	Number of Trucks
1	1	1	1	1	1
2	1	1	2	1	1
3	1	1	3	1	1

Table 3.86: Assignment of customers to locations and total demand

Location	Customer	Total Demand
3	1,13	6
4	2,12,14	13
5	3,11,15	20
6	4,10,16	24
7	5,9,17	16
8	6,8,18,20	21
9	7,19	16

Table 3.87 shows the results of the Variable X_{ujk}, the Number of delivery boxes are chosen based on the demand, as identified in Table 86. Table 3.88 depicts the total capacity which is calculated by the type and sum of delivery boxes being delivered to the specific location. The below results are for model 4 and show that in each case, the capacity is enough to cater to the total demand.

Table 3.87: X_{ujk}, Number of delivery boxes transported, model 4 vs. model 5

Model 4 (phase 1)				**Model 5**			
u	**J**	**k**	**Number of Delivery boxes**	**u**	**j**	**k**	**Number of Delivery boxes**
1	3	1	1	1	3	1	1
1	4	1	1	1	4	1	1
1	5	2	1	1	5	2	1
1	6	1	2	1	6	1	2
1	7	2	1	1	7	2	1
1	8	1	2	1	8	1	2
1	9	1	2	1	9	2	1

Table 3.88: Comparison of Capacities and Demand at delivery location (Model 4)

j	Total Number of D.Bs	D.B type	Total Capacity	Total Demand
3	1	1	15	6
4	1	1	15	13
5	1	2	20	20
6	2	1	30	24
7	1	2	20	16
8	2	1	30	21
9	2	1	30	16

Similarly, in the case of model 5, Table 3.89, shows that the total capacity is again sufficient to cater all demands.

Table 3.89: Comparison of Capacities and Demand at delivery location (Model 5)

j	Total Number of D.Bs	D.B type	Total Capacity	Total Demand
3	1	1	15	6
4	1	1	15	13
5	1	2	20	20
6	2	1	30	24
7	1	2	20	16
8	2	1	30	21
9	1	2	20	16

However, it is worth noting that even though the total demands are the same, the distribution of delivery boxes was not in the two cases. Table 3.90, below, compares the capacity of the UCCs to show that the capacity constraints were not violated, in both the circumstances and this result was the same for both the models.

Table 3.90: Comparison of UCC capacity and load

u	Total Parcels	Company	Capacity of UCC
1	116	1,2,3	200
2	0	0	200

Models 4 and 5 also take in to account the routing of the delivery vehicles from the UCCs to the respective delivery locations. Figures 3.19 and 3.20 show the path of vehicles in each scenario. Each of the vehicles starts and end at their respective UCCs. Figure 3.19 shows the routing of vehicle “1” from UCC “1” and back for model 4. Since we assumed the capacity of the vehicles to be large enough, it can perform the entire delivery journey in a single run as shown below.

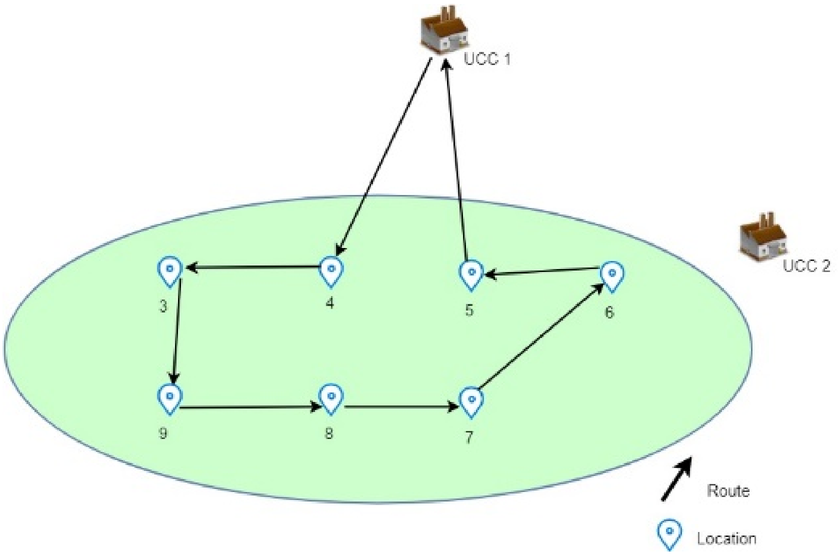

Figure 3.19: Model 4, vehicle “1” route

Figure 3.20 shows the routing of vehicle “1” as calculated by model 5. When we compare models 4 and 5, we can see that in both cases that the routes are a bit different. While, Table 3.91 compares the total costs in both the cases, after adjusting out the dummy costs. We can see that model 5 arrives at lower overall value, based on the same parameters as model 4.

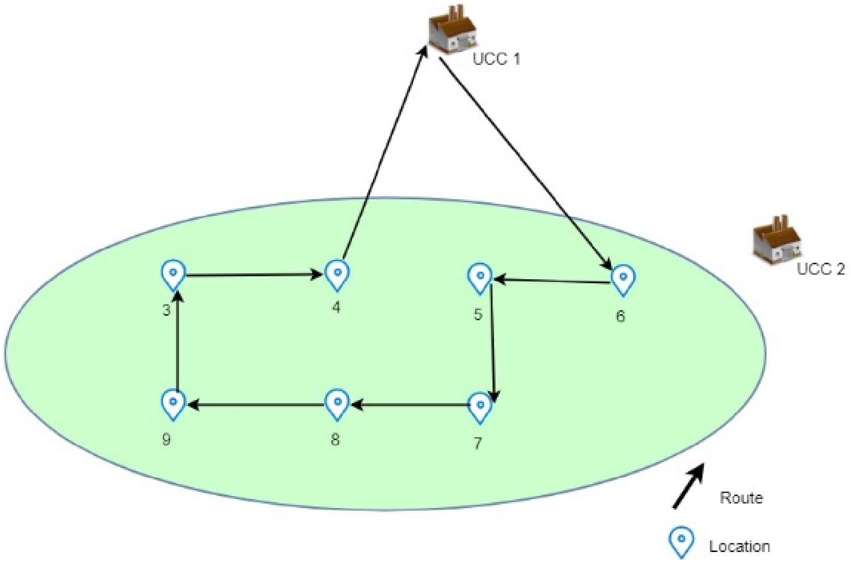

Figure 3.20: Model 5, vehicle "1" route

Table 3.91: Comparison of the objective functions (model 4 vs model 5)

Model 4	**Model 5**
586.5(4a) + 42.2(4b) – 7(Dummy Cost) = 621.7	736 - (42 + 130) (Dummy Cost) = 564

3.5. Comparison of Limitations between Model 4 and 5

Model 4 and 5 are quite similar since they have most of the equations in common. However, one of the most common inherent difference is that the two phases in model 4 allow it to remain linear while model 5 is non-linear. Both models have their limitations that are described in Table 3.92. Due to the size limitations of model 5, model 4 (Two-Phase Approach) is used for case study and sensitivity analysis in the subsequent chapters.

Table 3.92: Comparison of Model Limitations

Model 4 (Two-Phase Approach)	Model 5 (Consolidated Model)
Two phases mean manual intervention is required, to take the solution from the first phase and use it in the next stage	The Model is run once and requires no human response to produce final Solution, except the first input.
Model is Linear which makes it produce the solution faster.	The model is Nonlinear, due to which the solve time is relatively larger.

It can solve for up to ~1,260,000 Total Variables due to solver size restrictions.	The model cannot solve for more than ~10,000 variables due to solver size restrictions.
Due to two stages of solution, the solution is only near optimum	Comparatively, the Solution is closer to optimum since the total cost was lower when compared to the two stage solution.
The Solution is Near-Optimum due to the Two Phases	The Solution is Near-Optimum due to Non-linearity.

Chapter 4. Case Study and Sensitivity Analysis

4.1. Case Study using the Two-Phase Approach

For the case study we are using model 4 since it is a linear model. The linearity of model 4 allows us to be able to solve larger sized problems easily and faster.

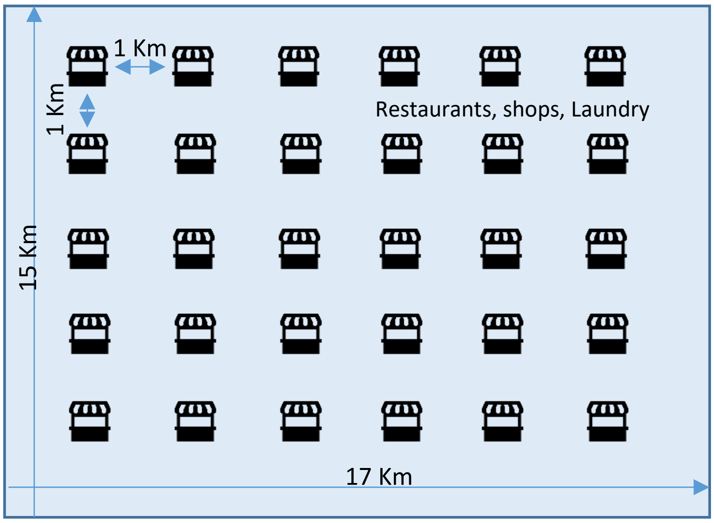

Figure 4.1: Test case Area for 500 Customers and 250 Locations

4.1.1. Sample data description

1. Demand: We assumed a random demand distribution from zero to four parcels, for each of the five-hundred customers ordering from the three companies.
2. The distance between potential location and customer: In the test scenario, we assumed a potential location to be available within every kilometer, with a total of two-hundred and fifty potential locations, we randomly distributed all the customers in this region. A potential location can be assumed to be a restaurant (as shown in Figure 4.1) or a saloon or a local grocery shop or any place where the people go to every day.
3. The distance between company and urban consolidation center (UCC): In a realistic environment, the companies could be long distances away from the UCCs, since the UCCs are usually located closer to the Cities. While the companies could either be in the industrial areas or other cities. However, since

our focus is primarily on the last mile, we kept the distance very low to minimize the effect on the objective function due to extra-large distances.

4. The capacity of cargo delivery boxes: The cargo delivery boxes were assumed to be of three types/sizes, each with a different leasing cost. We assumed that the smallest CDB could hold up to thirty parcels, the medium CDB, thirty-five parcels and the largest CDB could hold up to forty parcels.
5. The capacity of the urban consolidation centers (UCC): For the model to work, the sum of the capacities of all the UCCs must be more than the total demand hence we kept the capacity of each of the two UCCs to be able to manage two-thousand parcels at max.
6. Allocation of vehicles to UCCs: Each UCC has its fleet of vehicles. While the nature of the vehicles has not been taken into consideration. We have assumed that each vehicle can carry up to ten delivery boxes (of any size) at a time.

4.1.2. Case study result discussion. Table 4.1 shows that companies “1”, “2” and “3”, all deliver to UCC “1”, while for each case only ten trucks are required to make the deliveries.

Table 4.1: TRiu Number of Trucks going from “i” to “u”

i	u	Number of Trucks
1	1	10
2	1	10
3	1	10

Based on the result of the variable Q_{jl}, the model assigned the five-hundred customers to only forty-three locations, which implies that they were all within a kilometer distance of a location. The results of X_{ujk} show that type “1’ of the delivery box was enough to fulfill the delivery requirements. However, the capacity for type 1 was thirty which is relatively higher compared to analysis in the validation cases. Moreover, the average number of delivery boxes used at each of the locations is three, while the maximum number of delivery boxes used was five.

The usage of one type of delivery box is an advantage, concerning leasing, but it comes at the cost of delivering many delivery boxes. Also, it would also require a higher commitment from shopkeepers to store that many delivery boxes, unless we fix a limit to the maximum number of delivery boxes that can go to each location. We analyzed this aspect in the sensitivity analysis. Moreover, we also studied the effect of decreasing the capacity of each type of delivery box sensitivity analysis.

4.2. Sensitivity Analysis

For the Sensitivity analysis, we chose the above Case Study as the Base Model on which we studied the effects of variances in the parameters. The core focus of the Analysis is to see the sensitivity of the overall cost to the key parameters. The Summary of the result from the base model can be seen from below Table 4.2.

Table 4.2: Case Study Model Result Summary

Base Model - Parameters	
Customers	500
Locations Available	250
Customer Footprint Area	255 Km²
CDB Sizes for type 1, 2 and 3	30,40,50
Max DB allowed at each Location	10
Max Distance to travel	1 Km
Results	
Total Locations Open	43
Solve time	8 min
Obj Function	965
Maximum D.Bs used	5
Avg. Number of D.Bs used	2.5
D.B Types used	Type 1 only
Total D.Bs used	114

4.2.1. Customer footprint. We refer to the area coverage by the customers as the customer footprint. Potential locations such as restaurants, laundry or grocery shops are spread across fifteen by seventeen-kilometer area, i.e. Two Fifty-kilometer square of land as shown in Figure 4.1. In the base model, the customers were scattered across the entire region, while in this analysis we try limiting the customers to smaller areas to see how it affects the solution. We tested the model on four different customer footprints, two-hundred, one-hundred and fifty, one hundred, and lastly on just fifty kilometers square. Table 4.3, shows the results of the sensitivity of objective function to the scattering of the customers.

Although this reduces the cost of the delivery point, it does increase the total delivery boxes at each of the locations. For example, in the case where the customer footprint is fifty-kilometer square, only twelve places are open, and most of the places store up to ten Delivery boxes to fulfill the capacity requirements

Table 4.3: Sensitivity Analysis on Customer Footprint

Customer Footprint (Km²)	# of `Locations Open	Total CDBs used	Max D.B # of DBs	Total Cost
255	43	114	5	965
200	45	114	5	972
150	28	111	6	857
100	27	114	6	853
50	12	100	10	735

.

This number could be unpractical for the shop owners. Nevertheless, it would be easier to make the deliveries to a lesser amount of locations if they are closer to each other. Figure 4.2 visualizes the trend of objective functions against the customer footprint.

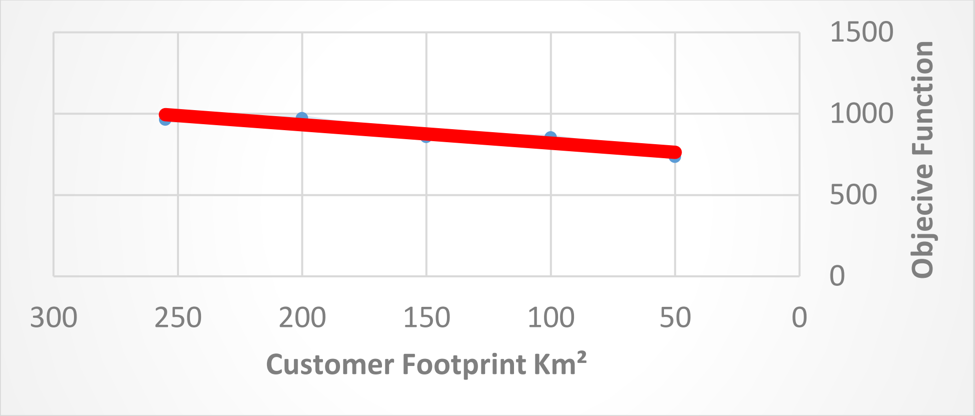

Figure 4.2: Trend of Objective Function vs. Customer Footprint

4.2.2. Cargo delivery box type. The Delivery box type refers to the Capacity that every kind of delivery box has. We have three types of delivery boxes with the capacity to carry thirty, forty and fifty parcels. These capacities are relatively larger and seem less practical concerning delivery and storage. Therefore, the model was tested for smaller sizes and to see how the solution would be affected. For this we assumed the customer footprint to be constant according to the base model. We examined two cases against the base model, in each case, the capacities of each of the CDB types is decreased by ten parcels. Table 4.4 summarizes the results.

Table 4.4: Sensitivity Analysis on CDB Type Capacities

CDB Sizes for Types 1-3	# of Locations Open	Total # of CDBs used	CDB Type Used	Avg. # of CDBs required	Total Cost
30,40,50	43	114	Type 1 Only	3	965
20,30,40	36	136	Type 1 Only	4	965
10,20,30	63	123	All Types	2	1123

From Table 4.4, we can see that change in the capacity of the delivery boxes dramatically affects the overall solution in many ways. In the last case, a higher number of delivery boxes are required to satisfy the capacity requirements, where initially we required fewer. The model uses only the type "1" of the delivery box to fulfill all requirements. However, in the second case where another ten parcels decrease the capacity, all kinds of delivery boxes are being utilized. In the base model, an Average of three delivery boxes is required per location, while for the second case, we needed on average four per location and in the last case, the average is two delivery boxes. The trade-off here is between using a single type of delivery box delivered to lesser locations, compared to delivering multiple types, but with a lower average number of CDBs. A Lower number would be practical for the shop owners in a real case scenario, but this would require delivering to a high number of locations. Figure 4.3 visualizes the trend on the Objective Function.

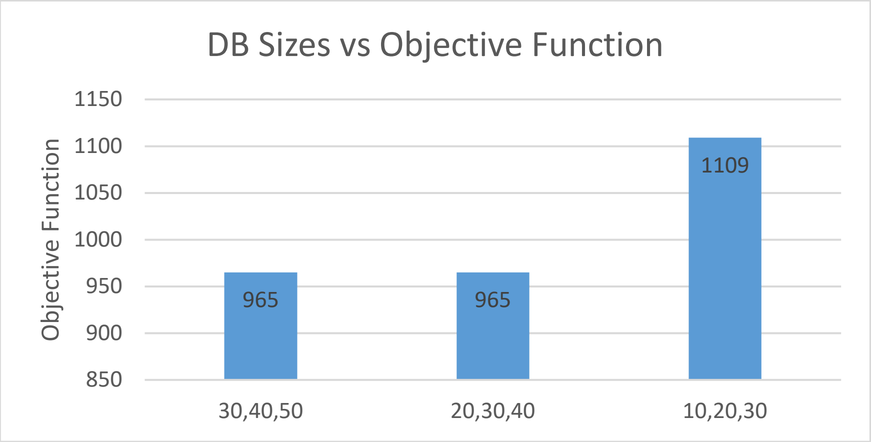

Figure 4.3: Trend of Objective Function vs. Db Sizes

4.2.3. Maximum cargo delivery box cap at each location. The maximum number of delivery boxes that a location can accept is something that is of great concern since it depends on the shopkeepers. In our case, we are assuming a constant number across all locations, but in real life, each shop keeper will have their preferences concerning the D.B.s. Therefore, the max capacity has to be more practically viable even for small shops that do not have enough space for too many D.Bs. Initially, we had set the maximum capacity for all the shops to be ten cargo delivery boxes, that we decreased the capacity for each case. Table 4.5 shows the results.

Table 4.5: Sensitivity Analysis on Max CDBs Allowed at each Location

Max Cap at each Location	# of Locations Open	Total # of CDBs used	CDB Type Used	Avg. # of CDBs	Highest # of CDBs	Total Cost
10	43	114	Type 1	2.65	5	965
5	43	114	Type 1	2.65	5	965
3	41	106	Mix	2.58	3	919
2	66	100	Mix	1.5	2	951

In the base model, we set the maximum capacity to ten D.B.s for each location. However, the maximum delivered was only five D.B.s. When we decreased the maximum capacity to five, the result did not change much. In the following case, we reduced the capacity further to three; the model reacts by using more types of delivery boxes to capture higher demand using fewer delivery boxes. When we cut the capacity again to two D.B.s, a lot more locations are opened to fulfill the deliveries. Figure 4.4 visualizes the effect on Objective function.

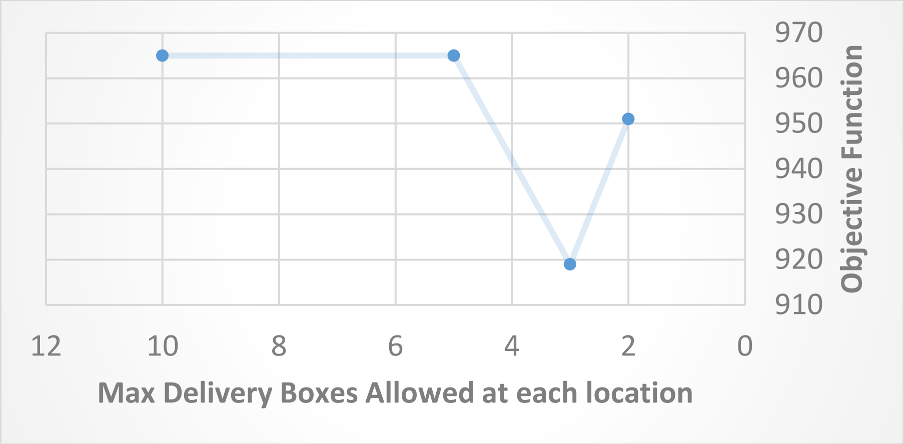

Figure 4.4: Trend of Objective Function vs. Max capacity at the pickup Locations

The figure shows that the optimum approach would be to limit the capacity of locations to three Delivery boxes. The trade-off here is that now more types of D.B.s are required to cater the demand, while previously, only one kind was enough. The advantage of having different sizes is that fewer locations are needed to cover the deliveries.

4.2.4. Overall sensitivity analysis. The impact of each of these parameters on the Objective function is compared to show the Sensitivity to these. We plotted the percentage difference for each of the parameters against the Percentage difference caused in the Objective function in Figure 4.5. The steepness of the plot reflects the Sensitivity of the Parameter.

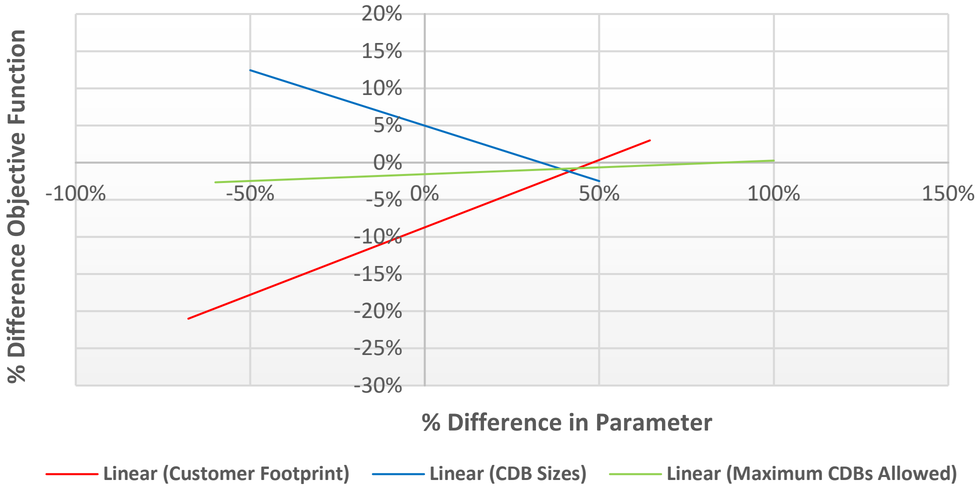

Figure 4.5: Sensitivity Analysis Spider-Plot

According to Figure 4.5, the Sensitivity analysis shows that the Objective function is highly sensitive to the Customer footprint and CDB sizes. While, the Maximum CDB limit at each location, does not have a significant effect on the Objective function, comparatively. In each case, we highlighted certain trade-offs for each of the cases.

Chapter 5. Conclusion and Future Work

The rise of the e-commerce industry signifies the importance of highlighting and mitigating the common inefficiencies in the last mile, that decrease the profitability of the overall supply chain and cause customer dissatisfaction. The causes of high costs included complicated travel routings, traffic congestions, fragmented deliveries and customer unavailability. In this thesis, we propose a solution that could eliminate complex routing under time constraints and minimize the effects of traffic on the delivery schedule and routings, while making the delivery easier for both customer and the shipper.

The proposed solution is based on the concept of cargo delivery boxes and consolidation of deliveries for multiple customers located in a specific area. We consolidate the shipments from different companies at the urban consolidation centers, and from there the deliveries are sorted, batched and loaded into cargo delivery boxes which in turn are packed into delivery trucks. We proposed four different models, where model 1 gave the base of the model, model 2 focused on the missed Delivery scenario, model 3 accounted for the capacitated deliveries scenario which makes it more realistic. The routing of the delivery vehicle from the UCC to Delivery Location was introduced in two phases, and then a consolidated model was proposed. Due to solver limitations concerning nonlinear models, we were unable to use the final consolidated model on a higher number of customers. Therefore, the Two-Phase Approach was run. We used the model, to study five hundred customers spread across two fifty-kilometer square areas. We tested the Model for sensitivity and the highest impact was found to be from the Customer footprint, across the study area. While the CDB sizes were also identified to be critical to the Objective function and rest of the Solution, the sensitivity Analysis helped us derive the following Key Managerial Insights:

- Customer Footprint has a significant effect on the overall Cost.
- It is beneficial to have many Customers spread in a smaller area. However, small Customer footprint would require a higher number of CDBs at each location.
- It is better to have CDBs in the Size range of twenty, thirty and forty parcels. However, this would be at the expense of transportation of a more significant number, less flexible to handle.

- CDB Type “1” is sufficient to make all deliveries if the maximum limit per location is three CDBs. However, this would require a higher number of locations to be operating in total.
- The routing is optimized when the number of locations open is less, and the number of CDBs is low.

Future research needs to focus on the environmental aspects to show that this model does or does not have a positive impact compared to the average Last mile delivery scenario. In a practical case, an outlier number of customers would opt for direct delivery, regardless of the associated cost. In this case, a framework could be proposed that could run two separate models in coordination with each other, where one part manages the cargo delivery boxes while the other manages direct delivery using the time windows, for the remainder of customers. Another approach could be to deliver all packages using cargo delivery boxes and then using the Cargo delivery bikes or crowd-sourced delivery for making direct home deliveries (attended) for the outlier number of customers who prefer the direct delivery regardless of the cost.

The core limitations of this study include the problem size in terms of the number of variables due to the usage of GAMS software. We were able to solve for a maximum of five hundred customer using the two-stage model, assuming there are three companies, three delivery box types, two UCCs and two hundred fifty potential locations. The second limitation is the assumption of the demand variation which is currently assumed to be randomly distributed. A study would be required to use the actual demand and the actual number of distributors involved in a real-life scenario. Lastly, we assumed the parcels to be all of the same size and type, the model doesn’t consider items of a large size for example the furniture items, neither does it consider grocery items that would require specialized equipment for example refrigerated delivery boxes.

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Appendix - A

Model 1:

```
Free Variable zz total costs;
Integer Variables X, W;
Binary variable Y, Q, T;
Equations obj, eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10;
obj.. zz =e= sum((j), F(j)*Y(j)) + sum((u,j,k), C(k)*X(u,j,k)) + sum((i,u,j,l), E(i,u)*
R(i,u)*W(i,u,j,l)) + sum((u,j,k), G(u,j)*H(u,j)*X(u,j,k));
eq1(j,l).. Q(j,l)*D(j,l) =l= 500 * Y(j);
eq2(j).. sum((i,l), Q(j,l)* P(i,l)) =l= sum((u,k), S(k)* X(u,j,k));
eq3(l).. sum((j), Q(j,l)) =e= 1 ;
eq4(j,l)..sum((i,u), W(i,u,j,l)) =e= sum((i), P(i,l)*Q(j,l));
eq5(i,u,j,l)..W(i,u,j,l) =l= Q(j,l)* O;
eq6(u,j).. sum((i,l), W(i,u,j,l)) =l= Sum(k, S(k)*X(u,j,k));
eq7(j,k).. sum((u), X(u,j,k)) =l= MaxDB ;
eq8(i,l)..sum((u,j), W(i,u,j,l)) =l= P(i,l) ;
eq9(i,u,j,l)..T(u,j)* O =g= W(i,u,j,l);
eq10(u,j)..sum(k, X(u,j,k)) =l= T(u,j)* O ;
model Osama /all/;
Option SOLPRINT=OFF, LIMCOL=0, LIMROW=0;
Solve osama using mip  minimizing zz;
Option W:0:0:1, X:0:0:1, Y:0:0:1, Q:0:0:1;
Display W.l, X.l, Y.l, Q.l;
```

Model 2:

```
Free Variable zz total costs;
Integer Variables  X, W;
Binary variable Y, Q, T;
Equations obj, eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10, eq11;
obj.. zz =e= sum((j,m), F(j,m)*Y(j,m)) + sum((u,j,k,m), C(k)*X(u,j,k,m)) +
sum((i,u,j,l,m), E(i,u)* R(i,u)*W(i,u,j,l,m)) + sum((u,j,k,m),
G(u,j)*H(u,j)*X(u,j,k,m));
eq1(j,l,m).. Q(j,l,m)*D(j,l,m) =l= 500 * Y(j,m);
eq2(j,m).. sum((i,l), Q(j,l,m)* P(i,l,m)) =l= sum((u,k), S(k)* X(u,j,k,m));
eq3(l,m).. sum((j), Q(j,l,m)) =e= 1 ;
eq4(j,l,m)..sum((i,u), W(i,u,j,l,m)) =e= sum((i), P(i,l,m)*Q(j,l,m));
eq5(i,u,j,l,m)..W(i,u,j,l,m) =l= Q(j,l,m)* O ;
eq6(u,j,m).. sum((i,l), W(i,u,j,l,m)) =l= Sum(k, S(k)*X(u,j,k,m));
eq7(j,k,m).. sum((u), X(u,j,k,m)) =l= MaxDB ;

eq8(i,l,m)..sum((u,j), W(i,u,j,l,m)) =l= P(i,l,m) ;
eq9(i,u,j,l,m)..T(u,j,m)* O =g= W(i,u,j,l,m) ;
```

```
eq10(u,j,m)..sum(k, X(u,j,k,m)) =l= T(u,j,m) *O ;
eq11(j,u,m)..sum(k, S(k)* X(u,j,k,m)) =g= sum((i,l), N(u,l,m-1)* W(i,u,j,l,m-1)) +
sum((i,l), W(i,u,j,l,m)) ;
model Osama /all/;
Option SOLPRINT=OFF, LIMCOL=0, LIMROW=0;
Solve osama using mip  minimizing zz;
Option W:0:0:1, X:0:0:1, Y:0:0:1, Q:0:0:1;
Display W.l, X.l, Y.l, Q.l;
```

Model 3:

```
Free Variable zz total costs;
Integer Variables  TR, W, X, VAN;
Binary variable Y, Q, T, YT;
Equations obj, eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10, eq11, eq12, eq13,
eq14, eq15;
obj.. zz =e= sum((j), F(j)*Y(j)) + sum((j,k,u), C(k)*X(u,j,k)) + sum((i,u), TR(i,u)*
CT(i,u)*R(i,u)) + sum((u,j), VAN(u,j)*CP(u,j)*H(u,j));
eq1(j,l).. Q(j,l)*D(j,l) =l= 0.5 * Y(j);
eq2(j).. sum((i,l), Q(j,l)* P(i,l)) =l= sum((u,k), S(k)* X(u,j,k));
eq3(l).. sum((j), Q(j,l)) =e= 1 ;
eq4(j,l)..sum((i,u), W(i,u,j,l)) =e= sum((i), P(i,l)*Q(j,l));
eq5(i,u,j,l)..W(i,u,j,l) =l= Q(j,l)* O ;
eq6(u,j).. sum((i,l), W(i,u,j,l)) =l= Sum(k, S(k)* X(u,j,k)) ;
eq7(j,k).. sum((u), X(u,j,k)) =l= MaxDB ;
eq8(i,l)..sum((u,j), W(i,u,j,l)) =l= P(i,l) ;
eq9(i,u,j,l)..T(u,j)* O =g= W(i,u,j,l) ;
eq10(u,j)..sum(k, X(u,j,k)) =l= T(u,j)*O ;
eq11(i,u)..TR(i,u)* MTr =g= sum((j,l), W(i,u,j,l)) ;
eq12(u)..sum((i,j,l), W(i,u,j,l)) =l= CAP(u) ;
eq13(u,j)..sum(k, X(u,j,k)) =l= sum(k, VAN(u,j)* MaxV(k));
eq14(i,u)..TR(i,u) =l= YT(i,u) * O ;
eq15(i)..sum(u, YT(i,u)) =l= 1 ;
model Osama /all/ ;
Option SOLPRINT=OFF, LIMCOL=0, LIMROW=0;
Solve osama using mip  minimizing zz;
Option TR:0:0:1, W:0:0:1, VAN:0:0:1, X:0:0:1, Y:0:0:1, Q:0:0:1;
Display TR.l, W.l, Y.l, Q.l, X.l, VAN.l;
```

Model 4 phase 1:

```
Free Variable zz total costs;
Integer Variables  TR, W, X;
Binary variable Y, Q, T, YT;
```

```
Equations obj, eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10, eq11, eq12, eq13,
eq14, eq15, eq16;
obj.. zz =e= sum((j), F(j)*Y(j)) + sum((j,k,u), C(k)*X(u,j,k)) + sum((i,u), TR(i,u)*
CT(i,u)*R(i,u))+ sum((u,j), T(u,j)*CP(u,j)*H(u,j));
eq1(j,l).. Q(j,l)*D(j,l) =l= 0.5 * Y(j);
eq2(j).. sum((i,l), Q(j,l)* P(i,l)) =l= sum((u,k), S(k)* X(u,j,k));
eq3(l).. sum((j), Q(j,l)) =e= 1 ;
eq4(j,l).. sum((i,u), W(i,u,j,l)) =e= sum((i), P(i,l)*Q (j,l));
eq5(i,u,j,l).. W(i,u,j,l) =l= Q(j,l)* O ;
eq6(u).. sum((i,j,l), W(i,u,j,l)) =l= Sum((j,k), S(k)* X(u,j,k)) ;
eq7(j,k).. sum((u), X(u,j,k)) =l= MaxDB ;
eq8(i,l)..sum((u,j), W(i,u,j,l)) =l= P(i,l) ;
eq9(i,u,j,l)..T(u,j)* O =g= W(i,u,j,l) ;
eq10(u,j)..sum(k, X(u,j,k)) =l= T(u,j)*O ;
eq11(i,u)..TR(i,u)* MTr =g= sum((j,l), W(i,u,j,l)) ;
eq12(u)..sum((i,j,l), W(i,u,j,l)) =l= CAP(u) ;
eq13(u,j)..sum(k, X(u,j,k)) =l= T(u,j)* MaxV ;
eq14(u,j).. sum((i,l), W(i,u,j,l)) =l= Sum(k, S(k)* X(u,j,k)) ;
eq15(i,u)..TR(i,u) =l= YT(i,u) * O ;
eq16(i)..sum(u, YT(i,u)) =l= 1 ;
model Osama /all/ ;
Option SOLPRINT=OFF, LIMCOL=0, LIMROW=0;
Solve osama using mip  minimizing zz;
Option TR:0:0:1, W:0:0:1, T:0:0:1, X:0:0:1, Y:0:0:1, Q:0:0:1;
Display TR.l, W.l, Y.l, Q.l, X.l, T.l;
$onlisting
```

Model 4 phase 2:

```
alias (a,b,e) ;
set J(b) /3*5/ ;
set U(a) /1,2/ ;
Free Variable zz total costs;
Integer Variable S;
Binary variable V;
Equations obj, eq1, eq2, eq3, eq4, eq5, eq6, eq7;
obj..zz =e= sum((a,b,m), D(a,b)*V(a,b,m));
eq1(J)..sum((a,m), V(a,J,m)) =g=  sum(u, T(u,j));
eq2(a,J,m)..S(j,m) =g= S(a,m) + (sum((u,k), X(u,j,k))* V(a,j,m)) - 10000*(1-
V(a,j,m));
eq3(a,m)..sum(b, V(a,b,m)) =e= sum(b, V(b,a,m));
eq4(U,J,m)..-T(U,J) + sum(e, V(U,e,m) + V(e,J,m)) =l= 1;
eq5(u,m)..sum(b, V(u,b,m)) =l=  N(u,m);
eq6(a,m).. V(a,a,m) =e= 0;
eq7(u,j,m)..sum((k), X(u,j,k)) =l= S(j,m);
```

```
eq8(j,m)..S(j,m) =l= MaxV(m);
eq9(u,J)..sum((a,m), V(a,J,m)) =g= T(u,j);
model Osama /all/;
Option SOLPRINT=OFF, LIMCOL=0, LIMROW=0;
Solve osama using mip  minimizing zz;
Option  V:0:0:1;
Display V.l;
```

Model 5:

```
alias (a,b,e);
set j(b) /3*5/ ;
set u(a) /1*2/ ;

Free Variable zz total costs;
Integer Variables  TR, W, X, CU;
Binary variable Y, Q, T, YT, V;
Equations obj, eq1, eq2, eq3, eq4, eq5, eq6, eq7, eq8, eq9, eq10, eq11, eq12, eq13,
eq14, eq15, eq16, eq17, eq18, eq19, eq20, eq21,
eq22, eq23, eq24, eq25, eq26;
obj.. zz =e= sum((j), F(j)*Y(j)) + sum((j,k,u), C(k)*X(u,j,k)) + sum((i,u), TR(i,u)*
CT(i,u)*RI(i,u))+ sum((a,b,r), DVC*V(a,b,r)*H(a,b)) ;
eq1(j,l).. Q(j,l)*D(j,l) =l= 0.5 * Y(j);
eq2(j).. sum((i,l), Q(j,l)* P(i,l)) =l= sum((u,k), S(k)* X(u,j,k));
eq3(l).. sum((j), Q(j,l)) =e= 1 ;
eq4(j,l).. sum((i,u), W(i,u,j,l)) =e= sum((i), P(i,l)*Q(j,l));
eq5(i,u,j,l).. W(i,u,j,l) =l= Q(j,l)* O ;
eq6(u).. sum((i,j,l), W(i,u,j,l)) =l= Sum((j,k), S(k)* X(u,j,k)) ;
eq7(j,k).. sum((u), X(u,j,k)) =l= MaxDB ;
eq8(i,l)..sum((u,j), W(i,u,j,l)) =l= P(i,l) ;
eq9(i,u,j,l)..T(u,j)* O =g= W(i,u,j,l) ;
eq10(u,j)..sum(k, X(u,j,k)) =l= T(u,j)*O ;
eq11(i,u)..TR(i,u)* MTr =g= sum((j,l), W(i,u,j,l)) ;
eq12(u)..sum((i,j,l), W(i,u,j,l)) =l= CAP(u) ;
eq13(u,j,k)..X(u,j,k) =l= T(u,j)* MaxV ;
eq14(u,j).. sum((i,l), W(i,u,j,l)) =l= Sum(k, S(k)* X(u,j,k)) ;
eq15(i,u)..TR(i,u) =l= YT(i,u) * O ;
eq16(i)..sum(u, YT(i,u)) =l= 1 ;
eq17(j)..sum((a,r), V(a,j,r)) =g= sum(u, T(u,j)) ;
eq18(a,j,r)..CU(j,r) =g= CU(a,r) + (sum((u,k), X(u,j,k))* V(a,j,r)) - 10000*(1-V(a,j,r))
;
eq19(b,r)..sum(a, V(a,b,r)) - sum(a, V(b,a,r)) =e= 0 ;
eq20(u,j,r)..-T(U,J) + sum(e, V(U,e,r) + V(e,J,r)) =l= 1 ;
eq21(u,r)..sum(b, V(u,b,r)) =l= N(u,r) ;
eq22(a,r).. V(a,a,r) =e= 0 ;
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eq23(j,r)..sum((u,k), X(u,j,k)) =l= CU(j,r);
eq24(j,r)..CU(j,r) =l= MaxV ;
eq25(u,J)..sum((a,r), V(a,J,r)) =g= T(u,j);
eq26(j).. sum(u, T(u,j)) =l= sum(l, Q(j,l)) *O  ;model Osama /all/ ;
Option SOLPRINT=OFF, LIMCOL=0, LIMROW=0;
Solve osama using minlp  minimizing zz;
Option TR:0:0:1, W:0:0:1, T:0:0:1, X:0:0:1, Q:0:0:1, V:0:0:1;
Display TR.l, W.l, Q.l, X.l, T.l, V.l;
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Vita

Osama Saqib Qazi was born in 1993, in Karachi, Pakistan. He received his primary and secondary education in Abu Dhabi, UAE. He received his B.Sc. degree in Mechanical Engineering from the University of Nottingham in 2014.

In February 2015, he joined the Engineering System Management master's program in the American University of Sharjah as a graduate teaching assistant. During his master's study, he worked part-time in the logistics department in Henkel, he also worked as a Project Coordinator with Exceed; working with clients like the Dept. of Urban Planning and Municipalities and The Executive Council in Dubai. His research interests are in Operations research, Data analytics, Logistics planning and Vehicle routing.