EFFECT OF FLEXURAL CFRP SHEETS AND PLATES ON SHEAR RESISTANCE OF REINFORCED CONCRETE BEAMS

by

Waleed Nawaz

A Thesis Presented to the Faculty of the American University of Sharjah College of Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Science in Civil Engineering

Sharjah, United Arab Emirates

May 2014
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Approval Signatures

We, the undersigned, approve the Master’s Thesis of Waleed Nawaz

Thesis Title: Effect of Flexural CFRP Sheets and Plates on Shear Resistance of Reinforced Concrete Beams.

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Abstract

Ageing of reinforced concrete (RC) structures has captured the attention of a number of researchers to find different materials and techniques to strengthen and retrofit deteriorated structures. Carbon Fiber Reinforced Polymer (CFRP) composite plates and sheets are widely used to externally strengthen RC beams in flexure and shear. The conventional method of strengthening RC beams in shear is by externally bonding CFRP laminates to the beam’s vertical sides via epoxy adhesives. However, in certain applications, the sides of the beam might not be accessible for shear strengthening. This study aims at investigating the contribution of longitudinal CFRP reinforcement on the shear strength of shear deficient RC beams. To achieve this objective, nineteen beams were cast without transverse reinforcement in the shear span and tested under four-point bending. The specimens were divided into three groups with different longitudinal steel reinforcement ratios. Each group has one control un-strengthened beam and five beams strengthened at their soffit with CFRP plates or sheets. An equivalent longitudinal reinforcement ratio was computed based on the modular ratio of the CFRP and steel reinforcement and ranged from 0.14 to 2.29%. The load and mid-span displacement values, strain gauges readings at different discrete locations along the beams’ shear and mid-spans were recorded until failure. The specimens failed in shear as a result of a diagonal-tension crack as expected. The strengthened specimens showed a significant increase in the load-carrying shear capacity over the control specimens. The increase in the concrete shear capacity for beams strengthened with sheets and plates ranged from 10 to 70% and 30 to 151%, respectively, over the control specimens. It was concluded that CFRP composite plates and sheets, when externally bonded to the soffit of simply supported beams, will enhance both the flexural and shear capacity of such beams. In addition, the concrete shear capacity of the tested specimens was predicted using the ACI 318-08 and CSA 2004 simplified and detailed shear design provisions. The results indicated that CSA 2004 shear design provisions, which are based on the modified compression field theory, yielded the closest agreement with the obtained experimental data.

Keywords: Shear Strengthening, Deficient Beam, CFRP, Reinforced Concrete
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Chapter 1: Introduction

1.1. Background

Many existing reinforced concrete (RC) structures are in severe state of deterioration due to construction faults, carbonation, chloride attack, increase in live load, and corrosion of steel reinforcement. The statistical report from the US Department of Transportation (USDOT) indicated that several bridges in the USA are structurally deficient and in need of repair and strengthening [1].

Structural RC members such as slabs and beams can fail in flexure or shear. However, shear failure is sudden, brittle and more catastrophic compared to flexural failures. A number of shear failures occurred over the last few decades in the USA. Examples include, the shear failure of RC girders in the Air Force warehouse [2] and the collapse of a bridge in Quebec [2] due to the shear deficient concrete slab not containing shear reinforcement.

Due to the complex behavior of shear failures and variability of data obtained from testing shear deficient RC slabs and beams, the design codes of practice apply a larger strength reduction factor (safety factor) to the nominal shear strength of RC members as compared to flexural strength reduction factors applied to the nominal moment strength of such members.

Researchers and engineers use different materials such as steel plates and fiber-reinforced polymer (FRP) composite plates and sheets to externally strengthen structural RC members in flexure and shear. Strengthening of structural members such as slabs, beams and columns using FRP composite material such as carbon (CFRP) has gained tremendous acceptance over the last two decades due to its high strength to weight ratio, high stiffness, light weight, flexibility, ease of installation and resistance to corrosion as compared to other materials [3-13].

The technique of externally strengthening RC slabs and beams in flexure by bonding CFRP plates and sheets to the beam’s tensile surface (soffit) via epoxy adhesives had shown a tremendous enhancement in the load-carrying capacity and stiffness of the strengthened specimens. A detailed literature review on strengthening RC beams in flexure with external FRP laminates is discussed in the proceeding chapter of this thesis. Extensive experimental and numerical research studies that have been conducted on
strengthened RC beams in flexure showed an increase in the flexural capacity of the strengthened beam specimens with CFRP laminates up to 100% over the control unstrengthened specimens when externally bonded to the tensile surface of such beams [3]. Thus, CFRP external flexural longitudinal reinforcement plays the same role as that of internal steel reinforcement in increasing the moment strength of RC members.

It is well-known that the internal flexural longitudinal steel reinforcement affects the concrete shear strength ($V_c$) in RC beams. Tests [14] have shown that as the internal flexural steel reinforcement in RC beams increases, $V_c$ increases. The flexural steel reinforcement ratio ($\rho$) is a major variable in predicting $V_c$ of RC beams in most design codes of practice. Thus, it could be expected that external flexural longitudinal CFRP laminates would have the same influence as that of internal steel reinforcement in enhancing the concrete shear strength ($V_c$) of RC beams. However, the literature is lacking information about the contribution of flexural CFRP composite plates or sheets on the shear strength of RC beams. Accordingly, this study aims at investigating experimentally the effect of longitudinal CFRP composite plates or sheets on the shear strength of RC beams when externally attached to the beam’s soffit. This will also examine the contribution of the combined steel and CFRP longitudinal reinforcement ratio to the shear capacity of RC beams. This might also resolve the issues in the construction industry when the sides of concrete beams are not accessible for conventional shear strengthening.

1.2. Research Significance

Strengthening of deteriorated structures using CFRP is gaining popularity over the years due to its high strength to weight ratio, high stiffness, light weight and resistance to corrosion as compared to other materials [3-13]. The conventional method of strengthening RC beams in shear is by externally bonding CFRP laminates to the vertical sides of the beams via epoxy adhesives. However, in certain applications, the sides of the beam might not be accessible for shear strengthening. Test results had shown that the amount of internal longitudinal flexural reinforcement has a significant effect on the shear capacity of RC beams, especially to the concrete shear strength contribution ($V_c$). Since the flexural CFRP longitudinal external reinforcement has the same effect in
increasing the moment capacity of RC beams as that of the internal steel reinforcement, it is also expected that it will enhance the shear capacity of such beams. Unlike the use of longitudinal external CFRP reinforcement on the flexural strengthening of RC beams and the use of external side bonded vertical/inclined CFRP reinforcement on the shear strengthening of RC beam which received a great attention by investigators the use of longitudinal external CFRP reinforcement on the shear strengthening of RC beams received very slight attention, if any. Therefore and to the best of my knowledge, the literature is relatively lacking information on this regard compared to other CFRP strengthening mechanisms. Thus, the importance of this research is to investigate the contribution of longitudinal CFRP reinforcement in the form of composite plates or sheets on the shear strength of shear deficient RC beams. This study presents experimental results on nineteen RC shear deficient simply supported beams with different amounts of internal steel reinforcement ratios and externally strengthened with longitudinal CFRP plates or sheets bonded to the soffit of the strengthened specimens. Such novel technique might be a feasible solution for strengthening RC beams in shear when the vertical sides of RC beams are not accessible for conventional shear strengthening.

1.3. Research objectives

The main objectives of this study are to:

1. Investigate experimentally the effect of external longitudinal CFRP reinforcement on the shear strength of RC beams.
2. Study the effect of combined internal steel and external CFRP longitudinal reinforcement ratios on the shear strength RC beams.
3. Study the shear strength contribution of flexural CFRP sheets or plates when attached to the beams tensile surface (soffit) using epoxy adhesives.
4. Investigate the modes of failure of the strengthened specimens.
5. Predict the shear strength of the strengthened RC beams using the shear design provisions of the ACI 318-11 and CSA (2004) design codes of practice.
6. Predict the shear strength of the strengthened specimens using published shear strength models based on the neutral axis depth.
1.4. History of Shear Design

Shear failure of RC beams is a complex phenomenon and is affected by numerous variables at the same time. Over the past century, plenty of concrete shear strength equations and analytical models were developed based on experimental results to capture the influence of these variables. In 1935, Hardy Cross [15] stated that there is no credibility of experimental data unless it is supported by an adequate theory. A number of empirical equations were developed over the time, but equations mostly lack adequate theories behind them. Hooke’s in 1678 developed a plane section theory [16], and it is commonly used to calculate the flexure strength of reinforced concrete (RC) members. There was no such theory for shear strength of RC beams. Therefore, for the last five decades, researchers were attempting to develop a comparable theory on shear behavior of RC beams. The lower bound theory [2] and theory of variable angle truss [2] were developed for RC beams with significant amount of transverse reinforcement. These theories were incorporated in different design codes; however, these codes used empirical procedures for beams without transverse reinforcement. In the 1950s, Whitney and Hognestad [2] developed an ultimate shear design method that gained more attention due to shear failure in Air force warehouse. Researchers diverted their attention towards shear design after the failure of the air force warehouse. During 1960s and 1970s, the ACI-ASCE committee 326 [17] improved the shear design provisions based on tests conducted on a large number of RC beams.

Over the years, the American Concrete Institute (ACI) shear design provisions became more complex due to the number of equations used in the design. It was noticed that the number of equations used for shear design in the ACI code was increasing over the years [17]. Until 1963, the ACI318 code had only four equations for shear design; whereas, ACI 318-95 had more than 40 equations for shear analysis and design [17]. All these equations were based on certain experimental data available at that time.

Over the last five decades, various types of research investigations had been conducted on RC members without transverse reinforcement. Mostly, RC members were subjected to four point bending tests. In the 1960s, Kani et al. [18] conducted several tests on slabs without shear reinforcement and developed the term “Size effect” in shear. In the
1970s, Fenwick and Paulay [19] discovered that the greater percentage of shear force is carried by aggregate interlock. In the 1980s, the modified compression field theory (MCFT) was developed and incorporated in the Canadian CSA code. Numerous other theories and models were also developed over the years and some selected ones are summarized below.

1.4.1. Truss Analogy

Ritter [20] presented a 45 degrees truss model to calculate the shear strength in RC beams. Several building codes around the world are based on this model; furthermore, the authors recommended the use of a truss to establish the distribution of forces in a cracked beam. The model assumes that the behavior of RC beams after cracking becomes similar to that of a truss, as shown in Figure 1. Moreover, it assumes that the beam developed tension forces in the bottom flange and compression forces in the top flange; whereas, the concrete between the inclined cracks is in compression and the stirrups are in tension.

This model ignores the contribution of tensile stresses in cracked concrete and, therefore, eliminates the need for diagonal tension members. The imaginary truss model was created with an upper and lower longitudinal chords, where the upper and lower chords represent the compression and tension zones, respectively. The diagonal members in an imaginary truss represent the concrete between the cracks, and the vertical members represent the ties or stirrups. This model assumes that shear force is resisted by stirrups only, and the diagonal cracks would occur at an angle of 45 degrees. Based on this
model, Equation 1 was developed to compute the contribution of the shear reinforcement to the shear strength of RC beams

\[ V_s = \frac{A_v f_y d \cot \theta}{s} \]  

(1)

Where \( A_v \) is the area of shear reinforcement in mm\(^2\), \( f_y \) is the yield strength of the stirrups in MPa, \( d \) is the depth of the cross-section in mm\(^2\), \( s \) is the spacing between the stirrups in mm and \( \theta \) is the angle of inclination of diagonal compressive stresses.

Numerous experimental and analytical research investigations were done to validate the accuracy of Equation (1). It was found that stress in stirrup was less than stress calculated from Equation (1) [21]. Experimental results showed a difference in stress; therefore, plenty of empirical modifications to Ritter [20] truss model have been proposed. Researchers proposed the empirical concrete shear strength contribution term \((V_c)\) to account for the difference between the stresses [21]. Various expressions have been developed over the years to calculate the concrete contribution \((V_c)\) on the shear strength of RC beams, but till today, it is an empirical equation based on experimental data without being supported by an adequate theory. Further research investigations were conducted to find the angle of inclination of diagonal compressive stresses. Results showed that the angle of inclination of diagonal cracks can vary from 25 to 65 degrees [21]. Ritter [20] 45 degrees truss model is still retained in the ACI code for shear design; however, for torsional design, the angle of inclination could be taken as low as 30 degrees [22].

1.4.2. Modified Compression Field Theory

During 1970s and 1980s, extensive analytical and experimental research has been conducted to understand the shear behavior of reinforced concrete beams [22]. Researchers mainly focused their attention to provide an adequate theory for the shear behavior of RC beams. The compression field theory (CFT) was developed in 1978 by Mitchell and Collins [22] that provided a more rational approach based on formulation in terms of compatibility, stress-strain relationships and equilibrium of forces. CFT used stain condition in the web to determine the angle of inclination of diagonal compressive
stresses [22]. The Canadian Standards Association (CSA, 1984) included the CFT approach for shear design of RC beams [22]. However, the CFT approach did not take into account the tensile stresses in concrete which led to the formation of the modified compression field theory (MCFT) in 1986 by Bentz et al. [23].

It took several years to develop an adequate theory because all the experimental results were based on three or four point bending tests, and it was fairly challenging to form a theoretical model by incorporating these results. The membrane element tester was a very innovative testing machine used to test RC elements in pure shear or shear combined with axial stresses [2]. These experiments were difficult to perform, but the results were easy to interpret. Fifteen equations were used [23] in the MCFT based on stress-strain, geometric condition and equilibrium, as shown in Figure 2.

\[ \text{Figure 2: Equations used in Modified Compression Field Theory (MCFT) [23]} \]

Membrane elements were taken as part of RC structure of the same thickness. These elements contained grid of reinforcement in the x and z directions shown in Figure
Longitudinal and vertical reinforcements have stresses $f_x$ and $f_z$, coinciding by the angle $\theta$. The MCFT shows that diagonal cracked concrete has tensile stresses $f_1$ associated with the tensile strain $\varepsilon_1$ and compressive stresses $f_2$ associated with tensile strain $\varepsilon_2$. To understand the relationship between the diagonal compressive stresses $f_2$ and strain $\varepsilon_2$, Bentz et al. [23] tested thirty RC elements in an innovative testing machine. They found that the diagonal cracked concrete was weaker and softer in compression than the same concrete in a standard cylinder test. In addition, they found that the diagonal compressive stress was not only a function of the compressive strain $\varepsilon_2$, but also a function of the tensile strain $\varepsilon_1$. Equations 13 and 14 in Figure 2 show the compressive and tensile stress-strain relationships. They show that the diagonal compressive and tensile stress decrease with the increase in the tensile strain. Moreover, these relations indicate that even after diagonal cracking occurs, tensile stresses still exist in the concrete between the cracks, and these cracks increase the ability to resist shear [21]. A major assumption of the MCFT is that in cracked concrete, the average direction of compressive stress is related to the direction of compressive strain, and the diagonal cracks are inclined in this direction [2]. In order to understand the relationships between different compressive and tensile stress-strain in cracked concrete, there is a need to understand the transmission of stresses across the cracks and the mechanism of shear resistance.

Beams failing in shear usually have more longitudinal reinforcement $\rho_x$ in $x$-direction as compared to transverse reinforcement $\rho_z$ in the $z$-direction. Beams with minimum or without transverse reinforcement depend on the ability of a crack to transmit shear. Shear failures occur due to diagonal cracks, and cracking usually occurs between the cement and aggregate paste. Equation 15 in Figure 2 shows the shear stress on a crack, and it depends on three factors. These factors are spacing between the cracks, aggregate interlock and compressive strength of concrete. The width of the diagonal crack is related to the tensile strain $\varepsilon_1$ and crack spacing parameter $s_0$ shown in Equation 9 of Figure 2. The equations shown in Figure 2 are very complicated to solve by hand; therefore, simplified equations and a procedure were needed for the shear design of RC beams. In 1996, Bentz et al. [23] created the Simplified Modified Compression Field Theory (SMCT). This theory assumes that shear stress remains constant over the depth of
the beam. Equations used in the SMCT provided more accurate results with less complication as compared to MCFT [23]; this will be discussed in the following section.

1.5. Shear Strength Models

1.5.1. American Concrete Institute (ACI 318-11)

In a simply supported RC beam, there are some sections which have a large bending moment or small shear force and other sections that have a large shear force or small bending moments. Usually, large bending moments occur at midspan, and large shear forces occur near the supports. In case of a large shear force or small bending moment, there will be few flexural cracks corresponding to an average shear stress value. Diagonal cracks occur when the diagonal tensile stresses in the vicinity of the neutral axis exceed the tensile strength of concrete. However, the ultimate shear strength varies between $0.29 \sqrt{f'_c}$ and $0.4 \sqrt{f'_c}$ [24], where $f'_c$ is the concrete compressive strength in MPa. After numerous tests were conducted to study the shear and diagonal tension of RC beams, it was found that in regions of large shear or small moments, diagonal cracks initiated at an average shear force $V_c$ of

$$V_c = 0.29 \sqrt{f'_c b_w d}$$ (2)

Where $f'_c$ is the compressive strength of the concrete in MPa, $b_w$ is the width of the concrete section in mm, and $d$ is the depth of the section in mm. In regions of large bending moments or small shear, flexural cracks are formed. At a later loading stage, some diagonal cracks develop because the diagonal tensile stresses at the upper end of such cracks exceed the tensile strength of concrete [24]. In case of large bending moments, the nominal shear force $V_c$, at which diagonal tension cracks would develop, is given as

$$V_c = 0.16 \sqrt{f'_c b_w d}$$ (3)

It is apparent from Equations (2) and (3) that Equation (3) is more than half of Equation (2), which means that the large bending moment reduces the shear stress where cracking occurs [24]. The following Equation has been suggested by the ACI 318-11 [25]
guidelines to predict the nominal shear strength of RC beams at which diagonal crack is expected to initiate.

\[ V_c = [0.16\sqrt{f'_c} + 17\rho_w\frac{V_u}{M_u}]b_w d \leq 0.29\sqrt{f'_c} b_w d \]  
(ACI Eq. 11-5)  

Where \( f'_c \) is the compressive strength of the concrete in MPa, \( b_w \) is the width of the concrete section in mm, \( \rho_w \) is the longitudinal flexural reinforcement ratio, \( V_u \) is the ultimate shear force in N (newton), \( M_u \) is the ultimate moment in N-mm and \( d \) is the depth of the section in mm. It is clear from Equation (4) that if \( M_u \) is large, the second term of Equation (4) becomes small and the shear strength approaches \( 0.16 \sqrt{f'_c} \). If \( M_u \) value is small, the second term of Equation (4) becomes large and the upper limit of \( 0.29 \sqrt{f'_c} \) controls. As an alternate to the above Equation, the ACI 318-11 [25] provisions also permit engineers to use the following simplified formula to predict the concrete shear strength contribution of RC beams.

\[ V_c = 0.17\sqrt{f'_c} b_w d \]  
(ACI Equation 11-3)  

(5a)

Where \( f'_c \) is the compressive strength of the concrete in MPa, \( b_w \) is the width of the concrete section in mm, and \( d \) is the depth of the section in mm. In the British system of units, Equation (5a) is presented as:

\[ V_c = 2\sqrt{f'_c} b_w d \]  
(ACI Equation 11-3)  

(5b)

Where \( f'_c \) is the compressive strength of the concrete in psi, \( b_w \) is the width of the concrete section in inches, and \( d \) is the depth of the section in inches.

1.5.2. Canadian Standard Association (CSA,2004)

Shear design provision in the Canadian code, CSA 2004 [26], is based on the simplified modified compression field theory (SMCFT). Shear strength of concrete \( V_c \) depends on the \( \beta \) and \( \theta \) variables. These factors in results depend on the strain \( \varepsilon_x \) at the mid depth of the section [26]. Aggregate interlock that governs the crack width is also
related to the longitudinal strain $\varepsilon_x$. Equation (6) presents the concrete contribution, $V_c$, on the shear strength of RC beams.

$$V_c = \beta \sqrt{f'_c b_w d_v}$$  \hspace{1cm} (CSA Equation 11-6)  \hspace{1cm} (6)

Where

$\beta =$ factor for the contribution of the tensile stresses in cracked concrete.

$f'_c =$ Compressive strength of concrete in MPa.

$b_w =$ Width of the cross-section in mm.

$d_v =$ Shear depth taken as the greater of 0.9$d$ or 0.72 $h$ in mm.

The ability of the crack to transmit shear depends on crack width, aggregate interlock and concrete compressive strength. Equations (7) and (8) show the contribution of these parameters. The first term in Equation (7) models the strain effect, and the second term models the aggregate size effect. Equation (7) is used to calculate the tensile stress factor $\beta$ that accounts for the longitudinal strain at mid-section and the equivalent crack spacing parameter [26].

$$\beta = \frac{0.40}{(1 + 1500\varepsilon_x)} \frac{1300}{(1000 + s_{ze})}$$  \hspace{1cm} (CSA Equation 11-11)  \hspace{1cm} (7)

where

$\varepsilon_x =$ Longitudinal strain in the web (mm/mm)

$s_{ze} =$ Equivalent crack spacing parameter in mm

To account for size effect and crack spacing, Equation (8) was developed to account for the maximum aggregate size.

$$s_{ze} = \frac{35s_s}{15 + a_g}$$  \hspace{1cm} (CSA Equation 11-10)  \hspace{1cm} (8)

where

$s_s =$ Crack spacing parameter in mm.
\( a_s = \text{Maximum aggregate size in mm.} \)

The angle of inclination of diagonal compressive stresses also depends on the axial strain in the web, as shown in Equation (9). Higher values of \( \theta \) lead to higher tensile stresses; consequently, the beam will fail at a lower shear stress.

\[
\theta = 29 + 7000\varepsilon_s \quad \text{(CSA Equation 11-12)}
\]

\( \varepsilon_s = \frac{M_f / d_v + V_f}{2(E_sA_s)} \quad \text{(CSA Equation 11-13)} \]

where

\( M_f \) = Moment at a particular section in N.mm.

\( V_f \) = Ultimate shear force calculated at a distance \( d_v \) in N.

\( E_s \) = Modulus of elasticity of steel in MPa.

\( A_s \) = Area of steel on tension side in \( \text{mm}^2 \)

1.5.3. Shear strength of concrete based on neutral axis depth model

Tureyen and Frosch [27] investigated the effect of FRP bars on nine large scale RC beam specimens without transverse reinforcement. Three different types of FRP reinforcement, including two types of glass (GFRP) bars, aramid (AFRP) bars, and two types of steel reinforcement with varying yield strengths, respectively, were used in the experimental program. All the tested beam specimens were simply supported by longitudinal reinforcement ratios varying between 0.36% to 2%, respectively. Experimental results showed that specimens reinforced with tensile reinforcement of equal axial stiffness exhibited similar shear strengths in terms of the load-carrying capacity. Additionally, the results indicated that ACI 318-11 shear design provisions resulted in unconservative computation of shear strength; whereas, the equation based on
neutral axis depth resulted in very conservative shear strength estimates. Therefore, it was concluded from this research investigation that the ACI 318-11 [25] shear design provisions should be re-evaluated for RC beams with a reinforcement ratio less than 1%.

Frosch [14] also investigated the contribution of concrete in shear resistance and presented a model to compute the concrete shear strength, $V_c$ of RC, beams based on the location of the neutral axis depth upon the initiation of the shear crack. There are several factors that affect the shear strength of concrete, but one of the most important factors is the flexural reinforcement ratio, $\rho_{\text{eff}}$. As the reinforcement ratio increases as shown in Figure 3, the shear strength of concrete in RC beams also increases [14]. This occurs due to the increase in the neutral axis depth as the longitudinal flexural reinforcement ratio increases. Accordingly, more concrete is available above the neutral axis to resist the tensile forces that lead to an increase in the concrete shear strength.

![Figure 3: Shear strength of RC beams [14]](image)

It is clear from Figure 3 that for RC specimens with low flexural reinforcement ratio, the coefficient 2 on the y-axis of Figure 3, which is equivalent to SI coefficient of 0.17 in the shear strength equation of the ACI 318-11 code (Equation 5, ACI Equation 11-3) may become unconservative [14].
Reassessment of shear strength provided by concrete has been conducted by Frosch [14], and a new model was developed. The proposed model assumes that the uncracked concrete above the neutral axis is the primary contributor to the shear strength of concrete, as shown in Figure 4 for a section taken at a crack or between the cracks, respectively.

![Shear stress distribution at a crack and between the cracks](image)

Figure 4: Shear stress distribution at a crack and between the cracks [14]

Considering this model and the distribution of shear stress at a crack, the following shear design strength expression Equation (11) was proposed by Frosch [14].

\[
V_{cr} = 5\sqrt{f'_c b_w c}
\]  

\[(11a)\]

Where \(f'_c\) is the compressive strength of concrete in psi, \(b_w\) is the width of the concrete section in inches, and \(c\) is the neutral axis depth of the section in inches. In the SI system of units, Equation (11a) is presented as:

\[
V_{cr} = \frac{2}{5} \sqrt{f'_c b_w c}
\]  

\[(11b)\]

Where \(f'_c\) is the compressive strength of concrete in MPa, \(b_w\) is the width of the concrete section in mm, and \(c\) is the neutral axis depth of the section in mm.
There are plenty of benefits of using Equation (11). Firstly, it is consistent with the assumption used in flexural theory that the concrete below the neutral axis is cracked, and it will not contribute to shear resistance [14]. Secondly, it is a very conservative expression because it provides a low bound of the shear strength for a wide range of longitudinal reinforcement ratios as shown in Figure 5.

![Figure 5: Proposed model results [14]](image)

It is apparent from Figure 5 that the expression reduces the variability and scattering as compared to ACI 318-11 expression [14]. This expression is also useful for low reinforcement ratio and mostly for ratios in the range between 1 and 1.5%, respectively as shown in Figure 5.

Frosch [28] investigated the effect of size on the shear strength of RC beams with minimum shear reinforcement. Concrete shear strength decreases as the depth of the beam increases, and this trend is known as the size effect. In this research, two large scale concrete beams with minimum shear reinforcement were tested with a/d ratio of 3. Barros and Dias [29] pointed out that this type of a/d ratio provides lower bound estimates on the shear strength of RC beams. Experimental results showed that the beam size did not affect the capability of the transverse reinforcement to provide shear resistance.
Moreover, it was concluded from this research that the beam size did not affect the post cracking behavior and shear strength of the tested specimens.

Tureyen and Frosch [30] presented the proposed shear design strength model (Equation 11) and its application to beams reinforced with both steel and FRP bars, respectively. The proposed model (Equation 11) was tested by predicting the experimental results of 370 specimens from the open literature, and it showed conservative results over a wide range of variables affecting the shear strength. The proposed equation is applicable to both types of reinforcement (FRP or steel) since it accounts for the elastic modulus of the flexural reinforcement that affects the location of the neutral axis depth of the cracked section. It was concluded that the proposed model (Equation 11) is conservative for large set of data but reduces the variability of the predicted results of the tested data in the open literature.
Chapter 2: Literature Review and Shear Strength behavior of RC Beams

2.1. Literature Review on Shear Strengthening of RC Beams

Composite fiber-reinforced polymer (FRP) materials received great attention over the last few years in strengthening reinforced concrete (RC) beams in flexure and shear. This is mainly due to its various distinctive characteristics including its light weight, high to weight ratio, ease of application and resistance to corrosion. Several experimental and numerical research investigations had been conducted over the last two decades on shear strengthening of RC beams using FRP laminates. This section summarizes selected studies related to external shear strengthening RC beams using different FRP composite materials and techniques.

Uji [31] conducted an experimental study on eight RC shear deficient beams externally strengthened using CFRP laminates. Two different types of wrapping schemes, including fully wrapped or bonded to the vertical sides of the strengthened beam specimens, were investigated. Experimental results showed that the application of CFRP composite laminates substantially improved the load-carrying capacity of the strengthened specimens. Furthermore, the experimental results indicated that the shear force carried by the CFRP laminate is a function of the bond area with the adjacent concrete surface.

Sulaimani et al. [32] conducted an experimental study on sixteen RC beam specimens deficient in shear and strengthened by fiberglass plate bonding (FGPB). Prior to strengthening, the beams were damaged till the appearance of the first shear crack and then repaired using different techniques. The main objective of the study was to check the effectiveness of different repairing schemes, such as U-wraps in the shear span, FGPB strips and continuous FGPB plates (FGPB shear wings) bonded to the sides of the beam’s web. Experimental results showed that all shear repairing schemes increased the load-carrying capacity and stiffness of the strengthened beam specimens. The U-Wrap technique showed more increase in shear capacity as compared to other repairing schemes and was sufficient to cause flexural failure of such beams. However, the
specimens that were strengthened with FGPB strips and shear wings showed a similar increase in the load-carrying capacity and failed in shear.

Triantafillou [33] investigated the effect of CFRP composite strips attached to the vertical sides of eleven shear deficient RC beams. The beams were loaded in four-point bending and failed due to brittle tensile cracks and diagonal cracking. Debonding of the CFRP strips from the concrete substrate was also observed at the diagonal crack at the onset of failure. The strengthened beam specimens showed an increase in the shear capacity in the range from 65 to 95%, respectively, over the control unstrengthened beam specimens.

Khalifa el al. [34] conducted an experimental investigation on nine full scale continuous RC beams with two spans strengthened in shear with externally bonded CFRP composite sheets. The investigated variables in this study were the percentage of shear reinforcement, amount of CFRP sheets and wrapping schemes. Experimental results showed that the contribution of CFRP sheets in shear strengthening was significant, and the increase in the load-carrying capacity was in the range from 22 to 135 %, respectively, over the control unstrengthened beam. Test results also showed that the contribution of CFRP shear reinforcement was more significant for the strengthened beams without internal stirrups compared to those with internal shear reinforcement.

Taljsten [35] investigated the effect of CFRP laminates on strengthening shear deficient RC beams. Seven shear deficient RC beams were tested to investigate the effect of CFRP when attached to the sides of the beam at 0, 90 and 45 degrees, respectively, measured from the longitudinal beam’s axis. Test results showed an increase in the shear capacity of the strengthened specimens in the range from 98 to 169%, respectively, over the control specimen. Experimental results in this research also showed that the orientation of the CFRP sheets has a significant effect on the load-carrying capacity of RC beams. Shear crack is usually formed at an angle of 45 degrees; therefore, test results indicated that the beam specimen that was strengthened with CFRP composite sheets at 45 degrees proved to be more effective compared to the other wrapping schemes.
Diagana et al. [36] tested ten RC beams deficient in shear and strengthened externally with carbon fiber fabrics (CFF). The main purpose of the study was to investigate the effect of CFF and wrapping scheme on the shear strength of RC beams. The ten beams consisted of two control beams and eight beams strengthened with CFF strips. The eight strengthened beams were divided into two groups based on the U-shape and closed ring shape strip schemes. In each group, the specimens were strengthened with CFF strips in the form of U-shape, closed ring, vertical strips and inclined strips at 45° from the longitudinal axis of the member with different spacing. Experimental results showed that there was a gain in the ultimate load-carrying capacity as the spacing between the strips reduced. Furthermore, the results pointed out that CFF strips in the form of closed ring were more effective as compared to the U-wrap strengthening scheme. The results also indicated that CFF strips inclined at 45° in the form of U-wrap showed more shear contribution compared to the other strengthening schemes because the strips were not subjected to a twisting force in the compressive region of the tested beam.

Sim et al. [37] conducted an investigation on ten RC shear deficient beams strengthened with externally bonded carbon plates (CFRP), glass sheets (GFRP) and carbon sheets (CFS). The contribution of GFRP and CFS composite sheets on the shear capacity of the tested specimens when bonded to the beam’s web or full wrapped. In addition, the CFRP strips were bended to the beams’ web at 45 and 90 degrees, respectively from the longitudinal axis of the beam specimen. The main objective of this research was to study the effect of the orientation when different types of composite materials were bonded to the beams. Test results showed that the shear capacity of all strengthened specimens increased by almost 54% over the control specimens. CFS material orientation at 45 degree angle showed higher increase (73%) in capacity as compared to other two materials. Fully wrapped beam with CFS also showed an increase by about 27% in the load carrying capacity.

Barros and Dias [38] studied the effect of near surface mounted (NSM) and externally bonded reinforcement (EBR) on a four groups of shear deficient RC beams
with different depths and longitudinal reinforcement ratios. Each group of beam specimens contained one beam without any shear reinforcement and the remaining beams were reinforced with different types of shear reinforcement, such as steel stirrups, CFRP strips and CFRP laminates. Shear reinforcement was attached to the tension side and on the lateral faces of the beams using the NSM and EBR techniques. Experimental results showed that NSM strengthening technique was the most effective. The strengthened beam specimens with EBR and NSM showed an increase of 54% to 83% compared to the control beam specimen, respectively. Moreover, the test results pointed out that failure of beam strengthened by the NSM technique was less brittle as compared to that of the EBR technique.

Jayaprakash et al. [39] conducted an experimental study on the shear strengthening capacity and failure modes of rectangular RC beams bonded externally with bi-directional CFRP composites. A total of sixteen beams without shear reinforcement had been tested. Six specimens were precracked and repaired with CFRP strips. The CFRP strips act like shear reinforcement similar to internal steel stirrups. Six other specimens strengthened initially without preloading or precracking, and the remaining four specimens served as unstrengthened control beam specimens. The experimental results showed that the overall increase in the load-carrying capacity of the CFRP strengthened beam specimens varied between 11% and 139% over the control beams. The results also showed that the beams strengthened with CFRP strips increased the shear strength of precracked or initially strengthened beams, and also controlled the debonding of the strip from the adjacent concrete surfaces. This study showed that the bi-directional CFRP strips are more economical than the uni-directional strips. In addition, it also indicated that the shear capacity of the strengthened beam specimens is affected by the amount of longitudinal tensile reinforcement ratio. The shear strength of the strengthened beam specimens was increased by about 76% when the longitudinal tensile reinforcement ratio increased by 56%. The study also showed that the spacing between CFRP strips affects the shear capacity of the precracked or initially strengthened beam specimens.
Abu- Obeidah et al. [40] and Abdalla et al. [41] carried out an experimental study on two shear deficient beams strengthened with externally bonded aluminum plates, in addition to a control unstrengthened beam specimen. No transverse reinforcement was provided in the shear span of the specimens. The first specimen was strengthened with structural aluminum plates bonded to the vertical sides of the beam’s web with a spacing of 130 mm, while the second specimen was strengthened with two aluminum plates bonded on the sides at an angle of 10 degrees from the longitudinal axis of the member. Both strengthened specimens showed an increase in the load carrying capacity of 23.6 and 80.4%, respectively over the control specimen. The researchers also developed a finite element model that was capable of capturing the response of the tested specimens with high accuracy. It can be concluded from this research that structural aluminum plates could be used as valid external strengthening materials, and the orientation of such plates has a major effect on the load carrying capacity of shear deficient beams.

From the literature search, the effect of external longitudinal FRP reinforcement together with the equivalent longitudinal reinforcement ratio computed based on the modular ratio of the CFRP and steel reinforcement on the shear capacity of shear deficient beams have not been investigated. This study represents experimental results on the contribution of flexural CFRP composite plates or sheets on the shear strength of shear deficient RC beams. The steel and external longitudinal CFRP reinforcement ratios will be also varied to investigate their effect on the shear strength of RC beams.

2.2. Shear Strength Behavior of RC Beams

It is highly vital to design RC beams for shear and flexure, but the shear failure behavior in RC beams is somewhat different as compared to flexural failure. Shear failures in RC beams are more catastrophic as compared to failures in bending since they occur suddenly in a brittle mode and; thus, require a larger design factor of safety. RC beams usually fail in flexure before their shear strength is reached, because the tensile strength of concrete is less than their shearing strength [24]. A number of researches had been done in the past century on the shear strength of concrete, but the explanations and the variability of test results were ambiguous. However, very few researchers have been able to determine the resistance of concrete to pure shearing stress. In order to determine
the contribution of concrete in shear strengthening, the shear transfer mechanism for cracked and uncracked sections of RC beams should be studied.

Several types of shear cracks developed in RC beams, such as web-shear cracks and flexural-shear cracks. These types of shear cracks are usually inclined in nature. In addition to shear cracks, diagonal flexural tension cracks usually develop in loaded RC beams. These types of diagonal cracks start from the bottom (tension side) of the beam and travel upward towards the neutral axis.

There are three major factors [42] that contribute to the shear resistance of RC beams, as listed below and shown in Figure 6. Such factors will be discussed in the following subsections.

1. Shear resistance of uncracked concrete ($V_c$)
2. Interlocking action of aggregates ($V_a$).
3. Dowel Action of steel reinforcement ($V_d$)

![Figure 6: Shear transfer mechanism of RC beams](image)

**2.2.1. Shear resistance of uncracked concrete**

In RC beams, as the load starts increasing, flexural cracks start to develop and certain amount of shear is carried by the concrete in the compression zone [42]. However, as soon as the first crack develops according to flexural theory, the concrete below the neutral axis does not contribute to shear resistance [14]. The uncracked compression zone above the neutral axis will contribute to the shear resistance of concrete. The position of neutral axis after flexural cracking in beams is mainly dependent on the elastic modulus of concrete and longitudinal reinforcement ratio. However, the shear carried by the
uncracked compression zone can be represented by the compressive strength of concrete [42] since the elastic modulus of concrete is a function of its compressive strength.

2.2.2. Interlocking action of aggregate

A large portion of the shear force is carried across the cracks by aggregate interlock at the initiation of shear cracks. The width of the cracks and concrete compressive strength among other variables contribute to this mechanism [42]. The crack width becomes smaller at failure, as the longitudinal reinforcement increases. It is very obvious that the interlocking force increases as the compressive strength of concrete increases. The size of the aggregate also affects the interlocking action of the aggregates.

2.2.3. Dowel action

The resistance of the longitudinal steel reinforcement to frictional forces is usually called dowel action [24]. When the shear displacement occurs along the cracks, the shear is transferred by means of dowel action of the longitudinal bars [42]. There are various factors that contribute to the dowel action; for instance, the spacing of longitudinal bars, the flexural rigidity of longitudinal bars and the strength of surrounding concrete [42].

2.3. Shear Failure Modes

2.3.1. Diagonal tension failure

This type of shear failure usually occurs when the shear span to depth ratio (a/d) is between 2.5 and 6 [43]. A diagonal crack usually occurs as an extension of flexural cracks and will propagate towards the beam’s compression zone. Beam failure occurs as a result of the crack in the top compression zone, and splitting of concrete would also occur in the compression zone as shown in Figure 7.
2.3.2. Shear tension failure

Shear tension failure is very similar to diagonal tension failure. In this type of failure, the crack travels along the longitudinal reinforcement and causes a loss of bond between the reinforcement and concrete [43]. Therefore, the beam will also fail as a result of the splitting of the concrete in the compression zone as shown in Figure 8.

Figure 7: Diagonal tension failure

Figure 8: Shear tension failure
2.3.3. Effect of a/d on modes of failure

The shear mode of failure depends on the a/d ratio. Figure 9 shows the effect of the a/d ratio on the mode of failure.

![Diagram showing the effect of a/d ratio on modes of failure](image)

Figure 9: Effect of the a/d ratio on mode of failure [42]

It is obvious from Figure 9 that the failure moment and mode depend on the a/d ratio. For a/d ratio values greater than 3, the inclined cracking load exceeds the shear compression load [42]. This leads to the formation of an inclined crack and results in the instability or failure of the beam. This type of failure is usually called “Diagonal Tension Failure”. For a/d values less than 3, the shear compression load exceeds the inclined cracking load; however, failure may occur by concrete crushing at the top compression face. This type of failure is usually called “Shear Compression Failure”. The shear transfer mechanism of RC beams is also affected by the a/d ratio. For slender beams with a/d values greater than 3, the shear force is carried by the uncracked concrete above the neutral axis, interlocking of aggregate and dowel action of the longitudinal reinforcement. However, for short beams with a/d values less than 3, the shear force is mainly resisted by arch action.
Chapter 3: Experimental Program

3.1. Test Specimens

Nineteen reinforced concrete beam specimens were designed, constructed, and tested. The specimens were divided into three groups. The difference in the groups is in the amount of internal longitudinal steel reinforcement. All beams were designed to ensure shear failure. All beams had a nominal width of 120 mm, a nominal length of 1840 mm, a nominal height of 240 mm and a shear span to depth ratio of 3.06. Beams were tested under four points bending. The designation of the beams is as follows: B1 stands for beams in group 1 which are reinforced with $2\Phi 12$ bars, B2 stands for beams in group 2 which are reinforced with $2\Phi 16$ bars, and UB stands for beams in group 3 which are unreinforced with steel bars. The letters S or P indicate whether the beam is strengthened with CFRP sheets or plates, respectively. The last numeral indicates the number of layers of CFRP sheets or plates.

3.1.1. Group one

This group contains seven beams reinforced with $2\Phi 12$ bars on the tension side. One beam, shown in Figure 10, is the control beam and designated as "B1". All other six beams were strengthened with different numbers of layers of CFRP sheets and plates attached between the supports as shown in Figure 11. CFRP sheets were attached to the full width of the beam (120mm); whereas, CFRP plates used were only 100 mm wide.

3.1.2. Group two

This group contains six beams reinforced with $2\Phi 16$ bars on the tension side. One beam, shown in Figure 12, is the control beam and is designated as "B2". All other five beams were strengthened with different numbers of layers of CFRP sheets and plates attached between the supports as shown in Figure 13. CFRP sheets were attached to the full width of the beam (120mm); whereas, CFRP plates used were only 100 mm wide.
**Figure 10**: Group one control specimen

**Figure 11**: Group one specimens details
Figure 12: Group two control specimen

Figure 13: Group two specimens details
3.1.3. Group three

This group contains six beams. One beam, shown in Figure 14, is the control beam and is designated as "UBS2". Since the beams in this group do not have steel flexural reinforcement, it was deemed necessary that the control specimen be strengthened with CFRP sheets in order to prevent a premature flexural failure. All other five beams were strengthened with different layers of CFRP sheets and plates attached between the supports. CFRP sheets were attached to the full width of the beam (120mm); whereas, CFRP plates used were only 100mm wide.

![Group three specimens details](image)

Figure 14: Group three specimens details

3.2. Materials

3.2.1. Concrete

Ready-mix concrete supplied by a local concrete company was used for all specimens. Concrete used has a specified 28-day cylindrical compressive strength of 20
MPa, and all specimens cast in the same batch. Ten concrete cylinders (100 by 200 mm) and 10 cubes (100x100x100mm) were cast on site simultaneously with all beam specimens and cured alongside the specimens. Two cylinders and one cube were tested during 28 days. Test setup for cube and cylinder crushing is shown in Figure 16. A typical mode of failure for cubes and cylinders is shown in Figures 15 and 17, respectively.

Figure 15: Failure shape of cube

Figure 16: Test setup
3.2.2. Steel bars

In this study, three representative reinforcing steel specimens were tested under tension to evaluate the stress-strain characteristics of the steel bars used. The diameter of the bars tested is 11.83 mm. The total length of the specimen tested is 300 mm with 100 mm gauge length. The bars were tested at a rate of 10 mm/min. Table 1 summarizes the mechanical properties of the reinforcing bars. The stress-strain response for the steel bars is shown in Figure 18.

Table 1: Steel bar properties

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$F_y$ (N/mm$^2$)</th>
<th>$E$ (GPa)</th>
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<tbody>
<tr>
<td>#1</td>
<td>588.5</td>
<td>199.9</td>
</tr>
<tr>
<td>#2</td>
<td>587.4</td>
<td>199.9</td>
</tr>
<tr>
<td>#3</td>
<td>595.1</td>
<td>200.4</td>
</tr>
<tr>
<td>Average</td>
<td><strong>590.3</strong></td>
<td><strong>199.9</strong></td>
</tr>
</tbody>
</table>

Figure 17: Failure shape of cylinder
3.2.3. Epoxy material

3.2.3.1. Sheets

Numerous studies show that the stress in FRP sheets or plates is transferred to reinforced concrete beam via adhesive. The bond behavior between CFRP and reinforced concrete beams is greatly affected by the strengthening technique, which depends upon the performance of the epoxy resin used. Several types of epoxy are commercially available with different mechanical and chemical properties. Usually, epoxy is a two part component liquid that is composed of resin and hardener. In this research, Sikadur-330 epoxy is used for bonding CFRP sheets to reinforced concrete beams. It is an adhesive and a two-part-component liquid that has a mixing ratio of 1:3. The two components are divided into Parts A and B, and they are mixed together until a light grey color emerges. As soon as the light grey color emerges, the adhesive must be used within 45 minutes, which is the time needed to dry it. The advantage of using epoxy is that no primer is needed, easy to mix, and it is suitable for dried concrete surfaces.
3.2.3.2. Plates

For the beams strengthened with FRP plates, an epoxy adhesive is used. In this study, Adesilex PG1 and PG2 are used for bonding the FRP plates to the soffit of the beams. This epoxy consists of two components, hardener and resin, which should be mixed with proportions of 1:3. The primer is a liquid applied on the dry concrete surface, before the epoxy adhesive is applied, in order to cover the voids on the concrete surface. The hardener and the resin should be mixed together until a gray color emerges, and epoxy should be used within an hour.

Figure 19: Primer

3.3. CFRP sheets and plates properties

Sheets and plates were bonded externally to the reinforced concrete beams using epoxy adhesive (Sikadur330 and Adesilex PG 1&2). A layer of epoxy adhesive was applied to the concrete surface before the bonding of CFRP sheet and plate. Epoxy was also placed on the voids in order to have an efficient bond between the concrete surface and the CFRP sheet and plate. Sheets and plates were placed well on the epoxy; however, another layer of epoxy was applied after the bonding of the plate.

The mechanical properties of the sheets and plates used in this study, as reported by the manufacturers, are shown in Table 2. Figure 20 shows the CFRP sheet and plate used in this study.
### Table 2: Mechanical Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness (mm)</th>
<th>Modulus of elasticity [GPa]</th>
<th>Ultimate tensile strength [MPa]</th>
<th>Elongation at failure [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carboplate</td>
<td>1.4</td>
<td>170</td>
<td>3100</td>
<td>2.00</td>
</tr>
<tr>
<td>SikaWrap®300 C</td>
<td>0.17</td>
<td>230</td>
<td>3900</td>
<td>1.5</td>
</tr>
<tr>
<td>Sikadur®-330</td>
<td>-</td>
<td>4.5</td>
<td>30</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Figure 20: CFRP sheet and plate

### 3.4. Test setup and instrumentation

All beams had a total span length of 1690 mm. They were tested under a four-point bending using Instron Universal Testing Machine (UTM). Rollers were used as supports at both ends. The load was applied to the beam using a hydraulic actuator with a capacity of 2000 kN, as shown in Figure 21. The loading rate applied on the beam was 2mm/min. Beam deflection was measured at mid span. In addition, six strain gages (three gages: top and bottom) with 5 mm length made by KYOWA were used per specimen in
order to measure the strain in the concrete and CFRP sheets and plates. Capability of the strain measurement of KYOWA strain gage was 5%. The strain gages locations are shown in Figure 22. Load, deflection and strain readings were continuously recorded during the test. Crack formations were also marked on the beams throughout the test.

Figure 21: Test Setup
Figure 22: Location of strain gauges in all specimens
3.5. Test Matrix

Table 3 shows the compressive strength, area of reinforcement, type of reinforcement (CFRP and Steel), modulus of elasticity, thickness of CFRP sheets & plates and number of layers of tested specimens of the specimens.

Table 3: Test matrix

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen Designation</th>
<th>Actual width (b)</th>
<th>$f_c$ at 28 days</th>
<th>$f_c$ at day of test</th>
<th>Number of Days</th>
<th>Area of Steel Reinforcement (A&lt;sub&gt;s&lt;/sub&gt;)</th>
<th>Type of CFRP reinforcement</th>
<th>Thickness (t)</th>
<th>Number of layers (n)</th>
<th>Width of layer (w&lt;sub&gt;f&lt;/sub&gt;)</th>
<th>$E_f$*</th>
<th>$f_{fu}$*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B1</td>
<td>120</td>
<td>19.4</td>
<td>19.0</td>
<td>38</td>
<td>219.6</td>
<td>Sheet</td>
<td>0.17</td>
<td>2</td>
<td>120</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B1S2</td>
<td>125</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>219.6</td>
<td>Sheet</td>
<td>0.17</td>
<td>3</td>
<td>120</td>
<td>230</td>
<td>3900</td>
</tr>
<tr>
<td></td>
<td>B1S3</td>
<td>125</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>219.6</td>
<td>Sheet</td>
<td>0.17</td>
<td>4</td>
<td>120</td>
<td>230</td>
<td>3900</td>
</tr>
<tr>
<td></td>
<td>B1S4</td>
<td>120</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>219.6</td>
<td>Sheet</td>
<td>0.17</td>
<td>5</td>
<td>120</td>
<td>230</td>
<td>3900</td>
</tr>
<tr>
<td></td>
<td>B1S5</td>
<td>122</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>219.6</td>
<td>Sheet</td>
<td>0.17</td>
<td>1</td>
<td>100</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B1P1</td>
<td>120</td>
<td>19.4</td>
<td>23.0</td>
<td>93</td>
<td>219.6</td>
<td>Plate</td>
<td>1.4</td>
<td>1</td>
<td>100</td>
<td>170</td>
<td>3100</td>
</tr>
<tr>
<td></td>
<td>B1P2</td>
<td>125</td>
<td>19.4</td>
<td>23.0</td>
<td>93</td>
<td>219.6</td>
<td>Plate</td>
<td>1.4</td>
<td>2</td>
<td>100</td>
<td>170</td>
<td>3100</td>
</tr>
<tr>
<td>2</td>
<td>B2</td>
<td>128</td>
<td>19.4</td>
<td>19.0</td>
<td>38</td>
<td>387.0</td>
<td>Sheet</td>
<td>0.17</td>
<td>2</td>
<td>120</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B2S2</td>
<td>126</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>387.0</td>
<td>Sheet</td>
<td>0.17</td>
<td>3</td>
<td>120</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B2S3</td>
<td>120</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>387.0</td>
<td>Sheet</td>
<td>0.17</td>
<td>4</td>
<td>120</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B2S4</td>
<td>128</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>387.0</td>
<td>Sheet</td>
<td>0.17</td>
<td>5</td>
<td>120</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B2P1</td>
<td>125</td>
<td>19.4</td>
<td>23.0</td>
<td>93</td>
<td>387.0</td>
<td>Plate</td>
<td>1.4</td>
<td>1</td>
<td>100</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B2P2</td>
<td>126</td>
<td>19.4</td>
<td>23.0</td>
<td>93</td>
<td>387.0</td>
<td>Plate</td>
<td>1.4</td>
<td>2</td>
<td>100</td>
<td>170</td>
<td>3100</td>
</tr>
<tr>
<td>3</td>
<td>UBS2</td>
<td>120</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>-</td>
<td>Sheet</td>
<td>0.17</td>
<td>2</td>
<td>120</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UBS3</td>
<td>120</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>-</td>
<td>Sheet</td>
<td>0.17</td>
<td>3</td>
<td>120</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UBS4</td>
<td>120</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>-</td>
<td>Sheet</td>
<td>0.17</td>
<td>4</td>
<td>120</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UBS5</td>
<td>120</td>
<td>19.4</td>
<td>21.0</td>
<td>52</td>
<td>-</td>
<td>Sheet</td>
<td>0.17</td>
<td>5</td>
<td>120</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UBP1</td>
<td>120</td>
<td>19.4</td>
<td>23.0</td>
<td>93</td>
<td>-</td>
<td>Plate</td>
<td>1.4</td>
<td>1</td>
<td>100</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UBP2</td>
<td>120</td>
<td>19.4</td>
<td>23.0</td>
<td>93</td>
<td>-</td>
<td>Plate</td>
<td>1.4</td>
<td>2</td>
<td>100</td>
<td>170</td>
<td>3100</td>
</tr>
</tbody>
</table>

* As reported by the manufacturer
Chapter 4: Results and Discussion

This chapter presents the test results of the experimental program carried out in this study. Load-deflection curves along with modes of failure and strain gages readings are also presented.

4.1. Overall specimen behavior

4.1.1. Load-deflection relationships

4.1.1.1. Group one

Table 4 presents a summary of the test results of group one specimens. It also presents the shear strength attained at first shear crack, deflection corresponding to first shear crack and the gain in shear capacity due to the application of the CFRP sheets and plates. Figures 23 and 24 show the load versus deflection of group one beams strengthened with different layers of CFRP sheets and plates, respectively. The load carrying capacity of all strengthened beams increased over the control specimen (B1), as shown in Figures 23 and 24. The beam strengthened with five layers of CFRP sheets (B1S5) and two layers of CFRP plates (B1P2) attained 70% and 76% increase over the control specimen (B1), respectively. In addition, beams strengthened with two, three and four layers of CFRP sheets attained 49%, 61%, and 66% increase over the control beam, respectively. Strengthened beam with one layer of CFRP plate (B1P1) attained 55% increase over the control specimen. The beam strengthened with two layers of CFRP sheets had the maximum deflection as compared to other strengthened specimens. Figures 23 and 24 also show the load at the formation of the first shear crack, and it is represented as a circle on the load-deflection curves. In most specimens, first shear crack formed at the ultimate load. The stiffness of the specimens increased as the number of layers of CFRP sheets and plates increased. Prior to cracking, all load-deflection curves were similar; however, after cracking, specimens had different degrees of stiffness depending on the number of CFRP layers. Figures 23 and 24 show that increasing the number of layers of the CFRP sheets and plates increased both the stiffness and the strength of the beams. Beam strengthened with five layers of CFRP sheets and two layers of CFRP plates have the maximum stiffness of almost 14(kN/mm). The beam
strengthened with five layers of CFRP sheets and the beam strengthened with two layers of CFRP plates are stiffer than other strengthened beams, as can be seen in Figures 23 and 24. All specimens did not exhibit any ductility, and typical shear failures were observed. Figures 23 and 24 also show that the shear strength of the strengthened specimens increased due to the externally bonded flexural CFRP sheets and plates. Moreover, it was observed that the percent increase in shear strength diminishes as the amount of CFRP flexural reinforcement increases. The highest deformation was observed in the beam strengthened with two sheets of CFRP (B1S2).

Table 4: Experimental results of group one specimens

<table>
<thead>
<tr>
<th>Specimen Designation</th>
<th>Shear Strength ( V_c ) (kN)</th>
<th>Load ( P_{exp} ) (kN)</th>
<th>Deflection (mm)</th>
<th>Stiffness (kN/mm)</th>
<th>Load Capacity percent increase over B1 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1(Control)</td>
<td>19.70</td>
<td>39.39</td>
<td>4.84</td>
<td>8.96</td>
<td>-</td>
</tr>
<tr>
<td>B1S2</td>
<td>29.39</td>
<td>58.78</td>
<td>7.45</td>
<td>9.81</td>
<td>49</td>
</tr>
<tr>
<td>B1S3</td>
<td>31.71</td>
<td>63.42</td>
<td>6.43</td>
<td>11.24</td>
<td>61</td>
</tr>
<tr>
<td>B1S4</td>
<td>32.78</td>
<td>65.56</td>
<td>5.54</td>
<td>12.36</td>
<td>66</td>
</tr>
<tr>
<td>B1S5</td>
<td>33.55</td>
<td>67.09</td>
<td>5.33</td>
<td>14.05</td>
<td>70</td>
</tr>
<tr>
<td>B1P1</td>
<td>30.45</td>
<td>60.90</td>
<td>6.78</td>
<td>11.02</td>
<td>55</td>
</tr>
<tr>
<td>B1P2</td>
<td>34.62</td>
<td>69.24</td>
<td>5.08</td>
<td>14.20</td>
<td>76</td>
</tr>
</tbody>
</table>
Figure 23: Load versus Deflection (Group one specimens strengthened with sheets)

Figure 24: Load versus Deflection (Group one specimens strengthened with plates)
4.1.1.2. Group two

Table 5 presents the summary of experimental results of group two specimens strengthened with sheets and plates. Furthermore, it presents the shear strength attained at first shear crack, deflection corresponding to first shear crack, stiffness of the beam and the gain in shear capacity due to CFRP sheets and plates. Figures 25 and 26 show the load-deflection curve of control and strengthened specimens in group two. Load-deflection curve in Figures 25 and 26 indicates that all specimens behave in a same way before cracking; however, the increase in load and cracking lead to change in stiffness. Beam strengthened with four layers of CFRP sheets and two layers of CFRP plates has the maximum stiffness due to higher reinforcement ratio as compared to other specimens. Figures 25 and 26 also show the load, represented as a circle on the load-deflection curve, at which the first shear crack formed. Load carrying capacity of all strengthened beams increased over the control specimen (B2). Beam strengthened with two and three layers of CFRP sheets shows 10% and 25% increase over the control beam. Strengthened beam with one layer of CFRP plate shows 30% increase over the control specimen. However, the beam strengthened with four layers of CFRP sheets and two layers of CFRP plates shows 31% increase over the benchmark specimen (B2) shown in Figures 25 and 26. Shear capacity or load capacity of all strengthened specimens increases due to the increase in the longitudinal reinforcement ratio. All specimens did not exhibit any ductility and typical shear failures were observed. It can be concluded from Figures 25 and 26 that as the reinforcement ratio increases, the ductility of the specimen decreases. Control beam (B2) shows higher ductility as compared to other specimens strengthened with sheets and plates. Experimental results show that increasing layers of CFRP sheets and plates seems quite effective in enhancing the stiffness. Moreover, Figures 25 and 26 indicate that the stiffness of the beam also increases with the reinforcement ratio; therefore, beams with highest reinforcement ratio, such as B2S4 and B2P2, are much stiffer as compared to other strengthened beams. Beam strengthened with four layers of CFRP sheets and two layers of CFRP plates has the maximum stiffness of 14.5 and 12.9 (kN/mm), respectively. It can also be concluded from Figures 25 and 26 that the flexural CFRP sheets and plates contributed to the shear capacity of RC beams. The higher the reinforcement ratio, the higher will be the shear capacity.
Table 5: Experimental results of group two specimens

<table>
<thead>
<tr>
<th>Specimen Designation</th>
<th>Shear Strength $V_c$ (kN)</th>
<th>Load $P_{exp}$ (kN)</th>
<th>Deflection (mm)</th>
<th>Stiffness (kN/mm)</th>
<th>Load Capacity percent increase over B2 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2(Control)</td>
<td>27.96</td>
<td>55.91</td>
<td>5.96</td>
<td>11.32</td>
<td>-</td>
</tr>
<tr>
<td>B2S2</td>
<td>30.84</td>
<td>61.67</td>
<td>5.81</td>
<td>11.40</td>
<td>10</td>
</tr>
<tr>
<td>B2S3</td>
<td>34.96</td>
<td>69.91</td>
<td>6.78</td>
<td>12.02</td>
<td>25</td>
</tr>
<tr>
<td>B2S4</td>
<td>36.61</td>
<td>73.21</td>
<td>5.28</td>
<td>14.49</td>
<td>31</td>
</tr>
<tr>
<td>B2P1</td>
<td>36.44</td>
<td>72.88</td>
<td>6.18</td>
<td>11.54</td>
<td>30</td>
</tr>
<tr>
<td>B2P2</td>
<td>36.76</td>
<td>73.52</td>
<td>5.54</td>
<td>12.86</td>
<td>31</td>
</tr>
</tbody>
</table>

Figure 25: Load versus Deflection (Group two specimens strengthened with sheets)
4.1.1.3. Group three

Table 6 presents a summary of experimental results of group three specimens strengthened with sheets and plates. It also shows the shear strength attained at first shear crack, deflection corresponding to first shear crack, stiffness of the beams and the gain in shear capacity due to CFRP sheets and plates. Load-deflection curve of unreinforced beam strengthened in shear using flexural CFRP sheets and plates is shown in Figures 27 and 28. These Figures show the load at which the first shear crack formed and that is represented as a circle on the load-deflection curve. It can be concluded from Figures 27 and 28 that the ultimate shear capacity of all strengthened specimens increases; however, specimens UBS5 and UBP2 proved to be more effective in increasing the load capacity. UBS5 and UBP2 show 65% and 151% increase over the control specimen (UBS2). Beam strengthened with three and four layers of CFRP sheets shows 59% and 60% increase over the control beam (UBS2). Strengthened beam with one layer of CFRP plate shows 94% increase over the control specimen. Figures 27 and 28 also show that none of the layers of CFRP sheets and plates are certainly effective in enhancing the stiffness of the
strengthened beams. Beams with the highest number of layers, such as UBS5 and UBP2, have the highest stiffness as compared to other specimens. Beam strengthened with four layers of CFRP sheets and two layers of CFRP plates has the maximum stiffness of almost 13 (kN/mm). All specimens did not exhibit any ductility, and typical shear failures were observed; however, UBS2 shows the highest ductility. Load-deflection curve also shows that unreinforced beam strengthened with flexural CFRP plates shows more contribution to the shear capacity as compared to beams strengthened with CFRP sheets.

Table 6: Experimental results of group three specimens

<table>
<thead>
<tr>
<th>Specimen Designation</th>
<th>Shear Strength $V_c$ (kN)</th>
<th>Load $P_{exp}$ (kN)</th>
<th>Deflection (mm)</th>
<th>Stiffness (kN/mm)</th>
<th>Load Capacity percent increase over UBS2 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UBS2</td>
<td>10.63</td>
<td>21.26</td>
<td>4.10</td>
<td>7.91</td>
<td>-</td>
</tr>
<tr>
<td>UBS3</td>
<td>16.85</td>
<td>33.70</td>
<td>5.54</td>
<td>10.13</td>
<td>59</td>
</tr>
<tr>
<td>UBS4</td>
<td>17.02</td>
<td>34.04</td>
<td>4.67</td>
<td>11.36</td>
<td>60</td>
</tr>
<tr>
<td>UBS5</td>
<td>17.50</td>
<td>35.01</td>
<td>3.45</td>
<td>13.07</td>
<td>65</td>
</tr>
<tr>
<td>UBP1</td>
<td>20.66</td>
<td>41.33</td>
<td>6.07</td>
<td>10.38</td>
<td>94</td>
</tr>
<tr>
<td>UBP2</td>
<td>26.64</td>
<td>53.29</td>
<td>5.45</td>
<td>13.06</td>
<td>151</td>
</tr>
</tbody>
</table>
Figure 27: Load versus Deflection (Group three specimens strengthened with sheets)

Figure 28: Load versus Deflection (Group three specimens strengthened with plates)
4.1.2. Observation of cracking, failure mode and strain gages results

This section presents the cracking behavior of all tested specimens in different groups. It also presents the different modes of shear failures observed during the test. Strain gages results at different locations along the beam length are also presented in this section.

4.1.2.1. Group one

The observed shear failure mechanism, cracking and shear strength are dependent on the tensile stress, which is a combination of flexural and shear stresses. A typical “diagonal tension” failure was observed with minor flexural cracks developing at mid-span for all specimens in group one as shown in Table 7. First, a flexural crack developed at mid-span where the bending moment is the highest. As the load increased, more flexural cracks started to develop away from the beams mid-span. These flexural cracks were vertical in direction, but as the load increased, they changed from flexural cracks to flexural shear cracks. These flexural-shear cracks were inclined at certain angle and propagated towards the loading point. The formation of these cracks resulted in a diagonal tension failure. Table 7 shows the typical mode of failure for the control specimen and for the beams strengthened with CFRP sheets and plates. The CFRP sheets and plates delayed the formation of the shear and flexural cracks. No sign of sheets or plates delamination was observed up to failure for all specimens in group one.

4.1.2.1.1. B1S2 strain results

Figure 29 shows the load versus strain curve for group one specimen strengthened with two layers of CFRP sheets, and Figure 29 shows the maximum strain in the CFRP (bottom) and concrete (top). The location of the strain gauges is already presented in chapter 3. It is shown in Figure 29 that strain in the CFRP sheets increases with the load and tends to decrease after reaching the ultimate load of almost 60 kN. Maximum \( \mu \)strain (micro strain) in the CFRP sheet and concrete reached up to 290 \( \mu \)strain and 100 \( \mu \)strain, respectively. No compression failure was observed in the concrete. Maximum strain in the CFRP sheet was less than the ultimate strain; therefore, no debonding or delamination
was observed during the test. Neutral axis depth is calculated using top and bottom strains, and was found to be 66.45 mm from the extreme compression fiber.

Table 7: Mode of failure of group one specimens

<table>
<thead>
<tr>
<th>Mode of failures</th>
<th>Control specimen B1</th>
<th>Strengthened specimen with sheets</th>
<th>Strengthened specimen with plates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode of failures</td>
<td>Control specimen B1</td>
<td>Strengthened specimen with sheets</td>
<td>Strengthened specimen with plates</td>
</tr>
</tbody>
</table>

Figure 29: Load versus Strain (Specimen B1S2)
4.1.2.1.2. B1S3 strain results

Figure 30 shows the load versus strain curves for specimen B1S3. Strain gauge 5 is mounted on the CFRP at the soffit of the beam, and strain gauge 8 is mounted on the concrete at the top of the beam. Both gauges were located near the supports of the beam. The maximum strain in the CFRP sheet and in the concrete reached approximately 1720 µstrain and 426 µstrain, respectively. At failure, the strain in the concrete at midspan was 370 µstrain. No compression failure was observed in the concrete. In addition, the maximum strain in the CFRP sheet was considerably less than the ultimate strain of 15000 µstrain; therefore, no debonding or delamination was observed during the test. Assuming that plane sections remain plane, the strain gauge data was further used to estimate the depth of the neutral axis at failure.

Figure 30: Load versus Strain (Specimen B1S3)
4.1.2.1.3. B1S5 strain results

Figure 31 shows the load versus strain curve, the graph showed the maximum strain in CFRP and concrete measured using strain gauges. It can be observed from Figure 31 that the strain in CFRP increases with the load, however, strain in the CFRP started to decrease after reaching an ultimate load of 65 kN. Figure 31 shows that the maximum strain in the CFRP and concrete reached up to 2166 µstrain and 714 µstrain, respectively. No compression failure was observed in the concrete. Maximum strain in CFRP sheet was less than the ultimate strain, therefore no debonding or delamination was observed during the test. Neutral axis depth is calculated using top and bottom strains and it was found to be 82.5 mm from the extreme compression fiber.

4.1.2.2. Group two

The failure mode of beams in group two is shown in Table 8. All the beams failed in shear due to one major diagonal tension crack. This crack formed from the loading point till the support and few horizontal cracks also developed near the support. Additionally, these horizontal cracks affect the bond between the reinforcement and the
surrounding concrete. Beams failed in shear with no sign of ductility and diagonal tension failure was observed. Moreover, flexural cracks developed at the mid-span in highest moment region, and these cracks were vertical in direction. Table 8 shows the typical mode of failure for control specimen and the beams strengthened with sheets and plates. Contribution of CFRP sheets and plates delayed the formation of the shear and flexural cracks.

Table 8: Mode of failure of group two specimens

<table>
<thead>
<tr>
<th>Mode of failures</th>
<th>Control specimen B2</th>
<th>Strengthened specimen with sheets</th>
<th>Strengthened specimen with plates</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.1.2.2.1. B2S4 strain results

Figure 32 displays the load versus the strain curve for specimen B2S4. It shows the maximum strain in CFRP and concrete measured using strain gauges. It can be observed from Figure 32 that the strain in CFRP increases when the load increases; however, strain in the CFRP started to decrease after reaching an ultimate load of 72 kN. Figure 32 shows that the maximum strain in the CFRP and concrete was 1612 μstrain and 542 μstrain, respectively. No compression failure was observed in the concrete. The maximum strain in CFRP sheet was less than the ultimate strain of 15000 μstrain;
therefore, no debonding or delamination was observed during the test. Neutral axis depth is calculated using top and bottom strains and assuming linear variation of strain.

![Figure 32: Load versus Strain (Specimen B2S4)](image)

4.1.2.3. Group three

It was observed that major flexural cracks developed at the mid-span as the control and strengthened specimens were loaded; however, these cracks were vertical in direction and propagating towards the neutral axis. When flexural cracks entered into the shear span, they changed their direction and became diagonal in nature. These diagonal cracks propagated towards the loading point compelling the beam to fail. As a result of all these cracks, the beam failed in shear due to diagonal tension crack shown in Table 9. All the specimens didn’t exhibit any sign of ductility and typical shear failure were observed. All specimens in this group had more flexural cracks at the mid-span as compared to other specimens in group 1 and 2. Table 9 also shows the typical mode of failure for the beam strengthened with sheets and plates.
Table 9: Mode of failure of group three specimens

<table>
<thead>
<tr>
<th>Mode of failures</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Strengthened specimen with sheets</td>
<td><img src="image1.jpg" alt="Image" /></td>
</tr>
<tr>
<td>Strengthened specimen with plates</td>
<td><img src="image2.jpg" alt="Image" /></td>
</tr>
</tbody>
</table>
Chapter 5: Analytical Predictions

5.1. Predicted shear strength

Concrete shear strength of RC beams is affected by different variables, such as compressive strength of concrete, longitudinal reinforcement ratio and effective depth of the member. In order to evaluate the concrete shear strength of beam strengthened with flexural CFRP plates and sheets, effective depth and longitudinal reinforcement ratios are incorporated in the ACI 318-11 and CSA (2004) shear design equations mentioned in Chapter 2. The method presented in Figure 33 shows the calculation for effective depth and longitudinal reinforcement ratios.

![Figure 33: Effective depth](image)

Considering the first moment of the areas about the effective centroid between the steel and CFRP reinforcement, the effective centroid is at distance $x$ away from the bottom of the beam’s cross section.

$$n_1 A_s (38 - x) = n_2 A_f (x)$$

(12)
where
\[ n_1 = \frac{E_s}{E_e}, n_2 = \frac{E_f}{E_e} \]

let,
\[ n_3 = \frac{E_f}{E_{st}} \]

\[ \Rightarrow \]
\[ \frac{E_s}{E_e} A_s (38 - x) = \frac{E_f}{E_e} n_2 A_f (x) \]
\[ E_s A_s (38 - x) = E_f A_f (x) \]

\[ \Rightarrow \]
\[ x = \frac{38}{1 + n_3 \frac{A_s}{A_s}} \]

and
\[ d_{eff} = h - x \]

The effective reinforcement ratio, \( \rho_{eff} \) is calculated as follows:
\[ \rho_{eff} = \rho_s + n_3 \rho_f \]

where
\[ \rho_s = \frac{A_s}{bd_{eff}} \]

and
\[ \rho_f = \frac{A_s}{bd_{eff}} \]

Table 10 shows the predicted shear strength of concrete based on ACI 318-11 design code (Equations 11-3 and 11-5), CSA design code (Equations 11-6) and the model suggested by Frosch [14]. It also indicates the flexure strength of tested specimens and all the flexure loads that are higher than the shear loads. It can be concluded from Table 10 that all beams failed in shear before reaching the ultimate flexure strength. Effective reinforcement ratio and depth were calculated based on the equations mentioned above, and these factors (effective depth and longitudinal reinforcement ratio) were incorporated in the shear design equations. Sample calculations, using ACI 318-11, CSA (2004) and the model suggested by Frosch [14] are shown in the Appendix A.
Table 10: Predicted Shear strength of concrete based on design codes

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen designation (ID)</th>
<th>Effective depth $d_{\text{eff}}$</th>
<th>Effective reinforcement ratio $\rho_{\text{eff}}$</th>
<th>Flexure strength $P_{\text{flexure}}$ (kN)</th>
<th>Shear strength $P_{\text{exp}}$ (kN)</th>
<th>ACI 318-11 Eq.(11-5) $V_c$</th>
<th>ACI 318-11 Eq.(11-3) $V_c$</th>
<th>CSA Eq.(11-6) $V_c$</th>
<th>Neutral axis depth $c$ (mm)</th>
<th>$\frac{2}{5}\sqrt{f_c b_w c}$ $V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>B1S2</td>
<td>209</td>
<td>1.03</td>
<td>94</td>
<td>59</td>
<td>29</td>
<td>21</td>
<td>21</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>B1S3</td>
<td></td>
<td>212</td>
<td>1.11</td>
<td>101</td>
<td>63</td>
<td>32</td>
<td>22</td>
<td>21</td>
<td>23</td>
<td>22</td>
</tr>
<tr>
<td>B1S4</td>
<td></td>
<td>213</td>
<td>1.22</td>
<td>103</td>
<td>66</td>
<td>33</td>
<td>21</td>
<td>20</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>B1S5</td>
<td></td>
<td>215</td>
<td>1.29</td>
<td>108</td>
<td>67</td>
<td>34</td>
<td>22</td>
<td>20</td>
<td>24</td>
<td>24</td>
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<tr>
<td>B1P1</td>
<td></td>
<td>215</td>
<td>1.31</td>
<td>98</td>
<td>61</td>
<td>30</td>
<td>22</td>
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<td>24</td>
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<td>B1P2</td>
<td></td>
<td>222</td>
<td>1.65</td>
<td>131</td>
<td>69</td>
<td>35</td>
<td>25</td>
<td>22</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>B2</td>
<td></td>
<td>202</td>
<td>1.50</td>
<td>116</td>
<td>56</td>
<td>28</td>
<td>20</td>
<td>19</td>
<td>23</td>
<td>23</td>
</tr>
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<td>B2S2</td>
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<td>31</td>
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<td>26</td>
<td>26</td>
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<td>B2S3</td>
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<td>1.71</td>
<td>115</td>
<td>70</td>
<td>35</td>
<td>23</td>
<td>21</td>
<td>27</td>
<td>24</td>
</tr>
<tr>
<td>B2S4</td>
<td></td>
<td>210</td>
<td>1.81</td>
<td>116</td>
<td>73</td>
<td>37</td>
<td>23</td>
<td>21</td>
<td>27</td>
<td>25</td>
</tr>
<tr>
<td>B2P1</td>
<td></td>
<td>211</td>
<td>1.92</td>
<td>126</td>
<td>73</td>
<td>36</td>
<td>24</td>
<td>21</td>
<td>28</td>
<td>26</td>
</tr>
<tr>
<td>B2P2</td>
<td></td>
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<td>137</td>
<td>74</td>
<td>37</td>
<td>25</td>
<td>22</td>
<td>30</td>
<td>29</td>
</tr>
<tr>
<td>UBS2</td>
<td></td>
<td>240</td>
<td>0.14</td>
<td>39</td>
<td>21</td>
<td>11</td>
<td>21</td>
<td>22</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>UBS3</td>
<td></td>
<td>240</td>
<td>0.21</td>
<td>57</td>
<td>34</td>
<td>17</td>
<td>22</td>
<td>22</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>UBS4</td>
<td></td>
<td>240</td>
<td>0.28</td>
<td>73</td>
<td>34</td>
<td>17</td>
<td>22</td>
<td>22</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>UBS5</td>
<td></td>
<td>240</td>
<td>0.35</td>
<td>80</td>
<td>35</td>
<td>18</td>
<td>22</td>
<td>22</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>UBP1</td>
<td></td>
<td>240</td>
<td>0.49</td>
<td>44</td>
<td>41</td>
<td>21</td>
<td>23</td>
<td>23</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>UBP2</td>
<td></td>
<td>240</td>
<td>0.97</td>
<td>77</td>
<td>53</td>
<td>27</td>
<td>24</td>
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<td>23</td>
<td>22</td>
</tr>
</tbody>
</table>
5.2. Experimental Analysis

5.2.1. Specimens strengthened with CFRP sheets

Tables 11, 12 and 13 present the ratio of experimentally measured shear strength to predicted shear strength evaluated based on different design codes, such as ACI 318-11 and CSA (2004) for specimens strengthened with CFRP sheets. Shear strength based on neutral axis depth is also computed to check the effectiveness of this parameter. Moreover, statistical analyses were conducted for each group and presented in tables 11, 12 and 13. Comparison of performance of these specimens for each of the three groups is given below.

5.2.1.1. Group 1S (sheets) specimens with moderate longitudinal reinforcement ratio

Table 11 presents the ratio of measured shear strength to predicted shear strength for specimens with moderate longitudinal reinforcement (2Φ12) and strengthened with multiple layers of CFRP sheets. It is observed from Table 12 that CSA provides the most accurate predictions compared to other methods. The mean of the ratio of CSA prediction is 1.33 with standard deviation of ±0.15. Prediction of ACI 318 detailed equation (ACI 318-11, Equations 11-5) shows slightly better predictions than ACI simplified equation (ACI 318-11, Equations 11-3). Shear strength prediction based on the model suggested by Frosch [14] is the least accurate and the most conservative among the presented methods.
Table 11: Comparison of shear strength of Group 1S specimens strengthened with CFRP sheets

<table>
<thead>
<tr>
<th>ID</th>
<th>ACI 318-11 Eq. (11-5)</th>
<th>ACI 318-11 Eq. (11-3)</th>
<th>CSA Eq. (11-6)</th>
<th>$\frac{2}{5} \sqrt{f_{c}b_{w}c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>1.08</td>
<td>1.11</td>
<td>1.06</td>
<td>1.48</td>
</tr>
<tr>
<td>B1S2</td>
<td>1.40</td>
<td>1.44</td>
<td>1.35</td>
<td>1.87</td>
</tr>
<tr>
<td>B1S3</td>
<td>1.48</td>
<td>1.54</td>
<td>1.39</td>
<td>1.93</td>
</tr>
<tr>
<td>B1S4</td>
<td>1.56</td>
<td>1.64</td>
<td>1.44</td>
<td>1.99</td>
</tr>
<tr>
<td>B1S5</td>
<td>1.55</td>
<td>1.64</td>
<td>1.41</td>
<td>1.94</td>
</tr>
<tr>
<td>Mean</td>
<td>1.41</td>
<td>1.47</td>
<td>1.33</td>
<td>1.84</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.20</td>
<td>0.22</td>
<td>0.15</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Figure 34 shows a graphical representation of the ratio of measured shear strength to predicted shear strength of Group 1S specimens strengthened with sheets. As indicated, both ACI 318 Equations (11-3 and 11-5) predicted have approximately the same values of shear strength for the specimens in Group 1S. Therefore, the use of the detailed ACI 318-11, Equation 11-5 that includes the longitudinal reinforcement ratio has no advantage over the use of the simplified ACI 318-11, Equation 11-3. It is also shown that the concrete contribution is underestimated by all design codes used in the analysis. Furthermore, shear strength of specimens based on neutral axis depth showed over conservative results with experimentally measured value almost twice (1.94) of the predicted one for specimen B1S5. Figure 34 also indicates the contribution of longitudinal reinforcement ratio to the shear capacity of the beam.
5.2.1.2. Group 2S (sheets) specimens with high longitudinal reinforcement ratio

Table 12 presents the ratio of measured shear strength to predicted shear strength for specimens with high longitudinal reinforcement (2Φ16) and strengthened with multiple layers of CFRP sheets. It is observed from Table 12 that CSA provides the most accurate predictions by far compared to other methods. The mean of the ratio of CSA prediction is 1.27 with very small standard deviation of ±0.08. Moreover, ACI 318 detailed Equation (ACI 318-11, Eq. 11-5) shows better predictions than ACI simplified Equation (ACI 318-11, Eq. 11-3). Additionally, shear strength prediction based on the model suggested by Frosch [14] is the least accurate and the most conservative among the presented methods.
Table 12: Comparison of shear strength of Group 2S specimens strengthened with CFRP sheets

<table>
<thead>
<tr>
<th>ID</th>
<th>(V_{\text{test}}/V_{\text{pred}})</th>
<th>(V_{\text{test}}/V_{\text{pred}})</th>
<th>(V_{\text{test}}/V_{\text{pred}})</th>
<th>(V_{\text{test}}/V_{\text{pred}})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ACI 318-11 Eq. (11-5)</td>
<td>ACI 318-11 Eq. (11-3)</td>
<td>CSA Eq. (11-6)</td>
<td>(2/5 \sqrt{f'c b_v^c})</td>
</tr>
<tr>
<td>B2</td>
<td>1.37</td>
<td>1.48</td>
<td>1.20</td>
<td>1.61</td>
</tr>
<tr>
<td>B2S2</td>
<td>1.40</td>
<td>1.52</td>
<td>1.21</td>
<td>1.63</td>
</tr>
<tr>
<td>B2S3</td>
<td>1.52</td>
<td>1.66</td>
<td>1.31</td>
<td>1.75</td>
</tr>
<tr>
<td>B2S4</td>
<td>1.59</td>
<td>1.75</td>
<td>1.35</td>
<td>1.81</td>
</tr>
<tr>
<td>Mean</td>
<td>1.47</td>
<td>1.60</td>
<td>1.27</td>
<td>1.70</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.10</td>
<td>0.13</td>
<td>0.08</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Figure 35 shows a graphical representation of the ratio of measured shear strength to predicted shear strength of Group 2S specimens strengthened with sheets. ACI 318-11 (Equations 11-3 and 11-5) shows different prediction ratios. The use of the detailed ACI 318-11, Equations 11-5, that includes the longitudinal reinforcement ratio, shows more accurate results compared to the simplified ACI 318-11, Equations 11-3. This is expected since specimens of Group 2S have high longitudinal reinforcement ratio and, therefore, its contribution in shear reinforcement cannot be ignored. In addition, it is shown that concrete contribution is underestimated by the model suggested by Frosch [14] which is based on neutral axis depth with the average of measured to predicted ratio of 1.7 and around 1.8 for specimen strengthened with four layers of CFRP sheets.
Figure 35: Test to predicted ratio of group 2S specimens strengthened with CFRP sheets

5.2.1.3. Group 3S (sheets) specimens with no longitudinal reinforcement

Table 13 presents the ratio of measured shear strength to predicted shear strength for specimens with no longitudinal reinforcement and strengthened with multiple layers of CFRP sheets. It is observed from Table 13 that CSA provides the most accurate predictions by far compared to other methods. The mean of the ratio of CSA prediction is 1.03 with a standard deviation of ±0.12. Due to low equivalent reinforcement ratio of the specimens of this group, both ACI 318 Equations (11-3 and 11-5) predicted approximately the same values of shear strength for each specimen in Group 3S. Shear strength prediction based on the model suggested by Frosch [14] is the most conservative among the presented methods as was the case with Groups 1S and 2S.
Table 13: Comparison of shear strength of Group 3S specimens strengthened with CFRP sheets

<table>
<thead>
<tr>
<th>ID</th>
<th>$V_{\text{test}}/V_{\text{pred}}$</th>
<th>$V_{\text{test}}/V_{\text{pred}}$</th>
<th>CSA Eq. (11-6)</th>
<th>$2/5 \sqrt{f_{\text{c}} b_{\text{n}} c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>UBS2</td>
<td>0.50</td>
<td>0.47</td>
<td>0.89</td>
<td>1.26</td>
</tr>
<tr>
<td>UBS3</td>
<td>0.78</td>
<td>0.75</td>
<td>1.18</td>
<td>1.67</td>
</tr>
<tr>
<td>UBS4</td>
<td>0.78</td>
<td>0.76</td>
<td>1.06</td>
<td>1.48</td>
</tr>
<tr>
<td>UBS5</td>
<td>0.80</td>
<td>0.78</td>
<td>1.00</td>
<td>1.38</td>
</tr>
<tr>
<td>Mean</td>
<td>0.71</td>
<td>0.69</td>
<td>1.03</td>
<td>1.45</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.15</td>
<td>0.15</td>
<td>0.12</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Figure 36 shows a graphical representation of the ratio of measured shear strength to predicted shear strength of Group 3S specimens strengthened with multiple layers of sheets. It is observed that both ACI 318 Equations (11-3 and 11-5) overestimated the shear strength as it does not take into account the size effect. It is also shown that ACI equations performed better with the higher reinforcement ratio. CSA showed conservative results for all strengthened specimen except for UBS2 which was overestimated by CSA. Similarly, it is also shown in Figure 36 that ACI 318-11 overestimated concrete contribution for all specimens. Equations based on neutral axis depth also showed conservative and promising results.
5.2.1.4. Summary of strengthened specimen with sheets

It is observed from Tables 11, 12 and 13 that CSA provides the most accurate predictions compared to experimental results for all the three groups of specimens strengthened with different layers of CFRP sheets. The mean of the ratio of CSA prediction is 1.33 with standard deviation of ±0.15 for Group 1S, 1.27 with standard deviation of ±0.08 for Group 2S and 1.03 with standard deviation of ±0.12 for Group 3S. It is clear that, among the three groups, the most accurate CSA predictions of shear strength are for Group 3 (i.e., for specimens without longitudinal reinforcements). The second best performer among the presented methods is ACI detailed Equation (ACI 318-11, Equations 11-5), followed by ACI simplified Equation (ACI-318, Equations 11-3). The model suggested by Frosch [14] which is based on the neutral axis depth is the most conservative. For beams with no longitudinal reinforcement (Group 3), ACI 318 equations (detailed and simplified) overestimated the shear strength of the beams for all
specimens, and their predicted values improved with the increase in the number of layers of CFRP sheets. On the other hand, the model suggested by Frosch showed conservative predictions.

Figure 37 presents a summary for all beam specimens of the different groups strengthened with different layers of CFRP sheets. It shows that CSA performs better as compared to other design codes because it takes into account the amount of longitudinal reinforcement ratio, depth and spacing between the cracks. ACI results become unconservative for members with low reinforcement ratio. Concrete contribution is also underestimated by all the design codes. Equation based on neutral axis depth shows promising and conservative results; however, for some of the specimens in Groups 1 and 2, it shows over conservative results.

Figure 37: Test to predicted ratio of all specimens strengthened with CFRP sheets
5.2.2. Specimens strengthened with CFRP plates

Tables 14, 15 and 16 present the ratio of experimentally measured shear strength to predicted shear strength evaluated based on different design codes, such as ACI 318-11 and CSA (2004) for specimens strengthened with CFRP plates. Shear strength based on neutral axis depth is also computed to check the effectiveness of this parameter. Statistical analyses were also conducted for each group and presented in Tables 14, 15 and 16. Comparison of performance of these specimens for each of the three groups is given below.

5.2.2.1. Group 1P (plate) specimens with moderate longitudinal reinforcement ratio

Table 14 presents the ratio of measured shear strength to predicted shear strength for specimens with moderate longitudinal reinforcement (2Φ12) and strengthened with one or two layers of CFRP plates. It is observed from Table 14 that CSA provides the most accurate predictions compared to other methods. The mean of the ratio of CSA prediction is 1.19 with standard deviation of ±0.11. Prediction of ACI 318 detailed Equation (ACI 318-11, Equations 11-5) is very close to CSA prediction and slightly better than ACI simplified Equation (ACI 318-11, Equations 11-3) prediction. Shear strength prediction according to the equation based on the neutral axis depth is the least accurate and the most conservative among the presented methods.

Figure 38 shows a graphical representation of the ratio of measured shear strength to predicted shear strength of Group 1P specimens strengthened with plates. As indicated, both ACI 318 Equations (11-3 and 11-5) predicted have approximately the same values of shear strength for the two specimens in Group 1P. Moreover, it is observed that, because the effective longitudinal reinforcement ratio is increasing, the concrete contribution to shear strength is underestimated by all design codes used in the analysis. Equation based on neutral axis depth shows over conservative results for specimens strengthened with plates. Shear strength of specimens based on neutral axis depth showed over conservative result with experimentally measured value almost twice (1.74) of the predicted one for specimen B1P1.
Table 14: Comparison of shear strength of Group 1P specimens strengthened with CFRP plates

<table>
<thead>
<tr>
<th>ID</th>
<th>ACI 318-11 Eq. (11-5)</th>
<th>ACI 318-11 Eq. (11-3)</th>
<th>CSA Eq. (11-6)</th>
<th>$\frac{2}{5} \sqrt{f_{c'} b w c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>1.08</td>
<td>1.11</td>
<td>1.06</td>
<td>1.48</td>
</tr>
<tr>
<td>B1P1</td>
<td>1.38</td>
<td>1.46</td>
<td>1.26</td>
<td>1.74</td>
</tr>
<tr>
<td>B1P2</td>
<td>1.42</td>
<td>1.55</td>
<td>1.23</td>
<td>1.68</td>
</tr>
<tr>
<td>Mean</td>
<td>1.29</td>
<td>1.37</td>
<td>1.19</td>
<td>1.63</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.18</td>
<td>0.23</td>
<td>0.11</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Figure 38: Test to predicted ratio of Group 1P specimens strengthened with CFRP plates
5.2.2.2. Group 2P (plate) specimens with moderate longitudinal reinforcement ratio

Table 15 presents the ratio of measured shear strength to predicted shear strength for specimens with high longitudinal reinforcement (2Φ16) and strengthened with multiple layers of CFRP plates. It is observed from Table 15 that CSA provides the most accurate predictions by far compared to other methods. The mean of the ratio of CSA prediction is 1.24 with very small standard deviation of ±0.06. ACI 318 detailed Equation (ACI 318-11, Equations 11-5) shows better predictions than ACI simplified Equation (ACI 318-11, Equations 11-3). Shear strength prediction based on the neutral axis depth equation is the least accurate and the most conservative among the presented methods.

Table 15: Comparison of shear strength of Group 2P specimens strengthened with CFRP plates

<table>
<thead>
<tr>
<th>ID</th>
<th>$\frac{V_{test}}{V_{pred}}$</th>
<th>ACI 318-11 Eq. (11-5)</th>
<th>ACI 318-11 Eq. (11-3)</th>
<th>CSA Eq. (11-6)</th>
<th>$\frac{2}{5} \sqrt{f'_c b_w c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2</td>
<td>1.37</td>
<td>1.48</td>
<td>1.20</td>
<td>1.61</td>
<td></td>
</tr>
<tr>
<td>B2P1</td>
<td>1.55</td>
<td>1.71</td>
<td>1.31</td>
<td>1.76</td>
<td></td>
</tr>
<tr>
<td>B2P2</td>
<td>1.46</td>
<td>1.67</td>
<td>1.21</td>
<td>1.60</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.46</td>
<td>1.62</td>
<td>1.24</td>
<td>1.66</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.09</td>
<td>0.13</td>
<td>0.06</td>
<td>0.09</td>
<td></td>
</tr>
</tbody>
</table>

Figure 39 shows a graphical representation of the ratio of measured shear strength to predicted shear strength of Group 2P specimens strengthened with Plates. ACI 318-11 Equations (11-3 and 11-5) show different prediction ratios. The use of the detailed ACI 318-11, Equations 11-5, that includes the longitudinal reinforcement ratio, shows more accurate results compared to the simplified ACI 318-11, Equations 11-3. This is expected since specimens of Group 2P have high longitudinal reinforcement ratio and; therefore, their contribution in shear reinforcement cannot be ignored. ACI equations show conservative result because both equations do not take into account size effect. It is also
shown that concrete contribution is underestimated by the equation which is based on the neutral axis depth with the average of measured to predicted ratio of 1.6 and around 1.76 for specimen strengthened with one layer of CFRP plate.

Figure 39: Test to predicted ratio of Group 2P specimens strengthened with CFRP plates

5.2.2.3. Group 3P (plate) specimens with no longitudinal reinforcement

Table 16 presents the ratio of measured shear strength to predicted shear strength for specimens with no longitudinal reinforcement and strengthened with one and two layers of CFRP plates. It is observed from Table 16 that CSA provides the most accurate predictions by far compared to other methods. The mean of the ratio of CSA prediction is 1.06 with a standard deviation of ±0.15. Due to low equivalent reinforcement ratio of the specimens of this group, both ACI 318 Equations (11-3 and 11-5) predicted approximately the same values of shear strength for each specimen in Group 3P. Shear strength prediction based on the equation of the neutral axis depth is the most conservative among the presented methods, as was the case with Groups 1P and 2P.
Table 16: Comparison of shear strength of Group 3P specimens strengthened with CFRP plates

<table>
<thead>
<tr>
<th>ID</th>
<th>$V_{test}/V_{pred}$</th>
<th>$V_{test}/V_{pred}$</th>
<th>CSA Eq.(11-6)</th>
<th>$\frac{2}{5}\sqrt{f_c b_w c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>UBS2</td>
<td>0.50</td>
<td>0.47</td>
<td>0.89</td>
<td>1.26</td>
</tr>
<tr>
<td>UBP1</td>
<td>0.90</td>
<td>0.89</td>
<td>1.14</td>
<td>1.56</td>
</tr>
<tr>
<td>UBP2</td>
<td>1.10</td>
<td>1.15</td>
<td>1.14</td>
<td>1.51</td>
</tr>
<tr>
<td>Mean</td>
<td>0.83</td>
<td>0.84</td>
<td>1.06</td>
<td>1.45</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.31</td>
<td>0.34</td>
<td>0.15</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Figure 40 shows a graphical representation of the ratio of measured shear strength to predicted shear strength of Group 3P specimens strengthened with one and two layers of plates. It is observed that both ACI 318 Equations (11-3 and 11-5) overestimated the shear strength because it does not take into account the size effect. It is also shown that ACI equations performed better with the higher equivalent reinforcement ratio. CSA showed accurate and conservative results for both specimens (UBP1 and UBP2), because it takes into account the effect of crack width, effect of size and the stress-strain at the crack. It is also shown in Figure 40 that ACI 318-11 overestimated concrete contribution for specimen with one plate (UBP1) and underestimated it for specimen with two plates (UBP2). The equation based on neutral axis depth also showed conservative results for both specimens.
5.2.2.4. Summary of strengthened specimens with plate

It is observed from Tables 14, 15 and 16 that CSA provides the most accurate predictions compared to experimental results for all the three groups of specimens strengthened with one and two layers of CFRP plates. The mean of the ratio of CSA prediction is 1.19 with standard deviation of ±0.11 for Group 1P, 1.24 with standard deviation of ±0.06 for Group 2P and 1.06 with standard deviation of ±0.15 for Group 3P. It is clear that, among the three groups, the most accurate CSA predictions of shear strength are for Groups 1P and 3P (i.e., for specimens with low or no longitudinal reinforcement ratio). The second best performer among the presented methods is ACI detailed Equation (ACI 318-11, Equations 11-5) followed by ACI simplified Equation (ACI-318, Eq. 11-3). The model suggested by Frosch [14], which is based on neutral axis depth, is the least accurate and the most conservative. For beams with no longitudinal reinforcement (Group 3P), ACI 318 equations (detailed and simplified) overestimated the shear strength of the beams for specimens with one plate and underestimated it for specimens with two plates. In other words, their predicted values improved with the increase in the number of layers of CFRP plates. Nonetheless, Frosch equation [14] showed conservative estimate all through.
Figure 41 shows a summary of the measured to predict ratios for all strengthened specimens in different groups based on various longitudinal reinforcement ratios. CSA yields the closest results as compared to other design codes because it takes into account large variety of parameters affecting the shear strength. ACI shows conservative results for all tested specimen except UBP1, which has a low reinforcement ratio. ACI equations performed better with higher longitudinal reinforcement ratio, because the crack width decreases as the longitudinal reinforcement increases. The equation based on neutral axis depth also shows satisfactory results; however, for some of the specimen, it shows over-conservative results.

Figure 41: Test to predicted ratio of all specimens strengthened with CFRP plates
5.2.3. Sheets and Plates

5.2.3.1. Normalized shear strength

Normalized shear strength of all the specimens strengthened with layers of CFRP sheets and plates in different groups is shown in Figures 42 and 43. Effective reinforcement ratio for all the specimens is shown in Table 10, and effective reinforcement concept was used to convert the area of steel and CFRP to an equivalent area based on the modular ratio. It also allows the comparison between the shear strength of reinforced concrete beam strengthened with different materials. Figures 42 and 43 indicate that the shear strength of reinforced concrete beam increases with the increase in the effective reinforcement ratio. Shear strength of strengthened beams in groups one and three increases at a faster rate for lower reinforcement ratio. Group two has higher reinforcement ratio; nonetheless, the increase in shear capacity is not considerably high as compared to other groups. Both Figures show that shear strength of reinforced concrete beam is a function of longitudinal reinforcement ratio. Shear strength increases more linearly for beams strengthened with plates as compared to others strengthened with sheets. Along the same lines, flexural plate shows more concrete contribution to the shear capacity as compared to sheets. Shear strength becomes stable as the reinforcement ratio reaches the maximum value as shown in Figures 42 and 43.
Figure 42: Normalized shear strength of specimen strengthened with CFRP sheets

Figure 43: Normalized shear strength of specimen strengthened with CFRP plates
5.2.3.2. Effective reinforcement ratio versus test to predict ratio

Measured shear strength to predict shear strength ratios for all specimens strengthened with sheets and plates are shown in Figures 44. CSA code proves to be more accurate but less conservative for all the tested specimens strengthened with different materials. However, ACI and Frosch equation show less accurate prediction and thus more conservative results. The ratio of measured shear strength to predicted shear strength increases as the reinforcement ratio increases for each group because shear strength is a function of longitudinal reinforcement ratio. All measured to predicted ratios for CSA (2004) range between 1.0 and 1.5; whereas, almost all measured to predicted ratios for Frosch equation are between 1.5 and 2.0. However, ACI 318-11 shows more dispersion, and its ratios are between 1.0 and 1.75. The equation which is based on the neutral axis depth shows over conservative results since it does not take into account the parameters, such as crack spacing and crack width.

![Graph showing test to predicted ratio versus effective reinforcement ratio](image)

Figure 44: Test to predicted ratio versus effective reinforcement ratio
5.2.3.3. CSA

CSA shows accurate predictions for a wide range of data except for the specimen UBS2, which is overestimated by CSA code. All shear strengths calculated using CSA code were below the experimental data, which shows the validity of this code, as shown in Figure 45. CSA proves to be more accurate than other codes because it takes into account the tensile stresses in cracked concrete, crack spacing and aggregate size. These parameters affect the shear strength of concrete; however, they are missing in all other design codes (ACI). Shear strength for reinforced concrete beam in CSA code is based on an adequate theory (MCFT); whereas, all other codes are based on empirical equations.

![Figure 45: V_{test}/V_{cal} ratio of all strengthened specimens with using CSA code](image)
5.2.3.3.1. Tensile stress factor versus angle of inclination

A negative relation is shown between the tensile stress factor ($\beta$) and the angle of inclination ($\theta$) for diagonal compressive stresses in Figures 46 and 47. Tensile stresses in the web increase with the angle of inclination; as a result, it increases the longitudinal strain in the web. Higher value of longitudinal strain leads to a lower value for shear stress. The higher the angle of inclination of diagonal compressive stresses, the lower the shear strength is. Longitudinal reinforcement ratio increases the tensile stress factor and; as a result, the angle of inclination decreases. Group 2 beams have higher reinforcement ratio; therefore, they have lower angle of inclination.

Figure 46: Tensile stress factor vs. angle for diagonal compressive stresses (Sheets)
5.2.3.3.2. Angle of inclination versus longitudinal strain

There is a linear positive relation between the angle of inclination ($\theta$) and longitudinal strain ($\varepsilon_x$) in the web, as shown in Figures 48 and 49. Strain in the web increases as the angle of inclination increases, because it increases the tensile strain in the diagonally cracked concrete; as a result, it reduces the compressive and tensile stresses in the cracked concrete. A lower value of tensile and compressive stresses leads to a higher value of angle of inclination and lower value of shear stress. Group 3 beams have lower reinforcement ratio; hence, they have higher angle of inclination.
Figure 48: Angle for diagonal compressive stresses vs. longitudinal strain (sheets)

Figure 49: Angle for diagonal compressive stresses vs. longitudinal strain (Plates)
5.2.3.3. **Tensile stress factor versus longitudinal strain**

Equations for the shear strength of reinforced concrete beam in CSA code are based on modified compression field theory. Shear strength in RC beams depends on the tensile stress factor ($\beta$) in the cracked concrete and the longitudinal strain ($\varepsilon_x$) in the web. There is a negative relation between tensile stress factor and longitudinal strain, and they are inversely proportional to each other, as shown in Figures 50 and 51. The higher the reinforcement ratio, the higher the tensile stress factor and the lower longitudinal strain are. As the longitudinal reinforcement ratio increases, tensile strain in the diagonally cracked concrete decreases due to small crack spacing and; as a result, it increases the tensile stress in the diagonally cracked concrete. Moreover, it helps the cracked concrete to transfer the shear stress between the cracks and it consequently increases the tensile stress factor. Shear strength of cracked beam increases as the tensile stress factor increases.

![Graph of Tensile Stress Factor versus Longitudinal Strain](image)

Figure 50: Tensile stress factor versus longitudinal strain in web (Sheets)
5.2.3.4. ACI 318-11

Figures 52 and 53 show that both ACI equations predicted almost the same shear strength, and they overestimated the shear strength of specimens with low reinforcement ratio. Shear strength is over-estimated by ACI because it does not take into account the size effect, axial stiffness of bars and tensile stress factor. ACI equations show better results for specimens strengthened with plates as compared to sheets. Additionally, these equations have higher standard deviation as compared to other design equations.
Figure 52: $V_{\text{test}}$ to $V_{\text{cal}}$ ratio of all strengthened specimens using ACI Eq.11-5

Figure 53: $V_{\text{test}}$ to $V_{\text{cal}}$ ratio of all strengthened specimens using ACI Eq. 11-3
5.2.3.5. Strain data

Table 17 presents a summary of experimental and predicted results for neutral axis depth. Neutral axis depth is shown graphically in Figure 54; it shows that predicted neutral axis depth is highly close to experimentally computed data. Based on the strain gauge data, neutral axis depth was computed experimentally. It can be concluded from Figure 54 that shear strength of concrete is also a function of neutral axis depth. Table 17 shows that as the longitudinal reinforcement ratio increases, the depth of the neutral axis also increases.

Table 17: Predicted and Experimental strain

<table>
<thead>
<tr>
<th>Specimen Designation (ID)</th>
<th>Predicted neutral axis depth c (mm)</th>
<th>Experimental neutral axis depth c (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1S2</td>
<td>68.65</td>
<td>66.45</td>
</tr>
<tr>
<td>B1S3</td>
<td>71.58</td>
<td>72.84</td>
</tr>
<tr>
<td>B1S5</td>
<td>77.28</td>
<td>82.5</td>
</tr>
<tr>
<td>B2S4</td>
<td>86.84</td>
<td>84.69</td>
</tr>
</tbody>
</table>

Figure 54: Experimental versus predicted neutral axis depth
5.2.3.6. **Comparative shear strength**

Numerous parameters are affecting the shear strength of the RC beams, at the same time, without transverse reinforcement. Parameters, such as effective reinforcement ratio, width of the beam, effective depth of the cross section and compressive strength of concrete, affect the shear strength. Shear strength of specimens is comparable based on the multiplication of different parameters shown in Table 18. Specimens with the same multiplication factor, such as B1P1 and B2, are comparable and their load deflection curve is shown in Figure 55. Both specimens failed at almost the same load, and their load deflection curves behaved in a very similar manner. In a similar manner, B2S3 and B1P2 have almost the same multiplication factor; therefore, their shear strengths are comparable. Load-deflection curve of both specimens is shown in Figure 56, and both specimens failed at a load of 70kN; however, the ultimate deflection was different for both specimens.

**Table 18:** Shear strength comparison

<table>
<thead>
<tr>
<th>Specimen Designation</th>
<th>$\rho_{eff}$ (%)</th>
<th>Width ($b$) (mm)</th>
<th>$d_{eff}$ (mm)</th>
<th>$f_c'$ (MPa)</th>
<th>$\rho_{eff}(b)(d_{eff})(\sqrt{f_c'})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1P1</td>
<td>1.31</td>
<td>120</td>
<td>215.35</td>
<td>22.57</td>
<td>1608.28</td>
</tr>
<tr>
<td>B2</td>
<td>1.5</td>
<td>128</td>
<td>202</td>
<td>18.59</td>
<td>1672.21</td>
</tr>
<tr>
<td>B2S3</td>
<td>1.71</td>
<td>130</td>
<td>208.25</td>
<td>21</td>
<td>2121.45</td>
</tr>
<tr>
<td>B1P2</td>
<td>1.65</td>
<td>125</td>
<td>221.76</td>
<td>22.57</td>
<td>2172.91</td>
</tr>
</tbody>
</table>
Figure 55: Load-deflection curve of B2 and B1P1

Figure 56: Load-deflection curve of B2S3 and B1P2


Chapter 6: Summary and Conclusion

The work presented in this study addressed the shear strengthening issue of RC beams using flexural CFRP sheets and CFRP plates bonded externally to the tensile surface (i.e., soffit) of the beam. Beams were divided into three groups based on the steel flexural reinforcement ratios. Each group has one control un-strengthened specimen while all other specimens were strengthened with varying amounts of CFRP sheets and CFRP plates. Each group has six specimens except group one which has seven specimens. Shear strength of RC beams is affected by numerous parameters including longitudinal reinforcement ratio, maximum aggregate size, concrete strength, size, and shear span to depth ratio (a/d). However, the main variable that was investigated in this study is the longitudinal reinforcement ratio. No transverse reinforcement was provided in all specimens in order to assess the contribution of concrete shear strength ($V_c$). Four point bending test was conducted on all specimens with shear span to depth ratio of 3.06. All beams failed in shear due to diagonal tension crack and load-deflection curve along with the strain gauges reading were recorded. Shear strength from experimental results were also compared with shear strength prediction from different design codes, such as ACI318-11, CSA (2004) and the model suggested by Frosch [14]. Based on the experimental results and analyses, the findings of this study can be summarized as follows:

1. All strengthened specimens in each group showed an increase in the shear capacity over the control specimens which supports the hypothesis that the shear capacity of RC beam is a function of CFRP flexural reinforcement.

2. The increase in shear strength over the control specimens was in the range of 49-76% for Group 1, 10-31% for Group 2, and 59-151% for Group 3.

3. Specimens with lower internal longitudinal reinforcement ratio shows higher increase in shear capacity when strengthened with CFRP sheets and plates as compared to specimens with higher internal longitudinal reinforcement. Thus shear strengthening RC beams using the presented technique is more effective for lightly reinforced beams.
4. As the number of layers of longitudinal CFRP sheets and plates increased, the concrete shear strength ($V_c$) of the specimens also increased in each group.

5. The percent increase in concrete shear strength ($V_c$) decreases as the number of layers of CFRP sheets and plates increases.

6. Diagonal shear cracks became steeper, and post cracking stiffness also increased as the amount of longitudinal reinforcement ratio increased. However, CFRP sheets and plates delayed the formation of flexural and shear cracks.

7. ACI 318-11 equations become unconservative for specimens with low reinforcement ratio because cracks width tend to decrease as the longitudinal reinforcement ratio increases. ACI 318-11 equations overestimated the shear capacity of the specimens in Group 3 (low reinforcement ratio).

8. CSA (2004) code which is based on the MCFT provided the most accurate estimates of shear strength for all specimens in each group as compared to other presented models. The mean ratio of the test to predicted shear strength for beams strengthened with CFRP sheets are $1.33, 1.27$ and $1.03$ with standard deviation of ±0.15, ±0.08, and ±0.12 in group one, two and three, respectively. The mean ratio of the test to predicted shear strength for beams strengthened with CFRP plates based on CSA (2004) code are $1.19$, $1.24$ and $1.06$ with standard deviation of ±0.11, ±0.06, and ±0.15 in group one, two and three, respectively.

9. The shear strength predicted by the equation which is based on the neutral axis depth is the most conservative among all presented models. This equation could be used as a lower bound estimate of concrete shear strength.
References


Appendix A

Control beam (B1)

Figure 57: Cross section detailing of control beam (B1)

Taking moment about neutral axis

\[(bx)(\frac{x}{2}) + (2n - 1)(A_y)(x - d) = (nA_y)(d - x)\]

\[(bx)(\frac{x}{2}) + (2n - 1)(A_y)(x - d^\prime) = (nA_y)(d - x)\]

\[n = \frac{E_s}{E_c}\]

\[n = \frac{E_s}{E_c}\]

\[E_c = 4700\sqrt{f_c}\]

\[E_c = 4700\sqrt{18.59}\]

\[E_c = 20264.57\text{Mpa}\]

\[n = \frac{20000}{20264.57}\]
\[ n = 9.87 \]
\[
(120x)\left(\frac{x}{2}\right) + (2(9.87) - 1)(2(49.6))(x - 37) = (9.87)(2(109.8))(202 - x)
\]
\[ 60x^2 + (18.74)(99.2)(x - 37) = (9.87)(219.6)(202 - x) \]
\[ 60x^2 + 1859x - 68783 = 437825.30 - 2167.45x \]
\[ 60x^2 + 1859x + 2167.45x - 68783 - 437825.30 = 0 \]
\[ 60x^2 + 4026.45x - 506608.3 = 0 \]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[ x = \frac{-4026.45 \pm \sqrt{4026.45^2 - 4(60)(-506608.3)}}{2(60)} \]
\[ x = \frac{-4026.45 \pm \sqrt{137798291.6}}{2(60)} \]
\[ x = \frac{-4026.45 \pm 11738.76}{2(60)} \]
\[ x = \frac{-4026.45 + 11738.76}{2(60)} \]
\[ x = \frac{-4026.45 + 11738.76}{2(60)} \]
\[ x = 64.26\text{mm} \]

**ACI 318-08 equations:**

\[ V_c = 0.17\sqrt{f_c b_w d_{eff}} \quad (\text{ACI 318-11, Equation 11-3}) \quad (5a) \]
\[ V_c = 0.17\sqrt{18.59(120)(202)} \]
\[ V_c = 0.17\sqrt{18.59(120)(202)} \]
\[ V_c = 17767.29N \]
\[ V_c = 17.77kN \]

\[ V_c = [0.16\sqrt{f_c} + 17\rho_{eff}\frac{V_{u d_{eff}}}{M_u}]b_w d_{eff} \leq 0.29\sqrt{f_c} b_w d_{eff} \quad (\text{ACI 318-11, Equation11-5}) \quad (4) \]
Section taken at d/2 away from the loading point

\[ \rho_{\text{eff}} = \rho_s + n_3 \rho_f \]

Control specimen therefore \( \rho_f \) will be zero.

\[ \rho_s = \frac{A_s}{bd_{\text{eff}}} \]

\[ \rho_s = \frac{2(109.8)}{(120)(202)} \]

\[ \rho_s = 0.91\% \]

\[ \rho_{\text{eff}} = 0.91 + 0 \]

\[ \rho_{\text{eff}} = 0.91 \]

\[ \frac{V_u d_{\text{eff}}}{M_u} = \frac{(P / 2)(202)}{(P / 2)(a - d_{\text{eff}}/2)} \]

\[ \frac{V_u d_{\text{eff}}}{M_u} = \frac{(P / 2)(202)}{(P / 2)(620 - 202/2)} \]

\[ \frac{V_u d_{\text{eff}}}{M_u} = 0.3892 \]

\[ V_c = \left[ 0.16 \sqrt{f_c} + 17 \rho_{\text{eff}} \frac{V_u d_{\text{eff}}}{M_u} b_w d_{\text{eff}} \right] \leq 0.29 \sqrt{f_c} b_w d_{\text{eff}} \]

\[ V_c = \left[ 0.16 \sqrt{18.59} + 17(0.0091) (0.3892) \right] (120)(202) \leq 0.29 \sqrt{18.59}(120)(202) \]

\[ V_c = 18.18 \leq 30.31 \]

\[ V_c = 18.18 \text{kN} \]

\[ V_c = \frac{2}{5} \sqrt{f_c} b_w c \]

\[ V_c = \frac{2}{5} \sqrt{18.59}(120)(64.26) \]
\[ V_c = 13.30\text{kN} \]

**CSA 2004:**

\[ V_c = \beta \sqrt{f_c b_w d_v} \]

\[ d_v = \text{greater}(0.9d, 0.72h) \]

\[ d_v = \text{greater}(0.9(202), 0.72(240)) \]

\[ d_v = \text{greater}(181.8, 172.8) \]

\[ d_v = 181.8\text{mm} \]

\[ \beta = \frac{0.40}{(1 + 1500\varepsilon_x)} \frac{1300}{(1000 + s_{ce})} \]

\[ s_{ce} = \frac{35s_z}{15 + a_g} \]

\[ a_g = 20\text{mm} \]

\[ s_c = d_v = 181.8\text{mm} \]

\[ s_{ce} = \frac{35s_z}{15 + a_g} = \frac{35(181.8)}{15 + 20} \]

\[ s_{ce} = 181.8\text{mm} \]

\[ \varepsilon_x = \frac{M_f}{d_v} + \frac{V_f}{2(E_y A_y)} \]

\[ M_f = V_c (a - \frac{d_v}{2}) \]

\[ M_f = V_c (620 - \frac{181.8}{2}) \]  \hspace{1cm} (19)

\[ \varepsilon_x = \frac{M_f}{181.8 + V_c} + \frac{V_c}{2(200000)(219.6)} \]

\[ \beta = \frac{0.40}{(1 + 1500\varepsilon_x)} \frac{1300}{(1000 + 181.8)} \]
\[ V_c = \beta \sqrt{18.59(120)(181.8)} \]  

Assume \( V_c \) in Equation 19 and find the final \( V_c \) by using Equation 6, subtract both of them. Try the iteration procedure until the difference between both \( V_c \) become zero.

Assume \( V_c = 18.50 \text{ kN or } 18509.87 \text{ N} \)

\[ M_f = (18509.87)(620 - \frac{181.8}{2}) \]

\[ M_f = 9793572.2 \text{ N mm} \]

\[ \varepsilon_x = \frac{9793572.2/181.8 + 18509.87}{2(200000)(219.6)} \]

\[ \varepsilon_x = 0.000824(\text{mm/mm}) \]

\[ \beta = \frac{0.40}{(1 + 1500 \times 0.000824)} \frac{1300}{(1000 + 181.8)} \]

\[ \beta = 0.196 \]

\[ V_c = 0.196\sqrt{18.59(120)(181.8)} \]

\[ V_c = 18436.17 \text{ N} \]

\[ V_c = 18.50 \text{ kN} \]
Strengthened beam with sheets (B1S2)

Figure 58: Cross section detailing of strengthened beam (B1S2)

Taking moment about neutral axis

\[(bx)\left(\frac{x}{2}\right) + (2n_1 - 1)(A_i)(x - d') = (n_1A_i)(d - x) + (n_2A_i)(h - x)\]

\[n_1 = \frac{E_s}{E_c}\]

\[E_c = 4700\sqrt{f_c}\]

\[E_c = 4700\sqrt{21}\]

\[E_c = 21538.10\text{MPa}\]

\[n_1 = \frac{200000}{21538.10}\]

\[n_1 = 9.286\]
\[ n_2 = \frac{E_f}{E_c} \]

\[ n_2 = \frac{230000}{21538.10} \]

\[ n_2 = 10.67 \]

\[ (125x)(\frac{x}{2}) + (2(9.286) - 1)(2(49.6))(x - 37) = (9.286)(2(109.8))(202 - x) + (10.67)(0.17)(125)(2)(240 - x) \]

\[ 62.5x^2 + (17.57)(99.2)(x - 37) = (9.286)(219.6)(202 - x) + 453.47(240 - x) \]

\[ 62.5x^2 + 1742.94x - 64488.93 = 411919.53 - 2039.21x + 108832.8 - 453.47x \]

\[ 62.5x^2 + 1742.94x + 2039.21x + 453.47x - 64488.93 - 411919.53 - 108832.8 = 0 \]

\[ 62.5x^2 + 4235.62x - 585241.26 = 0 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-4235.62 \pm \sqrt{4235.62^2 - 4(62.5)(-585241.26)}}{2(62.5)} \]

\[ x = \frac{-4235.62 \pm \sqrt{164250791.8}}{2(62.5)} \]

\[ x = \frac{-4235.62 \pm 12816.03}{125} \]

\[ x = \frac{-4235.62 + 12816.03}{125} \]

\[ x = 68.65 mm \]
Figure 59: Effective depth

Take the moment about the centroid

\[ nA_s (38 - x) - nA_f (x) = 0 \]

\[ 9.286(219.6)(38 - x) - 10.67(0.17)(125)(2)(x) = 0 \]

\[ 77489.81 - 2039.20x - 453.47x = 0 \]
\[ 77489.81 = 2039.20x + 453.47x = 0 \]
\[ \frac{77489.81}{2039.20 + 453.47} = x \]
\[ x = 31.08\text{mm} \]
\[ d_{\text{eff}} = h - x \]
\[ d_{\text{eff}} = 240 - 31.08 \]
\[ d_{\text{eff}} = 208.91 \]

\[ V_c = 0.17\sqrt{f_c b_v d_{\text{eff}}} \]  
\[ (\text{ACI 318-11, Equation 11-3}) \]  \[ (5a) \]

\[ V_c = 0.17\sqrt{21(125)(208.91)} \]

\[ V_c = 0.17\sqrt{21(125)(208.91)} \]

\[ V_c = 20343.60\text{N} \]
\[ V_c = 20.34kN \]

\[ V_c = 0.16\sqrt{f_{c'}} + 17\rho_{eff} \frac{V_{ud}d_{eff}}{M_u} \leq 0.29\sqrt{f_{c'}} b_u d_{eff} \quad \text{(ACI 318-11, Equation 11-5)} \]  

Section taken at \( d/2 \) away from the loading point

\[ \rho_{eff} = \rho_s + n_3\rho_f \]

\[ n_3 = \frac{E_f}{E_s} \]

\[ n = \frac{230000}{200000} \]

\[ n = 1.15 \]

\[ \rho_s = \frac{A_s}{bd_{eff}} \]

\[ \rho_s = \frac{2(109.8)}{(125)(208.91)} \]

\[ \rho_s = 0.84\% \]

\[ \rho_f = \frac{A_f}{bd_{eff}} \]

\[ \rho_f = \frac{0.17 \times 125 \times 2}{125 \times 208.91} \]

\[ \rho_f = 0.163\% \]

\[ \rho_{eff} = 0.84 + 1.15(0.163) \]

\[ \rho_{eff} = 1.05\% \]

\[ \frac{V_{ud}d_{eff}}{M_u} = \frac{(P/2)(202)}{(P/2)(a - \frac{d_{eff}}{2})} \]
\[
\frac{V_u d_{eff}}{M_u} = \frac{(P/2)(208.91)}{(P/2)(620 - \frac{208.91}{2})}
\]

\[
\frac{V_u d_{eff}}{M_u} = 0.4052
\]

\[
V_c = [0.16\sqrt{f_c} + 17 \rho_{eff} \frac{V_u d}{M_u}] b_{w} d_{eff} \leq 0.29 \sqrt{f_c} b_{w} d_{eff} \quad \text{(ACI 318-11, Equation 11-5)} \quad (9)
\]

\[
V_c = [0.16\sqrt{21} + 17(0.01057)(0.4052)] 125(208.91) \leq 0.29 \sqrt{21}(125)(208.91)
\]

\[
V_c = 21.05 \leq 34.70
\]

\[
V_c = 21.05 \text{kN}
\]

\[
V_c = \frac{2}{5} \sqrt{f_c b_{w} c}
\]  

\[
V_c = \frac{2}{5} \sqrt{21(125)(68.65)}
\]

\[
V_c = 15.73 \text{kN}
\]

**CSA 2004:**

\[
V_c = \beta \sqrt{f_c b_{w} d_v}
\]

\[
d_v = \text{greater}(0.9d, 0.72h)
\]

\[
d_v = \text{greater}(0.9(208.91), 0.72(240))
\]

\[
d_v = \text{greater}(188.1, 172.8)
\]

\[
d_v = 188.1 \text{mm}
\]

\[
\beta = \frac{0.40}{(1 + 1500 \varepsilon_x)(1000 + s_{ce})}
\]

\[
s_{ce} = \frac{35s_z}{15 + a_g}
\]

\[
a_g = 20 \text{mm}
\]
\[ s_c = d_v = 188.1 \text{mm} \]

\[ s_{ce} = \frac{35s_c}{15 + a_s} = \frac{35(188.1)}{15 + 20} \]

\[ s_{ce} = 188.1 \text{mm} \]

\[ \epsilon_x = \frac{M_f / d_v + V_f}{2(E_A + E_f A_f)} \]

\[ M_f = V_c (a - d_v) \]

\[ M_f = V_c (620 - \frac{188.1}{2}) \] \hspace{1cm} (19)

\[ \epsilon_x = \frac{M_f / 188.1 + V_c}{2[(200000)(219.6) + (230000)(0.17 \times 125 \times 2)]} \]

\[ \beta = \frac{0.40}{(1 + 1500 \epsilon_x)} \frac{1300}{(1000 + 188.1)} \]

\[ V_c = \beta \sqrt{21}(125)(188.1) \] \hspace{1cm} (6)

Assume \( V_c \) in Equation 19 and find the final \( V_c \) by using Equation 6, subtract both of them. Try the iteration procedure until the difference between both \( V_c \) become zero.

Assume \( V_c = 21.84 \text{kN} \) or \( 21840.13 \text{N} \)

\[ M_f = (21840.13)(620 - \frac{188.1}{2}) \]

\[ M_f = 11487702 \text{N.mm} \]

\[ \epsilon_x = \frac{11487702 / 188.1 + 21840.13}{2[(200000)(219.6) + (230000)(0.17 \times 125 \times 2)]} \]

\[ \epsilon_x = 0.00077 \text{(mm/mm)} \]

\[ \beta = \frac{0.40}{(1 + 1500 \times 0.00077)} \frac{1300}{(1000 + 188.1)} \]

\[ \beta = 0.203 \]

\[ V_c = 0.203 \sqrt{21}(125)(188.1) \]
\[ V_c = 21842.8 N \]
\[ V_c = 21.84 kN \]
Vita

Waleed Nawaz was born on August 26, 1987, in Dubai, U.A.E. He studied and received his higher school certificate from Pakistani Islamia Higher Secondary School in Sharjah, U.A.E. He topped in two subjects in high school, and therefore he got the scholarship from American University Of Sharjah. He was awarded the Bachelor of Science in Civil Engineering from American University of Sharjah in Fall 2010. In February 2011, he joined the Masters of Science in Civil Engineering program with a concentration in structures in American University of Sharjah with a full time scholarship. He worked as a lab assistant for geotechnical and fluid mechanics course. He also worked as an instructor and grader for steel design, structural concrete design, statics and mechanics of material for architects and earthquake engineering. He also supervised six senior design groups and helped them in the structural design software like ETABS, SAP2000 and SAFE. He also worked as a graduate research assistant for one semester.