CYCLIC PRODUCTION SCHEDULING

WITH REMANUFACTURING

by

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Approval Signatures

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Dedication

To my family

for their endless love and support.

Abstract

The cyclic production scheduling problem is concerned with the development of a least-cost cyclic production schedule for a set of tasks required to produce a set of products over a given production cycle and according to a given delivery schedule. Recently, remanufacturing of used products has become an important area of research due to its environmental and economic benefits. Consequently, many organizations integrated manufacturing and remanufacturing operations to optimize their forward and reverse supply chain processes. There are some studies that have been conducted on cyclic production scheduling, remanufacturing, as well as cyclic production scheduling with remanufacturing systems, which have led to the development of different optimization approaches. But to the best of our knowledge, none of them considered cyclic deliveries in order to build cyclic production schedules integrating both manufacturing and remanufacturing. Therefore, the focus of this research is to develop a mixed integer linear programing model for an integrated cyclic production scheduling with remanufacturing system to satisfy a given cyclic delivery schedule while minimizing the total holding and setup costs under the assumption of a delivery cycle of one week discretized into smaller periods. A sensitivity analysis is conducted by varying the model parameters to study the limits of the developed model and illustrate the effect of these variations on the total holding and setup costs as well as the computational time. The results of the sensitivity analysis indicate that thirty minutes would be the best period length for the developed model. These results also validate the capability of the developed model in solving the problem at hand and finding the optimal, near optimal, and good feasible solutions for a period length equal to thirty minutes. Further, increasing the number of decision variables or increasing the amount of scheduled deliveries and returns will result in the computational time to increase exponentially. Also, varying the holding cost or increasing the return rate will result in an increase in the computational time. Finally, increasing the holding cost or increasing the return rate will result in an increase in the total holding and setup costs.

Search Terms: Cyclic Production Scheduling, Cyclic Delivery Schedules, Production Scheduling, Remanufacturing, Green Supply Chain, Reverse Logistics

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Abbreviations

AGV – Automated Guided Vehicle

CT – Computational Time

CPS – Cyclic Production Scheduling

CPSR – Cyclic Production Scheduling with Remanufacturing

ELSP – Economic Lot Scheduling Problem

ELSPR – Economic Lot Scheduling Problem with Returns

ELV – End-of-Life Vehicle

EuP – Eco-design of Energy-using Products

FMS – Flexible Manufacturing System

GA – Genetic Algorithm

GP – Green Productivity

GSCM – Green Supply Chain Management

HPP – Hedging Point Policy

JIT – Just-in-Time

LP – Linear Programming

MHPP – Multiple Hedging Point Policy

MIP – Mixed Integer Programming

MILP – Mixed Integer Linear Programming

NP-Hard – Non-Deterministic Polynomial-Time Hard

PL – Period Length or Period Duration

REACH – Registration, Evaluation, and Restriction of Chemicals

RoHS – Restriction of Hazardous Substance

SCM – Supply Chain Management

U – System Utilization

WIP – Work-in-Process

Chapter 1: Introduction

In this chapter, an overview of supply chain, green supply chain and reverse logistics, remanufacturing, and cyclic production scheduling (CPS) will be provided. In addition, the problem statement, the research objectives and significance, as well as the research methodology, will be highlighted.

1.1 Supply Chain Overview

Organizations have always been trying to reduce their cost and increase their profit by improving their internal processes. This has been going on to the point where it was not practically possible to do so internally anymore. As a result, they had to start looking to their external processes to find improvement opportunities which led to the creation of the term supply chain. A supply chain can be defined as "all parties involved, directly or indirectly, in fulfilling a customer request. The supply chain includes not only the manufacturer and supplier, but also the transporters, warehouses, retailers and even customers themselves" [1]. So as seen from the above, the stages that a commercial product goes through; starting from raw material until it reaches to the shelves, are called a supply chain. To fulfill a customer order, apart from the aforementioned parties, different departments and functions of the supply chain such as finance, marketing, purchasing, human resources, research and development, inventory and warehousing, production, and demand forecasting are required as well [1].

Nowadays, an organization tries to emerge with its suppliers and customers to reduce its cost and increase its profitability. Basically, the organization is trying to look for improvement opportunities, externally, to reduce its cost and increase its profit through better coordination with its different supply chain members. As a result, the profitability and losses of an organization is not going to be analyzed solely based on its performance; instead, it will be analyzed based on the performance of the supply chain that it belongs to [1].

A typical supply chain consists of five stages which are connected through the flow of product, information, and fund in both forward and backward directions. These stages are [1]:

- Suppliers

- Manufacturers
- Distributers
- Retailers
- Customers

For example, when a product is being purchased by a customer, the customer will pay the selling price (fund flows backward from the customer to the retailer) and receive the product (product flows forward from the retailer to the customer). These flows can be managed by one of the stages or an intermediary [1].

One of the main concerns of a supply chain is to reduce the cost and increase the profitability of all parties involved [1]. However, the development of environmental issues gradually led the existing supply chain concept to move toward sustainability in order to become more environmentally friendly [2].

1.2 Green Supply Chain and Reverse Logistics Overview

Due to the development of science and technology, as well as the increase in the population of mankind, the rate of consumption of the environment's resources has been increased. The increase in resource consumption has led to the development of some environmental problems and pressing issues, such as depletion of the ozone layer [2] due to the production of products containing chlorofluorocarbons (CFCs) [3], global warming [2] due to the emission of greenhouse gasses like carbon dioxide (CO_2) [4], and the increase in the produced amount of the hazardous wastes [2]. According to Wang and Gupta [5], some of these gasses and wastes, which are also called pollutants, can spread via air and water from one region to another. As a result, the pollution caused by them has become a global concern rather than local. These concerns have led to the development of environmental laws and regulations [2] by different countries such as the US [6], Germany [7], and even unions and international organizations such as United Nations (UN) [5], Asian Productivity Organization (APO) [2], and European Union (EU) [6] to reduce the emission of hazardous gasses and wastes by different industries. Some of these laws and regulations are [2, 8]:

- End-of-life vehicle (ELV)
- Restriction of hazardous substances (RoHS)
- Green productivity (GP)

- Eco-design of energy-using products (EuP)
- Registration, evaluation, and restriction of chemicals (REACH)
- Kyoto protocol

Environmental concerns, as well as the environmental laws and regulations, gradually led the existing supply chain management (SCM) concept to become more environmentally friendly and the term green supply chain management (GSCM) was introduced [3]. The GSCM can be defined as:

> The creation of coordinated supply chains through the voluntary integration of environmental considerations with key inter-organizational business systems designed to efficiently and effectively manage the material, information, and capital flows associated with the procurement, production, and distribution of products or services in order to meet stakeholder requirements and improve the profitability, competitiveness, and resilience of the organization over the short-term and long-term. [9]

In plain English, GSCM takes into account both the environmental and social factors, in addition to the traditional economic factors, so it will be able to produce and distribute goods in a sustainable way [10].

Further development of GSCM resulted in organizations and industries realizing a need for a reverse supply chain, apart from an efficient forward one, for the recovery of end-of-life [6] and used products [11]. Some of these organizations and industries were the automobile industries [11] such as General Motors [12], electronics industries [11] such as Dell, Kodak, and Xerox [12], as well as battery and glass industries [11]. Reverse logistics can be defined as:

> The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods, and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal. [13]

In other words, according to Alfonso-lizarazo *et al.* [10], reverse logistics is about collection of end-of-life and used products from any link of the supply chain in order to re-enter them to the forward supply chain. This is done in a way that is efficient so it will lead to financial and environmental benefits while generating added value or disposing of those products in the same way.

Reverse logistics can be used to capture the value of end-of-life and used products by recovering them, and that is why it is known to be one of the ways of pursuing sustainability [14]. Yet, sometimes due to the condition of these end-of-life or used products, disposing of these products might be the best option [12]. As a result, on the one hand, for the purpose of pursuing sustainability, design of a reverse supply chain can be considered as a positive move [14]. Subsequently, this led to the reverse supply chain to become an important area of research [6]. On the other hand, the economical and operational benefits such as; lowering cost [6, 10, 11, 12], improving coordination, improving customer service [6], generating positive profit, creating competitive advantage, reducing energy usage, reducing waste and managing its treatment [10], and improving customer satisfaction [10, 12] also led to the increase of its popularity. Moreover, reverse supply chain can result in creating new market opportunities [11], minimizing the environmental impacts [11, 12], increasing market share, adding value to the logistics network, pollution reduction [12], potential source of revenue, creating new jobs in the manufacturing sector, and mitigation of greenhouse gas emissions [14]. However, if organizations want to realize the above benefits, they have to develop and maintain their reverse logistics network which can cost up to 4% of their total logistics cost. This 4% was estimated to be about $35 billion in 2001 for the US alone [15].

Reverse logistics is composed of five processes. These processes are product acquisition, transportation (of the products to the plant for further processing), inspection and disposition, remanufacturing, and marketing [11]. Research shows that, product acquisition is the most important process among the other five processes, since it can affect the performance of the reverse supply chain significantly because economic, social, or environmental benefits can only be realized if the collected product quantity is large enough [14]. However, remanufacturing, as one of the five reverse logistics processes is also an important process since it is known to be one of the main recovery approaches [16].

1.3 Remanufacturing Overview

Remanufacturing can be defined as “recovering usable parts from discarded or retired products, recycling the unusable parts, and reassembling the recovered parts into usable components and/or products” [16]. Remanufacturing is the most attractive and desirable option for end-of-life [11] and used products [11, 12] due to two aspects. The first one is the environmental aspect [12], such as minimizing the negative environmental impacts [11], and the second one is the economical aspect, such as reducing the cost of production by 40% to 60% through reutilizing the product components [12], reducing the loss of value, and creating new market opportunities [11]. For example, cell phone remanufacturing did not exist in 2000, however, the global market value for remanufactured cell phones was about $240 million in 2007 [17]. Also, the market value for remanufactured information technology networking equipment in the United States increased from a very low amount in 2000 to approximately $2 billion in 2009 [17]. One of the other reasons that makes remanufacturing attractive is the total value of the returned products, which in the US alone was worth $100 billion in 2002 [14]. According to Baki *et al.* [18], another benefit of remanufacturing is improving competiveness. Finally, they have also discussed that remanufacturing can limit the environmental damage by reducing the usage of raw materials and replacing them by integration of waste back into the production cycle. This can result in reducing landfill and waste disposal, as well as reducing the required energy for production by 50% to 80%.

Apart from the above remanufacturing benefits, like any other process, remanufacturing also has its own challenges. For example, one of the remanufacturing challenges is to set an attractive price for the remanufactured products to guarantee enough sales [19]. According to Das and Dutta [11], in various industries such as automobiles and electronics, in order to capture the demand for the remanufactured products, they are offered with lower prices than the new products. Hence, the remanufactured products are offered as an alternative to the new products to the customers who do not wish to pay the full price and are attracted by that particular brand.

However, Xiong *et al.* [19] stated that, even if the price is attractive, there are other remanufacturing challenges that an organization can face. For example, on the one hand, if the amount of the returned products is high, the uncertainty of supply and demand can result in high inventory holding cost. On the other hand, if the amount of the returned products is low, the uncertainty of supply and demand can result in high lost-sale costs. Further, they have mentioned that the uncertainty of quality of remanufactured products can be another issue that can result in uncertainties in remanufacturing time and cost.

One of the other challenges of remanufacturing is production planning and scheduling of remanufactured products, as well as integration of manufacturing and remanufacturing production schedules together.

1.4 Cyclic Production Scheduling Overview

Production scheduling has existed for as long as the products have been produced. After the industrial revolution, production scheduling became more popular and later on it became an active research area starting from 1950s [20]. The purpose of production scheduling is to plan and control the activities to come up with a schedule for a product or a set of products in order to achieve a time, quality, and cost effective production [21]. There are different types of scheduling problems, such as job shop or open shop, flow shop, closed shop, and CPS problem [22]. However, CPS is a special type of scheduling problems and can be defined as follows:

> An infinite number of occurrences of some tasks required to produce numerous pieces of the same type. An occurrence is a single execution of a task. A set of tasks are executed repeatedly over a probable infinite time horizon. [23]

To simplify the above definition, certain set of tasks are performed repeatedly so that the system would be able to produce a certain set of products during each production cycle [24] where each production cycle will represent a time span such as one month, one week, one quarter, etc. [25].

Cyclic production schedules can result in several benefits, such as respecting the just-in-time (JIT) principles due to the synchronization of supply and demand and improving the apprenticeship [26]. Moreover it can increase efficiency of material handling, station utilization, and shop floor control [27]. It can also lead to economic benefits, such as cost reduction. For example, in a case study conducted in 2006, it was realized that using cyclic production can lead to about 30% cost reduction [28]. Respecting the JIT principles alone can result in waste elimination, quality improvement, decreasing lead time for different types of activities, and increasing skills and employee morals [29].

1.5 Problem Statement

Nowadays, organizations such as the ones in the automotive sector are becoming linked to their customers through contracts specifying cyclic delivery schedules. Moreover, many researchers demonstrated the benefits of implementing cyclic production schedules in terms of improving apprenticeship and increasing efficiency of material handling, station utilization, and shop floor control [25, 26, 27, 28, 29]. Some of these organizations also practice remanufacturing due to its environmental and economic benefits, such as reducing resource consumption that will result in minimizing the negative environmental impacts which at the same time reduces the production cost through reutilization of the product components [7, 11, 12, 14, 17, 18]. As a result, in this research, a mathematical model will be developed to integrate cyclic production scheduling with remanufacturing (CPSR) to satisfy a given cyclic delivery schedule.

Cyclic delivery schedule can be classified as a special type of dynamic demand pattern. In this study, the delivery cycle is a week which will be discretized into smaller periods. A cyclic delivery schedule for a particular product will be composed of several independent deliveries in a week. The independent deliveries are basically the independent demands for that particular product in a week. For example, a cyclic delivery schedule for product ‘A’ can be 200 units every Monday, 100 every Tuesday, and 700 every Saturday and this schedule will be repeated every week. Figure 1 illustrates this example.

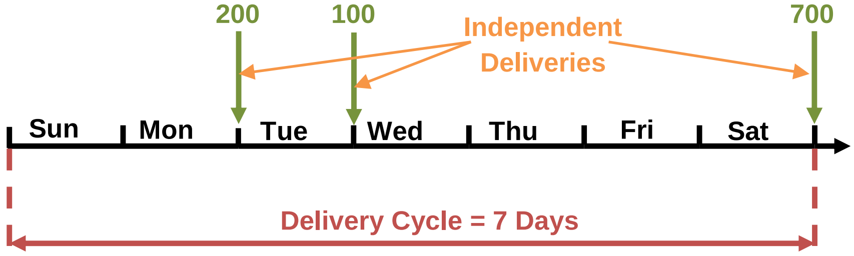

Figure 1: An Example of a Cyclic Delivery Schedule for Product ‘A’

In practice, the independent delivery quantities will vary between a minimum and a maximum value and the exact quantity will be known only a few days before each delivery. However, in this thesis the delivery quantities will be assumed as constant values calculated as an average of the minimum and maximum delivery quantities.

Later, sensitivity analysis will be performed on the developed model to evaluate the effect of varying the independent delivery quantities on the total holding and setup costs as well as the computational time. Apart from the demand, the return rate during a delivery cycle is going to be proportional to the demand rate and it will be assumed as a percentage of the demand during that delivery cycle. However, during the sensitivity analysis, the percentage of the returned products will be varied to illustrate the effect of this variation on the total holding and setup costs, as well as the computational time.

Another assumption made is that there is only one production line with finite capacity for manufacturing and remanufacturing. It is also assumed that the organization is producing n different products and that the manufactured and remanufactured products are identical. Therefore, the demand for each product can be satisfied from both manufactured and remanufactured products.

The objective of this study is to determine the least-cost cyclic production schedule that satisfies the demand and minimizes the total holding (serviceable and returns) and setup costs (manufacturing and remanufacturing) while integrating CPS with remanufacturing. In other words, the aim is to minimize the total holding and setup costs by developing a cyclic production schedule specifying when and how much to manufacture and remanufacture, as well as the proportion of the demand to be satisfied from manufactured and remanufactured products. Since the objective is to develop a cyclic production schedule, the levels of on-hand inventory for both returned and finished products at the beginning and end of each production cycle will remain the same. The amount of the on-hand inventory will be another decision variable to be determined by the developed model.

Therefore, the inputs and outputs of this system with the objective of minimizing the total inventory holding and setup costs are:

- Inputs (parameters):
    - Number of products.
    - Delivery cycle (7 days – one week).
    - Period length.
    - Cyclic delivery schedule (demand).

- Manufacturing and remanufacturing production rates per period (production capacity).
- Product return rate (r = percentage of the demand during the delivery cycle).
- Unit inventory holding cost for returned products per period.
- Unit inventory holding cost for finished products per period.
- Manufacturing and remanufacturing setup costs.
- Manufacturing and remanufacturing setup times.

- Outputs (decision variables):
  - Cyclic production schedule.
  - Levels of on-hand inventory at the beginning and end of each production cycle are equal.

As a result, the model under study can be viewed as a system that is provided in Figure 2.

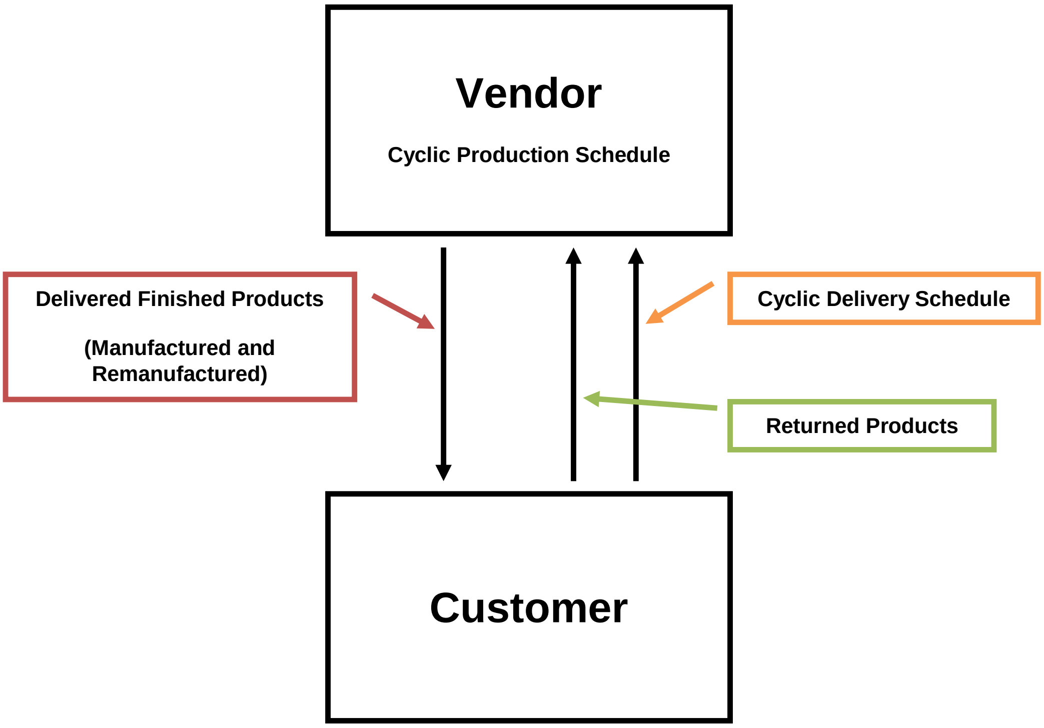

Figure 2: The System under Study

For illustrative purposes, Figure 3 depicts a possible solution (cyclic production schedule) for the cyclic delivery schedule of product ‘A’ that was presented earlier in Figure 1.

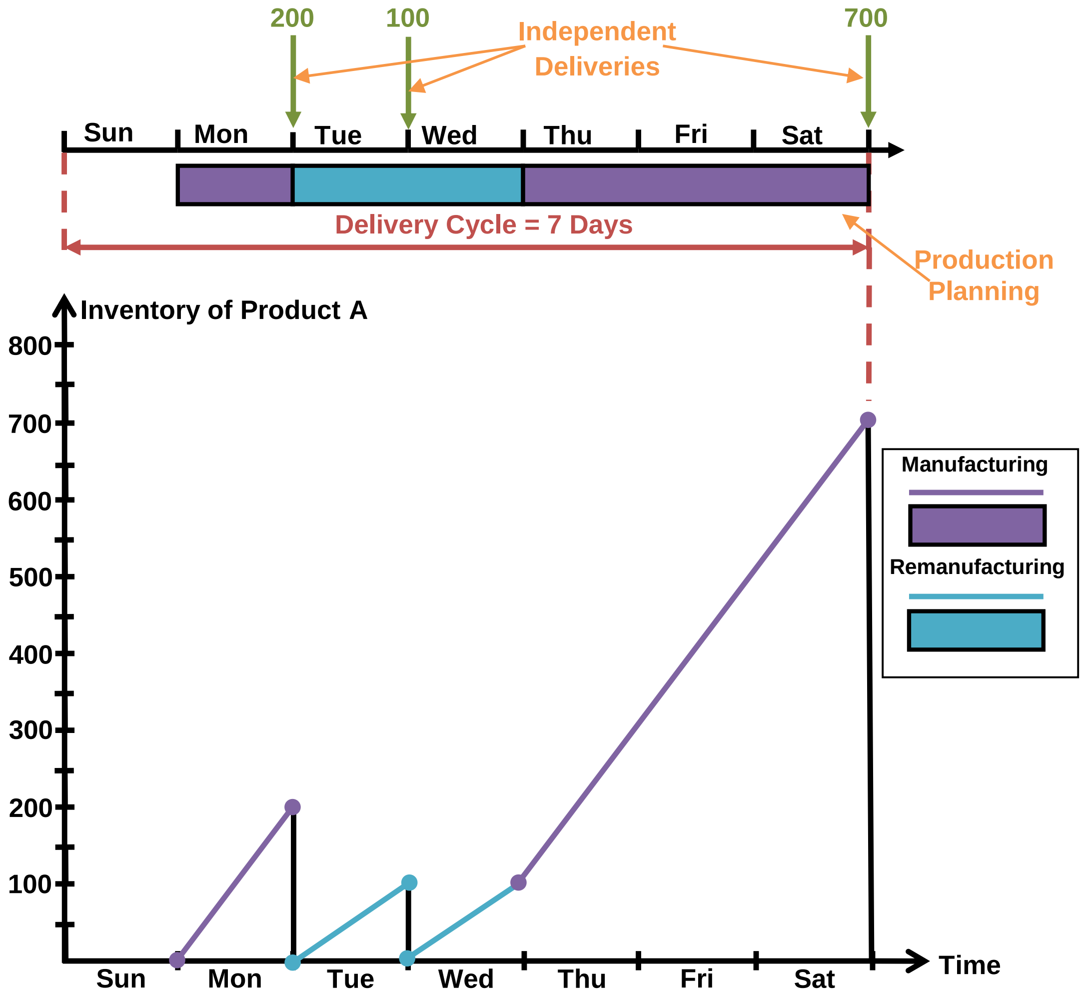

Figure 3: One Possible Cyclic Production Schedule (Solution) for Product 'A'

1.6 Research Objectives and Significance

The objective of this research is to develop a mathematical model integrating CPS with remanufacturing to satisfy a given cyclic delivery schedule. The research also aims to assess the benefits of such integration.

The significance of this research is about its contribution to the literature, since to the best of our knowledge, among the studies that have been conducted so far, none of them considered cyclic deliveries in order to build cyclic production schedules integrating both manufacturing and remanufacturing.

1.7 Research Methodology

In order to achieve the objectives of this research, the steps stated below will be followed.

Step 1. Conducting a literature review on remanufacturing, CPS, and integrated CPSR systems.

Step 2. Developing a mathematical model for an integrated CPSR system to satisfy a given cyclic delivery schedule for multiple products.

Step 3. Coding and solving the developed model using IBM ILOG optimization software, CPLEX Optimization Studio, version 12.5.1.0. The computer that the CPLEX software is installed on has the following specifications:

- Processor: Intel (R) Core (TM) i7-3770 CPU @ 3.40 Hz 3.40 Hz
- Installed Memory (RAM): 8.00 GB
- Operating System: Windows 7 Enterprise N – Service Pack 1
- System Type: 64-bit Operating System
- Hard Disk: 465 GB

Step 4. Performing sensitivity analysis by varying the model parameters to study the limits of the developed model and illustrate the effect of these variations on the total holding (serviceable and returns) and setup costs (manufacturing and remanufacturing), as well as the computational time.

1.8 Thesis Organization

The rest of this thesis is organized as follows. In Chapter 2, the related literature to remanufacturing, CPS, and integrated CPSR systems are reviewed. Chapter 3 presents the developed mathematical model along with a numerical example. The results of the sensitivity analysis conducted on the developed model are provided in Chapter 4. Finally, Chapter 5 concludes this thesis and suggests future research directions.

Chapter 2: Literature Review

In this chapter, the relevant literature about remanufacturing, CPS, as well as integrated CPSR systems, will be reviewed.

2.1 Remanufacturing

There have been a great deal of research conducted on remanufacturing based on different case studies [30]. The research on remanufacturing started around 1980s [31]. The focus of some of the research conducted is limited to aircraft applications and railroad locomotives and equipment. The rest are focused on firms remanufacturing different types of products such as copiers, automotive components, and personal computers [30].

One of the first studies on remanufacturing was conducted by Krupp [32] who developed a model for forecasting core obsolescence in an automotive component remanufacturing system. Later, the work was extended by the same researcher [33] leading to the development of a modified bill of materials for the same system.

Guide [34] performed another study on a remanufacturing facility with bottleneck resources. This study led to the development of a scheduling heuristic resulting in a stable flow-time at the expense of increasing the production time causing the work-in-process (WIP) inventory to increase.

Another remanufacturing model was developed by Jayaraman *et al.* [35]. They stated that remanufacturing firms need to find the following critical information based on the level of performance objective that the firm is planning to achieve: number and location of distribution and remanufacturing facilities, as well as amount and type of products to be stocked at each location. Accordingly, they used a deterministic model as a starting point and later expanded the model leading to development of a 0-1 mixed integer programming (MIP) model that could be solved for the location of different facilities, as well as optimal transshipment, production, and stocking quantities for different products at each location.

Other studies have also been performed on remanufacturing systems with the objective of developing production planning models. The models that have been developed so far can be classified into single-stage and multi-stage models. The difference between single-stage and multi-stage models is in the aggregation of their subsystems [36]. A typical remanufacturing system is composed of three subsystems: disassembly, processing, and reassembly [7]. In single-stage models, the three subsystems are aggregated as one while in multi-stage models the three subsystems are considered explicitly [36].

Some of the studies conducted on remanufacturing systems that resulted in the development of single-stage models were performed by Richter and Sombrutzki [37], Richter and Weber [38], and Richter and Gobsch [39]. Richter and Sombrutzki [37] performed a study on remanufacturing systems that led to the development of a single-stage production planning model which proposed solution algorithms using the reverse Wagner/Within model. This research was extended by Richter and Weber [38] where they assumed additional variability in the manufacturing and remanufacturing cost under the main framework of the reverse Wagner/Within model. Another extension was added by Richter and Gobsch [39] by taking into consideration the JIT framework.

There are some other studies done on remanufacturing systems that resulted in the development of multi-stage models such as Jayaraman [40] and Doh and Lee [36]. In the study performed by Jayaraman [40], a linear programming (LP) model minimizing the total unit cost of the remanufactured product was developed for the proposed multi-stage production planning model for remanufacturing systems.

Doh and Lee [36] proposed a generic multi-stage production planning model for remanufacturing systems with discrete time periods over a given planning horizon. Unlike the existing models in the literature, since they believed that setup costs and time are very important in a remanufacturing environment, they explicitly considered them even though they have added to the complexity of the problem. An MIP model was developed to determine the number of used products to be disassembled, disposed, and reassembled, as well as the number of parts to be reprocessed, disposed, and newly purchased with the objective of maximizing the profit over the given planning horizon. However, due to the complexity of the problem, the model could not be solved directly using a commercial software package. As a result, based on the LP techniques, two types of heuristics were developed, called LP relaxation and one to zero heuristics.

Development of the environmental laws and legislations, such as end-of-life vehicle (ELV) [2] and Kyoto protocol [8], also influenced the remanufacturing studies. Vlachos *et al.* [41] conducted a study on remanufacturing facilities where apart from economical values, it also took into account the environmental issues and legislations, such as the take-back obligation as well as the effect of those issues on customer demand. The goal of this study was to develop efficient capacity planning policies for remanufacturing facilities. As a result, for the purpose of studying the system behavior, a generic simulation model was developed which was used as an experimental tool for evaluating different long-term capacity policies using the profitability as the measure of the policy effectiveness.

Finally, there have also been some studies addressing stochastic remanufacturing systems. Gharbi *et al.* [42] performed a study on stochastic remanufacturing systems with the objective of controlling the production rate of remanufactured products. Then, the problem was formulated as a function of serviceable equipment inventory level that could minimize the repair as well as inventory or shortage costs over an infinite horizon. Based on the control theory, they proposed a near-optimal control policy, called multiple hedging point policy (MHPP). They have used the design of experiment, simulation modeling, and response surface methodology to find the parameters of that policy, as well as a close approximation of the optimal repair policy. They have also shown that their policy generates better solutions than the existing classical hedging point policy (HPP) in the literature.

2.2 Cyclic Production Scheduling

There has been a great deal of research conducted on CPS. The CPS has been around as part of the economic lot scheduling problem (ELSP) that is a problem of "accommodating cyclical production patterns when several products are made on a single facility" [43]. It is common in the ELSP literature to assume the demand and manufacturing rates as constants [44].

One of the first CPS methods in context of ELSP was developed by Graves *et al.* [45] for a re-entrant flow shop where production sequence of the operations in one cycle was fixed and the sequence was cyclically repeated. To find the best cyclic schedule, they had to solve an optimization problem, but due to its complexity a computerized heuristic algorithm was developed for the purpose of producing good schedules. However, the developed algorithm had some flaws, such as not being able to handle the operations at a multiple-channel facility, as well as a batch processing facility [46]. Later on, van Beek and van Putten [47] and Hahn *et al.* [48] introduced setup costs, setup times, and capacity as decision variables to the existing ELSP models extending them to incorporate JIT concepts resulting in some forms of cyclic schedules. To overcome the flaws of the CPS method developed by Graves *et al.* [45], a revised method was developed by Yura [46] aiming to maximize the production rate and to minimize the throughput time in a single-channel, multiple-channel, and batch processing facility.

As mentioned earlier, most of the ELSPs in the literature assume constant demand rates [46] but not all of them. Kelle *et al.* [49] developed an extension to the ELSP model for a production inventory scheduling problem at a large chemical company that differed from the existing ELSP models since it considered random demand. A heuristic procedure was used to find the optimal length of production cycles that while satisfying demand at the specified service level, it also minimized the sum of setup and inventory holding costs per unit of time.

According to Chung and Chan [50], since ELSP is known as a non-deterministic polynomial-time hard (NP-hard) problem, most heuristic algorithms were adopting two types of rounding-off methods for the production frequency of products. They state that, the two types of rounding-off methods used are the nearest integer and power-of-two approaches. Further, they discuss that if the production frequency varies, it will lead to different optimization results even for the same set of products since the production frequency of products is actually the number of times that such product has been produced during a cycle. They conclude that, to deal with this problem, genetic algorithms (GAs) were used to solve ELSPs.

Khouja *et al.* [51] conducted a study on ELSP and for the first time proposed a GA to solve it. Recently, Chung and Chan's [50] study on ELSP led to the development of a two-level GA that outperformed the existing approaches in the literature. Later on Chan *et al.* [52] extended the application of the proposed GA to a multi-facility ELSP problem. To solve the problem, they divided it into two parts. The first part is a master problem that deals with the allocation of common items to facilities which was solved using integer programming. The second part is a set of sub-problems dealing with the determination of production frequencies, production schedules, and production lead time which were solved using GA.

However, apart from ELSP, there were other studies conducted on CPS. Munier [53] introduced linear precedence constraints between two generic tasks of a cyclic schedule while modeling them using a linear graph without any resource constraints. The goal was to characterize the optimal frequencies of the generic tasks. As a result of this study, an algorithm was developed showing that the longest paths of the graph provided the maximum values of the frequencies.

Zhang and Graves [54] conducted one of the first studies on CPS in a stochastic environment. They tried to consider uncertainties, such as random machine failures, that may cause the actual cyclic production schedule to deviate from what has been planned. They formulated the CPS problem of one or multiple machines into a convex program with the objective of minimizing the expected delays.

Another study of the stochastic CPS problem was done by Karabati and Tan [55] on synchronous assembly and production lines. Their objective was to maximize throughput with the assumption of variable processing time to make the problem more realistic. As a result of their study, two heuristic solution procedures were presented. Jodlbauer and Reitner [25] also studied a stochastic multi-item cyclic production system which led to the development of algorithms that maximized the service level while minimizing the total holding and setup costs.

CPS has also been used as an approach to solve other existing problems in the literature such as the flexible manufacturing systems (FMSs) which can be defined as "highly automated production systems, consisting of a computer-controlled integrated configuration of multipurpose workstations, storage buffers, and one or more automated guided vehicles (AGVs)" [56]. One of the first studies done on FMS, while dealing with it as a CPS problem, was performed by Gaalman *et al.* [57] which was later expanded by Nawijn [58]. Another study was done by Korbaa *et al.* [59] that led to the development of new CPS algorithm for FMSs based on progressive operations placing. They were aiming to maximize throughput while minimizing WIP inventory.

Apart from the algorithm, a controlled beam search approach was also developed to determine the schedule of the next operation at each step.

There are other applications of CPS models that have been developed in the literature. Brucker and Kampmeyer [60] have developed a general mixed integer linear programming (MILP) formulation for modeling and handling cyclic scheduling problems that could cover different cyclic versions of the job-shop, robotic cell, the single hoist scheduling, and tool transportation between the machines problems. The initial model was solved using CPLEX software. Later, for the expanded version of the model, a tabu search procedure was developed.

Pundoor and Chen [61] conducted a study, in a two-stage supply chain, on an integrated cyclic production and delivery scheduling problem. The system under study consisted of one or multiple suppliers, a warehouse, and a customer. Each of those suppliers have a different setup time and setup cost per production run. Further, each of those suppliers were producing a different product at a constant rate. All of the final products were first delivered to the warehouse and then sent to the customer while the demand of each product was also constant over time. They have considered two different policies. In the first policy, the production cycle time of each supplier and the delivery cycle time from that supplier to the warehouse is the same. In the second policy, the production cycle time of each supplier is an integer multiple of the delivery cycle time from that supplier to the warehouse. Also, the delivery cycle time from a supplier to the warehouse is an integer multiple of the delivery cycle time from the warehouse to the customer. The objective of their study was to satisfy the demand of all products without delays while minimizing the total inventory, production, and distribution costs. This was to be achieved through finding a joint cyclic production schedule for each supplier, as well as two cyclic delivery schedules, one from each supplier to the warehouse, and another from the warehouse to the customer. For the two policies, they have either found the exact optimal solution or used a heuristic algorithm to find a near optimum solution.

In another study performed by Bahroun and Campagne [26], the demand of each product follows a cyclic pattern and varies between a minimum and maximum quantity while the exact delivery quantities will be known a few days before the delivery. However, they assumed the demand to be constant (average value of the minimum and maximum quantity). The delivery dates are fixed and cyclic. For example, the cyclic delivery schedule for a particular product can be 300 units each Monday, 100 each Tuesday, and 600 each Friday and this schedule will be repeated every week. As a result of their study, they have proposed heuristic approaches to develop a cyclic production schedule in a medium term level that could satisfy the demand and minimize the total holding and setup costs. However, the nature of the problem in this level strongly depends on the ratio between holding and setup cost. On the one hand, if the setup costs are more important relative to holding costs, then the aim is to minimize the number of production runs while stocking more products. On the other hand, if the holding costs are more important relative to setup costs, then the aim is to minimize the inventory while producing more manufacturing orders. In their study, they assumed that the holding costs are more important. The main difference between their work and the existing literature is that they are trying to find a production cycle with different manufacturing orders and different lot sizes while considering cyclic deliveries with different quantities in different dates for each product. In this thesis, their study is being extended through integrating it with remanufacturing.

Recently, Bonfietti *et al.* [62] proposed a constraint programming approach based on modular precedence constraint and a global cumulative constraint along with filtering algorithms to solve CPS problems.

2.3 Integrated Cyclic Production Scheduling with Remanufacturing Systems

Recently, there have also been some studies conducted on integrating CPS with remanufacturing. The first study on a hybrid production line that integrated CPS with remanufacturing was performed by Tang and Teunter [44] for a company producing automotive components. They referred to the problem as economic lot scheduling problem with returns (ELSPR). Based on the policy of the company, they assumed a common cycle time for all products with one manufacturing and one remanufacturing lot for each of them during a cycle. They also assumed the demand and return rates for used products, as well as for the manufacturing and remanufacturing production rates to be constant. The manufacturing and remanufacturing processes were performed on the same product line. As a result, an MILP problem was formulated that provided a basis for an exact solution for a given cycle time. The application of the developed model indicated a 16% reduction in cost for one of their products compared to their current lot scheduling policy. Teunter *et al.* [63] extended Tang and Teunter's [44] model by proposing four simple heuristics. These heuristics can provide a quicker near-optimal solution compared to the complex and time consuming MILP formulation which was developed earlier. However, the results of the numerical study showed that the initial solution of the heuristics could result in poor performance if no improvement steps were taken. Accordingly, inserting slack time or swapping production sequences were applied to improve the initial solution.

Ouyang and Zhu [64] proposed a mathematical model for ELSPR under the same assumptions of common cycle approach and both manufacturing and remanufacturing processes being performed on the same product line, as well as constant demand and return rates. They also assumed that the product line has a limited manufacturing and remanufacturing capacity. The objective of the model was to find the production cycle that minimized the total cost.

Teunter *et al.* [65] studied the ELSPR under the same assumptions of common cycle approach and constant demand, return, manufacturing, and remanufacturing rates, but they assumed separate lines for manufacturing and remanufacturing processes. Another assumption they made was sequence independent setup costs and setup times. Their study led to the development of an algorithm that integrates a MIP formulation under the assumption of fixed cycle time with a cycle time search to find the optimal common cycle time policy. Their problem is more complex than the problem which was studied by Tang and Teunter [44] since they were facing more complex inventory patterns due to two complicating factors. The first factor was the demand rates since not all of them were smaller than the production rates. The second factor was that the intervals of manufacturing and remanufacturing could overlap. They used a case study data to evaluate their model. Based on the evaluation, they concluded that their model results in more cost reduction than the model which was proposed by Tang and Teunter [44]. However, in their model, they did not take into account the initial cost of setting up a new production line for remanufacturing processes.

All of the above studies on ELSPR were performed using, common cycle approach. However, there have also been some studies on ELSPR using basic period approach. The first study on ELSPR using the basic period approach was done by Chang *et al.* [66]. They assumed both manufacturing and remanufacturing processes being performed on the same product line, the demand and return rates were constant, and the setup costs and setup times of each of the products were independent of their production sequence. As a result of their study, they formulated a mathematical model and proposed a search algorithm with the objective of minimizing the average total cost by finding the optimal values for cyclic multiplier of each product, length of basic period, and production sequence of all lots. The cyclic multiplier of each product is basically an integer value that will be multiplied by the basic period to compute the production cycle of each product.

Zanoni *et al.* [67] performed another study on ELSPR using a basic period approach. They assumed both manufacturing and remanufacturing processes being performed on the same product line. They also assumed constant demand, return, manufacturing, and remanufacturing rates while the manufactured and remanufactured products have the same quality and satisfy the same demand stream. As a result of their study, they proposed an algorithm based on basic period approach claiming that it can result in cost savings and at the same time it is simple, easy, and requires shorter time to be implemented compared to the existing models in the literature.

There are some studies integrating dynamic lot sizing problem with remanufacturing, such as Teunter *et al.* [68], Schulz [69], Li *et al.* [18], and Baki *et al.* [70]. Even though these studies are similar to this research work since they consider a dynamic demand but they do not exploit the fact that the demand is following a cyclic pattern, they do not assume a finite capacity, and they do not aim to build a cyclic production schedule. However, in this research, it is assumed that the demand follows a cyclic pattern and is provided as a given as a cyclic delivery schedule. Also, in this research, it is assumed that the production line has a finite capacity for both manufacturing and remanufacturing. Indeed, all of the studies conducted on integrated dynamic lot sizing problem with remanufacturing assume the beginning inventory to be equal to zero or to a pre-established initial value. The objective of these studies is to determine in which periods they should produce or not in order to satisfy the demand while minimizing total holding and setup costs. However, in this research, the objective is to develop a least-cost cyclic production schedule where the levels of on-hand inventory at the beginning and end of the production cycle should be equal and should be determined. Further, the determined least-cost cyclic production schedule should satisfy the demand and minimize the total holding and setup costs while integrating CPS with remanufacturing.

There are also some studies conducted on capacitated dynamic lot sizing problem with remanufacturing such as Li *et al.* [71], Pan *et al.* [72], and Zhang *et al.* [73]. These studies are even more similar to this research work compared to the dynamic lot sizing problem with remanufacturing. These studies are more similar because, apart from a dynamic demand, they also consider a finite capacity. However, still they do not exploit the fact that the demand is following a cyclic pattern and they do not aim to build a cyclic production schedule. All of these studies also assume that the beginning inventory is equal to zero or to a pre-established initial value and their objective is to determine in which periods they should produce or not in order to satisfy the demand while minimizing total holding and setup costs. However, as mentioned earlier, the objective of this research is to develop a least-cost cyclic production schedule where the levels of on-hand inventory at the beginning and end of the production cycle should be equal and should be determined. Further, the determined least-cost cyclic production schedule should satisfy the demand and minimize the total holding and setup costs while integrating CPS with remanufacturing.

To conclude, to the best of our knowledge, among the studies that have been conducted so far, none of them considered cyclic deliveries in order to build cyclic production schedules integrating both manufacturing and remanufacturing. Therefore, the focus of this research is to develop a mathematical model integrating CPS with remanufacturing to satisfy a given cyclic delivery schedule. The developed model will determine the least-cost cyclic production schedule that satisfies the demand and minimizes the total holding (serviceable and returns) and setup costs (manufacturing and remanufacturing) while integrating CPS with remanufacturing.

Chapter 3: Mathematical Model

As it was discussed earlier, the objective of this study is to develop and solve a mathematical model that integrates CPS with remanufacturing to satisfy a given cyclic delivery schedule.

Accordingly, the solution of the developed model will determine the least-cost cyclic production schedule that satisfies the demand and minimizes the total holding (serviceable and returns) and setup costs (manufacturing and remanufacturing) while integrating CPS with remanufacturing.

Before presenting the model, the model assumptions, parameters, and decision variables will be introduced.

3.1 Model Assumptions

The formulation of the developed mathematical model is based on the following assumptions:

1. The delivery cycle is assumed to be equal to one week which is discretized into smaller periods (7 days per week and 8 hours per day; total of 56 hours per delivery cycle) since without discretizing the time, the development of the model will be very difficult.
2. The demand follows a cyclic pattern and is fulfilled using several independent constant delivery quantities as it was illustrated through Figure 1 in section 1.5.
3. The demand for each and every product must be satisfied without backlogs.
4. The return rate during a delivery cycle is proportional to the demand rate during that delivery cycle.
5. The levels of on-hand inventory for both returned and finished products at the beginning and end of each production cycle are equal.
6. There is only one production line with finite capacity for manufacturing and remanufacturing.
7. It is assumed that the organization is producing *n* different products and that the manufactured and remanufactured products are identical.

3.2 Problem Parameters

The following are the parameters of the developed mathematical model:

n Number of products i.e. $N = \{1, 2, \dots, n\}$.

t Number of periods per delivery cycle i.e. $T = \{1, 2, \dots, t\}$.

i Product index.

j, w Period indices.

PL Period length or period duration (minutes).

K_i^M Setup cost for manufacturing product *i*.

K_i^R Setup cost for remanufacturing product *i*.

MS_i^M Setup time (minutes) to manufacture product *i*.

MS_i^R Setup time (minutes) to remanufacture product *i*.

S_i^M Setup time (periods) to manufacture product $i = \left\lceil \frac{MS_i^M}{PL} \right\rceil$ where $\lceil x \rceil$ is the rounded up value of *x*.

S_i^R Setup time (periods) to remanufacture product $i = \left\lceil \frac{MS_i^R}{PL} \right\rceil$.

h_i^s Holding cost to carry a serviceable (manufactured or remanufactured) unit of product *i* inventory from period *j* to period *j+1*.

h_i^r Holding cost to carry a returned unit of product *i* inventory from period *j* to period *j+1*.

PC_i^M Production capacity of manufacturing product *i* per period.

PC_i^R Production capacity of remanufacturing product *i* per period.

D_{ij} Demand of product *i* to be delivered at the end period *j*.

R_{ij} Amount of returns of product *i* received at the beginning of period *j*.

r Return rate per delivery cycle.

3.3 Problem Decision Variables

The followings are the decision variables of the developed mathematical model:

I_{ij}^{s} Serviceable inventory level of product *i* at the end of period *j*.

I_{ij}^{r} Returned inventory level of product *i* at the end of period *j*.

Q_{ij}^{M} Quantity of product *i* manufactured in period *j*.

Q_{ij}^{R} Quantity of product *i* remanufactured in period *j*.

y_{ij}^{M} Binary variable equal to one when the production line is allocated to product *i* for its setup or manufacturing in period *j* and zero otherwise.

y_{ij}^{R} Binary variable equal to one when the production line is allocated to product *i* for its setup or remanufacturing in period *j* and zero otherwise.

x_{ij}^{M} Binary variable equal to one when setup for manufacturing product *i* started in period *j* (or *w*) and zero otherwise.

x_{ij}^{R} Binary variable equal to one when setup for remanufacturing product *i* started in period *j* (or *w*) and zero otherwise.

3.4 Model Formulation

Using the above assumptions and notations, the objective function can be written as follows:

$$Min \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{t} K_i^M x_{ij}^M}_{\substack{\text{Manufacturing}\\ \text{Setup Cost}}} + \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{t} K_i^R x_{ij}^R}_{\substack{\text{Remanufacturing}\\ \text{Setup Cost}}} + \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{t} h_i^s I_{ij}^s}_{\substack{\text{Serviceable}\\ \text{Holding Cost}}} + \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{t} h_i^r I_{ij}^r}_{\substack{\text{Returns}\\ \text{Holding Cost}}} \qquad (1)$$

The objective function (1) is to minimize the total holding (serviceable and returns) and setup costs (manufacturing and remanufacturing).

Subject to:

- **Inventory balance constraints for the first period of the delivery cycle for both serviceable and returns.**

$$I_{i1}^{s} = I_{it}^{s} + Q_{i1}^{M} + Q_{i1}^{R} - D_{i1} \qquad \forall\, i \in N \qquad (2)$$

$$I_{i1}^{r} = I_{it}^{r} - Q_{i1}^{R} + R_{i1} \qquad \forall\, i \in N \qquad (3)$$

Constraints (2) and (3) are inventory balance equations for serviceable and returns respectively for the first period of the delivery cycle only, assuming that the level of on-hand inventory at the beginning and end of the delivery cycle to be equal.

Constraint (3) also ensures that all of the returned products during a delivery cycle are remanufactured during the same cycle. This constraint is added to preserve the cyclic nature of the model through preventing the accumulation of the returned products inventory in the long run.

- **Inventory balance constraints for the remaining periods of the delivery cycle for both serviceable and returns.**

$$I_{ij}^{s} = I_{i,j-1}^{s} + Q_{ij}^{M} + Q_{ij}^{R} - D_{ij} \qquad \forall\, i \in N \quad \text{and} \quad \forall\, j \in T : j \neq 1 \qquad (4)$$

$$I_{ij}^{r} = I_{i,j-1}^{r} - Q_{ij}^{R} + R_{ij} \qquad \forall\, i \in N \quad \text{and} \quad \forall\, j \in T : j \neq 1 \qquad (5)$$

Constraints (4) and (5) are inventory balance equations for serviceable and returns respectively for the rest of the periods of the delivery cycle.

- **Manufacturing production capacity allocation constraints.**

$$Q_{ij}^{M} \leq \left(y_{ij}^{M} \times PC_{i}^{M}\right) \qquad \forall\, i \in N \quad \text{and} \quad \forall\, j \in T \qquad (6)$$

Constraint (6) ensures that the quantity manufactured in a period is zero if the production line is not allocated to any product for its setup or manufacturing. Otherwise, the quantity manufactured will be less than or equal to the manufacturing production capacity.

If S_{i}^{M} is equal to one, then constraint (7) will be used to ensure that the quantity manufactured in the period during which manufacturing setup occurs is less than or equal to a fraction of the manufacturing production capacity of that period. This fraction will depend on the remaining free time available which is determined based on the actual manufacturing setup time (MS_i^M) of a product.

$$Q_{ij}^M \leq PC_i^M \left(1 - \left(\left(1 - \left(S_i^M - \frac{MS_i^M}{PL} \right) \right) \times \sum_{w=j-S_i^M+1}^{j} x_{iw}^M \right) \right)$$

$$\forall\, i \in N \qquad \text{and} \quad \forall\, j \in T : \; j \geq S_i^M \quad (7)$$

To demonstrate the use of constraints (6) and (7) together, the following illustrative example is prepared. It is assumed that for this example, CPLEX found the optimal solution where part of it is presented in Table 1. For this example, PL is equal to one hour resulting in 56 periods per delivery cycle, PC_i^M is equal to 100 units, MS_i^M is equal to 45 minutes, and accordingly S_i^M will be equal to one period.

Table 1: Partial Solution of the Illustrative Example Prepared to Demonstrate the Use of Constraints (6) and (7) Together

j	1	2	3	4	5	6	7	8
Q_{ij}^M	0	0	0	25	100	0	0	0
y_{ij}^M	0	0	0	1	1	0	0	0
x_{ij}^M or x_{iw}^M	0	0	0	1	0	0	0	0

Subsequently, in order to calculate Q_{ij}^M quantity for periods 3, 4, and 5 constraints (6) and (7) should be used.

- Calculating Q_{i3}^M quantity:

$$\left.\begin{array}{l} Q_{i3}^M \leq (0 \times 100) \quad \rightarrow \quad Q_{i3}^M \leq 0 \\ Q_{i3}^M \leq 100\left(1 - \left(\left(1 - \left(1 - \frac{45}{60} \right) \right) \times \sum_{w=3-1+1}^{3} x_{iw}^M \right) \right) \quad \rightarrow \quad Q_{i3}^M \leq 100 \end{array}\right\} Q_{i3}^M \leq 0$$

- Calculating Q_{i4}^M quantity:

$$\left.\begin{array}{l} Q_{i4}^M \leq (1 \times 100) \quad \rightarrow \quad Q_{i4}^M \leq 100 \\ Q_{i4}^M \leq 100\left(1 - \left(\left(1 - \left(1 - \frac{45}{60} \right) \right) \times \sum_{w=4-1+1}^{4} x_{iw}^M \right) \right) \quad \rightarrow \quad Q_{i4}^M \leq 25 \end{array}\right\} Q_{i4}^M \leq 25$$

- Calculating Q_{i5}^{M} quantity:

$$\left.\begin{array}{l} Q_{i5}^{M} \leq (1 \times 100) \quad \rightarrow \quad Q_{i5}^{M} \leq 100 \\ Q_{i5}^{M} \leq 100\left(1 - \left(\left(1 - \left(1 - \frac{45}{60}\right)\right) \times \sum_{w=5-1+1}^{5} x_{iw}^{M}\right)\right) \quad \rightarrow \quad Q_{i5}^{M} \leq 100 \end{array}\right\} Q_{i5}^{M} \leq 100$$

Figure 4 represents a partial cyclic production schedule prepared for this example based on the results presented in Table 1. Period 4 in Figure 4 is viewed as black stripes with blue background. This indicates that in period 4, both setup and manufacturing are taking place. In other words, the first 45 minutes is allocated to setup and the remaining 15 minutes is used for manufacturing.

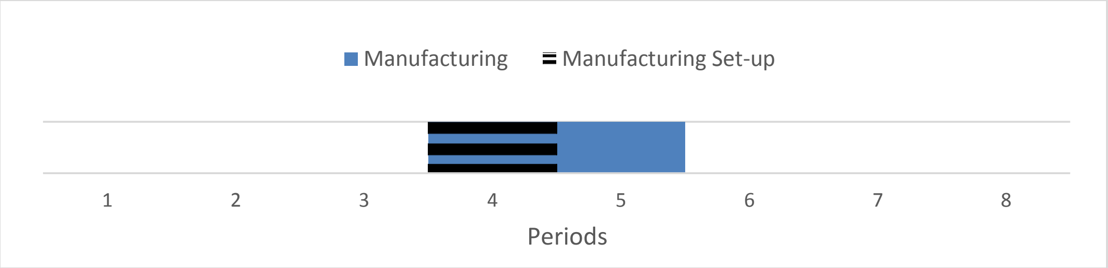

Figure 4: Partial Cyclic Production Schedule of the Illustrative Example Prepared to Demonstrate the Use of Constraints (6) and (7) Together

$$Q_{ij}^{M} \leq PC_{i}^{M}\left(1 - \sum_{w=j-S_{i}^{M}+2}^{j} x_{iw}^{M}\right) \qquad \forall\, i \in N : S_{i}^{M} > 1 \text{ and } \forall\, j \in T : j \geq S_{i}^{M} \quad (8)$$

If S_{i}^{M} is more than one and at the same time all periods of manufacturing setup takes place during the same cycle, then on the one hand, constraint (7) will be used. This constraint ensures that the quantity manufactured during the last period of manufacturing setup is less than or equal to a fraction of the manufacturing production capacity of that period. This fraction will depend on the remaining free time available which is determined based on the actual manufacturing setup time (MS_{i}^{M}) of a product. On the other hand, constraint (8) will be used to ensure that the quantity manufactured during the previous periods of manufacturing setup of that product is equal to zero.

To demonstrate the use of constraints (6), (7), and (8) together, the following illustrative example is prepared. It is assumed that for this example, CPLEX found the optimal solution where part of it is presented in Table 2. For this example, PL is equal to one hour resulting in 56 periods per delivery cycle, PC_i^M is equal to 100 units, MS_i^M is equal to 90 minutes, and accordingly S_i^M will be equal to two periods.

Table 2: Partial Solution of the Illustrative Example Prepared to Demonstrate the Use of Constraints (6), (7), and (8) Together

j	1	2	3	4	5	6	7	8
Q_{ij}^M	0	0	0	0	50	100	0	0
y_{ij}^M	0	0	0	1	1	1	0	0
x_{ij}^M or x_{iw}^M	0	0	0	1	0	0	0	0

Figure 5 represents a partial cyclic production schedule prepared for this example based on the results presented in Table 2. Period 4 in Figure 5 is viewed as black stripes with white background. This indicates that in period 4, only setup is taking place. However, period 5 is viewed as black stripes with blue background indicating the occurrence of both setup and manufacturing. In fact, in period 5, the first 30 minutes is allocated to setup and the remaining 30 minutes is used for manufacturing.

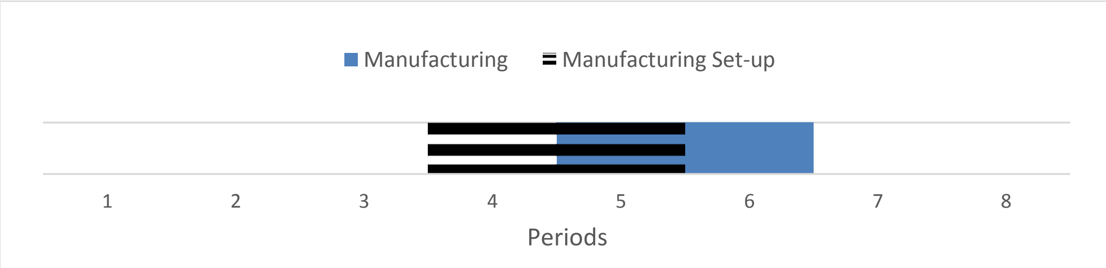

Figure 5: Partial Cyclic Production Schedule of the Illustrative Example Prepared to Demonstrate the Use of Constraints (6), (7), and (8) Together

Subsequently, in order to calculate Q_{ij}^M quantity for periods 3, 4, 5, and 6, constraints (6), (7), and (8) should be used.

- Calculating Q_{i3}^M quantity:

$$\left.\begin{array}{l} Q_{i3}^M \le (0 \times 100) \quad \rightarrow \quad Q_{i3}^M \le 0 \\ Q_{i3}^M \le 100\left(1-\left(\left(1-\left(2-\dfrac{90}{60}\right)\right) \times \displaystyle\sum_{w=3-2+1}^{3} x_{iw}^M\right)\right) \quad \rightarrow \quad Q_{i3}^M \le 100 \\ Q_{i3}^M \le 100\left(1-\displaystyle\sum_{w=3-2+2}^{3} x_{iw}^M\right) \quad \rightarrow \quad Q_{i3}^M \le 100 \end{array}\right\} Q_{i3}^M \le 0$$

- Calculating Q_{i4}^{M} quantity:

$$\left.\begin{array}{l} Q_{i4}^{M} \leq (1 \times 100) \quad \rightarrow \quad Q_{i4}^{M} \leq 100 \\ Q_{i4}^{M} \leq 100\left(1-\left(\left(1-\left(2-\frac{90}{60}\right)\right) \times \sum_{w=4-2+1}^{4} x_{iw}^{M}\right)\right) \quad \rightarrow \quad Q_{i4}^{M} \leq 50 \\ Q_{i4}^{M} \leq 100\left(1-\sum_{w=4-2+2}^{4} x_{iw}^{M}\right) \quad \rightarrow \quad Q_{i4}^{M} \leq 0 \end{array}\right\} Q_{i4}^{M} \leq 0$$

- Calculating Q_{i5}^{M} quantity:

$$\left.\begin{array}{l} Q_{i5}^{M} \leq (1 \times 100) \quad \rightarrow \quad Q_{i5}^{M} \leq 100 \\ Q_{i5}^{M} \leq 100\left(1-\left(\left(1-\left(2-\frac{90}{60}\right)\right) \times \sum_{w=5-2+1}^{5} x_{iw}^{M}\right)\right) \quad \rightarrow \quad Q_{i5}^{M} \leq 50 \\ Q_{i5}^{M} \leq 100\left(1-\sum_{w=5-2+2}^{5} x_{iw}^{M}\right) \quad \rightarrow \quad Q_{i5}^{M} \leq 100 \end{array}\right\} Q_{i5}^{M} \leq 50$$

- Calculating Q_{i6}^{M} quantity:

$$\left.\begin{array}{l} Q_{i6}^{M} \leq (1 \times 100) \quad \rightarrow \quad Q_{i6}^{M} \leq 100 \\ Q_{i6}^{M} \leq 100\left(1-\left(\left(1-\left(2-\frac{90}{60}\right)\right) \times \sum_{w=6-2+1}^{6} x_{iw}^{M}\right)\right) \quad \rightarrow \quad Q_{i6}^{M} \leq 100 \\ Q_{i6}^{M} \leq 100\left(1-\sum_{w=6-2+2}^{6} x_{iw}^{M}\right) \quad \rightarrow \quad Q_{i6}^{M} \leq 100 \end{array}\right\} Q_{i6}^{M} \leq 100$$

If S_i^M is more than one and at the same time manufacturing setup starts at the previous cycle and ends in the current one, then on the one hand, constraint (9) will be used. This constraint ensures that the quantity manufactured during the last period of manufacturing setup is less than or equal to a fraction of the manufacturing production capacity of that period. This fraction will depend on the remaining free time available which is determined based on the actual manufacturing setup time (MS_i^M) of a product. On the other hand, constraint (10) will be used to ensure that the quantity manufactured during the previous periods of manufacturing setup of that product is equal to zero.

$$Q_{ij}^M \leq PC_i^M \left(1 - \left(\left(1 - \left(S_i^M - \frac{MS_i^M}{PL} \right) \right) \times \left(\sum_{w=t-S_i^M+j+1}^{t} x_{iw}^M + \sum_{w=1}^{j} x_{iw}^M \right) \right) \right)$$

$$\forall\, i \in N \qquad \text{and} \quad \forall\, j \in T : j < S_i^M \quad (9)$$

$$Q_{ij}^M \leq PC_i^M \left(1 - \sum_{w=t-S_i^M+j+2}^{t} x_{iw}^M - \sum_{w=1}^{j} x_{iw}^M \right)$$

$$\forall\, i \in N : S_i^M > 1 \quad \text{and} \quad \forall\, j \in T : j < S_i^M \quad (10)$$

To demonstrate the use of constraints (6), (9), and (10) together, the following illustrative example is prepared. It is assumed that for this example, CPLEX found the optimal solution where part of it is presented in Table 3. For this example, PL is equal to one hour resulting in 56 periods per delivery cycle, PC_i^M is equal to 100 units, MS_i^M is equal to 195 minutes, and accordingly S_i^M will be equal to four periods.

Table 3: Partial Solution of the Illustrative Example Prepared to Demonstrate the Use of Constraints (6), (9), and (10) Together

j	53	54	55	56	1	2	3	4
Q_{ij}^M	0	0	0	0	0	75	100	0
y_{ij}^M	0	0	1	1	1	1	1	0
x_{ij}^M or x_{iw}^M	0	0	1	0	0	0	0	0

Figure 6 represents a partial cyclic production schedule prepared for this example based on the results presented in Table 3.

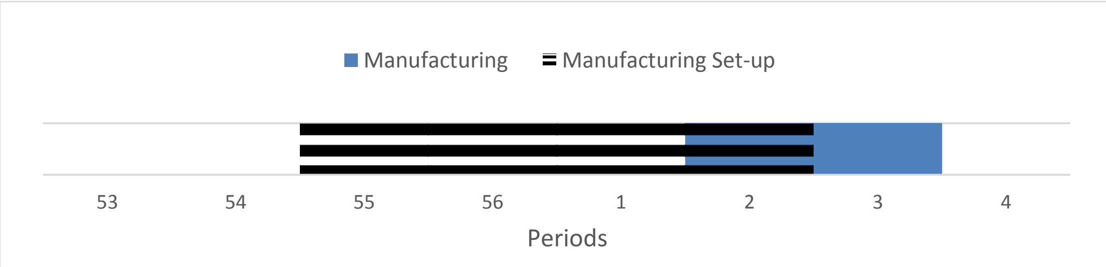

Figure 6: Partial Cyclic Production Schedule of the Illustrative Example Prepared to Demonstrate the Use of Constraints (6), (9), and (10) Together

Subsequently, in order to calculate Q_{ij}^M quantity for periods 1 and 2, constraints (6), (9), and (10) should be used.

- Calculating Q_{i1}^{M} quantity:

$$\left.\begin{array}{l} Q_{i1}^{M} \leq (1 \times 100) \quad \rightarrow \quad Q_{i1}^{M} \leq 100 \\ Q_{i1}^{M} \leq 100\left(1-\left(\left(1-\left(4-\frac{195}{60}\right)\right) \times\left(\sum_{w=54}^{56} x_{iw}^{M}+\sum_{w=1}^{1} x_{iw}^{M}\right)\right)\right) \quad \rightarrow \quad Q_{i1}^{M} \leq 75 \\ Q_{i1}^{M} \leq 100\left(1-\sum_{w=55}^{56} x_{iw}^{M}-\sum_{w=1}^{1} x_{iw}^{M}\right) \quad \rightarrow \quad Q_{i1}^{M} \leq 0 \end{array}\right\} Q_{i1}^{M} \leq 0$$

- Calculating Q_{i2}^{M} quantity:

$$\left.\begin{array}{l} Q_{i2}^{M} \leq (1 \times 100) \quad \rightarrow \quad Q_{i2}^{M} \leq 100 \\ Q_{i2}^{M} \leq 100\left(1-\left(\left(1-\left(4-\frac{195}{60}\right)\right) \times\left(\sum_{w=55}^{56} x_{iw}^{M}+\sum_{w=1}^{2} x_{iw}^{M}\right)\right)\right) \quad \rightarrow \quad Q_{i2}^{M} \leq 75 \\ Q_{i2}^{M} \leq 100\left(1-\sum_{w=56}^{56} x_{iw}^{M}-\sum_{w=1}^{2} x_{iw}^{M}\right) \quad \rightarrow \quad Q_{i2}^{M} \leq 100 \end{array}\right\} Q_{i2}^{M} \leq 75$$

- **Remanufacturing production capacity allocation constraints.**

$$Q_{ij}^{R} \leq \left(y_{ij}^{R} \times PC_{i}^{R}\right) \qquad \forall\, i \in N \qquad \text{and} \quad \forall\, j \in T \qquad (11)$$

Constraint (11) ensures that the quantity remanufactured in a period is zero if the production line is not allocated to any product for its setup or remanufacturing. Otherwise, the quantity remanufactured will be less than or equal to the remanufacturing production capacity.

If S_i^R is equal to one, then constraint (12) will be used to ensure that the quantity remanufactured in the period during which remanufacturing setup occurs is less than or equal to a fraction of the remanufacturing production capacity of that period. This fraction will depend on the remaining free time available which is determined based on the actual remanufacturing setup time (MS_i^R) of a product.

$$Q_{ij}^{R} \leq PC_{i}^{R}\left(1-\left(\left(1-\left(S_{i}^{R}-\frac{MS_{i}^{R}}{PL}\right)\right) \times \sum_{w=j-S_{i}^{R}+1}^{j} x_{iw}^{R}\right)\right)$$

$$\forall\, i \in N \qquad \text{and} \quad \forall\, j \in T:\ j \geq S_{i}^{R} \quad (12)$$

If S_i^R is more than one and at the same time all periods of remanufacturing setup takes place during the same cycle, then on the one hand, constraint (12) will be used. This constraint ensures that the quantity remanufactured during the last period of remanufacturing setup is less than or equal to a fraction of the remanufacturing production capacity of that period. This fraction will depend depending on the remaining free time available which is determined based on the actual remanufacturing setup time (MS_i^R) of a product. On the other hand, constraint (13) will be used to ensure that the quantity remanufactured during the previous periods of remanufacturing setup of that product is equal to zero.

$$Q_{ij}^R \leq PC_i^R \left(1 - \sum_{w=j-S_i^R+2}^{j} x_{iw}^R \right) \qquad \forall\, i \in N : \; S_i^R > 1 \;\text{ and }\; \forall\, j \in T : \; j \geq S_i^R \quad (13)$$

If S_i^R is more than one and at the same time remanufacturing setup starts at the previous cycle and ends in the current one, then on the one hand, constraint (14) will be used. This constraint ensures that the quantity remanufactured during the last period of remanufacturing setup is less than or equal to a fraction of the remanufacturing production capacity of that period. This fraction will depend on the remaining free time available which is determined based on the actual remanufacturing setup time (MS_i^R) of a product. On the other hand, constraint (15) will be used to ensure that the quantity remanufactured during the previous periods of remanufacturing setup of that product is equal to zero.

$$Q_{ij}^R \leq PC_i^R \left(1 - \left(\left(1 - \left(S_i^R - \frac{MS_i^R}{PL} \right) \right) \times \left(\sum_{w=t-S_i^R+j+1}^{t} x_{iw}^R + \sum_{w=1}^{j} x_{iw}^R \right) \right) \right)$$

$$\forall\, i \in N \qquad \text{and } \; \forall\, j \in T : \; j < S_i^R \quad (14)$$

$$Q_{ij}^R \leq PC_i^R \left(1 - \sum_{w=t-S_i^R+j+2}^{t} x_{iw}^R - \sum_{w=1}^{j} x_{iw}^R \right)$$

$$\forall\, i \in N : \; S_i^R > 1 \;\text{ and }\; \forall\, j \in T : \; j < S_i^R \quad (15)$$

- **Production line allocation constraint.**

$$\sum_{i=1}^{n} y_{ij}^{M} + \sum_{i=1}^{n} y_{ij}^{R} \leq 1 \qquad \forall\, j \in T \qquad (16)$$

Since there is only one production line, constraint (16) ensures that at any given period, the production line can be used for either manufacturing or remanufacturing a specific product.

- **Manufacturing setup time and cost constraints.**

$$y_{ij}^{M} \geq x_{ij}^{M} \qquad \forall\, i \in N \quad \text{and} \quad \forall\, j \in T \qquad (17)$$

$$x_{i1}^{M} \geq y_{i1}^{M} - y_{it}^{M} \qquad \forall\, i \in N \qquad (18)$$

$$x_{ij}^{M} \geq y_{ij}^{M} - y_{i,j-1}^{M} \qquad \forall\, i \in N \quad \text{and} \quad \forall\, j \in T : j \neq 1 \qquad (19)$$

Constraints (17), (18), and (19) ensure that the manufacturing setup cost for a product will be counted once per manufacturing production run of that specific product.

- **Remanufacturing setup time and cost constraints.**

$$y_{ij}^{R} \geq x_{ij}^{R} \qquad \forall\, i \in N \quad \text{and} \quad \forall\, j \in T \qquad (20)$$

$$x_{i1}^{R} \geq y_{i1}^{R} - y_{it}^{R} \qquad \forall\, i \in N \qquad (21)$$

$$x_{ij}^{R} \geq y_{ij}^{R} - y_{i,j-1}^{R} \qquad \forall\, i \in N \quad \text{and} \quad \forall\, j \in T : j \neq 1 \qquad (22)$$

Constraints (20), (21), and (22) ensure that the remanufacturing setup cost for a product will be counted once per remanufacturing production run of that specific product.

- **Non-negativity constraints.**

$$Q_{ij}^{M}\, , Q_{ij}^{R}\, , I_{ij}^{s}\, , I_{ij}^{r} \geq 0 \qquad \forall\, i \in N \quad \text{and} \quad \forall\, j \in T \qquad (23)$$

$$y_{ij}^{M}\, , y_{ij}^{R}, x_{ij}^{M}, x_{ij}^{R} \in \{0,1\} \qquad \forall\, i \in N \quad \text{and} \quad \forall\, j \in T \qquad (24)$$

Constraint (23) is to ensure that the values of the stated decision variables must be more than or equal to zero (non-negativity constraint). Constraint (24) is to ensure that the values of the stated binary decision variables can either be equal to zero or one.

Generally speaking, the model has *4nt* binary variables and *4nt* integer variables at most. With respect to constraints, it has $t + 20nt + (2n-2)\sum_{i=1}^{n}(S_i^M + S_i^R - 2) - t\sum_{i=1}^{n}(z_i^M S_i^M + z_i^R S_i^R)$ constraints at most where z_i^M (z_i^R) is a binary variable equal to one when S_i^M (S_i^R) is equal to one and zero otherwise.

3.5 Feasibility Check

Before using the model to solve a given problem, first it is required to check if the problem has a feasible solution or not. Equation (25) was developed for checking the feasibility of the problem. If this condition is not satisfied, then the problem has no feasible solution and there will be no point of using the model to solve the problem.

$$f = \frac{\sum_{i=1}^{n}\left(\overbrace{\left\lceil \frac{\sum_{j=1}^{t}\left(D_{ij} - R_{ij}\right)}{PC_i^M} + \frac{MS_i^M}{PL} \right\rceil}^{\textbf{Part One}} + \overbrace{\left\lceil \frac{\sum_{j=1}^{t} R_{ij}}{PC_i^R} + \frac{MS_i^R}{PL} \right\rceil}^{\textbf{Part Two}}\right)}{t} \leq 1 \qquad (25)$$

Part one in the above equation calculates the least number of periods that have to be allocated to setup and manufacturing of product i. Part two in the above equation calculates the least number of periods that have to be allocated to setup and remanufacturing of product i. Accordingly, the sum of part one and part two will represent the least number of periods allocated to satisfy (setup, manufacturing, and remanufacturing) the demand of product i. As a result, the numerator of equation (25) calculates the least total number of periods allocated to satisfy (setup, manufacturing, and remanufacturing) the demand of all products while the denominator represents the total number of periods available.

3.6 Numerical Example

In this section, an example is presented to illustrate the developed optimization model. This example is solved using three different period lengths (PL); two hours, one hour, and thirty minutes. As discussed earlier, the delivery cycle is assumed to be equal to one week. It is also assumed that the production line can be active seven days a week and eight hours per day. As a result, for solving this example, the delivery cycle will be divided into 28 periods when *PL* is two hours, 56 periods when *PL* is one hour, and 112 periods when *PL* is thirty minutes. In this example, it is assumed that the production line is used to produce (manufacture and remanufacture) three different products (*n*). The return rate (*r*) during the delivery cycle is assumed to be equal to 20% of the demand during that week.

Table 4 and Table 5 represent the scheduled deliveries (demand) and returns of each product during the delivery cycle respectively.

Table 4: Scheduled Deliveries

n	*i*	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
3	**1**	0	700	165	0	0	960	325	**2150**
	2	560	300	0	450	0	0	900	**2210**
	3	800	0	275	320	0	1050	0	**2445**

Table 5: Schedules Returns

n	*i*	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
3	**1**	0	148	0	52	0	95	135	**430**
	2	155	0	70	0	0	97	120	**442**
	3	173	0	0	120	100	96	0	**489**

As it is presented in the above tables, the total sum of scheduled returns for each product during the delivery cycle is equal to 20% of the total sum of scheduled deliveries for that product during that delivery cycle.

The scheduled deliveries and returns are due for delivery and receipt respectively at the end of the day that they are scheduled for.

On the one hand, as it was mentioned earlier, the scheduled deliveries (demand) are delivered at the end of the period that they are due for delivery. Thus, the scheduled deliveries for each day are due for delivery at the end of the last period of that day. For example, the demand for product 1 on Monday is 700 units. Assuming that the period length is equal to thirty minutes resulting in 112 periods per delivery cycle, then the scheduled delivery of product 1 on Monday must be satisfied at the end of period 32. Accordingly, the scheduled deliveries on Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday will correspond to the scheduled deliveries at the end of periods 16, 32, 48, 64, 80, 96, and 112 respectively. Therefore, the scheduled deliveries for the rest of the periods are equal to zero. However, the production can happen at any period.

On the other hand, the scheduled returns are received at the beginning of the period that they are due for receipt. As a result, the scheduled returns for each day are due for receipt at the beginning of the last period of that day. For example, the scheduled returns for product 1 on Monday is 148 units. Assuming that the period length is equal to thirty minutes resulting in 112 periods per delivery cycle, then the scheduled returns of product 1 on Monday must be received at the beginning of period 32. Accordingly, the scheduled returns on Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday will correspond to the scheduled returns at the beginning of periods 16, 32, 48, 64, 80, 96, and 112 respectively. Therefore, the scheduled returns for the rest of the periods are equal to zero.

The manufacturing and serviceable as well as the remanufacturing and returns input parameters for the example with 28 periods per delivery cycle are presented in Tables 6 and 7 respectively.

Table 6: Manufacturing and Serviceable Input Parameters for the Example with 28 Periods

n	i	K_i^M	h_i^s	PC_i^M	MS_i^M	S_i^M
3	1	\$150	\$1.400	700	90	1
	2	\$450	\$0.800	340	60	1
	3	\$250	\$2.000	900	140	2

Table 7: Remanufacturing and Returns Input Parameters for the Example with 28 Periods

n	i	K_i^R	h_i^r	PC_i^R	MS_i^R	S_i^R
3	1	\$85	\$0.600	240	45	1
	2	\$210	\$0.300	160	30	1
	3	\$150	\$1.200	540	90	1

The model was coded and the example was solved using the IBM ILOG optimization software; CPLEX Optimization Studio, version 12.5.1.0. The code can be found in Appendix A.

Before solving the model using CPLEX, the feasibility (*f*) was checked through equation (25). For the example with 28 periods, *f* was equal to 0.82 (82%) which is less than or equal to one (100%). Thus, the example with 28 periods was solved using CPLEX and the optimal solution was found in 1 minute and 2 seconds (computational time). The solution integrated CPS with remanufacturing through determining the least-cost cyclic production schedule that satisfied the demand for the minimum total holding (serviceable and returns) and setup costs (manufacturing and remanufacturing) of $27,190.20. The summary of the results are presented in Figures 7, 8, 9, and 10.

The manufacturing and serviceable as well as the remanufacturing and returns input parameters for the example with 56 periods per delivery cycle are presented in Tables 8 and 9 respectively.

Table 8: Manufacturing and Serviceable Input Parameters for the Example with 56 Periods

n	i	K_i^M	h_i^s	PC_i^M	MS_i^M	S_i^M
3	**1**	$150	$0.700	350	90	2
	2	$450	$0.400	170	60	1
	3	$250	$1.000	450	140	3

Table 9: Remanufacturing and Returns Input Parameters for the Example with 56 Periods

n	i	K_i^R	h_i^r	PC_i^R	MS_i^R	S_i^R
3	**1**	$85	$0.300	120	45	1
	2	$210	$0.150	80	30	1
	3	$150	$0.600	270	90	2

In this case, *f* was equal to 0.75 (75%). CPLEX solved this example and found the optimal solution in 3 minutes and 22 seconds with the minimum total holding and setup costs of $25,485.80. The summary of the results are presented in Figures 11, 12, 13, and 14.

The manufacturing and serviceable as well as the remanufacturing and returns input parameters for the example with 112 periods per delivery cycle are presented in Tables 10 and 11 respectively.

Table 10: Manufacturing and Serviceable Input Parameters for the Example with 112 Periods

n	i	K_i^M	h_i^s	PC_i^M	MS_i^M	S_i^M
3	**1**	\$150	\$0.350	175	90	3
	2	\$450	\$0.200	85	60	2
	3	\$250	\$0.500	225	140	5

Table 11: Remanufacturing and Returns Input Parameters for the Example with 112 Periods

n	i	K_i^R	h_i^r	PC_i^R	MS_i^R	S_i^R
3	**1**	\$85	\$0.150	60	45	2
	2	\$210	\$0.075	40	30	1
	3	\$150	\$0.300	135	90	3

In this case, f was equal to 0.71 (71%). CPLEX solved this example and found the optimal solution in 2 hours and 2 minutes and 38 seconds with the minimum total holding and setup costs of \$24,771.18. The summary of the results are presented in Figures 15, 16, 17, and 18.

As mentioned earlier, this example is solved based on three different period lengths. In order to be able to compare the results for the three different period lengths, it is required to convert the results into to the same period length. The accuracy of the results depends on the period length. Smaller period length will result in a more accurate solution. Accordingly, the results of the example with 28 and 56 periods are going to be converted to 112 periods. This conversion usually increases the value of the objective function. The increase in the objective function value is due to the decrease in the period length. For example, in the case of 28 periods, the period length is equal to two hours. In the periods that the delivery takes place, the inventory cost of the delivered quantity will not be taken into account. This is due to the fact that in this research, the inventory value is assumed to be equal to the amount of inventory at the end of each period. As a result, at the end of the period that delivery takes place, the demand has already been delivered and will not be counted as part of the on-hand inventory. In conclusion, when the example with 28 periods is converted into 112 periods, the amount of inventory equivalent to the delivered quantity will only be taken into account for thirty minutes compared to two hours. This results in an increase in the value of serviceable holding cost due to the remaining 90 minutes.

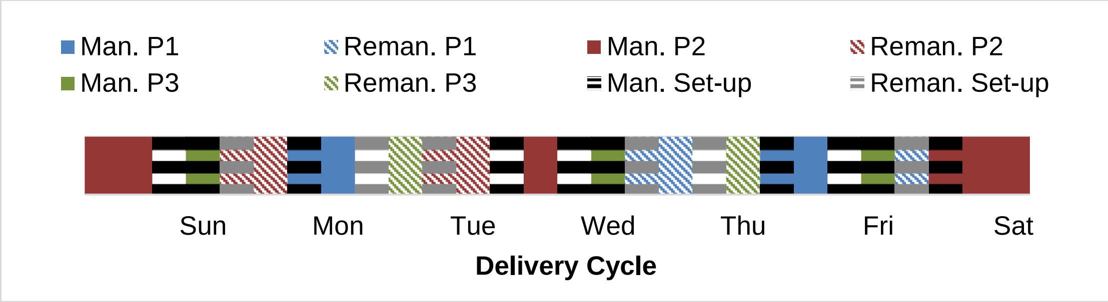

Figure 7: Cyclic Production Schedule for the Example with 28 Periods

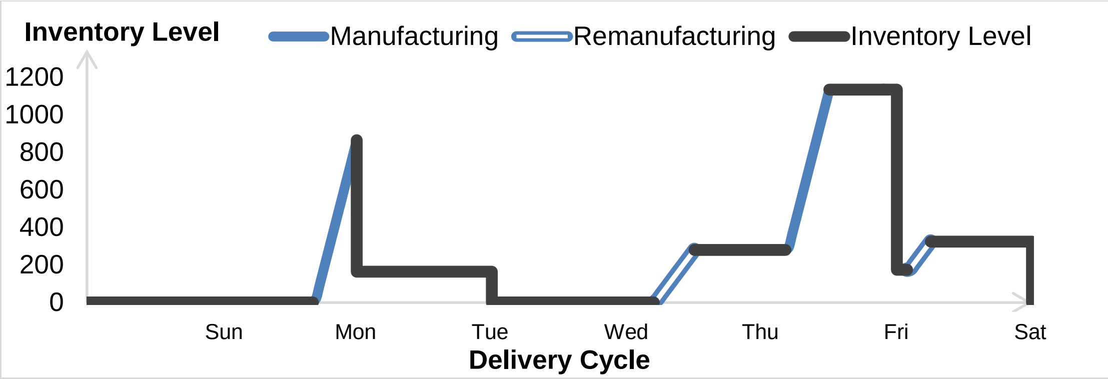

Figure 8: Inventory Evolution of Product 1 for the Example with 28 Periods

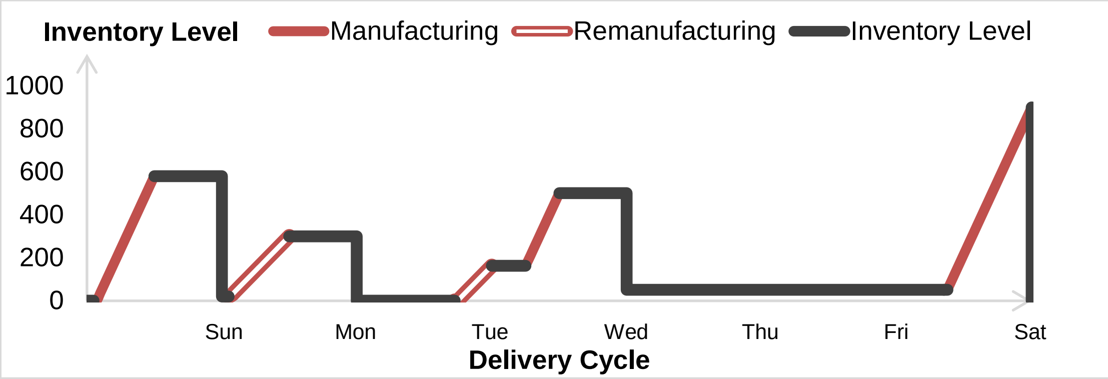

Figure 9: Inventory Evolution of Product 2 for the Example with 28 Periods

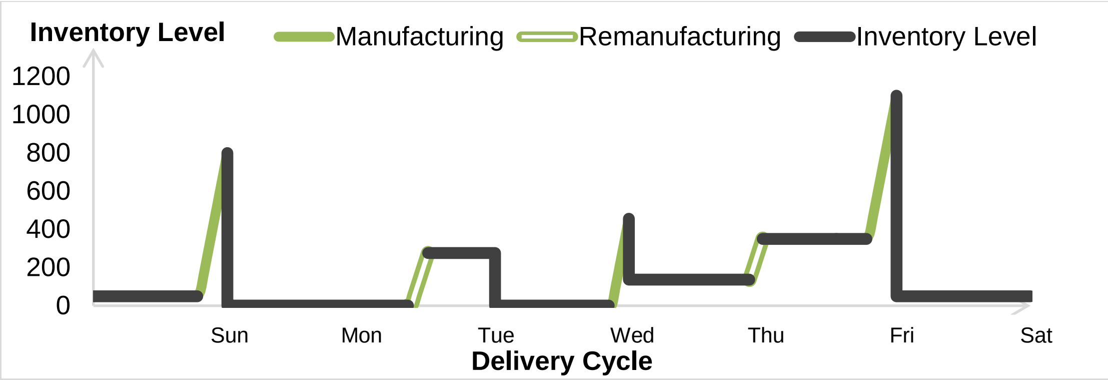

Figure 10: Inventory Evolution of Product 3 for the Example with 28 Periods

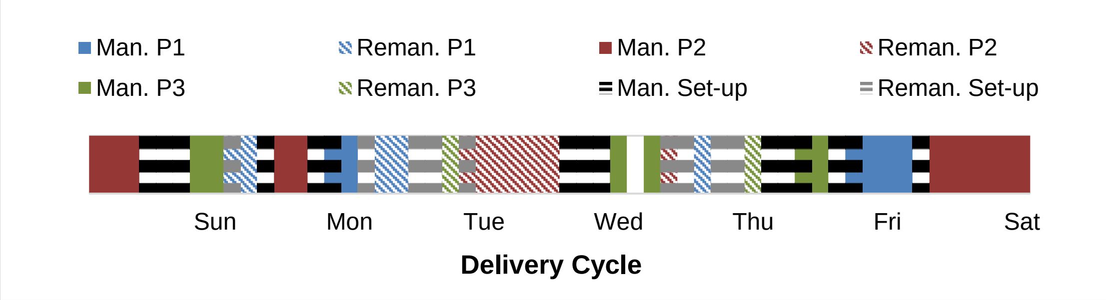

Figure 11: Cyclic Production Schedule for the Example with 56 Periods

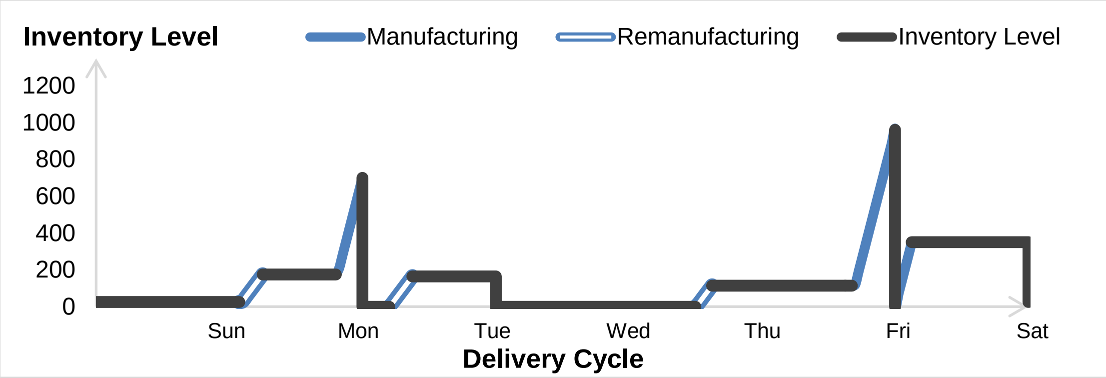

Figure 12: Inventory Evolution of Product 1 for the Example with 56 Periods

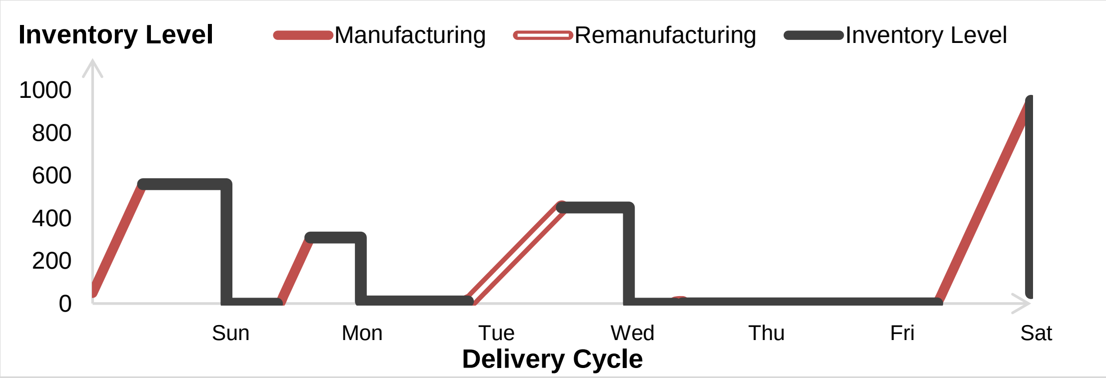

Figure 13: Inventory Evolution of Product 2 for the Example with 56 Periods

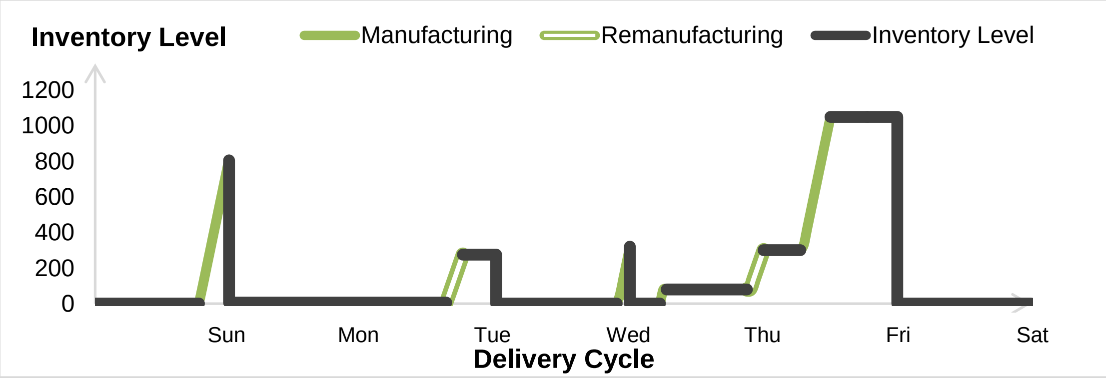

Figure 14: Inventory Evolution of Product 3 for the Example with 56 Periods

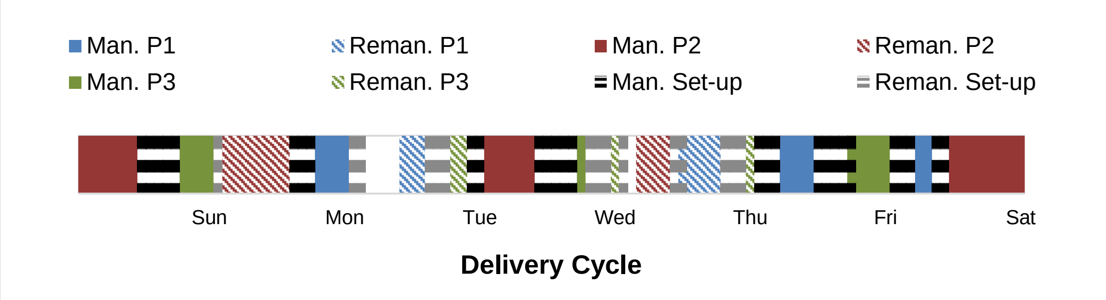

Figure 15: Cyclic Production Schedule for the Example with 112 Periods

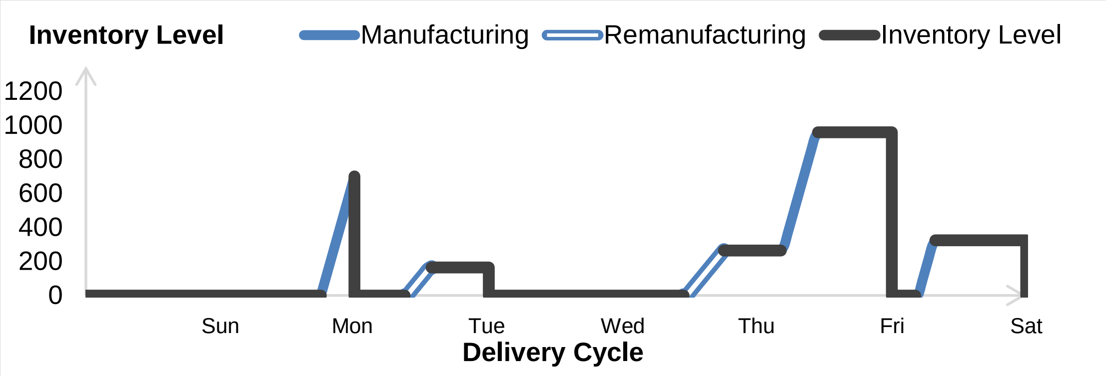

Figure 16: Inventory Evolution of Product 1 for the Example with 112 Periods

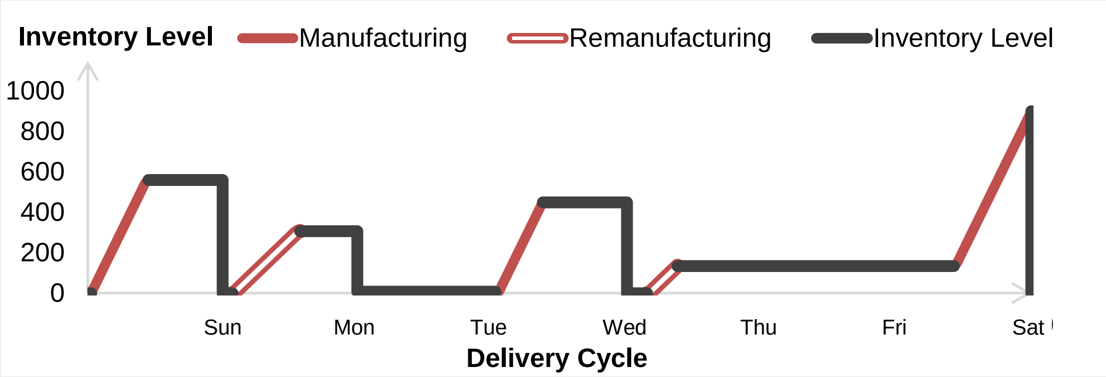

Figure 17: Inventory Evolution of Product 2 for the Example with 112 Periods

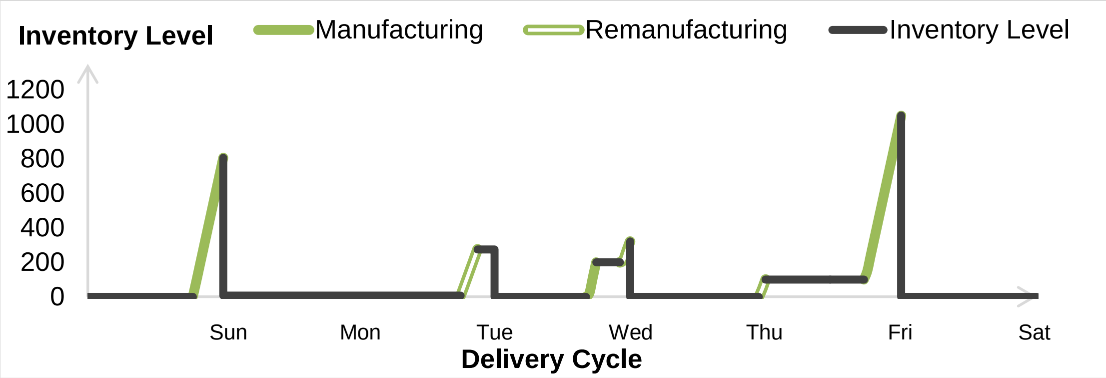

Figure 18: Inventory Evolution of Product 3 for the Example with 112 Periods

The following example is prepared to further illustrate the cost conversion from one period length to another. For this example, *PL* is equal to one hour resulting in 56 periods per delivery cycle. The objective is to convert the results of this example from 56 to 112 periods. Table 12 represents the input parameters of this example for both 56 and 112 periods.

Table 12: Input Parameters of the Illustrative Example Prepared for the Cost Conversion from 56 to 112 Periods

Parameters	56	112
PL	60	30
PC_i^M	100	50
MS_i^M	75	75
S_i^M	2	3
K_i^M	\$300	\$300
PC_i^R	50	25
MS_i^R	30	30
S_i^R	1	1
K_i^R	\$100	\$100
h_i^s	\$0.8	\$0.4
h_i^r	\$0.4	\$0.2

It is assumed that CPLEX have found the optimal solution. Partial results from period 1 to 8 corresponding to Sunday, which is the first day of the delivery cycle, as well as the scheduled deliveries and returns for the same timeframe are presented in Table 13.

Table 13: Partial Results as well as the Scheduled Deliveries and Returns for the First 8 Periods of the Illustrative Example Prepared for the Cost Conversion

j	1	2	3	4	5	6	7	8
D_{ij}	0	0	0	0	0	0	0	200
R_{ij}	0	0	0	0	0	0	0	25
I_{ij}^s	0	0	0	0	0	75	175	0
I_{ij}^r	0	0	0	0	0	0	0	0
Q_{ij}^M	0	0	0	0	0	75	100	0
Q_{ij}^R	0	0	0	0	0	0	0	25
y_{ij}^M	0	0	0	0	1	1	1	0
y_{ij}^R	0	0	0	0	0	0	0	1
x_{ij}^M	0	0	0	0	1	0	0	0
x_{ij}^R	0	0	0	0	0	0	0	1

In order to convert the cost, first the results should be converted from 56 periods to 112 periods. Thus, each of the 8 periods with the *PL* of 60 minutes presented in Table 13 should be divided into two periods resulting in total of 16 periods with the *PL* of 30 minutes.

After converting the *PL*, the scheduled deliveries and returns should be converted. For example, as it was presented in Table 13, 200 units of serviceable are scheduled for delivery at the end of period 8. Period 8 for the case of 56 periods corresponds to periods 15 and 16 for the case of 112 periods. Accordingly, end of period 8 corresponds to end of period 16 when converted. Therefore, that 200 units of serviceable scheduled for delivery at the end of period 8 should be delivered at the end of period 16 once conversion takes place. Also, 25 units of returns are scheduled for receipt at the beginning of period 8. Following the same concept, that 25 units of returns should be received at the beginning of period 15 once conversion takes place.

The next step is to convert the rest of the results. For example, 75 units of products are manufactured in period 6. Period 6 for the case of 56 periods corresponds to periods 11 and 12 for the case of 112 periods. Since the manufacturing production capacity is divided over two for the case of 112 periods, then that 75 units of products manufactured in period 6 will correspond to 25 units manufactured in period 11 and 50 units manufactured in period 12. The conversion of remanufactured products will also follow the same concept.

Finally, to convert the on-hand inventories of the serviceable and returns, the inventory balance constraints, constraints (2), (3), (4), and (5), which were presented in section 3.4, are used.

The conversion of on-hand inventories usually increases the converted value of the objective function. For example, the 25 units of products which were remanufactured during period 8, in the case of 56 periods, are now being remanufactured in period 16 due to the conversion to 112 periods. This will result in the on-hand inventory of returned products to increase by 25 units in period 15 (which increases the holding cost) compared to the original case where the on-hand inventory is equal to zero in period 8.

The same situation can also be observed for the serviceable on-hand inventory. For example, the 175 units of manufactured products delivered at the end of period 8 were not counted as part of the serviceable on-hand inventory in that period for the case of 56 periods. However, now due to the conversion to 112 periods, they are counted as part of the serviceable on-hand inventory for period 15, resulting in an increase in the holding cost of the serviceable on-hand inventory.

The results with respect to the setup cost do not need to be converted since the setup cost is not calculated per period. The final converted results are presented in Table 14. However, since the scheduled deliveries and returns, as well as the decision variables for the first 4 periods, are all equal to zero and do not affect the value of the objective function, they are not presented in Table 14.

Table 14: Converted Partial Results as well as the Scheduled Deliveries and Returns of the Illustrative Example Prepared for the Cost Conversion

j	***56 Periods***	**5**		**6**		**7**		**8**	
	112 Periods	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
D_{ij}		0	0	0	0	0	0	0	200
R_{ij}		0	0	0	0	0	0	25	0
I_{ij}^{s}		0	0	25	75	125	175	175	0
I_{ij}^{r}		0	0	0	0	0	0	25	0
Q_{ij}^{M}		0	0	25	50	50	50	0	0
Q_{ij}^{R}		0	0	0	0	0	0	0	25
y_{ij}^{M}		1	1	1	1	1	1	0	0
y_{ij}^{R}		0	0	0	0	0	0	1	1
x_{ij}^{M}		1	0	0	0	0	0	0	0
x_{ij}^{R}		0	0	0	0	0	0	1	0

The results are converted. At this instance, the original results in Table 13 and the converted results in Table 14 are used to calculate the original and the converted values of the objective function respectively. Accordingly, the objective function expression (1) which was presented in section 3.4 is used for this calculation.

- Original objective function value for 56 periods:

$$\sum_{i=1}^{1}\sum_{j=1}^{8}\$300x_{ij}^{M} + \sum_{i=1}^{1}\sum_{j=1}^{8}\$100x_{ij}^{R} + \sum_{i=1}^{1}\sum_{j=1}^{8}\$0.8I_{ij}^{s} + \sum_{i=1}^{1}\sum_{j=1}^{8}\$0.4I_{ij}^{r} = \$600$$

- Converted objective function value of 56 periods to 112 periods:

$$\sum_{i=1}^{1}\sum_{j=1}^{16}\$300x_{ij}^{M}+\sum_{i=1}^{1}\sum_{j=1}^{16}\$100x_{ij}^{R}+\sum_{i=1}^{1}\sum_{j=1}^{16}\$0.4I_{ij}^{s}+\sum_{i=1}^{1}\sum_{j=1}^{16}\$0.2I_{ij}^{r}=\$635$$

As expected, the conversion from 56 to 112 periods increased the converted objective function value by $35 compared to the original value due to the increase in the inventory holding cost.

Accordingly, for the numerical example presented in this section, the values of the objective function for both 28 and 56 periods are converted to 112 periods. Table 15 summarizes the results of this example for 28, 56, and 112 periods.

Table 15: Summary of Results for 3 Products

<table>
<tr><th>n</th><th>t</th><th>f</th><th>U</th><th>TC</th><th>Gap</th><th>TCC</th><th>IL</th><th>CT "HH:MM:SS"</th></tr>
<tr><td rowspan="7">3</td><td>28</td><td>82%</td><td>100%</td><td>$27,190.20</td><td>0%</td><td>$31,982.70</td><td>29.11%</td><td>"00:01:02"</td></tr>
<tr><td>56</td><td>75%</td><td>98%</td><td>$25,485.80</td><td>0%</td><td>$28,242.40</td><td>14.01%</td><td>"00:03:22"</td></tr>
<tr><td rowspan="5">112</td><td rowspan="5">71%</td><td>NA</td><td>$27,009.50</td><td>20%</td><td>$27,009.50</td><td>9.04%</td><td>"00:07:15"</td></tr>
<tr><td>NA</td><td>$25,253.50</td><td>15%</td><td>$25,253.50</td><td>1.95%</td><td>"00:08:19"</td></tr>
<tr><td>NA</td><td>$25,253.50</td><td>10%</td><td>$25,253.50</td><td>1.95%</td><td>"00:24:50"</td></tr>
<tr><td>NA</td><td>$25,253.50</td><td>5%</td><td>$25,253.50</td><td>1.95%</td><td>"01:28:49"</td></tr>
<tr><td>96%</td><td>$24,771.18</td><td>0%</td><td>$24,771.18</td><td>0%</td><td>"02:02:38"</td></tr>
</table>

In the above table, the headings represent:

- *n* Number of products.
- *t* Number of periods per delivery cycle.
- *f* Feasibility (Calculated using equation (25)).
- *U* System utilization (Calculated by dividing the number of periods during which the production line is busy over the total number of periods per delivery cycle).
- *TC* Objective function value obtained by CPLEX.
- *Gap* Relative MIP Gap Tolerance; which is defined by the *CPLEX Parameters Reference Manual* as “a relative tolerance on the gap between the best integer objective and the objective of the best node remaining” [74]. In other words, it is a relative tolerance on the gap

between the best feasible integer solution and the best existing lower bound. For example, setting the relative MIP gap tolerance to 0.05 results in CPLEX to stop as soon as it has found a feasible integer solution proven to be within five percent of optimal. The default value is equal 10^{-4} (which is assumed to be equal to zero). As a result, when *Gap* becomes zero, the solution is optimal. *Gap* is being calculated by CPLEX based on the following equation [74].

$$Gap = \frac{|Best\ Bound - Best\ Integer|}{(10^{-10} + |Best\ Integer|)} \tag{26}$$

- *TCC* CPLEX obtained total cost for the case of 28 and 56 periods being converted to 112 periods.
- *IL* Percentage of improvement (if positive) or loss (if negative) of the *TC* of 112 periods compared to the *TCC* of 28 and 56 periods. This value is obtained using equation (27).

$$IL = 100 \times \frac{TCC_{28\ or\ 56} - TC_{112}}{TC_{112}} \tag{27}$$

- *CT* Computational time (Hours:Minutes:Seconds).

In Table 15, *NA* stands for not applicable since in those cases the complete solution (values of decision variables) is not available and CPLEX only provides the values of *TC*, *Gap*, and *CT* in the Engine Log tab (Figure 19). According to one of the CPLEX user manuals called *Getting Started with the IDE*, “the Engine Log tab displays information from the solving engine on the solving process and on the objective function” [75].

	Node	Nodes Left	Objective	IInf	Best Integer	Cuts/ Best Bound	ItCnt	Gap
*	0+	0			22507.7500	6137.0305	1058	72.73%
	0	0	6137.7016	254	22507.7500	Cuts: 5	1058	72.73%
	0	0	6137.7016	254	22507.7500	Cuts: 6	1060	72.73%
*	0+	0			22506.2500	6137.7016	1060	72.73%
	0	2	6137.7016	254	22506.2500	6137.7016	1060	72.73%

Elapsed time = 2.29 sec. (299.98 ticks, tree = 0.01 MB, solutions = 2)

Figure 19: CPLEX Engine Log Tab

Figure 20 represents the relationship between *CT* and *TCC* for this example indicating that they have a negative relationship. The corresponding values of the points colored in blue, red, and green presented in Figure 20 are highlighted with the same color in Table 15.

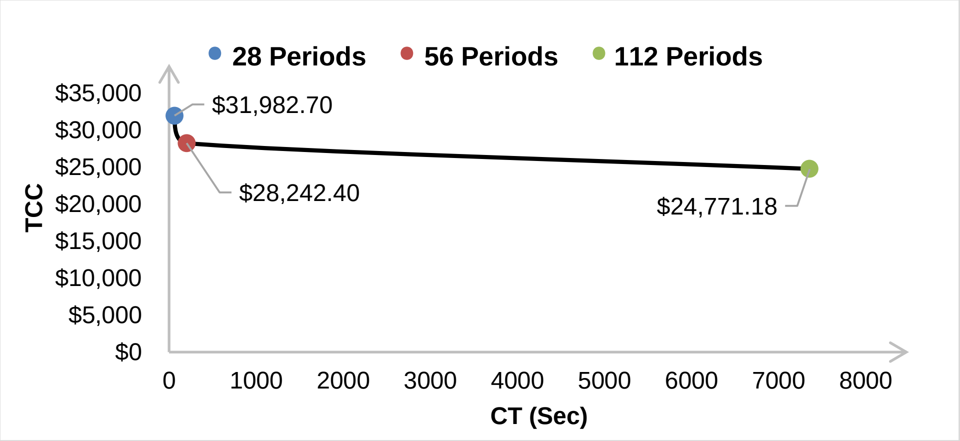

Figure 20: Time vs. Cost Tradeoffs for 3 Products with Respect to the Three Different Period Lengths

Chapter 4: Sensitivity Analysis

In this chapter, the results of the sensitivity analysis performed on the developed MILP model will be presented. The sensitivity analysis was performed in two phases. The first phase of the analysis was performed aiming to find the best *PL* to be used as the base for the second phase of the analysis. The sensitivity analysis was performed under the following assumptions:

- The delivery cycle is assumed to be equal to one week; 7 days per week and 8 hours per day (total of 56 hours per delivery cycle).
- The delivery cycle is discretized into smaller periods.
- All trials will run for a maximum of 24 hours (computational time). If CPLEX could not find the optimal solution within 24 hours, it will be stopped. This timeframe (24 hours) is assumed to be reasonable since the delivery schedule will not change very often.
- The remanufacturing production rate is assumed lower than the manufacturing production rate [63, 65, 71].
- The manufacturing setup time is assumed higher than the remanufacturing setup time [63, 64, 67].
- The manufacturing setup cost is assumed higher than the remanufacturing setup cost [64, 71].
- The holding cost of returned products are assumed lower than the holding cost of finished (manufactured and remanufactured) products [44, 63, 65, 66, 67, 71].

4.1 First Phase

As mentioned earlier, the aim of the first phase of the sensitivity analysis was to identify the best *PL*. The best *PL* will be chosen based on the tradeoff between computational time and objective function value.

Therefore, a total of 24 trials were performed while the number of products (n) were varied from 3 to 10. For each n, an example was solved using three different period lengths: two hours (28 periods), one hour (56 periods), and thirty minutes (112 periods). Further, for this phase, the return rate is assumed to be equal to 20% of the demand during the delivery cycle.

The scheduled deliveries, the scheduled returns, as well as the manufacturing and remanufacturing input parameters for the first phase of the sensitivity analysis are presented in Appendices B, C, D, and E respectively.

Table 16 represents a summary of the results obtained for the first phase of the sensitivity analysis. The detailed results can be found in Appendix F.

Table 16: Summary of the Results for the First Phase of the Sensitivity Analysis

n	*t*	*f*	*U*	*TC*	*Gap*	*TCC*	*IL*	*CT "HH:MM:SS"*
3	28	82%	100%	$27,190.20	0%	$31,982.70	29.11%	"00:01:02"
	56	75%	98%	$25,485.80	0%	$28,242.40	14.01%	"00:03:22"
	112	71%	96%	$24,771.18	0%	$24,771.18	0%	"02:02:38"
4	**28**	**82%**	**89%**	**$12,404.70**	**0%**	**$13,666.95**	**5.65%**	**"00:00:18"**
	56	**75%**	**88%**	**$12,510.45**	**0%**	**$12,994.65**	**0.45%**	**"00:36:32"**
	112	**73%**	**83%**	**$12,936.53**	**10.39%**	**$12,936.53**	**0%**	**"24:00:00"**
5	28	79%	100%	$22,564.12	0%	$24,473.77	10.28%	"00:01:54"
	56	75%	93%	$20,046.64	0%	$20,846.31	-6.07%	"02:15:48"
	112	71%	92%	$22,192.36	24%	$22,192.36	0%	"24:00:00"
6	**28**	**86%**	**96%**	**$19,302.60**	**0%**	**$20,716.50**	**21.51%**	**"00:02:08"**
	56	**75%**	**93%**	**$17,152.85**	**7.07%**	**$17,895.69**	**4.96%**	**"24:00:00"**
	112	**72%**	**90%**	**$17,049.77**	**27.79%**	**$17,049.77**	**0%**	**"24:00:00"**
7	28	86%	96%	$18,602.63	0%	$19,815.89	3.91%	"00:01:54"
	56	75%	96%	$16,580.19	6.50%	$17,147.39	-10.08%	"24:00:00"
	112	71%	94%	$19,069.96	47.82%	$19,069.96	0%	"24:00:00"
8	**28**	**89%**	**100%**	**$19,967.40**	**0%**	**$20,621.01**	**10.66%**	**"05:01:04"**
	56	**75%**	**95%**	**$17,343.35**	**22.56%**	**$17,799.67**	**-4.48%**	**"24:00:00"**
	112	**65%**	**90%**	**$18,634.79**	**48.88%**	**$18,634.79**	**0%**	**"24:00:00"**
9	28	96%	100%	$17,654.40	0%	$18,151.86	29.16%	"10:27:40"
	56	75%	95%	$13,984.50	14.41%	$14,222.33	1.20%	"24:00:00"
	112	68%	88%	$14,053.65	46.91%	$14,053.65	0%	"24:00:00"
10	**28**	**89%**	**100%**	**$17,962.40**	**0%**	**$18,317.08**	**22.24%**	**"00:01:43"**
	56	**75%**	**96%**	**$16,053.05**	**18.94%**	**$16,221.88**	**8.25%**	**"24:00:00"**
	112	**66%**	**92%**	**$14,985.09**	**49.67%**	**$14,985.09**	**0%**	**"24:00:00"**

In order to be able to compare the results of these trials, they should be in the same period length. Accordingly, the result of the trials performed for 28 and 56 periods were converted into 112 periods; using the same concept demonstrated in section 3.6, since as it was mentioned earlier, the accuracy of the solution obtained by the model will increase as the period length becomes smaller.

Based on the results presented in Table 16, the values of the objective function in the case of 112 periods has been improved compared to the converted objective function values of 28 and 56 periods for 13 trials (out of 16 trials performed for 28 and 56 periods). Sharp improvements could be very well observed in 8 of those trials which were highlighted in green color in Table 16.

These results were unexpected since for all of the trials (8 trials) performed for the case of 28 periods, the optimal solution was found in a relatively low computational time. However, still the results for 112 periods, considering that they were not optimal except for the case with 3 products, were improved compared to the converted objective function values of 28 periods.

The same improvement pattern was observed for the 8 trials performed for 56 periods, except for three cases, 5, 7, and 8 products, which were highlighted in red color in Table 16. For these three trials, the values of the objective function for the case of 112 periods did not improve compared to the converted values for 56 periods. However, for the cases with 9 and 10 products, even though the *Gap* was relatively large, sharp improvements were still observed.

Further, these results indicate that the model is capable of solving the problem at hand when the period duration is thirty minutes. For example, the model has even found the optimal solution for 3 products when the period length was equal to thirty minutes. Apart from that, the results of the trials with 112 periods also determine the capability of the developed model in finding near optimal solutions for trials with 4 and 6 products, finding very good solutions for trials with 9 and 10 products, and even finding feasible solutions for trials with 5, 7, and 8 products.

Accordingly, because of the observed improvements in the value of the objective function in trials with 112 periods and the higher accuracy in the results due to the smaller period length, thirty minutes (112 periods) was chosen as the best period duration to be used for the second phase of the sensitivity analysis. Figure 20, which was presented earlier in section 3.6, as well as Figures 21 to 27 presented below, describe the relationship between *CT* and *TCC* for the 24 trials performed during the first phase of the sensitivity analysis.

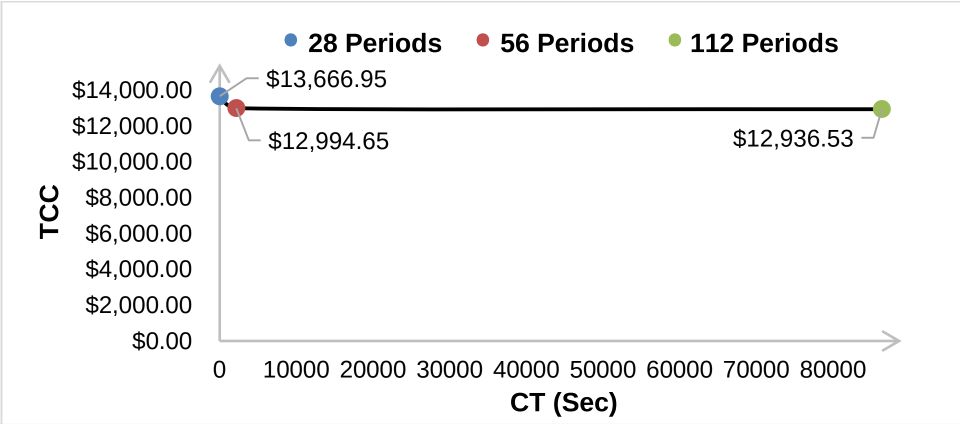

Figure 21: Time vs. Cost Tradeoffs for 4 Products with Respect to the Three Different Period Lengths

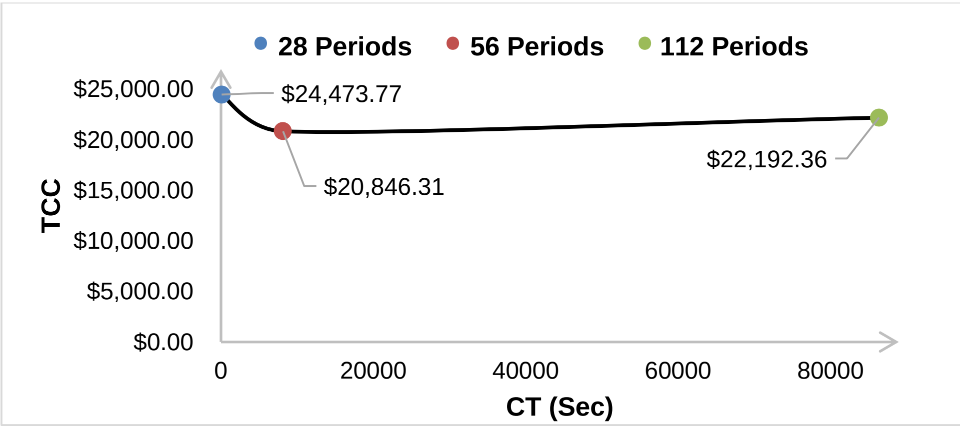

Figure 22: Time vs. Cost Tradeoffs for 5 Products with Respect to the Three Different Period Lengths

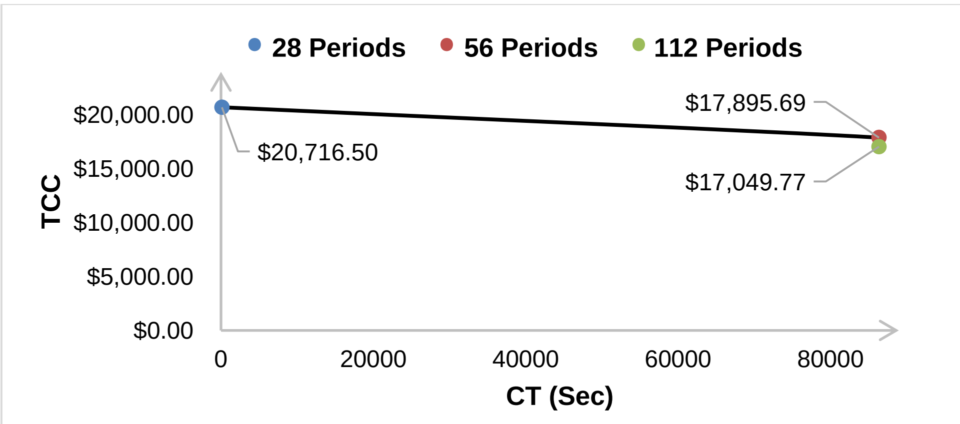

Figure 23: Time vs. Cost Tradeoffs for 6 Products with Respect to the Three Different Period Lengths

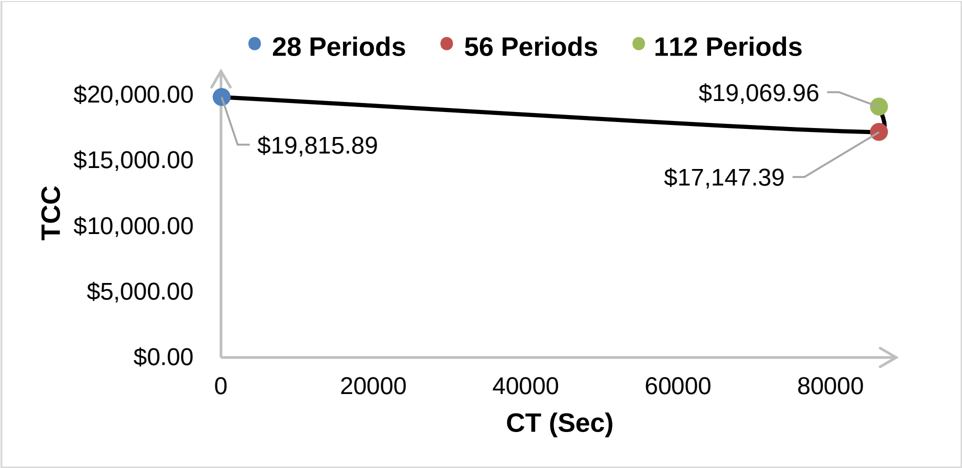

Figure 24: Time vs. Cost Tradeoffs for 7 Products with Respect to the Three Different Period Lengths

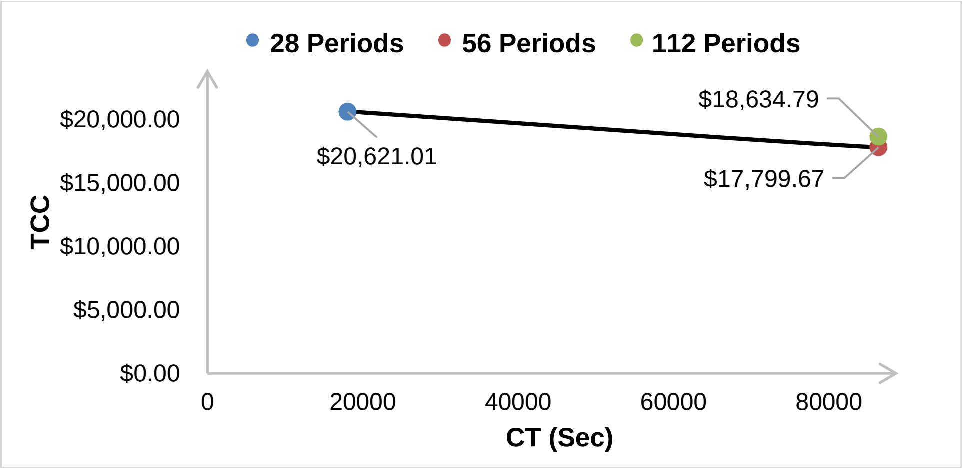

Figure 25: Time vs. Cost Tradeoffs for 8 Products with Respect to the Three Different Period Lengths

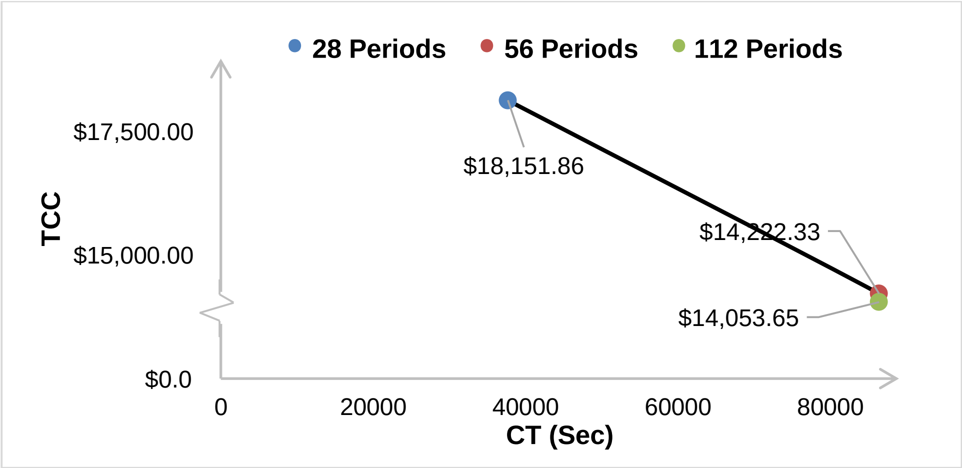

Figure 26: Time vs. Cost Tradeoffs for 9 Products with Respect to the Three Different Period Lengths

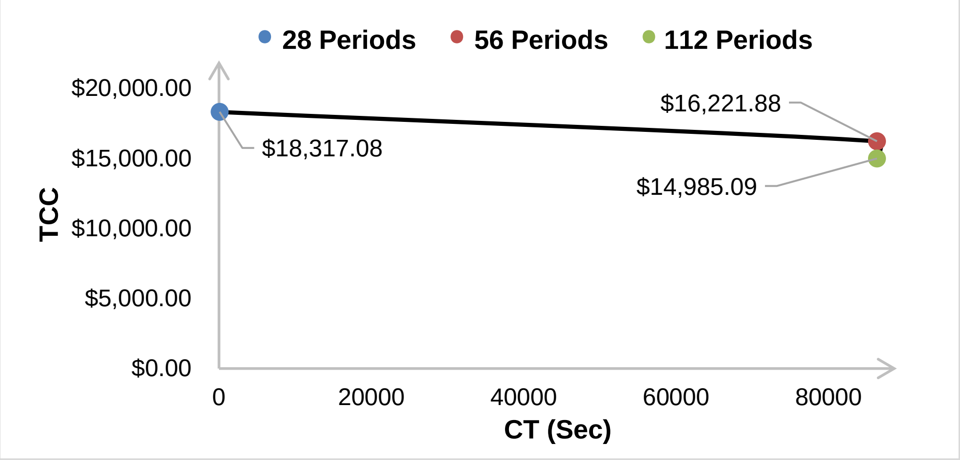

Figure 27: Time vs. Cost Tradeoffs for 10 Products with Respect to the Three Different Period Lengths

At this point, the best period duration is determined. Subsequently, to start the second phase of the sensitivity analysis, a reference case must be chosen from the trials which were conducted using 112 periods. Thus, the case with 3 products and 112 periods was chosen as the reference case (Table 17) since for that case, the optimal solution was found in a relatively low computational time.

Table 17: Reference Case

n	*t*	*f*	*U*	*TC*	*Gap*	*TCC*	*IL*	*CT* "HH:MM:SS"
3	112	71%	96%	$24,771.18	0%	$24,771.18	0%	"02:02:38"

4.2 Second Phase

In the second phase of the sensitivity analysis, the reference case will be used as the base and three parameters will be varied independently aiming to illustrate the effect of these variations on the computational time, as well as the total holding (serviceable and returns) and setup costs (manufacturing and remanufacturing). As a result, a total of 13 trials were performed. The first 4 trials were performed by varying the holding cost for both returned and finished products. The next 4 trials were performed by varying the return rate. The last 5 trials were performed by varying the feasibility through changing the scheduled deliveries and returns.

4.2.1 Holding cost variation. During these 4 trials, the holding cost was varied to illustrate the effect of these variations on the total holding and setup costs (*TC*) as well as computational time (*CT*).

During the first trial, the holding cost for both returned and finished products was divided by four. During the second trial, the holding cost for both returned and finished products was divided by two. During the third trial, the holding cost for both returned and finished products was multiplied by two. During the fourth and last trial, the holding cost for both returned and finished products was multiplied by four. Tables 18 and 19 represent the variation in the holding cost for finished and returned products respectively.

Table 18: Holding Cost Variation for Finished Products

n	*i*	Trial 1	Trial 2	Reference Case	Trial 3	Trial 4
		$h_i^s/4$	$h_i^s/2$	h_i^s	$2h_i^s$	$4h_i^s$
3	1	\$0.088	\$0.175	\$0.350	\$0.700	\$1.400
	2	\$0.050	\$0.100	\$0.200	\$0.400	\$0.800
	3	\$0.125	\$0.250	\$0.500	\$1.000	\$2.000

Table 19: Holding Cost Variation for Returned Products

n	*i*	Trial 1	Trial 2	Reference Case	Trial 3	Trial 4
		$h_i^r/4$	$h_i^r/2$	h_i^r	$2h_i^r$	$4h_i^r$
3	1	\$0.038	\$0.075	\$0.15	\$0.300	\$0.600
	2	\$0.019	\$0.038	\$0.08	\$0.150	\$0.300
	3	\$0.075	\$0.150	\$0.30	\$0.600	\$1.200

Table 20 represent a summary of results for the second phase of the sensitivity analysis with respect to the variations in the holding costs of returned and finished products. The detailed results can be found in Appendix G.

Table 20: Summary of Results for the Second Phase of the Sensitivity Analysis with Respect to the Variation in the Holding Cost of Returned and Finished Products

n	*t*	*f*	*Trials*	*Holding Cost Variation*	*U*	*TC*	*Gap*	*CT "HH:MM:SS"*
3	112	71%	1	¼	91%	\$8,302.77	0%	"18:59:23"
			2	½	91%	\$13,892.40	0%	"08:21:48"
			Ref. Case	1	96%	\$24,771.18	0%	"02:02:38"
			3	2	96%	\$46,402.35	0%	"07:00:06"
			4	4	96%	\$89,664.70	0%	"04:26:32"

Figure 28 represents the relationship between the variation in the holding cost and *CT* indicating that varying the holding cost values of both returned and finished products will result in an increase in *CT*. However, it seems that, decreasing the holding cost will increase *CT* sharply.

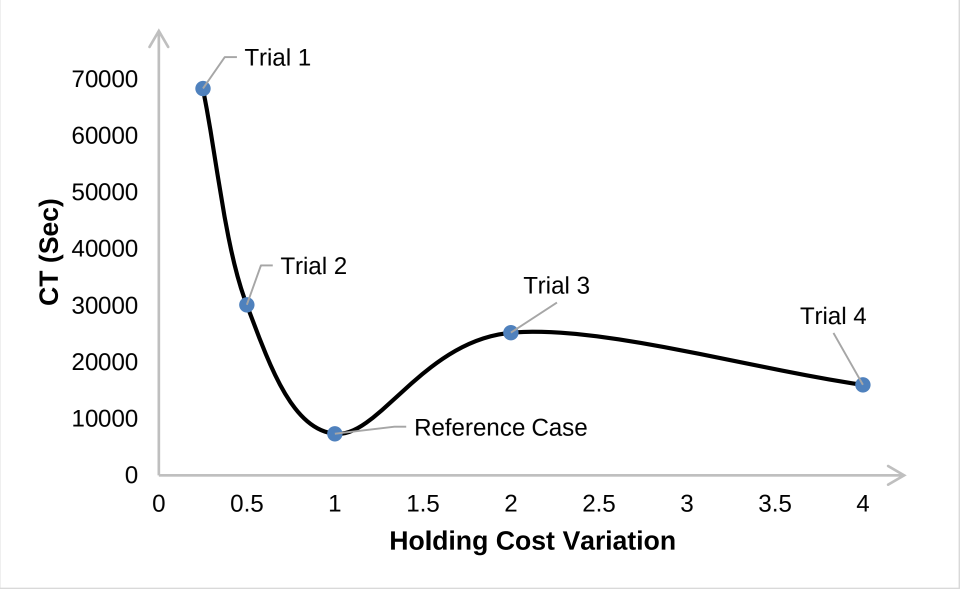

Figure 28: Holding Cost Variation vs. Time

Figure 29 represents the relationship between the variation in the holding cost and TC indicating that they have a strong positive linear relationship. In other words, an increase in the holding cost values will result in an increase in *TC*.

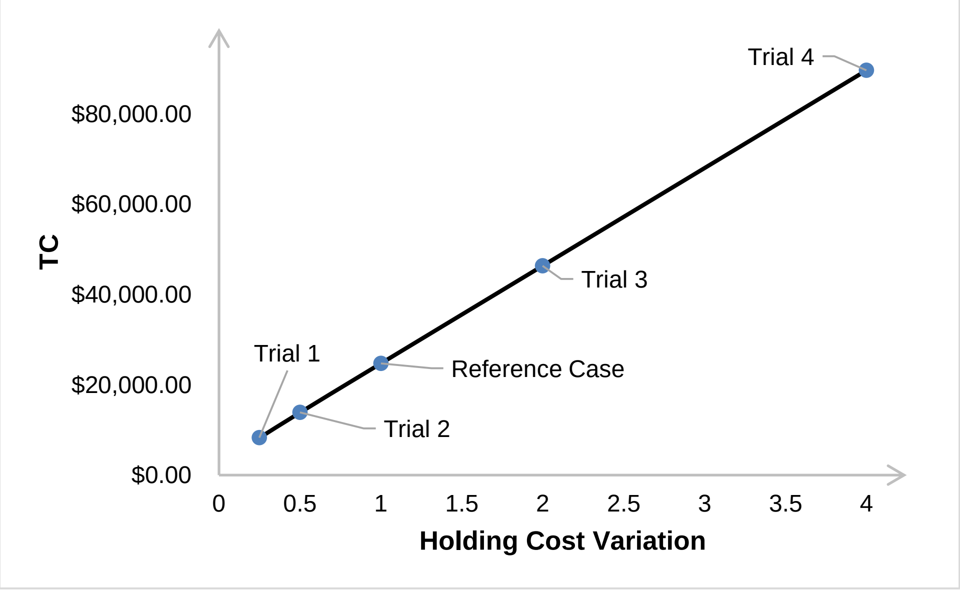

Figure 29: Holding Cost Variation vs. *TC*

4.2.2 Return rate variation. During these 4 trials, the return rate (*r*) was varied from 10% to 30%. The goal was to illustrate the effect of these variations on the total holding and setup costs (*TC*), as well as the computational time (*CT*). The change in the return rate results in the change in the amount of scheduled returns during the delivery cycle. The scheduled returns for each trial can be found in Appendix H.

Table 21 represent a summary of results for the second phase of the sensitivity analysis with respect to the variations in the return rate. The detailed results can be found in Appendix I.

Table 21: Summary of Results for the Second Phase of the Sensitivity Analysis with Respect to the Variation in the Return Rate

n	*t*	*Trials*	*f*	*r*	*U*	*TC*	*Gap*	*CT* "HH:MM:SS"
3	112	1	66%	10%	94%	$21,072.70	0%	"01:04:10"
		2	68%	15%	92%	$22,847.60	0%	"00:51:04"
		Ref. Case	71%	20%	96%	$24,771.18	0%	"02:02:38"
		3	73%	25%	100%	$27,350.20	0%	"10:05:31"
		4	77%	30%	100%	$30,199.15	4.18%	"24:00:00"

Figure 30 represents the positive relationship between the variation in the return rate and *CT* meaning that an increase in the return rate will result in an increase in *CT*. This also suggests a very sharp increase in *CT* as the return rate goes above 20%.

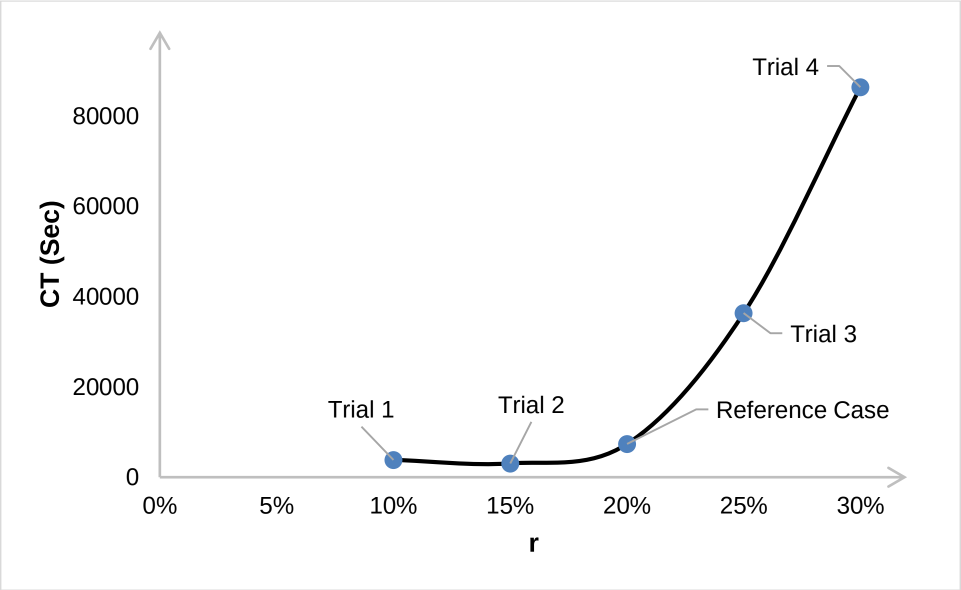

Figure 30: Return Rate Variation vs. Time

Figure 31 represents the positive relationship between the variation in the return rate and *TC* meaning that an increase in the return rate will result in an increase in *TC*. This increase could be due to the lower remanufacturing production capacity and the fact that all of the returned products should be remanufactured.

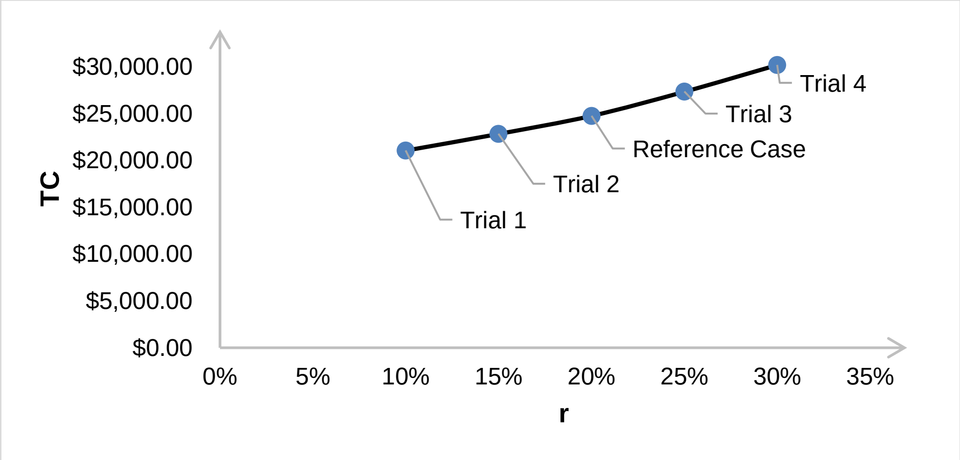

Figure 31: Return Rate Variation vs. *TC*

4.2.3 Feasibility variation. During these 5 trials, the feasibility (*f*) was varied from 75% to 95% through changing the amount of scheduled deliveries and returns. The goal was to illustrate the effect of these variations on the computational time. The scheduled deliveries and returns for each trial can be found in Appendices J and K respectively.

Table 22 represent a summary of results for the second phase of the sensitivity analysis with respect to the feasibility variations. CPLEX could not find any solution for the last 3 trials within the 24 hours timeframe: *f* equal to 85%, 90%, and 95%. The detailed results can be found in Appendix L.

Table 22: Summary of Results for the Second Phase of the Sensitivity Analysis with Respect to the Feasibility Variation

n	*t*	*Trials*	*f*	*U*	*TC*	*Gap*	*CT "HH:MM:SS"*
3	112	Ref. Case	71%	96%	$24,771.18	0%	"02:02:38"
		1	75%	98%	$26,457.15	0%	"04:13:16"
		2	80%	100%	$33,331.00	9.86%	"24:00:00"
		3	85%	NA	No Solution	NA	"24:00:00"
		4	90%	NA	No Solution	NA	"24:00:00"
		5	95%	NA	No Solution	NA	"24:00:00"

Figure 32 represents the positive relationship between the feasibility variation and *CT* meaning that an increase in *f* will result in an exponential increase in *CT*, since as *f* increases, it will be more difficult to find a solution.

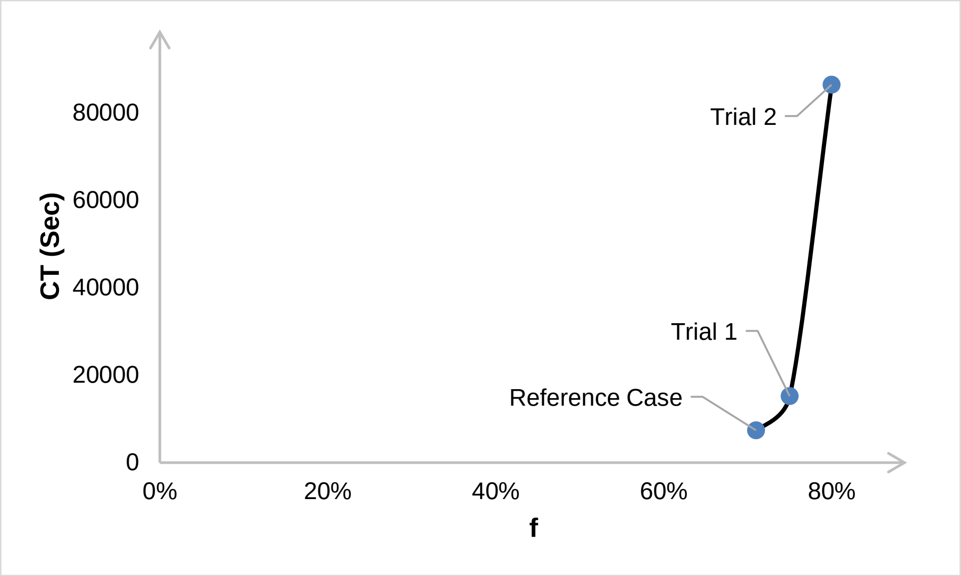

Figure 32: Feasibility Variation vs. Time

Chapter 5: Conclusion and Future Research Directions

5.1 Conclusion

This research addressed the integration of cyclic production scheduling (CSP) with remanufacturing. Nowadays, organizations, such as the ones in the automotive sector, are becoming linked to their customers through contracts specifying cyclic delivery schedules. Cyclic delivery schedules can be classified as a special type of dynamic demand patterns. Some of these organizations also practice remanufacturing due to its environmental and economic benefits. A comprehensive literature review revealed that, among the studies that have been conducted so far, none of them considered cyclic deliveries in order to build cyclic production schedules integrating both manufacturing and remanufacturing.

As a result, the objective of this research was to develop a mathematical model integrating CPS with remanufacturing to satisfy a given cyclic delivery schedule. The developed model determines the least-cost cyclic production schedule that satisfies the demand and minimizes the total holding (serviceable and returns) and setup costs (manufacturing and remanufacturing) while integrating CPS with remanufacturing. There are some similar studies in the literature where they consider a dynamic demand and a finite production capacity. But to the best of our knowledge, they do not exploit the fact that the demand is following a cyclic pattern and they do not aim to build a cyclic production schedule. However, in this research, it is assumed that the demand follows a cyclic pattern and is provided as a given as a cyclic delivery schedule. All of these studies also assume that the beginning inventory is equal to zero or to a pre-established initial value and their objective is to determine in which periods they should produce or not in order to satisfy the demand while minimizing total holding and setup costs. However, as mentioned earlier, the objective of this research is to develop a cyclic production schedule where the levels of on-hand inventory at the beginning and end of the production cycle should be equal and should be determined. Further, the determined least-cost cyclic production schedule should satisfy the demand and minimize the total holding and setup costs while integrating CPS with remanufacturing.

The next step was to code, solve, and perform a sensitivity analysis on the developed model using the IBM ILOG optimization software, CPLEX Optimization

Studio, version 12.5.1.0. After coding and solving the model using CPLEX, the sensitivity analysis was performed in two phases. During both phases of the sensitivity analysis, trials were run for a maximum of 24 hours (computational time). If CPLEX could not find the optimal solution within 24 hours, it was stopped. This timeframe (24 hours) was assumed to be reasonable since, the delivery schedule would not change very often.

The first phase of the analysis aimed at finding the best period duration. Accordingly, one example corresponding to each *n* (number of products) was designed and *n* was varied from 3 to 10 products. The delivery cycle was assumed to be equal to one week; 7 days per week and 8 hours per day (total of 56 hours per delivery cycle). Each of those 8 examples were solved for three different period lengths: two hours (28 periods), one hour (56 periods), and thirty minutes (112 periods).

Therefore, a total of 24 trials were conducted. In order to be able to compare the results of these trials, they had to be in the same period length. Accordingly, the result of the trials performed for 28 and 56 periods were converted into 112 periods since the accuracy of the solution obtained by the model will increase as the period length becomes smaller.

Based on the results of those trials, the values of the objective function in the case of 112 periods, were improved compared to the converted objective function values of 28 and 56 periods for 13 trials (out of 16 trials performed for 28 and 56 periods). Sharp improvements could be very well observed in 8 of those trials.

Actually, the obtained results of the first phase were unexpected since the results for the case of 112 periods were improved compared to the converted objective function values of 28 periods. This improvement have happened, even though, for all of the trials (8 trials) performed for the case of 28 periods, the optimal solution was found in a relatively low computational time. Apart from that, the results for the case of 112 periods were not optimal, except for the case with 3 products. The same pattern was observed for the 8 trials performed for 56 periods, except for three cases; 5, 7, and 8 products. For example, for the case of 56 periods with 9 and 10 products, sharp improvements were observed.

Based on these results, it can be concluded that the model is capable of solving the problem at hand and even finding the optimal solution (the case with 3 products) when the period duration is thirty minutes. Apart from that, this proves the capability of the developed model in finding near optimal solutions for trials with 4 and 6 products, finding very good solutions for trials with 9 and 10 products, and even finding feasible solutions for trials with 5, 7, and 8 products when the period length is thirty minutes.

As a result, thirty minutes (112 periods) was chosen as the best period duration to be used for the second phase of the sensitivity analysis. Subsequently, to start the second phase, a reference case had to be chosen from the trials which were conducted using 112 periods. Thus, the case with 3 products and 112 periods was chosen as the reference case since for that case, the optimal solution was found in a relatively low computational time.

For the second phase of the sensitivity analysis, a total of 13 trials were designed and performed through the variation of three different parameters to illustrate the effect of these variations on the total holding (serviceable and returns) and setup costs (manufacturing and remanufacturing), as well as the computational time.

- The first 4 trials were performed to illustrate the effect of varying the holding cost of both returned and finished products on the total holding and setup costs, as well as the computational time. The results indicated that varying the holding cost values of both returned and finished products will result in an increase in the computational time. Also, increasing the holding cost values will result in an increase in the total holding and setup costs.
- The next 4 trials were performed to illustrate the effect of varying the return rate on the total holding and setup costs, as well as the computational time. The results indicated that an increase in the return rate will result in an increase in the total holding and setup cost, as well as the computational time.
- The last 5 trials were performed to illustrate the effect of varying the feasibility (through changing the scheduled deliveries and returns) on the computational time. The results indicated that increasing feasibility will cause the computational time to increase exponentially.

In conclusion, assuming that the delivery cycle is equal to one week, 7 days per week and 8 hours per day, thirty minutes (112 periods) would be the best period duration for the developed MILP model. This decision is based on the results of the first phase of the sensitivity analysis indicating the observed improvements in the values of the objective function for the case of 112 periods, compared to the converted objective function values of 28 and 56 periods. Furthermore, these results validate the capability of the developed model in solving the problem at hand and finding the optimal, near optimal, and good feasible solutions for the period duration of thirty minutes.

Based on the results of the second phase of the sensitivity analysis, the increase in the number of decision variables, which depends on the number of products as well as the number of periods, will result in the computational time to increase exponentially. Further, the increase in the feasibility (*f*), regardless of the number of decision variables, will also result in the computational time to increase exponentially. The reason behind this exponential increase is due to the lower chance of finding a feasible solution. Also, varying the holding cost or increasing the return rate will result in an increase in the computational time. Finally, increasing the holding cost or increasing the return rate will result in an increase in the total holding and setup costs. Therefore, all of these parameters must be taken into consideration for the developed model to function as expected.

5.2 Future Research Directions

The model presented in this research was developed under several assumptions which were mentioned earlier. For future research directions, the model can be expanded through modifying some of the aforementioned assumptions. For example, allowing backlogs and adding subcontracting to the model, increasing the length of the delivery cycle, increasing number of production lines and/or having separate production lines for manufacturing and remanufacturing, and not having the manufacturing and remanufacturing products be identical. Furthermore, improving the current result, and even performing a study on increasing the number of products, adding the inventory for raw materials and considering transportation from both suppliers and customers point of view for raw materials, finished products (manufactured and remanufactured), as well as returned products, can also be considered as future research directions.

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Appendix A

CPLEX Code

```
// Parameters

	// Number of products (n)
	// Product index (i)
	int n=...;
	range N=1..n;

	// Number of periods per delivery cycle (t)
	// Period index (j or w)
	int t=...;
	range T=1..t;

	// Period length or period duration (minutes)
	float PL=...;

	// Setup cost for manufacturing product i
	float KM[N]=...;

	// Setup cost for remanufacturing product i
	float KR[N]=...;

	// Setup time (minutes) to manufacture product i
	int MSM[N]=...;

	// Setup time (minutes) to remanufacture product i
	int MSR[N]=...;

	// Setup time (periods) to manufacture product i; rounded up
	value of (MSM[N]/PL)
	int SM[N]=...;

	// Setup time (periods) to remanufacture product i; rounded up
	value of (MSR[N]/PL)
	int SR[N]=...;

	// Holding cost to carry a serviceable (manufactured or
	remanufactured) unit of product i inventory from period j to
	period j+1
	float Hs[N]=...;

	// Holding cost to carry a returned unit of product i inventory
	from period j to period j+1
	float Hr[N]=...;

	// Production capacity of manufacturing product i per period
	float PCM[N]=...;

	// Production capacity of remanufacturing product i per period
	float PCR[N]=...;

	// Demand of product i to be delivered at the end period j
	int D[N][T]=...;
```

CPLEX Code (continued)

```
	// Amount of returns of product i received at the beginning of
	period j
	int R[N][T]=...;


// Decision Variables

	// Serviceable inventory level of product i at the end of
	period j
	dvar int+ Is[N][T];

	// Returned inventory level of product i at the end of period j
	dvar int+ Ir[N][T];

	// Quantity of product i manufactured in period j
	dvar int+ QM[N][T];

	// Quantity of product i remanufactured in period j
	dvar int+ QR[N][T];

	// Binary variable equal to one when the production line is
	allocated to product i for its setup or manufacturing in period
	j and zero otherwise
	dvar boolean YM[N][T];

	// Binary variable equal to one when the production line is
	allocated to product i for its setup or remanufacturing in
	period j and zero otherwise
	dvar boolean YR[N][T];

	// Binary variable equal to one when setup for manufacturing
	product i started in period j (or w) and zero otherwise
	dvar boolean XM[N][T];

	// Binary variable equal to one when setup for remanufacturing
	product i started in period j (or w) and zero otherwise
	dvar boolean XR[N][T];


// Objective Function - (1)

	dexpr float TotalCost = sum(i in N, j in T) KM[i]*XM[i][j]+
	                        sum(i in N, j in T) KR[i]*XR[i][j]+
	                        sum(i in N, j in T) Hs[i]*Is[i][j]+
	                        sum(i in N, j in T) Hr[i]*Ir[i][j];
	minimize TotalCost;


// Constrains

 subject to {

	// (2)
	forall(i in N)
	  Is[i][1] == Is[i][t]+QM[i][1]+QR[i][1]-D[i][1];
```

CPLEX Code (continued)

```
// (3)
forall(i in N)
  Ir[i][1] == Ir[i][t]-QR[i][1]+R[i][1];

// (4)
forall(i in N, j in T : j!=1)
  Is[i][j] == Is[i][j-1]+QM[i][j]+QR[i][j]-D[i][j];

// (5)
forall(i in N, j in T : j!=1)
  Ir[i][j] == Ir[i][j-1]-QR[i][j]+R[i][j];

// (6)
forall(i in N, j in T)
  QM[i][j] <= YM[i][j]*PCM[i];

// (7)
forall(i in N, j in T : j>=SM[i])
  QM[i][j] <= PCM[i]*(1-((1-(SM[i]-(MSM[i]/PL)))*
              (sum(w in j-SM[i]+1..j) XM[i][w])));

// (8)
forall(i in N : SM[i]>1, j in T : j>=SM[i])
  QM[i][j] <= PCM[i]*(1-sum(w in j-SM[i]+2..j) XM[i][w]);

// (9)
forall(i in N, j in T : j<SM[i])
  QM[i][j] <= PCM[i]*(1-((1-(SM[i]-(MSM[i]/PL)))*
              ((sum(w in t-SM[i]+j+1..t) XM[i][w])+
              (sum(w in 1..j) XM[i][w]))));

// (10)
forall(i in N : SM[i]>1, j in T : j<SM[i])
  QM[i][j] <= PCM[i]*(1-(sum(w in t-SM[i]+j+2..t) XM[i][w])-
              (sum(w in 1..j) XM[i][w]));

// (11)
forall(i in N, j in T)
  QR[i][j] <= YR[i][j]*PCR[i];

// (12)
forall(i in N, j in T : j>=SR[i])
  QR[i][j] <= PCR[i]*(1-((1-(SR[i]-(MSR[i]/PL)))*
              (sum(w in j-SR[i]+1..j) XR[i][w])));

// (13)
forall(i in N : SR[i]>1, j in T : j>=SR[i])
  QR[i][j] <= PCR[i]*(1-sum(w in j-SR[i]+2..j) XR[i][w]);

// (14)
forall(i in N, j in T : j<SR[i])
  QR[i][j] <= PCR[i]*(1-((1-(SR[i]-(MSR[i]/PL)))*
              ((sum(w in t-SR[i]+j+1..t) XR[i][w])+
              (sum(w in 1..j) XR[i][w]))));
```

CPLEX Code (continued)

```
        // (15)
        forall(i in N : SR[i]>1, j in T : j<SR[i])
          QR[i][j] <= PCR[i]*(1-(sum(w in t-SR[i]+j+2..t) XR[i][w])-
                     (sum(w in 1..j) XR[i][w]));
        // (16)
        forall(j in T)
          sum(i in N) YM[i][j]+sum(i in N) YR[i][j] <= 1;

        // (17)
        forall(i in N, j in T)
          YM[i][j] >= XM[i][j];

        // (18)
        forall(i in N)
          XM[i][1] >= YM[i][1]-YM[i][t];

        // (19)
        forall(i in N, j in T : j!=1)
          XM[i][j] >= YM[i][j]-YM[i][j-1];

        // (20)
        forall(i in N, j in T)
          YR[i][j] >= XR[i][j];

        // (21)
        forall(i in N)
          XR[i][1] >= YR[i][1]-YR[i][t];

        // (22)
        forall(i in N, j in T : j!=1)
          XR[i][j] >= YR[i][j]-YR[i][j-1];

 }



// Data

        SheetConnection my_sheet("Thesis.xlsx");

        n from SheetRead(my_sheet,"n");
        t from SheetRead(my_sheet,"t");
        PL from SheetRead(my_sheet,"PL");
        KM from SheetRead(my_sheet,"KM");
        KR from SheetRead(my_sheet,"KR");
        MSM from SheetRead(my_sheet,"MSM");
        MSR from SheetRead(my_sheet,"MSR");
        SM from SheetRead(my_sheet,"SM");
        SR from SheetRead(my_sheet,"SR");
        Hs from SheetRead(my_sheet,"Hs");
        Hr from SheetRead(my_sheet,"Hr");
        PCM from SheetRead(my_sheet,"PCM");
        PCR from SheetRead(my_sheet,"PCR");
        D from SheetRead(my_sheet,"Demand");
        R from SheetRead(my_sheet,"Return");
```

CPLEX Code (continued)

```
Is to SheetWrite(my_sheet,"Is");
Ir to SheetWrite(my_sheet,"Ir");
QM to SheetWrite(my_sheet,"QM");
QR to SheetWrite(my_sheet,"QR");
YM to SheetWrite(my_sheet,"YM");
YR to SheetWrite(my_sheet,"YR");
XM to SheetWrite(my_sheet,"XM");
XR to SheetWrite(my_sheet,"XR");
TotalCost to SheetWrite(my_sheet,"TC");
```

Appendix B

Scheduled Deliveries for the First Phase of the Sensitivity Analysis

n	*i*	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
3	**1**	0	700	165	0	0	960	325	**2150**
	2	560	300	0	450	0	0	900	**2210**
	3	800	0	275	320	0	1050	0	**2445**
4	**1**	0	350	110	0	0	0	200	**660**
	2	260	100	0	0	0	0	310	**670**
	3	550	0	130	0	0	190	0	**870**
	4	0	0	145	75	110	0	0	**330**
5	**1**	0	350	235	0	0	0	200	**785**
	2	486	200	0	0	0	0	324	**1010**
	3	550	0	175	0	0	250	0	**975**
	4	0	0	155	90	320	0	0	**565**
	5	0	475	0	225	0	0	260	**960**
6	**1**	0	350	150	0	0	0	200	**700**
	2	225	65	0	0	0	0	90	**380**
	3	525	0	275	0	0	50	0	**850**
	4	0	0	230	45	125	0	0	**400**
	5	0	275	0	375	0	0	150	**800**
	6	310	120	0	0	0	205	0	**635**
7	**1**	0	250	70	0	0	0	110	**430**
	2	225	100	0	0	0	0	75	**400**
	3	175	0	125	0	0	150	0	**450**
	4	0	0	205	50	140	0	0	**395**
	5	0	350	0	100	0	0	150	**600**
	6	485	180	0	0	0	265	0	**930**
	7	0	0	225	0	0	0	75	**300**
8	**1**	0	50	150	0	0	0	300	**500**
	2	225	100	0	0	0	0	75	**400**
	3	175	0	275	0	0	150	0	**600**
	4	0	0	205	50	45	0	0	**300**
	5	0	350	0	100	0	0	150	**600**
	6	50	20	0	0	0	130	0	**200**
	7	0	0	125	0	0	0	75	**200**
	8	0	120	0	0	130	0	200	**450**

Scheduled Deliveries for the First Phase of the Sensitivity Analysis (continued)

n	*i*	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
9	**1**	0	50	150	0	0	0	100	**300**
	2	225	100	0	0	0	0	75	**400**
	3	175	0	175	0	0	150	0	**500**
	4	0	0	205	50	45	0	0	**300**
	5	0	250	0	100	0	0	150	**500**
	6	50	20	0	0	0	130	0	**200**
	7	0	0	25	0	0	0	75	**100**
	8	0	120	0	0	50	0	160	**330**
	9	175	0	20	0	90	0	0	**285**
10	**1**	0	50	150	0	0	0	300	**500**
	2	225	100	0	0	0	0	75	**400**
	3	175	0	275	0	0	150	0	**600**
	4	0	0	205	50	45	0	0	**300**
	5	0	350	0	100	0	0	150	**600**
	6	50	20	0	0	0	130	0	**200**
	7	0	0	25	0	0	0	75	**100**
	8	0	120	0	0	130	0	200	**450**
	9	120	0	20	0	110	0	0	**250**
	10	0	60	0	40	0	0	100	**200**

Appendix C

Scheduled Returns for the First Phase of the Sensitivity Analysis

n	*i*	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
3	**1**	0	148	0	52	0	95	135	**430**
	2	155	0	70	0	0	97	120	**442**
	3	173	0	0	120	100	96	0	**489**
4	**1**	0	45	0	0	0	55	32	**132**
	2	78	0	15	0	0	41	0	**134**
	3	37	0	0	76	0	61	0	**174**
	4	0	16	0	0	35	15	0	**66**
5	**1**	0	75	0	0	0	35	47	**157**
	2	92	0	33	0	0	77	0	**202**
	3	39	0	0	76	0	80	0	**195**
	4	0	27	0	0	69	17	0	**113**
	5	0	0	62	80	0	0	50	**192**
6	**1**	0	60	0	0	0	35	45	**140**
	2	35	0	21	0	0	20	0	**76**
	3	85	0	0	55	0	30	0	**170**
	4	0	15	0	0	40	25	0	**80**
	5	0	0	25	90	0	0	45	**160**
	6	65	0	0	0	25	0	37	**127**
7	**1**	0	30	0	0	0	35	21	**86**
	2	35	0	25	0	0	20	0	**80**
	3	45	0	0	30	0	15	0	**90**
	4	0	15	0	0	39	25	0	**79**
	5	0	0	25	70	0	0	25	**120**
	6	75	0	0	0	65	0	46	**186**
	7	40	0	20	0	0	0	0	**60**
8	**1**	0	40	0	0	0	35	25	**100**
	2	35	0	25	0	0	20	0	**80**
	3	55	0	0	45	0	20	0	**120**
	4	0	10	0	0	30	20	0	**60**
	5	0	0	25	70	0	0	25	**120**
	6	5	0	0	0	25	0	10	**40**
	7	0	0	40	0	0	0	0	**40**
	8	30	0	0	25	0	35	0	**90**

Scheduled Returns for the First Phase of the Sensitivity Analysis (continued)

n	*i*	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
9	**1**	0	20	0	0	0	25	15	**60**
	2	35	0	25	0	0	20	0	**80**
	3	45	0	0	35	0	20	0	**100**
	4	0	10	0	0	30	20	0	**60**
	5	0	0	25	50	0	0	25	**100**
	6	5	0	0	0	25	0	10	**40**
	7	0	0	20	0	0	0	0	**20**
	8	16	0	0	15	0	35	0	**66**
	9	0	35	0	0	0	0	22	**57**
10	**1**	0	40	0	0	0	35	25	**100**
	2	35	0	25	0	0	20	0	**80**
	3	55	0	0	45	0	20	0	**120**
	4	0	10	0	0	30	20	0	**60**
	5	0	0	25	70	0	0	25	**120**
	6	5	0	0	0	25	0	10	**40**
	7	0	0	20	0	0	0	0	**20**
	8	30	0	0	25	0	35	0	**90**
	9	0	25	0	0	0	0	25	**50**
	10	0	0	25	0	0	15	0	**40**

Appendix D

Manufacturing and Serviceable Input Parameters for the First Phase of the Sensitivity Analysis

n	i	t	K_i^M	h_i^s	PC_i^M	MS_i^M	S_i^M
3	1	28	$150	$1.400	700	90	1
3	2	28	$450	$0.800	340	60	1
3	3	28	$250	$2.000	900	140	2
3	1	56	$150	$0.700	350	90	2
3	2	56	$450	$0.400	170	60	1
3	3	56	$250	$1.000	450	140	3
3	1	112	$150	$0.350	175	90	3
3	2	112	$450	$0.200	85	60	2
3	3	112	$250	$0.500	225	140	5
4	1	28	$150	$1.600	240	75	1
4	2	28	$320	$1.000	200	30	1
4	3	28	$400	$1.500	170	105	1
4	4	28	$275	$0.800	300	135	2
4	1	56	$150	$0.800	120	75	2
4	2	56	$320	$0.500	100	30	1
4	3	56	$400	$0.750	85	105	2
4	4	56	$275	$0.400	150	135	3
4	1	112	$150	$0.400	60	75	3
4	2	112	$320	$0.250	50	30	1
4	3	112	$400	$0.375	42	105	4
4	4	112	$275	$0.200	75	135	5
5	1	28	$150	$2.000	360	45	1
5	2	28	$420	$1.600	400	75	1
5	3	28	$250	$1.100	390	135	2
5	4	28	$300	$0.700	540	105	1
5	5	28	$500	$1.400	490	160	2
5	1	56	$150	$1.000	180	45	1
5	2	56	$420	$0.800	200	75	2
5	3	56	$250	$0.550	195	135	3
5	4	56	$300	$0.350	270	105	2
5	5	56	$500	$0.700	245	160	3
5	1	112	$150	$0.500	90	45	2
5	2	112	$420	$0.400	100	75	3
5	3	112	$250	$0.275	97	135	5
5	4	112	$300	$0.175	135	105	4
5	5	112	$500	$0.350	122	160	6

Manufacturing and Serviceable Input Parameters for the First Phase of the Sensitivity Analysis (continued)

n	*i*	*t*	K_i^M	h_i^s	PC_i^M	MS_i^M	S_i^M
6	1	28	$150	$0.500	340	40	1
	2		$100	$1.800	400	80	1
	3		$275	$1.100	360	60	1
	4		$410	$0.800	270	140	2
	5		$500	$1.340	640	100	1
	6		$340	$2.000	420	160	2
	1	56	$150	$0.250	170	40	1
	2		$100	$0.900	200	80	2
	3		$275	$0.550	180	60	1
	4		$410	$0.400	135	140	3
	5		$500	$0.670	320	100	2
	6		$340	$1.000	210	160	3
	1	112	$150	$0.125	85	40	2
	2		$100	$0.450	100	80	3
	3		$275	$0.275	90	60	2
	4		$410	$0.200	67	140	5
	5		$500	$0.335	160	100	4
	6		$340	$0.500	105	160	6
7	1	28	$150	$2.000	330	75	1
	2		$110	$1.500	440	60	1
	3		$310	$0.880	560	35	1
	4		$245	$1.840	350	105	1
	5		$460	$0.920	630	145	2
	6		$290	$0.760	312	95	1
	7		$500	$1.380	490	165	2
	1	56	$45	$1.000	165	40	2
	2		$60	$0.750	220	25	1
	3		$100	$0.440	280	15	1
	4		$80	$0.920	175	70	2
	5		$200	$0.460	315	85	3
	6		$140	$0.380	156	60	2
	7		$180	$0.690	245	75	3
	1	112	$150	$0.500	82	75	3
	2		$110	$0.375	110	60	2
	3		$310	$0.220	140	35	2
	4		$245	$0.460	87	105	4
	5		$460	$0.230	157	145	5
	6		$290	$0.190	78	95	4
	7		$500	$0.345	122	165	6

Manufacturing and Serviceable Input Parameters for the First Phase of the Sensitivity Analysis (continued)

n	i	t	K_i^M	h_i^s	PC_i^M	MS_i^M	S_i^M
8	1	28	$150	$0.800	630	75	1
	2		$125	$2.000	490	50	1
	3		$210	$1.700	740	85	1
	4		$365	$1.200	820	145	2
	5		$425	$1.100	630	170	2
	6		$500	$1.490	500	100	1
	7		$195	$1.390	590	120	1
	8		$300	$1.650	760	45	1
	1	56	$150	$0.400	315	75	2
	2		$125	$1.000	245	50	1
	3		$210	$0.850	370	85	2
	4		$365	$0.600	410	145	3
	5		$425	$0.550	315	170	3
	6		$500	$0.745	250	100	2
	7		$195	$0.695	295	120	2
	8		$300	$0.825	380	45	1
	1	112	$150	$0.200	157	75	3
	2		$125	$0.500	122	50	2
	3		$210	$0.425	185	85	3
	4		$365	$0.300	205	145	5
	5		$425	$0.275	157	170	6
	6		$500	$0.373	125	100	4
	7		$195	$0.348	147	120	4
	8		$300	$0.413	190	45	2

Manufacturing and Serviceable Input Parameters for the First Phase of the Sensitivity Analysis (continued)

n	*i*	*t*	K_i^M	h_i^s	PC_i^M	MS_i^M	S_i^M
9	**1**	**28**	$150	$2.000	620	75	1
	2		$100	$1.600	530	60	1
	3		$230	$0.400	760	80	1
	4		$450	$1.300	770	145	2
	5		$320	$1.500	1000	110	1
	6		$110	$1.000	700	120	1
	7		$285	$0.720	580	180	2
	8		$500	$0.950	460	135	2
	9		$340	$1.800	440	60	1
	1	**56**	$150	$1.000	310	75	2
	2		$100	$0.800	265	60	1
	3		$230	$0.200	380	80	2
	4		$450	$0.650	385	145	3
	5		$320	$0.750	500	110	2
	6		$110	$0.500	350	120	2
	7		$285	$0.360	290	180	3
	8		$500	$0.475	230	135	3
	9		$340	$0.900	220	60	1
	1	**112**	$150	$0.500	155	75	3
	2		$100	$0.400	132	60	2
	3		$230	$0.100	190	80	3
	4		$450	$0.325	192	145	5
	5		$320	$0.375	250	110	4
	6		$110	$0.250	175	120	4
	7		$285	$0.180	145	180	6
	8		$500	$0.238	115	135	5
	9		$340	$0.450	110	60	2

Manufacturing and Serviceable Input Parameters for the First Phase of the Sensitivity Analysis (continued)

n	i	t	K_i^M	h_i^s	PC_i^M	MS_i^M	S_i^M
10	1	28	$150	$1.700	1480	75	1
	2		$100	$1.200	1180	60	1
	3		$370	$0.900	1700	85	1
	4		$210	$0.600	790	145	2
	5		$340	$1.800	1420	100	1
	6		$460	$0.400	1320	90	1
	7		$500	$1.400	780	130	2
	8		$175	$0.640	1220	180	2
	9		$265	$1.100	960	60	1
	10		$420	$2.000	760	105	1
	1	56	$150	$0.850	740	75	2
	2		$100	$0.600	590	60	1
	3		$370	$0.450	850	85	2
	4		$210	$0.300	395	145	3
	5		$340	$0.900	710	100	2
	6		$460	$0.200	660	90	2
	7		$500	$0.700	390	130	3
	8		$175	$0.320	610	180	3
	9		$265	$0.550	480	60	1
	10		$420	$1.000	380	105	2
	1	112	$150	$0.425	370	75	3
	2		$100	$0.300	295	60	2
	3		$370	$0.225	425	85	3
	4		$210	$0.150	197	145	5
	5		$340	$0.450	355	100	4
	6		$460	$0.100	330	90	3
	7		$500	$0.350	195	130	5
	8		$175	$0.160	305	180	6
	9		$265	$0.275	240	60	2
	10		$420	$0.500	190	105	4

Appendix E

Remanufacturing and Returns Input Parameters for the First Phase of the Sensitivity Analysis

n	i	t	K_i^R	h_i^r	PC_i^R	MS_i^R	S_i^R
3	1	28	$85	$0.600	240	45	1
	2		$210	$0.300	160	30	1
	3		$150	$1.200	540	90	1
	1	56	$85	$0.300	120	45	1
	2		$210	$0.150	80	30	1
	3		$150	$0.600	270	90	2
	1	112	$85	$0.150	60	45	2
	2		$210	$0.075	40	30	1
	3		$150	$0.300	135	90	3
4	1	28	$75	$0.400	80	40	1
	2		$155	$0.600	120	15	1
	3		$175	$0.900	100	60	1
	4		$135	$0.200	90	80	1
	1	56	$75	$0.200	40	40	1
	2		$155	$0.300	60	15	1
	3		$175	$0.450	50	60	1
	4		$135	$0.100	45	80	2
	1	112	$75	$0.100	20	40	2
	2		$155	$0.150	30	15	1
	3		$175	$0.225	25	60	2
	4		$135	$0.050	22	80	3
5	1	28	$50	$0.470	220	15	1
	2		$90	$1.200	290	35	1
	3		$120	$0.400	200	70	1
	4		$160	$0.200	290	60	1
	5		$200	$0.700	160	80	1
	1	56	$50	$0.235	110	15	1
	2		$90	$0.600	145	35	1
	3		$120	$0.200	100	70	2
	4		$160	$0.100	145	60	1
	5		$200	$0.350	80	80	2
	1	112	$50	$0.118	55	15	1
	2		$90	$0.300	72	35	2
	3		$120	$0.100	50	70	3
	4		$160	$0.050	72	60	2
	5		$200	$0.175	40	80	3

Remanufacturing and Returns Input Parameters for the First Phase of the Sensitivity Analysis (continued)

n	i	t	K_i^R	h_i^r	PC_i^R	MS_i^R	S_i^R
6	1	28	$60	$0.125	180	15	1
	2		$40	$0.400	220	45	1
	3		$110	$0.500	170	35	1
	4		$135	$0.200	90	70	1
	5		$150	$0.660	400	25	1
	6		$85	$1.400	200	85	1
	1	56	$60	$0.063	90	15	1
	2		$40	$0.200	110	45	1
	3		$110	$0.250	85	35	1
	4		$135	$0.100	45	70	2
	5		$150	$0.330	200	25	1
	6		$85	$0.700	100	85	2
	1	112	$60	$0.031	45	15	1
	2		$40	$0.100	55	45	2
	3		$110	$0.125	42	35	2
	4		$135	$0.050	22	70	3
	5		$150	$0.165	100	25	1
	6		$85	$0.350	50	85	3
7	1	28	$45	$0.500	190	40	1
	2		$60	$0.870	320	25	1
	3		$100	$0.430	310	15	1
	4		$80	$1.350	220	70	1
	5		$200	$0.250	330	85	1
	6		$140	$0.125	190	60	1
	7		$180	$0.690	230	75	1
	1	56	$45	$0.250	95	40	1
	2		$60	$0.435	160	25	1
	3		$100	$0.215	155	15	1
	4		$80	$0.675	110	70	2
	5		$200	$0.125	165	85	2
	6		$140	$0.063	95	60	1
	7		$180	$0.345	115	75	2
	1	112	$45	$0.125	47	40	2
	2		$60	$0.218	80	25	1
	3		$100	$0.108	77	15	1
	4		$80	$0.338	55	70	3
	5		$200	$0.063	82	85	3
	6		$140	$0.031	47	60	2
	7		$180	$0.173	57	75	3

Remanufacturing and Returns Input Parameters for the First Phase of the Sensitivity Analysis (continued)

n	*i*	*t*	K_i^R	h_i^r	PC_i^R	MS_i^R	S_i^R
8	**1**	**28**	$65	$0.210	330	35	1
	2		$50	$0.450	220	25	1
	3		$85	$0.650	370	50	1
	4		$105	$0.600	480	75	1
	5		$165	$0.310	400	90	1
	6		$185	$0.840	260	55	1
	7		$125	$0.790	320	60	1
	8		$145	$0.470	360	15	1
	1	**56**	$65	$0.105	165	35	1
	2		$50	$0.225	110	25	1
	3		$85	$0.325	185	50	1
	4		$105	$0.300	240	75	2
	5		$165	$0.155	200	90	2
	6		$185	$0.420	130	55	1
	7		$125	$0.395	160	60	1
	8		$145	$0.235	180	15	1
	1	**112**	$65	$0.053	82	35	2
	2		$50	$0.113	55	25	1
	3		$85	$0.163	92	50	2
	4		$105	$0.150	120	75	3
	5		$165	$0.078	100	90	3
	6		$185	$0.210	65	55	2
	7		$125	$0.198	80	60	2
	8		$145	$0.118	90	15	1

Remanufacturing and Returns Input Parameters for the First Phase of the Sensitivity Analysis (continued)

n	i	t	K_i^R	h_i^r	PC_i^R	MS_i^R	S_i^R
9	1	28	$65	$0.800	270	20	1
	2		$50	$0.470	240	15	1
	3		$80	$0.200	490	25	1
	4		$175	$0.800	370	75	1
	5		$140	$0.600	490	35	1
	6		$55	$0.650	280	30	1
	7		$125	$0.330	280	90	1
	8		$200	$0.400	250	60	1
	9		$160	$1.150	200	15	1
	1	56	$65	$0.400	135	20	1
	2		$50	$0.235	120	15	1
	3		$80	$0.100	245	25	1
	4		$175	$0.400	185	75	2
	5		$140	$0.300	245	35	1
	6		$55	$0.325	140	30	1
	7		$125	$0.165	140	90	2
	8		$200	$0.200	125	60	1
	9		$160	$0.575	100	15	1
	1	112	$65	$0.200	67	20	1
	2		$50	$0.118	60	15	1
	3		$80	$0.050	122	25	1
	4		$175	$0.200	92	75	3
	5		$140	$0.150	122	35	2
	6		$55	$0.163	70	30	1
	7		$125	$0.083	70	90	3
	8		$200	$0.100	62	60	2
	9		$160	$0.288	50	15	1

Remanufacturing and Returns Input Parameters for the First Phase of the Sensitivity Analysis (continued)

n	i	t	K_i^R	h_i^r	PC_i^R	MS_i^R	S_i^R
10	1	28	$85	$1.000	800	20	1
	2		$50	$0.290	640	15	1
	3		$105	$0.440	630	25	1
	4		$125	$0.185	300	65	1
	5		$150	$0.730	750	55	1
	6		$185	$0.125	380	40	1
	7		$200	$0.800	290	60	1
	8		$60	$0.200	600	90	1
	9		$135	$0.250	420	25	1
	10		$170	$0.500	330	70	1
	1	56	$85	$0.500	400	20	1
	2		$50	$0.145	320	15	1
	3		$105	$0.220	315	25	1
	4		$125	$0.093	150	65	2
	5		$150	$0.365	375	55	1
	6		$185	$0.063	190	40	1
	7		$200	$0.400	145	60	1
	8		$60	$0.100	300	90	2
	9		$135	$0.125	210	25	1
	10		$170	$0.250	165	70	2
	1	112	$85	$0.250	200	20	1
	2		$50	$0.073	160	15	1
	3		$105	$0.110	157	25	1
	4		$125	$0.046	75	65	3
	5		$150	$0.183	187	55	2
	6		$185	$0.031	95	40	2
	7		$200	$0.200	72	60	2
	8		$60	$0.050	150	90	3
	9		$135	$0.063	105	25	1
	10		$170	$0.125	82	70	3

Appendix F

Detailed Results for the First Phase of the Sensitivity Analysis

n	*t*	*f*	*U*	*TC*	*Gap*	*TCC*	*IL*	*CT* "HH:MM:SS"
3	28	82%	100%	$27,190.20	0%	$31,982.70	29.11%	"00:01:02"
	56	75%	98%	$25,485.80	0%	$28,242.40	14.01%	"00:03:22"
	112	71%	NA	$27,009.50	20%	$27,009.50	9.04%	"00:07:15"
			NA	$25,253.50	15%	$25,253.50	1.95%	"00:08:19"
			NA	$25,253.50	10%	$25,253.50	1.95%	"00:24:50"
			NA	$25,253.50	5%	$25,253.50	1.95%	"01:28:49"
			96%	$24,771.18	0%	$24,771.18	0%	"02:02:38"
4	28	82%	89%	$12,404.70	0%	$13,666.95	5.65%	"00:00:18"
	56	75%	NA	$13,518.50	20%	NA	NA	"00:03:21"
			NA	$13,233.60	15%	NA	NA	"00:06:23"
			NA	$12,707.00	10%	NA	NA	"00:09:24"
			NA	$12,534.85	5%	NA	NA	"00:21:45"
			88%	$12,510.45	0%	$12,994.65	0.45%	"00:36:32"
	112	73%	NA	$13,398.15	20%	$13,398.15	3.57%	"00:36:44"
			NA	$13,004.58	15%	$13,004.58	0.53%	"02:22:23"
			83%	$12,936.53	10.39%	$12,936.53	0%	"24:00:00"
5	28	79%	100%	$22,564.12	0%	$24,473.77	10.28%	"00:01:54"
	56	75%	NA	$21,656.49	20%	NA	NA	"00:09:24"
			NA	$20,506.04	15%	NA	NA	"00:15:00"
			NA	$20,435.09	10%	NA	NA	"00:36:40"
			NA	$20,383.85	5%	NA	NA	"01:31:56"
			93%	$20,046.64	0%	$20,846.31	-6.07%	"02:15:48"
	112	71%	92%	$22,192.36	24%	$22,192.36	0%	"24:00:00"
6	28	86%	96%	$19,302.60	0%	$20,716.50	21.51%	"00:02:08"
	56	75%	NA	$17,332.30	20%	NA	NA	"00:55:42"
			NA	$17,231.60	15%	NA	NA	"03:23:32"
			NA	$17,152.85	10%	NA	NA	"11:24:58"
			93%	$17,152.85	7.07%	$17,895.69	4.96%	"24:00:00"
	112	72%	90%	$17,049.77	27.79%	$17,049.77	0%	"24:00:00"
7	28	86%	96%	$18,602.63	0%	$19,815.89	3.91%	"00:01:54"
	56	75%	NA	$16,688.80	20%	NA	NA	"00:11:22"
			NA	$16,580.19	15%	NA	NA	"00:43:07"
			NA	$16,580.19	10%	NA	NA	"04:37:29"
			96%	$16,580.19	6.50%	$17,147.39	-10.08%	"24:00:00"
	112	71%	94%	$19,069.96	47.82%	$19,069.96	0%	"24:00:00"

Detailed Results for the First Phase of the Sensitivity Analysis (continued)

n	*t*	*f*	*U*	*TC*	*Gap*	*TCC*	*IL*	*CT* "HH:MM:SS"
8	28	89%	100%	$19,967.40	0%	$20,621.01	10.66%	"05:01:04"
	56	75%	95%	$17,343.35	22.56%	$17,799.67	-4.48%	"24:00:00"
	112	65%	90%	$18,634.79	48.88%	$18,634.79	0.00%	"24:00:00"
9	28	96%	100%	$17,654.40	0%	$18,151.86	29.16%	"10:27:40"
	56	75%	NA	$14,185.00	20%	NA	NA	"01:31:59"
			NA	$13,984.50	15%	NA	NA	"17:18:56"
			95%	$13,984.50	14.41%	$14,222.33	1.20%	"24:00:00"
	112	68%	88%	$14,053.65	46.91%	$14,053.65	0%	"24:00:00"
10	28	89%	100%	$17,962.40	0%	$18,317.08	22.24%	"00:01:43"
	56	75%	NA	$16,062.25	20%	NA	NA	"12:10:21"
			96%	$16,053.05	18.94%	$16,221.88	8.25%	"24:00:00"
	112	66%	92%	$14,985.09	49.67%	$14,985.09	0%	"24:00:00"

Appendix G

Detailed Results for the Second Phase of the Sensitivity Analysis with Respect to the Variation in the Holding Cost of Returned and Finished Products

n	*t*	*f*	*Trials*	*Holding Cost Variation*	*U*	*TC*	*Gap*	*CT "HH:MM:SS"*
3	112	71%	1	¼	NA	$8,618.98	20%	"00:11:45"
					NA	$8,605.73	15%	"00:58:37"
					NA	$8,555.40	10%	"04:28:49"
					NA	$8,331.52	5%	"13:02:57"
					91%	$8,302.77	0%	"18:59:23"
			2	½	NA	$14,356.45	20%	"00:05:18"
					NA	$14,264.95	15%	"00:18:16"
					NA	$14,264.95	10%	"01:39:15"
					NA	$14,176.60	5%	"05:43:13"
					91%	$13,892.40	0%	"02:02:38"
			Ref. Case	1	NA	$27,009.50	20%	"00:07:15"
					NA	$25,253.50	15%	"00:08:19"
					NA	$25,253.50	10%	"00:24:50"
					NA	$25,253.50	5%	"01:28:49"
					96%	$24,771.18	0%	"02:02:38"
			3	2	NA	$48,221.80	20%	"00:06:08"
					NA	$47,958.75	15%	"00:23:33"
					NA	$47,958.75	10%	"02:01:52"
					NA	$47,178.30	5%	"05:13:31"
					96%	$46,402.35	0%	"07:00:06"
			4	4	NA	$98,335.80	20%	"00:20:23"
					NA	$93,331.00	15%	"00:24:42"
					NA	$91,111.00	10%	"00:52:27"
					NA	$90,733.60	5%	"02:46:24"
					96%	$89,664.70	0%	"04:26:32"

Appendix H

Scheduled Returns for the Second Phase of the Sensitivity Analysis with Respect to the Variation in the Return Rate

n	*i*	Trials	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
3	**1**	1	0	74	0	26	0	48	67	**215**
	2		78	0	35	0	0	48	60	**221**
	3		87	0	0	60	50	48	0	**245**
	1	2	0	111	0	39	0	71	102	**323**
	2		116	0	53	0	0	73	90	**332**
	3		130	0	0	90	75	72	0	**367**
	1	Ref. Case	0	148	0	52	0	95	135	**430**
	2		155	0	70	0	0	97	120	**442**
	3		173	0	0	120	100	96	0	**489**
	1	3	0	185	0	65	0	119	169	**538**
	2		194	0	88	0	0	121	150	**553**
	3		216	0	0	150	125	120	0	**611**
	1	4	0	222	0	78	0	142	203	**645**
	2		233	0	105	0	0	145	180	**663**
	3		260	0	0	180	150	144	0	**734**

Appendix I

Detailed Results for the Second Phase of the Sensitivity Analysis with Respect to the Variation in the Return Rate

n	***t***	***Trials***	***f***	***r***	***U***	***TC***	***Gap***	***CT "HH:MM:SS"***
3	112	1	66%	10%	NA	$22,258.53	20%	"00:02:00"
					NA	$21,836.58	15%	"00:05:46"
					NA	$21,826.58	10%	"00:21:14"
					NA	$21,206.25	5%	"00:36:54"
					94%	$21,072.70	0%	"01:04:10"
		2	68%	15%	NA	$24,024.65	20%	"00:01:24"
					NA	$23,939.98	15%	"00:04:30"
					NA	$23,599.40	10%	"00:14:02"
					NA	$23,167.30	5%	"00:25:49"
					92%	$22,847.60	0%	"00:51:04"
		Ref. Case	71%	20%	NA	$27,009.50	20%	"00:07:15"
					NA	$25,253.50	15%	"00:08:19"
					NA	$25,253.50	10%	"00:24:50"
					NA	$25,253.50	5%	"01:28:49"
					96%	$24,771.18	0%	"02:02:38"
		3	73%	25%	NA	$29,169.20	20%	"00:20:19"
					NA	$28,596.30	15%	"00:58:42"
					NA	$28,219.20	10%	"03:11:03"
					NA	$27,424.95	5%	"05:14:42"
					100%	$27,350.20	0%	"10:05:31"
		4	77%	30%	NA	$31,604.15	20%	"00:26:24"
					NA	$31,337.90	15%	"01:34:48"
					NA	$31,337.90	10%	"11:29:24"
					NA	$30,199.15	5%	"22:07:31"
					100%	$30,199.15	4.18%	"24:00:00"

Appendix J

Scheduled Deliveries for the Second Phase of the Sensitivity Analysis with Respect to the Feasibility Variation

n	*i*	Trials	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
3	**1**	Ref. Case	0	700	165	0	0	960	325	**2150**
	2		560	300	0	450	0	0	900	**2210**
	3		800	0	275	320	0	1050	0	**2445**
	1	1	0	700	400	0	0	960	325	**2385**
	2		560	350	0	450	0	0	900	**2260**
	3		800	0	325	560	0	1050	0	**2735**
	1	2	0	1050	400	0	0	960	325	**2735**
	2		560	350	0	450	0	0	1050	**2410**
	3		800	0	425	560	0	1250	0	**3035**
	1	3	0	1225	400	0	0	960	325	**2910**
	2		560	350	0	600	0	0	1050	**2560**
	3		800	0	425	560	0	1450	0	**3235**
	1	4	0	1225	600	0	0	960	325	**3110**
	2		560	540	0	800	0	0	1050	**2950**
	3		800	0	525	560	0	1450	0	**3335**
	1	5	0	1225	600	0	0	960	500	**3285**
	2		590	540	0	800	0	0	1050	**2980**
	3		800	0	525	760	0	1450	0	**3535**

Appendix K

Scheduled Returns for the Second Phase of the Sensitivity Analysis with Respect to the Feasibility Variation

n	*i*	Trials	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
3	**1**	Ref. Case	0	148	0	52	0	95	135	**430**
	2		155	0	70	0	0	97	120	**442**
	3		173	0	0	120	100	96	0	**489**
	1	1	0	148	0	59	0	110	160	**477**
	2		155	0	80	0	0	97	120	**452**
	3		203	0	0	140	108	96	0	**547**
	1	2	0	148	0	89	0	110	200	**547**
	2		165	0	80	0	0	107	130	**482**
	3		208	0	0	175	128	96	0	**607**
	1	3	0	148	0	89	0	110	235	**582**
	2		185	0	85	0	0	112	130	**512**
	3		238	0	0	175	138	96	0	**647**
	1	4	0	158	0	99	0	130	235	**622**
	2		235	0	85	0	0	120	150	**590**
	3		238	0	0	185	148	96	0	**667**
	1	5	0	178	0	94	0	140	245	**657**
	2		232	0	85	0	0	124	155	**596**
	3		268	0	0	185	158	96	0	**707**

Appendix L

Detailed Results for the Second Phase of the Sensitivity Analysis with Respect to the Feasibility Variation

n	***t***	***Trials***	***f***	***U***	***TC***	***Gap***	***CT "HH:MM:SS"***
3	112	Ref. Case	71%	NA	$27,009.50	20%	"00:07:15"
				NA	$25,253.50	15%	"00:08:19"
				NA	$25,253.50	10%	"00:24:50"
				NA	$25,253.50	5%	"01:28:49"
				96%	$24,771.18	0%	"02:02:38"
		1	75%	NA	$29,705.75	20%	"00:25:17"
				NA	$28,151.80	15%	"00:59:56"
				NA	$27,138.75	10%	"01:21:16"
				NA	$26,618.05	5%	"02:43:03"
				98%	$26,457.15	0%	"04:13:16"
		2	80%	NA	$34,653.40	20%	"01:05:28"
				NA	$33,672.20	15%	"04:03:35"
				NA	$33,331.00	10%	"22:47:07"
				100%	$33,331.00	9.86%	"24:00:00"
		3	85%	NA	No Solution	NA	"24:00:00"
		4	90%	NA	No Solution	NA	"24:00:00"
		5	95%	NA	No Solution	NA	"24:00:00"

Vita

Armin Aminipour was born in 1989, in Shiraz, Islamic Republic of Iran. Mr. Aminipour moved to the United Arab Emirates in 2007. He graduated with Bachelor of Science in Industrial Engineering and Management from the University of Sharjah in January 2013.

Mr. Aminipour subsequently began a Master's program in Engineering Systems Management at the American University of Sharjah and is currently working toward his degree. He was a graduate teaching assistant at the American University of Sharjah for two years.